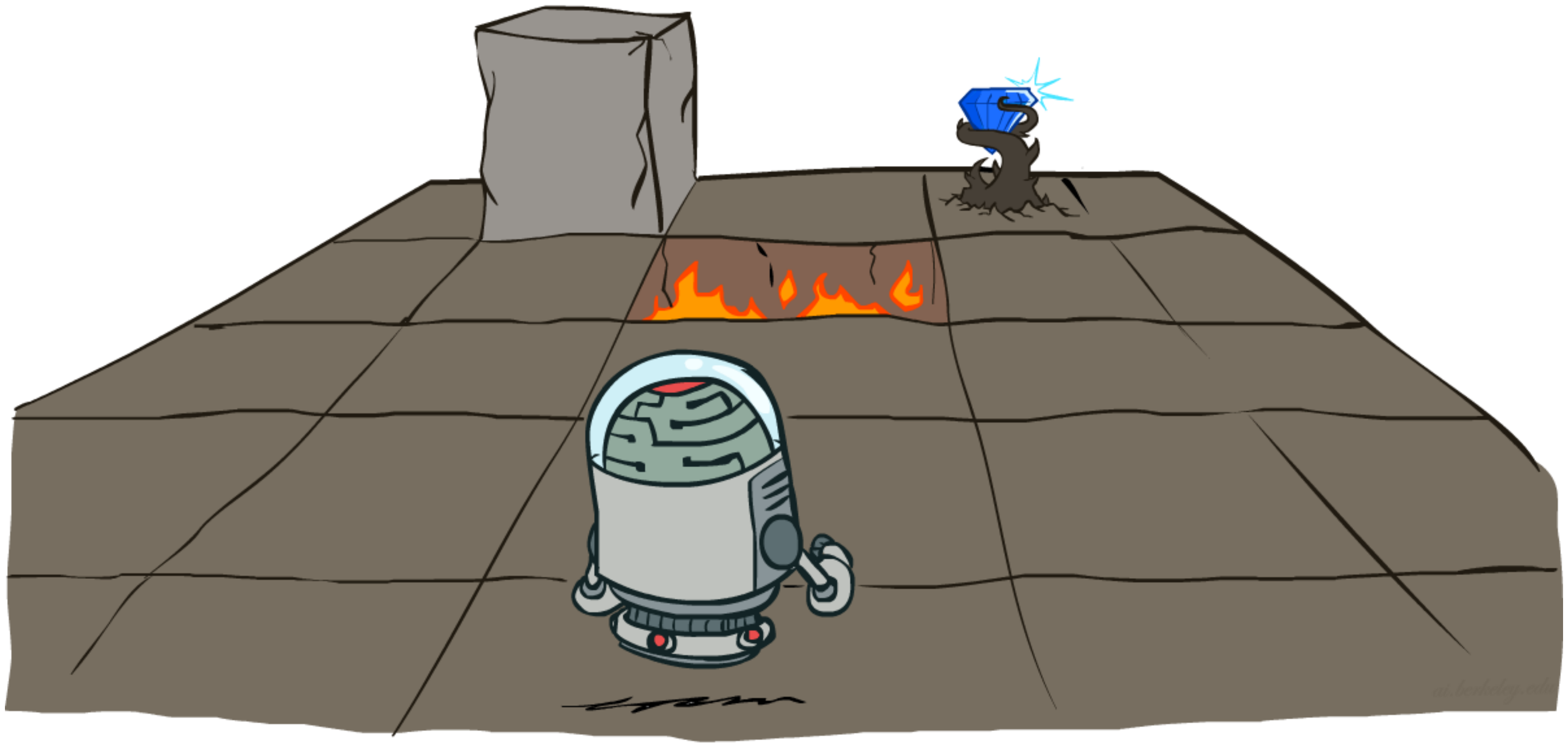
Announcements

- **Homework 2** due **today (Sept 19)** at 11:59pm PT
- **Project 2** due **this Friday (Sept 22)** at 11:59pm PT

