Announcements

- **Midterm**
  - Next TUESDAY, 10/25, 5-8pm
  - Prep page is on the web (practice exams, etc)
  - Topical review sessions: see prep page
  - Overall review: in class Thursday
  - If you have a conflict, we should already know about it!

- **Written 3**
  - Due this Friday but fixes not due until NEXT Friday

- **P1, P2, W1 in glookup**

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CS 188: Artificial Intelligence
Fall 2011

Lecture 16: Bayes Nets IV
10/18/2011

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Approximate Inference

Simulation has a name: sampling

Sampling is a hot topic in machine learning, and it’s really simple

Basic idea:
- Draw N samples from a sampling distribution S
- Compute an approximate posterior probability
- Show this converges to the true probability P

Why sample?
- Learning: get samples from a distribution you don’t know
- Inference: getting a sample is faster than computing the right answer (e.g. with variable elimination)
Prior Sampling

- This process generates samples with probability:
  \[ S_{PS}(x_1 \ldots x_n) = \prod_{i=1}^{n} P(x_i | \text{Parents}(X_i)) = P(x_1 \ldots x_n) \]
  \( \ldots \text{i.e. the BN's joint probability} \)

- Let the number of samples of an event be \( N_{PS}(x_1 \ldots x_n) \)

- Then
  \[ \lim_{N \to \infty} \hat{P}(x_1, \ldots, x_n) = \lim_{N \to \infty} N_{PS}(x_1, \ldots, x_n)/N \]
  \[ = S_{PS}(x_1, \ldots, x_n) \]
  \[ = P(x_1 \ldots x_n) \]

- I.e., the sampling procedure is consistent
Example

- First: Get a bunch of samples from the BN:
  - +c, -s, +r, +w
  - +c, +s, +r, +w
  - -c, +s, +r, -w
  - +c, -s, +r, +w
  - -c, -s, -r, +w

- Example: we want to know P(W)
  - We have counts <+w:4, -w:1>
  - Normalize to get approximate P(W) = <+w:0.8, -w:0.2>
  - This will get closer to the true distribution with more samples
  - Can estimate anything else, too
  - What about P(C| +w)?  P(C| +r, +w)?  P(C| -r, -w)?
  - Fast: can use fewer samples if less time (what’s the drawback?)

Rejection Sampling

- Let’s say we want P(C)
  - No point keeping all samples around
  - Just tally counts of C as we go

- Let’s say we want P(C| +s)
  - Same thing: tally C outcomes, but ignore (reject) samples which don’t have S=+s
  - This is called rejection sampling
  - It is also consistent for conditional probabilities (i.e., correct in the limit)
Sampling Example

- There are 2 cups.
  - The first contains 1 penny and 1 quarter
  - The second contains 2 quarters

- Say I pick a cup uniformly at random, then pick a coin randomly from that cup. It’s a quarter (yes!). What is the probability that the other coin in that cup is also a quarter?

Likelihood Weighting

- Problem with rejection sampling:
  - If evidence is unlikely, you reject a lot of samples
  - You don’t exploit your evidence as you sample
  - Consider \( P(B|+a) \)

- Idea: fix evidence variables and sample the rest

- Problem: sample distribution not consistent!
- Solution: weight by probability of evidence given parents
Likelihood Weighting

- Sampling distribution if $z$ sampled and $e$ fixed evidence

$$S_{WS}(z, e) = \prod_{i=1}^{l} P(z_i|\text{Parents}(Z_i))$$

- Now, samples have weights

$$w(z, e) = \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i))$$

- Together, weighted sampling distribution is consistent

$$S_{WS}(z, e) \cdot w(z, e) = \prod_{i=1}^{l} P(z_i|\text{Parents}(Z_i)) \prod_{i=1}^{m} P(e_i|\text{Parents}(E_i))$$

$$= P(z, e)$$
Likelihood Weighting

- Likelihood weighting is good
  - We have taken evidence into account as we generate the sample
  - E.g. here, W’s value will get picked based on the evidence values of S, R
  - More of our samples will reflect the state of the world suggested by the evidence
- Likelihood weighting doesn’t solve all our problems
  - Evidence influences the choice of downstream variables, but not upstream ones (C isn’t more likely to get a value matching the evidence)
- We would like to consider evidence when we sample every variable

