Functional Dependencies

CS 186, Spring 2006, Lecture 21
R&G Chapter 19

Science is the knowledge of consequences, and dependence of one fact upon another.

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(1588-1679)

Review: Database Design

- Requirements Analysis
  - user needs; what must database do?
- Conceptual Design
  - high level descr (often done w/ER model)
- Logical Design
  - translate ER into DBMS data model
- Schema Refinement
  - consistency,normalization
- Physical Design - indexes, disk layout
- Security Design - who accesses what

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The Evils of Redundancy

- ***Redundancy*** **is at the root of several problems associated with relational schemas:**
  - *redundant storage, insert/delete/update anomalies*
- **Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.**
- **Main refinement technique: *decomposition***
  - replacing ABCD with, say, AB and BCD, or ACD and ABD.
- **Decomposition should be used judiciously:**
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Functional Dependencies (FDs)

- **A functional dependency $X \rightarrow Y$ holds over relation schema R if, for every allowable instance *r* of R:**

$$t1 \in r,\ t2 \in r,\ \pi_X(t1) = \pi_X(t2) \text{ implies } \pi_Y(t1) = \pi_Y(t2)$$

(where t1 and t2 are tuples;X and Y are sets of attributes)

- **In other words: $X \rightarrow Y$ means**
  Given any two tuples in *r*, if the X values are the same, then the Y values must also be the same. (but not vice versa)
- **Can read "$\rightarrow$" as "determines"**

FD's Continued

- **An FD is a statement about *all* allowable relations.**
  - Must be identified based on semantics of application.
  - Given some instance *r1* of R, we can check if *r1* violates some FD *f*, but we cannot determine if *f* holds over R.
- **Question: How related to keys?**
- **if "K $\rightarrow$ all attributes of R" then K is a *superkey* for R**
  (does not require K to be *minimal*.)
- **FDs are a generalization of keys.**

Example: Constraints on Entity Set

- **Consider relation obtained from** Hourly_Emps:
  Hourly_Emps (*ssn*, *name*, *lot*, *rating*, *wage_per_hr*, *hrs_per_wk*)
- We sometimes denote a relation schema by listing the attributes: e.g., SNLRWH
- This is really the *set* of attributes {S,N,L,R,W,H}.
- Sometimes, we refer to the set of *all attributes* of a relation by using the relation name. e.g., "Hourly_Emps" for SNLRWH

What are some FDs on Hourly_Emps?

ssn is the key: S → SNLRWH
rating determines *wage_per_hr*: R → W
lot determines *lot*: L → L ("trivial" dependnency)

Problems Due to R → W

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Hourly_Emps

- ***Update anomaly***: **Can we modify W in only the 1st tuple of SNLRWH?**
- ***Insertion anomaly***: **What if we want to insert an employee and don't know the hourly wage for his or her rating? (or we get it wrong?)**
- ***Deletion anomaly***: **If we delete all employees with rating 5, we lose the information about the wage for rating 5!**

Detecting Reduncancy

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
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Hourly_Emps

Q: Why was R → W problematic, but S→W not?

Decomposing a Relation

- **Redundancy can be removed by "chopping" the relation into pieces.**
- **FD's are used to drive this process.**
  R → W is causing the problems, so decompose SNLRWH into what relations?

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
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Hourly_Emps2

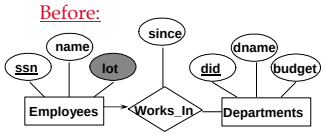
R	W
8	10
5	7

Wages

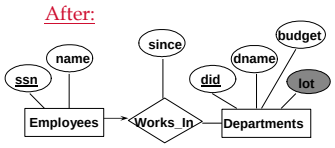
Refining an ER Diagram

- **1st diagram becomes:**
  Workers(S,N,L,D,Si)
  Departments(D,M,B)
  - Lots associated with workers.
- **Suppose all workers in a dept are assigned the same lot: D → L**
- **Redundancy; fixed by:**
  Workers2(S,N,D,Si)
  Dept_Lots(D,L)
  Departments(D,M,B)
- **Can fine-tune this:**
  Workers2(S,N,D,Si)
  Departments(D,M,B,L)

