1 Probability

Use the probability table to calculate the following values:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$P(X_1, X_2, X_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

1. $P(X_1 = 1, X_2 = 0)$

2. $P(X_3 = 0)$

3. $P(X_2 = 1|X_3 = 1)$

4. $P(X_1 = 0|X_2 = 1, X_3 = 1)$

5. $P(X_1 = 0, X_2 = 1|X_3 = 1)$
2  D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.

1. $U \perp \perp X$
2. $U \perp \perp X|T$
3. $V \perp W|Y$
4. $V \perp W|T$
5. $T \perp Y|V$
6. $Y \perp Z|W$
7. $Y \perp Z|T$

3  Variable Elimination

Using the same Bayes Net (shown below), we want to compute $P(Y|+z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: $X, T, U, V, W$.

Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V, W), P(+z|X)$$
(a) When eliminating $X$ we generate a new factor $f_1$ as follows, which leaves us with the factors:

$$f_1(+z|T) = \sum_x P(x|T)P(+z|x) \quad P(T), P(U|T), P(V|T), P(W|T), P(Y|V, W), f_1(+z|T)$$

(b) When eliminating $T$ we generate a new factor $f_2$ as follows, which leaves us with the factors:

(c) When eliminating $U$ we generate a new factor $f_3$ as follows, which leaves us with the factors:

(d) When eliminating $V$ we generate a new factor $f_4$ as follows, which leaves us with the factors:

(e) When eliminating $W$ we generate a new factor $f_5$ as follows, which leaves us with the factors:

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

(g) What is the size of the largest factor that gets generated during the above process?

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?