CS 188: Artificial Intelligence

Markov Decision Processes I

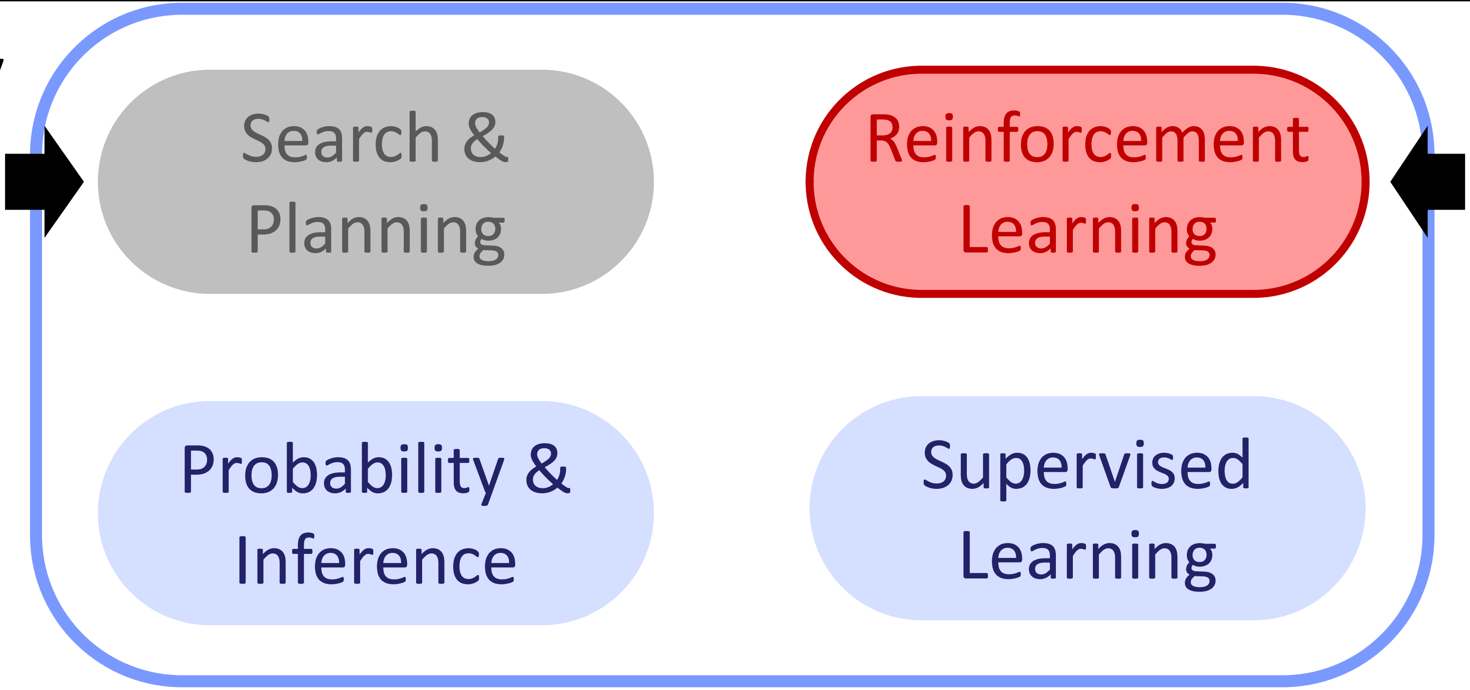


University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

Preview of Next Two Weeks

How to act only in current situation

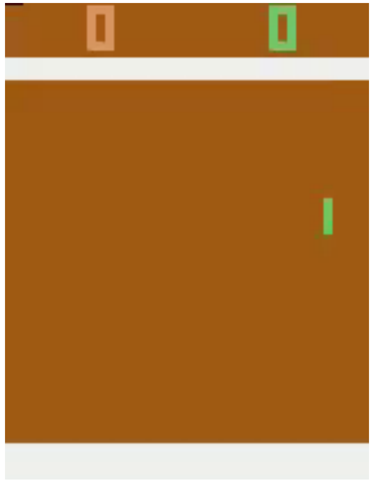


Learn reflex / policy to act in *any* situation

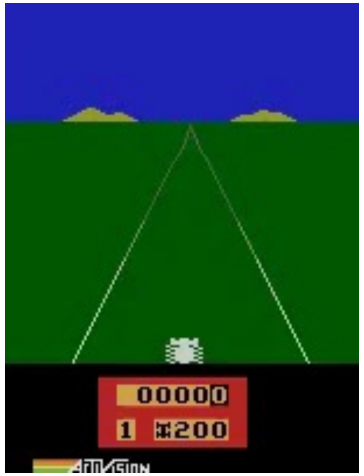
- This week: *value iteration* and *policy iteration*
  - Assumes we can query model of the world
- Next week: learning from trial and error
  - Learn only from interactions with the world