Markov Chain Monte Carlo*

- Idea: instead of sampling from scratch, create samples that are each like the last one.
- Procedure: resample one variable at a time, conditioned on all the rest, but keep evidence fixed. E.g., for P(B|+c):

- Properties: Now samples are not independent (in fact they’re nearly identical), but sample averages are still consistent estimators!
- What’s the point: both upstream and downstream variables condition on evidence.
Decision Networks

- MEU: choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, cannot have parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)

Decision Networks

- Action selection:
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action
Example: Decision Networks

Umbrella = leave
EU(leave) = \sum_{w} P(w)U(leave, w)
= 0.7 \cdot 100 + 0.3 \cdot 0 = 70

Umbrella = take
EU(take) = \sum_{w} P(w)U(take, w)
= 0.7 \cdot 20 + 0.3 \cdot 70 = 35

Optimal decision = leave
MEU(\alpha) = \max_{\alpha} EU(\alpha) = 70

Decisions as Outcome Trees

- Almost exactly like expectimax / MDPs
- What’s changed?
Evidence in Decision Networks

- Find \( P(W|F=bad) \)
  - Select for evidence
  - First we join \( P(W) \) and \( P(bad|W) \)
  - Then we normalize

**Example: Decision Networks**

\[
\text{Umbrella} = \text{leave} \\
\text{EU}(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad}) U(\text{leave}, w) \\
= 0.34 \cdot 100 + 0.66 \cdot 0 = 34
\]

\[
\text{Umbrella} = \text{take} \\
\text{EU}(\text{take}|\text{bad}) = \sum_w P(w|\text{bad}) U(\text{take}, w) \\
= 0.34 \cdot 20 + 0.66 \cdot 70 = 53
\]

Optimal decision = take

\[
\text{MEU}(F = \text{bad}) = \max_w \text{EU}(a|\text{bad}) = 53
\]
Decisions as Outcome Trees

Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2
Value of Information

- Assume we have evidence $E=e$. Value if we act now:
  \[ \text{MEU}(e) = \max_a \sum_s P(s|e) \, U(s, a) \]

- Assume we see that $E' = e'$. Value if we act then:
  \[ \text{MEU}(e, e') = \max_a \sum_s P(s|e, e') \, U(s, a) \]

- BUT $E'$ is a random variable whose value is unknown, so we don't know what $e'$ will be

- Expected value if $E'$ is revealed and then we act:
  \[ \text{MEU}(e, E') = \sum_{e'} P(e'|e) \text{MEU}(e', e') \]

- Value of information: how much MEU goes up by revealing $E'$ first then acting, over acting now:
  \[ VPI(E'|e) = \text{MEU}(e, E') - \text{MEU}(e) \]

VPI Example: Weather

MEU with no evidence
\[ \text{MEU}(a) = \max_a \text{EU}(a) = 70 \]

MEU if forecast is bad
\[ \text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53 \]

MEU if forecast is good
\[ \text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95 \]

Forecast distribution

\[
\begin{array}{c|c|c|c}
F & P(F) & \text{MEU}(F|e) & \text{MEU}(F|e') \\
\hline
\text{good} & 0.59 & 0.59 \cdot 95 & 0.59 \cdot (95) + 0.41 \cdot (53) - 70 \\
\text{bad} & 0.41 & 0.41 \cdot 53 & 0.59 \cdot (95) + 0.41 \cdot (53) - 70 \\
\end{array}
\]

\[ VPI(E'|e) = \left( \sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e) \]
VPI Properties

- Nonnegative
  \[ \forall E', e : \text{VPI}(E'|e) \geq 0 \]
- Nonadditive – consider, e.g., obtaining \( E_j \) twice
  \[ \text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e) \]
- Order-independent
  \[ \text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \]
  \[ = \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \]

Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn’t order either one. What’s the value of knowing which it is?

- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What’s the value of knowing which?

- You’re playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?