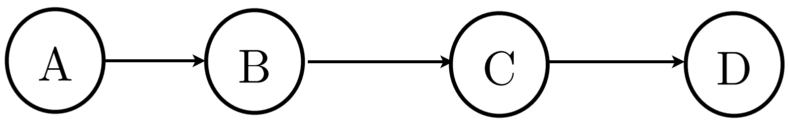
CS 188 Fall 2023 — Regular Discussion 9 Solutions

Q1. Bayes' Nets Sampling

Assume the following Bayes' net, and the corresponding distributions over the variables in the Bayes' net:



$P(A)$	
$-a$	$3/4$
$+a$	$1/4$

$P(B\|A)$		
$-a$	$-b$	$2/3$
$-a$	$+b$	$1/3$
$+a$	$-b$	$4/5$
$+a$	$+b$	$1/5$

$P(C\|B)$		
$-b$	$-c$	$1/4$
$-b$	$+c$	$3/4$
$+b$	$-c$	$1/2$
$+b$	$+c$	$1/2$

$P(D\|C)$		
$-c$	$-d$	$1/8$
$-c$	$+d$	$7/8$
$+c$	$-d$	$5/6$
$+c$	$+d$	$1/6$

(a) You are given the following samples:

$+a$	$+b$	$-c$	$-d$
$+a$	$-b$	$+c$	$-d$
$-a$	$+b$	$+c$	$-d$
$-a$	$-b$	$+c$	$-d$

$+a$	$-b$	$-c$	$+d$
$+a$	$+b$	$+c$	$-d$
$-a$	$+b$	$-c$	$+d$
$-a$	$-b$	$+c$	$-d$

(i) Assume that these samples came from performing Prior Sampling, and calculate the sample estimate of $P(+c)$.

$5/8$

(ii) Now we will estimate $P(+c \mid +a, -d)$. Above, clearly cross out the samples that would **not** be used when doing Rejection Sampling for this task, and write down the sample estimate of $P(+c \mid +a, -d)$ below.

$2/3$

(b) Using Likelihood Weighting Sampling to estimate $P(-a \mid +b, -d)$, the following samples were obtained. Fill in the weight of each sample in the corresponding row.

Sample				Weight
$-a$	$+b$	$+c$	$-d$	$P(+b \mid -a)P(-d \mid +c) = 1/3 * 5/6 = 5/18 = 0.277$
$+a$	$+b$	$+c$	$-d$	$P(+b \mid +a)P(-d \mid +c) = 1/5 * 5/6 = 5/30 = 1/6 = 0.17$
$+a$	$+b$	$-c$	$-d$	$P(+b \mid +a)P(-d \mid -c) = 1/5 * 1/8 = 1/40 = 0.025$
$-a$	$+b$	$-c$	$-d$	$P(+b \mid -a)P(-d \mid -c) = 1/3 * 1/8 = 1/24 = 0.042$

(c) From the weighted samples in the previous question, estimate $P(-a \mid +b, -d)$.

$\frac{5/18+1/24}{5/18+5/30+1/40+1/24} = 0.625$

(d) Which query is better suited for likelihood weighting, $P(D \mid A)$ or $P(A \mid D)$? Justify your answer in one sentence.

$P(D \mid A)$ is better suited for likelihood weighting sampling, because likelihood weighting conditions only on upstream evidence.

(e) Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is $A = +a$. Clearly fill in the circle(s) of the sequence(s) below that could have been generated by Gibbs Sampling.

Sequence 1

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$+d$
3 :	$+a$	$-b$	$+c$	$+d$

Sequence 2

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$-a$	$-b$	$-c$	$+d$

Sequence 3

1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$-d$

Sequence 4

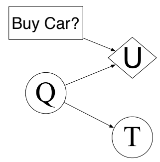
1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$+b$	$-c$	$+d$

Gibbs sampling updates one variable at a time and never changes the evidence.

The first and third sequences have at most one variable change per row, and hence could have been generated from Gibbs sampling. In sequence 2, the evidence variable is changed. In sequence 4, the second and third samples have both B and D changing.

2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ($Q = +q$) or bad quality ($Q = -q$). A test (T) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test: T = pass or T = fail. The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



1. Calculate the expected net gain from buying the car, given no test.

$$\begin{aligned} EU(\text{buy}) &= P(Q = +q) \cdot U(+q, \text{buy}) + P(Q = \neg q) \cdot U(\neg q, buy) \\ &= .7 \cdot 500 + 0.3 \cdot -200 = 290 \end{aligned}$$

2. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

$$P(T = \text{pass}|Q = +q) = 0.9$$
$$P(T = \text{pass}|Q = \neg q) = 0.2$$

Calculate the probability that the car will pass (or fail) its test, and then the probability that it is in good (or bad) shape given each possible test outcome.

$$\begin{aligned} P(T = \text{pass}) &= \sum_q P(T = \text{pass}, Q = q) \\ &= P(T = \text{pass}|Q = +q)P(Q = +q) + P(T = \text{pass}|Q = \neg q)P(Q = \neg q) \\ &= 0.69 \\ P(T = fail) &= 0.31 \\ P(Q = +q|T = \text{pass}) &= \frac{P(T = \text{pass}|Q = +q)P(Q = +q)}{P(T = \text{pass})} \\ &= \frac{0.9 \cdot 0.7}{0.69} = \frac{21}{23} \approx 0.91 \\ P(Q = +q|T = \text{fail}) &= \frac{P(T = \text{fail}|Q = +q)P(Q = +q)}{P(T = \text{fail})} \\ &= \frac{0.1 \cdot 0.7}{0.31} = \frac{7}{31} \approx 0.22 \end{aligned}$$

3. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

$$\begin{aligned} EU(\text{buy}|T = pass) &= P(Q = +q|T = \text{pass})U(+q, \text{buy}) + P(Q = \neg q|T = \text{pass})U(\neg q, \text{buy}) \\ &\approx 0.91 \cdot 500 + 0.09 \cdot (-200) \approx 437 \\ EU(\text{buy}|T = \text{fail}) &= P(Q = +q|T = \text{fail})U(+q, \text{buy}) + P(Q = \neg q|T = \text{fail})U(\neg q, \text{buy}) \\ &\approx 0.22 \cdot 500 + 0.78 \cdot (-200) = -46 \\ EU(\neg\text{buy}|T = pass) &= 0 \\ EU(\neg\text{buy}|T = fail) &= 0 \end{aligned}$$

Therefore: $MEU(T = \text{pass}) = 437$ (with buy) and $MEU(T = \text{fail}) = 0$ (using $\neg$buy)

4. Calculate the value of (perfect) information of the test. Should the buyer pay for a test?

$$
\begin{aligned}
VPI(T) &= (\sum_t P(T = t)MEU(T = t)) - MEU(\phi) \\
&= 0.69 \cdot 437 + 0.31 \cdot 0 - 290 \approx 11.53
\end{aligned}
$$

You shouldn't pay for it, since the cost is \$50.