Examples of (Deep) Reinforcement Learning

2013: Playing Atari games

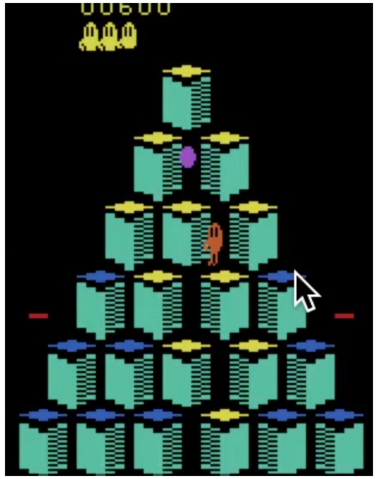


Pong



Enduro

Beamrider

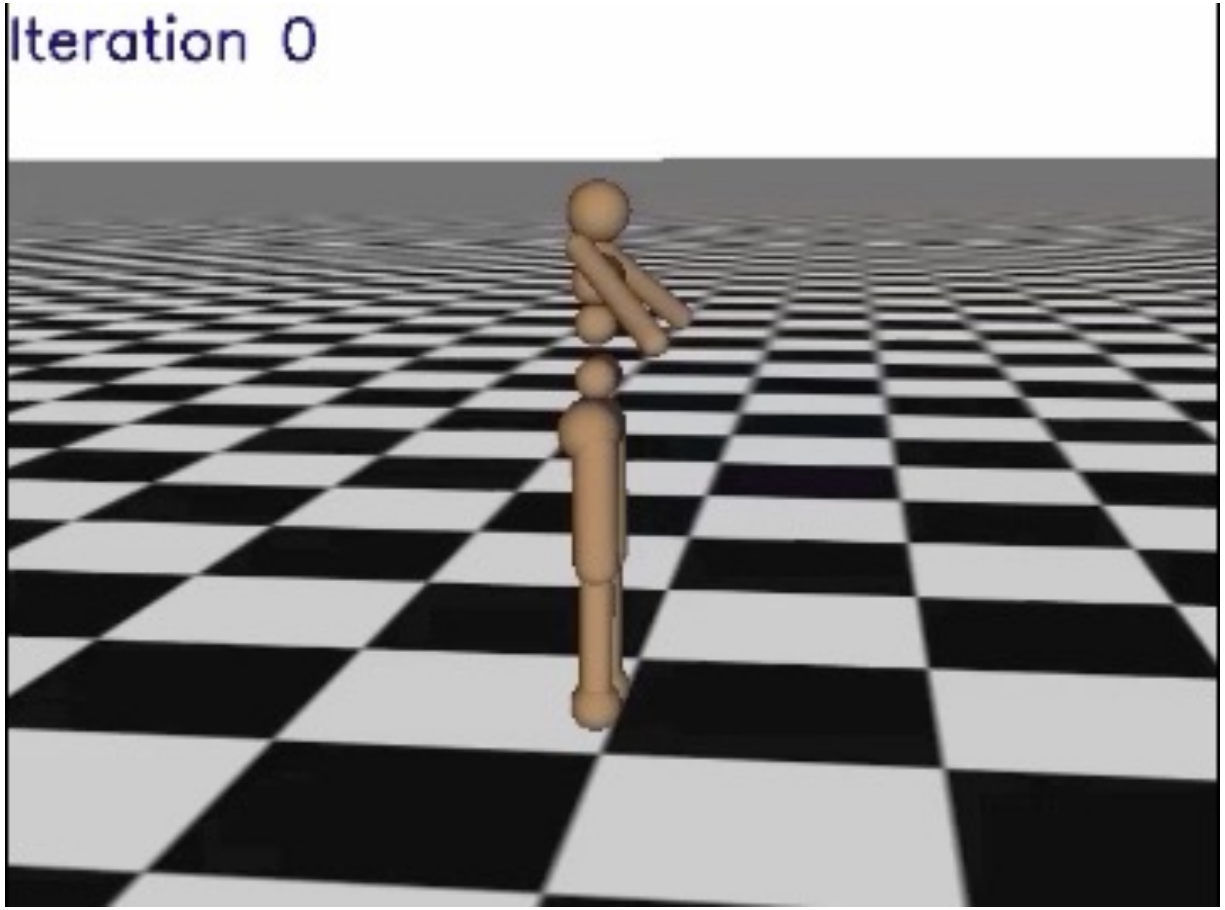


Q*bert

[Human-level control through deep reinforcement learning. Mnih et al. Nature 2015]