Before:



After:



Reasoning About FDs

- **Given some FDs, we can usually infer additional FDs:**
  title → *studio, star* implies *title* → *studio* and *title* → *star*
  title → *studio* and *title* → *star* implies *title* → *studio, star*
  title → *studio*, *studio* → *star* implies *title* → *star*

  But,
  title, star → *studio* does NOT necessarily imply that *title* → *studio* or that *star* → *studio*
- **An FD *f* is *implied by* a set of FDs *F* if *f* holds whenever all FDs in *F* hold.**
- **F^+ = *closure of F* is the set of all FDs that are implied by *F*. (includes "trivial dependencies")**

Rules of Inference

- **Armstrong's Axioms (X, Y, Z are sets of attributes):**
  - *Reflexivity*: If $X \supseteq Y$, then $X \rightarrow Y$
  - *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
  - *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- **These are *sound* and *complete* inference rules for FDs!**
  - i.e., using AA you can compute all the FDs in F+ and only these FDs.
- **Some additional rules (that follow from AA):**
  - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Example

- **Contracts(*cid,sid,jid,did,pid,qty,value*), and:**
  - C is the key: $C \rightarrow CSJDPQV$
  - Proj purchases each part using single contract: $JP \rightarrow C$
  - Dept purchases at most 1 part from a supplier: $SD \rightarrow P$
- **Problem: Prove that SDJ is a key for Contracts**
- **$JP \rightarrow C$, $C \rightarrow CSJDPQV$ imply $JP \rightarrow CSJDPQV$**
  (by transitivity) (shows that JP is a key)
- **$SD \rightarrow P$ implies $SDJ \rightarrow JP$** (by augmentation)
- **$SDJ \rightarrow JP$, $JP \rightarrow CSJDPQV$ imply $SDJ \rightarrow CSJDPQV$**
  (by transitivity) thus SDJ is a key.

Q: can you now infer that $SD \rightarrow CSDPQV$ (i.e., drop J on both sides)?

No! FD inference is not like arithmetic multiplication.

Attribute Closure

- **Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)**
- **Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs *F*. An efficient check:**
  - Compute *attribute closure* of X (denoted X^+) wrt *F*.
    X^+ = Set of all attributes A such that $X \rightarrow A$ is in F^+
    - $X^+ := X$
    - Repeat until no change: if there is in fd $U \rightarrow V$ in *F* such that U is in X^+, then add V to X^+
  - Check if Y is in X^+
  - Approach can also be used to find the keys of a relation.
    - If all attributes of R are in the closure of X then X is a superkey for R.
    - Q: How to check if X is a "candidate key"?

Attribute Closure (example)

- **R = {A, B, C, D, E}**
- **F = { $B \rightarrow CD$, $D \rightarrow E$, $B \rightarrow A$, $E \rightarrow C$, $AD \rightarrow B$ }**
- **Is $B \rightarrow E$ in F^+ ?**
  $B^+ = B$
  $B^+ = BCD$
  $B^+ = BCDA$
  $B^+ = BCDAE$ ... Yes!
  and B is a key for R too!
- **Is D a key for R?**
  $D^+ = D$
  $D^+ = DE$
  $D^+ = DEC$
  ... Nope!
- **Is AD a key for R?**
  $AD^+ = AD$
  $AD^+ = ABD$ and B is a key, so Yes!
- **Is AD a *candidate* key for R?**
  $A^+ = A$
  ... A not a key, so Yes!
- **Is ADE a *candidate* key for R?**
  ... No! AD is a key, so ADE is a superkey, but not a cand. key

Next Class...

- **Normal forms and normalization**
- **Table decompositions**