

# CS 188 Section Handout: Probability Basics

Let  $D \in \{d_0, d_1, d_2, d_3\}$  be one of four dice, where  $d_0$  is fair. Let  $R \in [1, 6]$  be the outcome of a die roll.

## 1 Joint probability assembly

A casino employee informs you (the inspector) about a dealer, Angelo: “He cheats a third of the time, using loaded die  $d_1$  for which the six-dotted side shows up five times as often as each of the other sides.”

$D$	$P(D)$
$d_0$	
$d_1$	

$R$	$D$	$P(R D)$
1	$d_0$	
2	$d_0$	
3	$d_0$	
4	$d_0$	
5	$d_0$	
6	$d_0$	
1	$d_1$	
2	$d_1$	
3	$d_1$	
4	$d_1$	
5	$d_1$	
6	$d_1$	

$R$	$D$	$P(R, D)$
1	$d_0$	
2	$d_0$	
3	$d_0$	
4	$d_0$	
5	$d_0$	
6	$d_0$	
1	$d_1$	
2	$d_1$	
3	$d_1$	
4	$d_1$	
5	$d_1$	
6	$d_1$	

## 2 Estimation

Your informant reports back with the following statistics about another dealer, Bert: “He uses two dice, both loaded. Using  $d_2$ , I watched him roll 15 sixes, 10 fives, 5 fours, 5 threes, 5 twos, and no ones. With  $d_3$ , I observed 5 sixes, 10 fives, 15 fours, 15 threes, 15 twos, and 20 ones.” We can infer the following about these dice (a *maximum likelihood estimate*).

$R$	$D$	$c(R, D)$
1	$d_2$	0
2	$d_2$	5
3	$d_2$	5
4	$d_2$	5
5	$d_2$	10
6	$d_2$	15
1	$d_3$	20
2	$d_3$	15
3	$d_3$	15
4	$d_3$	15
5	$d_3$	10
6	$d_3$	5

$R$	$D$	$P(R, D)$
1	$d_2$	0
2	$d_2$	1/24
3	$d_2$	1/24
4	$d_2$	1/24
5	$d_2$	1/12
6	$d_2$	1/8
1	$d_3$	1/6
2	$d_3$	1/8
3	$d_3$	1/8
4	$d_3$	1/8
5	$d_3$	1/12
6	$d_3$	1/24

$R$	$D$	$P(R D)$
1	$d_2$	0
2	$d_2$	1/8
3	$d_2$	1/8
4	$d_2$	1/8
5	$d_2$	1/4
6	$d_2$	3/8
1	$d_3$	1/4
2	$d_3$	3/16
3	$d_3$	3/16
4	$d_3$	3/16
5	$d_3$	1/8
6	$d_3$	1/16

### 3 Inference by enumeration

- What is  $P(R = 6 \mid \text{dealer} = A)$ ?
- What is  $P(R = 6 \mid \text{dealer} = B)$ ?
- What is  $P(D = d_0 \mid R = 6, \text{dealer} = A)$ ?
- What is  $P(D = d_0 \mid \text{dealer} = B)$ ?
- What is  $P(D = d_2 \mid \text{dealer} = B)$ ?

Suppose you know that A works some five nights per week and B works on some other two nights.

- What is  $P(D)$ ?

$D$	$P(D)$
$d_0$	
$d_1$	
$d_2$	
$d_3$	

- One night, you observe a dealer (A or B) roll a six. What is the probability that the die is loaded?

### 4 Sequence of independent events

You confront the suspected dealer and ask him to roll his current die three times in a row. He rolls the sequence  $S = (6, 2, 6)$

Calculate the following likelihood probabilities:

- $P(S \mid D = d_0) =$
- $P(S \mid D = d_1) =$
- $P(S \mid D = d_2) =$
- $P(S \mid D = d_3) =$

### 5 Bayes' Rule

Given the evidence you acquired previously, determine which die was rolled:

$$\hat{d} = \arg \max_d P(D = d \mid S)$$

Is the die loaded? Which dealer are you arresting?