Examples of (Deep) Reinforcement Learning

2015: Locomotion from trial and error



[Trust Region Policy Optimization. Schulman et al. ICLR 2015]

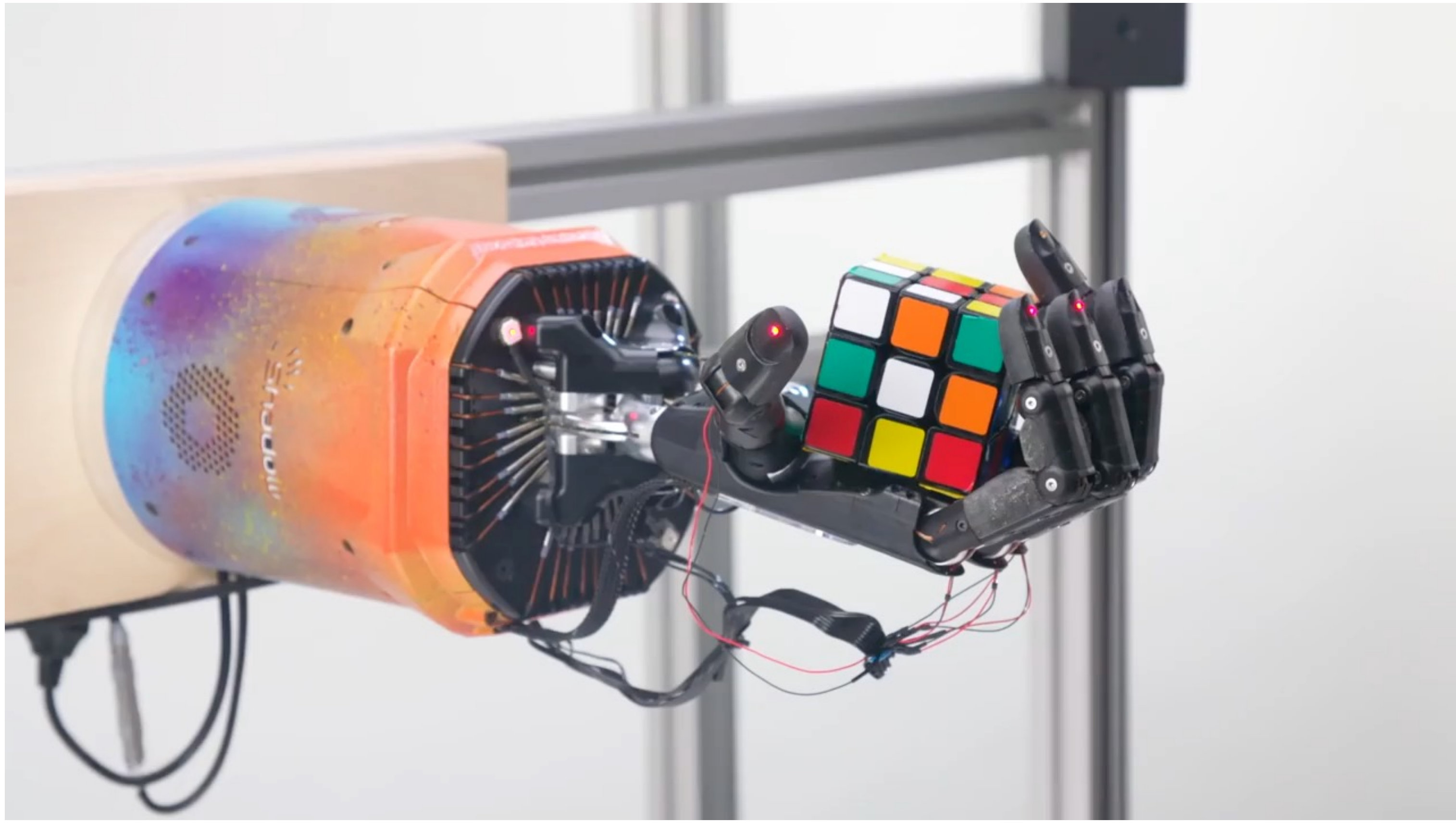
Examples of (Deep) Reinforcement Learning

