CS 188: Artificial Intelligence

Informed Search

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Today

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search

- Graph Search
Recap: Search
Recap: Search

- **Search problem:**
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test

- **Search tree:**
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)

- **Search algorithm:**
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans
Example: Pancake Problem

Cost: Number of pancakes flipped
BOUNDS FOR SORTING BY PREFIX REVERSAL

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For a permutation $\sigma$ of the integers from 1 to $n$, let $f(\sigma)$ be the smallest number of prefix reversals that will transform $\sigma$ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all $\sigma$ in (the symmetric group) $S_n$. We show that $f(n) \leq (5n + 5)/3$, and that $f(n) \geq 17n/16$ for $n$ a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n + 3$. 
Example: Pancake Problem

State space graph with costs as weights
General Tree Search

function Tree-Search(problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
end

Action: flip top two
Cost: 2

Path to reach goal:
Flip four, flip three
Total cost: 7
All these search algorithms are the same except for fringe strategies

- Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
- Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
- Can even code one implementation that takes a variable queuing object
Uniform Cost Search

- Strategy: expand lowest path cost

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location
Video of Demo Contours UCS Empty
Video of Demo Contours UCS Pacman Small Maze
Informed Search
Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing
Example: Heuristic Function

$h(x)$
Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place
Greedy Search
Example: Heuristic Function

$h(x)$
Greedy Search

- Expand the node that seems closest...

- What can go wrong?
Greedy Search

- **Strategy:** expand a node that you think is closest to a goal state
  - **Heuristic:** estimate of distance to nearest goal for each state

- **A common case:**
  - Best-first takes you straight to the (wrong) goal

- **Worst-case:** like a badly-guided DFS

[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]
Video of Demo Contours Greedy (Empty)
Video of Demo Contours Greedy (Pacman Small Maze)
A* Search
A* Search
Combining UCS and Greedy

- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy** orders by goal proximity, or *forward cost* $h(n)$

- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

Example: Teg Grenager
When should A* terminate?

- Should we stop when we enqueue a goal?  

- No: only stop when we dequeue a goal
Is A* Optimal?

- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!
Admissible Heuristics
Idea: Admissibility

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs.
A heuristic $h$ is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal.

**Examples:**

Coming up with admissible heuristics is most of what’s involved in using A* in practice.
Optimality of A* Tree Search
Optimality of A* Tree Search

Assume:
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:
- A will exit the fringe before B
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor \( n \) of A is on the fringe, too (maybe A!)
- Claim: \( n \) will be expanded before B
  1. \( f(n) \) is less or equal to \( f(A) \)

\[
f(n) = g(n) + h(n) \quad \text{Definition of f-cost}
\]
\[
f(n) \leq g(A) \quad \text{Admissibility of } h
\]
\[
g(A) = f(A) \quad h = 0 \text{ at a goal}
\]
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$

$g(A) < g(B)$  \hspace{1cm} B is suboptimal

$f(A) < f(B)$  \hspace{1cm} h = 0 at a goal
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$
  3. $n$ expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal
Properties of A*
Properties of A*
UCS vs A* Contours

- Uniform-cost expands equally in all “directions”

- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]
Video of Demo Contours (Empty) -- UCS
Video of Demo Contours (Empty) -- Greedy
Video of Demo Contours (Empty) – A*
Video of Demo Contours (Pacman Small Maze) – A*
Comparison

Greedy

Uniform Cost

A*
A* Applications
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...

[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]
Video of Demo Pacman (Tiny Maze) – UCS / A*
Video of Demo Empty Water Shallow/Deep – Guess Algorithm
Creating Heuristics

You got

Heuristic upgrade!
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics.

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available.

- Inadmissible heuristics are often useful too.
Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?
Heuristic: Number of tiles misplaced

Why is it admissible?

\( h(\text{start}) = 8 \)

This is a \textit{relaxed-problem} heuristic

<table>
<thead>
<tr>
<th>Average nodes expanded when the optimal path has...</th>
<th>...4 steps</th>
<th>...8 steps</th>
<th>...12 steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>112</td>
<td>6,300</td>
<td>( 3.6 \times 10^6 )</td>
</tr>
<tr>
<td>TILES</td>
<td>13</td>
<td>39</td>
<td>227</td>
</tr>
</tbody>
</table>

Statistics from Andrew Moore
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

- Total Manhattan distance

- Why is it admissible?

- \( h(\text{start}) = 3 + 1 + 2 + \ldots = 18 \)

<table>
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<th>MANHATTAN</th>
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<td>...4 steps</td>
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<tr>
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<td>39</td>
<td>25</td>
</tr>
<tr>
<td>...12 steps</td>
<td>227</td>
<td>73</td>
</tr>
</tbody>
</table>
How about using the *actual cost* as a heuristic?
- Would it be admissible?
- Would we save on nodes expanded?
- What’s wrong with it?

With A*: a trade-off between quality of estimate and work per node
- As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself
Semi-Lattice of Heuristics
Trivial Heuristics, Dominance

- Dominance: \( h_a \geq h_c \) if
  \[
  \forall n : h_a(n) \geq h_c(n)
  \]

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible
  \[
  h(n) = \max(h_a(n), h_b(n))
  \]

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic
Graph Search
Failure to detect repeated states can cause exponentially more work.
Graph Search

- In BFS, for example, we shouldn’t bother expanding the circled nodes (why?)
Graph Search

- Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but...
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

- How about optimality?
A* Graph Search Gone Wrong?

State space graph

Search tree

S (0+2)

A (1+4)

B (1+1)

C (2+1)

G (5+0)

C (3+1)

G (6+0)
Consistency of Heuristics

- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal
    \[ h(A) \leq \text{actual cost from } A \text{ to } G \]
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc
    \[ h(A) - h(C) \leq \text{cost}(A \text{ to } C) \]

- Consequences of consistency:
  - The f value along a path never decreases
    \[ h(A) \leq \text{cost}(A \text{ to } C) + h(C) \]
  - A* graph search is optimal
Optimality of A* Graph Search
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state $s$, nodes that reach $s$ optimally are expanded before nodes that reach $s$ suboptimally
  - Result: A* graph search is optimal
Optimality

- **Tree search:**
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- **Graph search:**
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems
A*: Summary
A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems
Tree Search Pseudo-Code

function TREE-SEARCH(problem, fringe) return a solution, or failure

    fringe ← INSERT(make-node(initial-state[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
end
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
        end
    end
Consider what A* does:

- Expands nodes in increasing total f value (f-contours)
  Reminder: \( f(n) = g(n) + h(n) = \text{cost to } n + \text{heuristic} \)
- Proof idea: the optimal goal(s) have the lowest f value, so it must get expanded first

There’s a problem with this argument. What are we assuming is true?
Optimality of A* Graph Search

Proof:

- New possible problem: some $n$ on path to $G^*$ isn’t in queue when we need it, because some worse $n'$ for the same state dequeued and expanded first (disaster!)
- Take the highest such $n$ in tree
- Let $p$ be the ancestor of $n$ that was on the queue when $n'$ was popped
- $f(p) < f(n)$ because of consistency
- $f(n) < f(n')$ because $n'$ is suboptimal
- $p$ would have been expanded before $n'$
- Contradiction!