## Q1. SpongeBob and Pacman (Search Formulation)

Pacman bought a car, was speeding in Pac-City, and SpongeBob wasn't able to catch him. Now Pacman has run out of gas, his car has stopped, and he is currently hiding out at an undisclosed location.

In this problem, you are on the SpongeBob side, tryin' to catch Pacman!
There are still $p$ SpongeBob cars in the Pac-city of dimension $m$ by $n$. In this problem, all SpongeBob cars can move, with two distinct integer controls: throttle and steering, but Pacman has to stay stationary. Once one SpongeBob car takes an action which lands him in the same grid as Pacman, Pacman will be arrested and the game ends.

Throttle: $t_{i} \in\{1,0,-1\}$, corresponding to $\{$ Gas, Coast, Brake $\}$. This controls the speed of the car by determining its acceleration. The integer chosen here will be added to his velocity for the next state. For example, if a SpongeBob car is currently driving at $5 \mathrm{grid} / \mathrm{s}$ and chooses Gas (1) he will be traveling at $6 \mathrm{grid} / \mathrm{s}$ in the next turn.
Steering: $s_{i} \in\{1,0,-1\}$, corresponding to \{Turn Left, Go Straight, Turn Right\}. This controls the direction of the car. For example, if a SpongeBob car is facing North and chooses Turn Left, it will be facing West in the next turn.
(a) Suppose you can only control 1 SpongeBob car, and have absolutely no information about the remainder of $p-1$ SpongeBob cars, or where Pacman stopped to hide. Also, the SpongeBob cars can travel up to 6 grid/s so $0 \leq v \leq 6$ at all times.
(i) What is the tightest upper bound on the size of state space, if your goal is to use search to plan a sequence of actions that guarantees Pacman is caught, no matter where Pacman is hiding, or what actions other SpongeBob cars take. Please note that your state space representation must be able to represent all states in the search space.
$\qquad$
(ii) What is the maximum branching factor? Your answer may contain integers, $m, n$.
$\square$
(iii) Which algorithm(s) is/are guaranteed to return a path passing through all grid locations on the grid, if one exists?

| $\square$ Depth First Tree Search | $\square$ Breadth First Tree Search |
| :--- | :--- |
| $\square$ Depth First Graph Search | $\square$ Breadth First Graph Search |

(iv) Is Breadth First Graph Search guaranteed to return the path with the shortest number of time steps, if one exists?
$\bigcirc$ Yes
$\bigcirc$ No
(b) Now let's suppose you can control all $p$ SpongeBob cars at the same time (and know all their locations), but you still have no information about where Pacman stopped to hide
(i) Now, you still want to search a sequence of actions such that the paths of $p$ SpongeBob car combined pass through all $m * n$ grid locations. Suppose the size of the state space in part (a) was $N_{1}$, and the size of the state space in this part is $N_{p}$. Please select the correct relationship between $N_{p}$ and $N_{1}$
$\bigcirc N_{p}=p * N_{1}$
$\bigcirc$
$N_{p}=p^{N_{1}}$$N_{p}=\left(N_{1}\right)^{p}$
None of the above
(ii) Suppose the maximum branching factor in part (a) was $b_{1}$, and the maximum branching factor in this part is $b_{p}$. Please select the correct relationship between $b_{p}$ and $b_{1}$
$\bigcirc b_{p}=p * b_{1}$
$\bigcirc b_{p}=p^{b_{1}}$
$\bigcirc b_{p}=\left(b_{1}\right)^{p}$
None of the above

## Q2. Search: Snail search for love

Scorpblorg the snail is looking for a mate. It can visit different potential mates based on a trail of ooze to nearby snails, and then test them for chemistry, as represented in the below graph, where each node represents a snail. In all cases, nodes with equal priority should be visited in alphabetical order.

(a) Simple search

In this part, assume that the only match for Scorpblorg is Squish (i.e. Squish is the goal state). Which of the following are true when searching the above graph?
(i) BFS Tree Search expands more nodes than DFS Tree Search

True
False
(ii) DFS Tree Search finds a path to the goal for this graphTrue False
(iii) DFS Graph Search finds the shortest path to the goal for this graph $\bigcirc$ True False
(iv) If we remove the connection from Cuddles $\rightarrow$ Alex, can DFS Graph Search find a path to the goal for the altered graph?No
(b) Third Time's A Charm

Now we assume that Scorpblorg's mate preferences have changed. The new criteria she is looking for in a mate is that she has visited the mate twice before (i.e. when she visits any state for the third time, she has found a path to the goal).
(i) What should the most simple yet sufficient new state space representation include?
$\square$ The current location of Scorpblorg
$\square$ The total number of edges travelled so far
$\square$ An array of booleans indicating whether each snail has been visited so far
$\square$ An array of numbers indicating how many times each snail has been visited so far
$\square$ The number of distinct snails visited so far
(ii) DFS Tree Search finds a path to the goal for this graphFalse
(iii) BFS Graph Search finds a path to the goal for this graph
$\bigcirc$ True
False
(iv) If we remove the connection from Cuddles $\rightarrow$ Alex, can DFS Graph Search finds a path to the goal for the altered graph?

Yes No

We continue as in part (b) where the goal is still to find a mate who is visited for the third time.
(c) Costs for visiting snails

Assume we are using Uniform cost search and we can now add costs to the actions in the graph.
(i) Can one assign (non-negative) costs to the actions in the graph such that the goal state returned by UCS (Tree-search) changes? $\bigcirc$ Yes $\bigcirc$
(ii) Can one assign (potentially negative) costs to the actions in the graph such that UCS (Tree-search) will never find a goal state? Yes No