2016: Playing Go (and beating human champion)

[Mastering the game of Go with deep neural networks and tree search. Silver et al. Nature 2016]

Examples of (Deep) Reinforcement Learning

2019: Robot manipulation

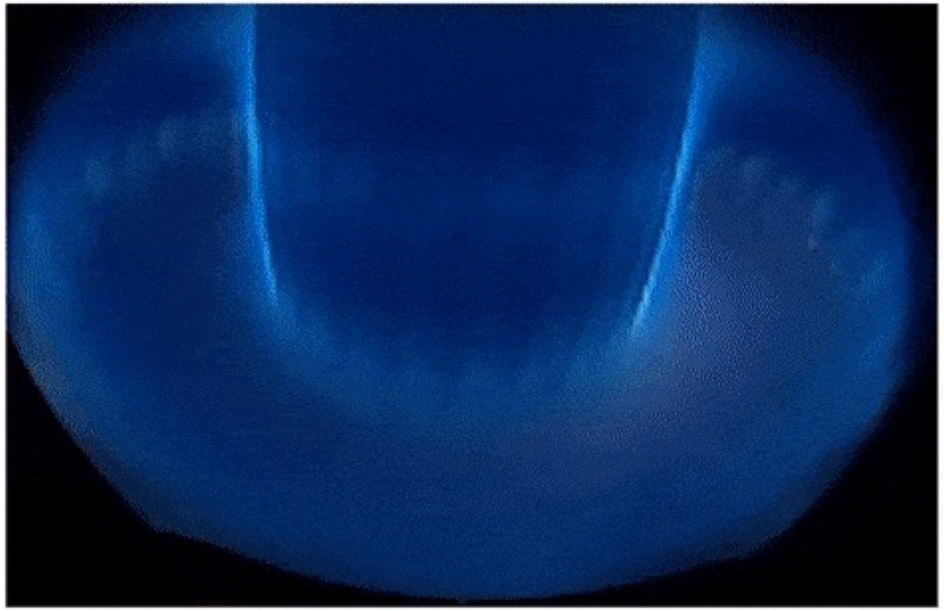


[Solving Rubik's cube with a robot hand. OpenAI. 2019]

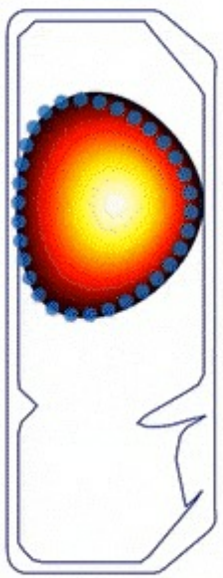
Examples of (Deep) Reinforcement Learning

2022: Nuclear fusion plasma control

Photo Credits: DeepMind and SPC/EPFL



View from inside the tokamak

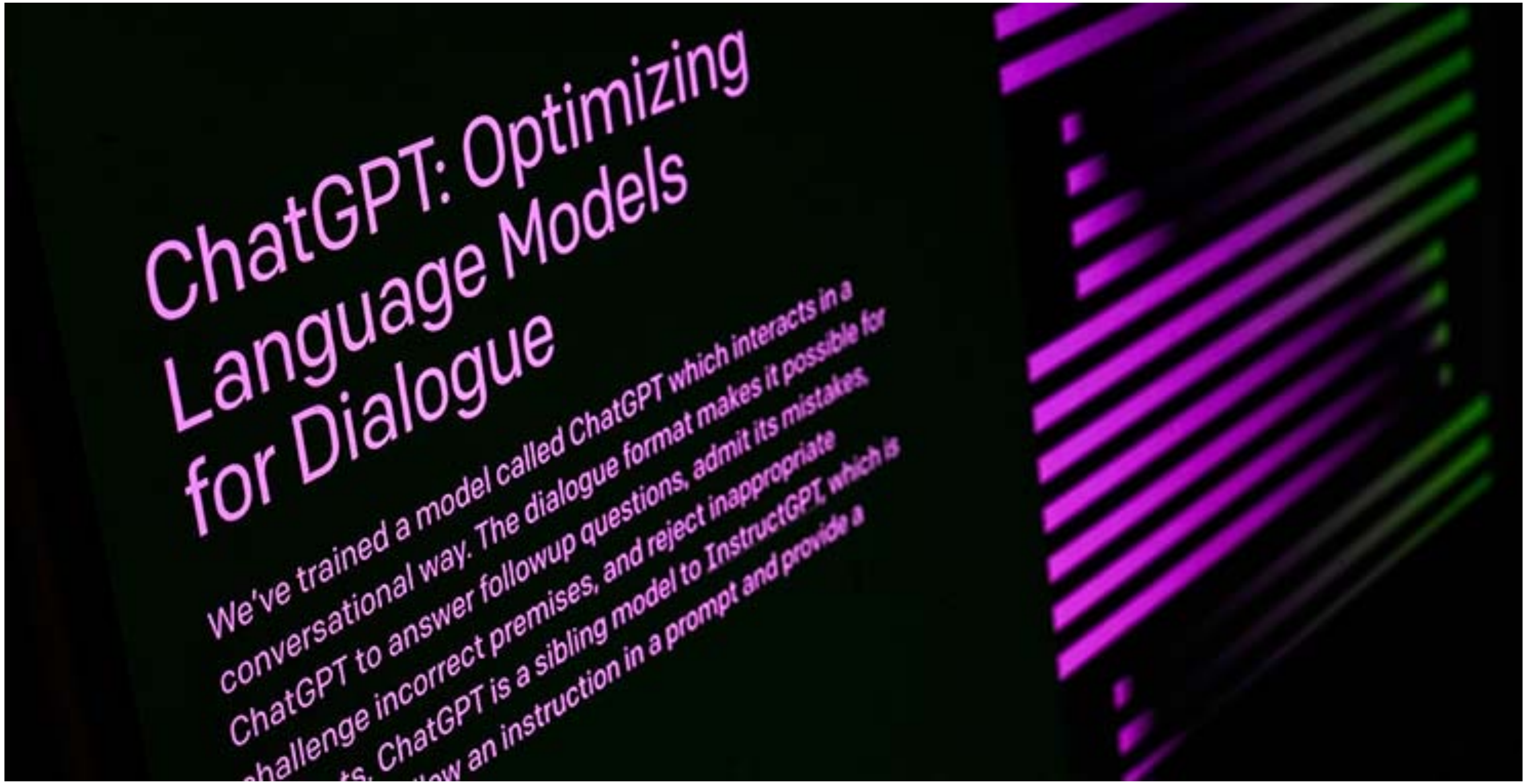


Plasma state reconstruction

[Magnetic control of tokamak plasmas through deep reinforcement learning. Degrave et al. Nature 2022]

Examples of (Deep) Reinforcement Learning

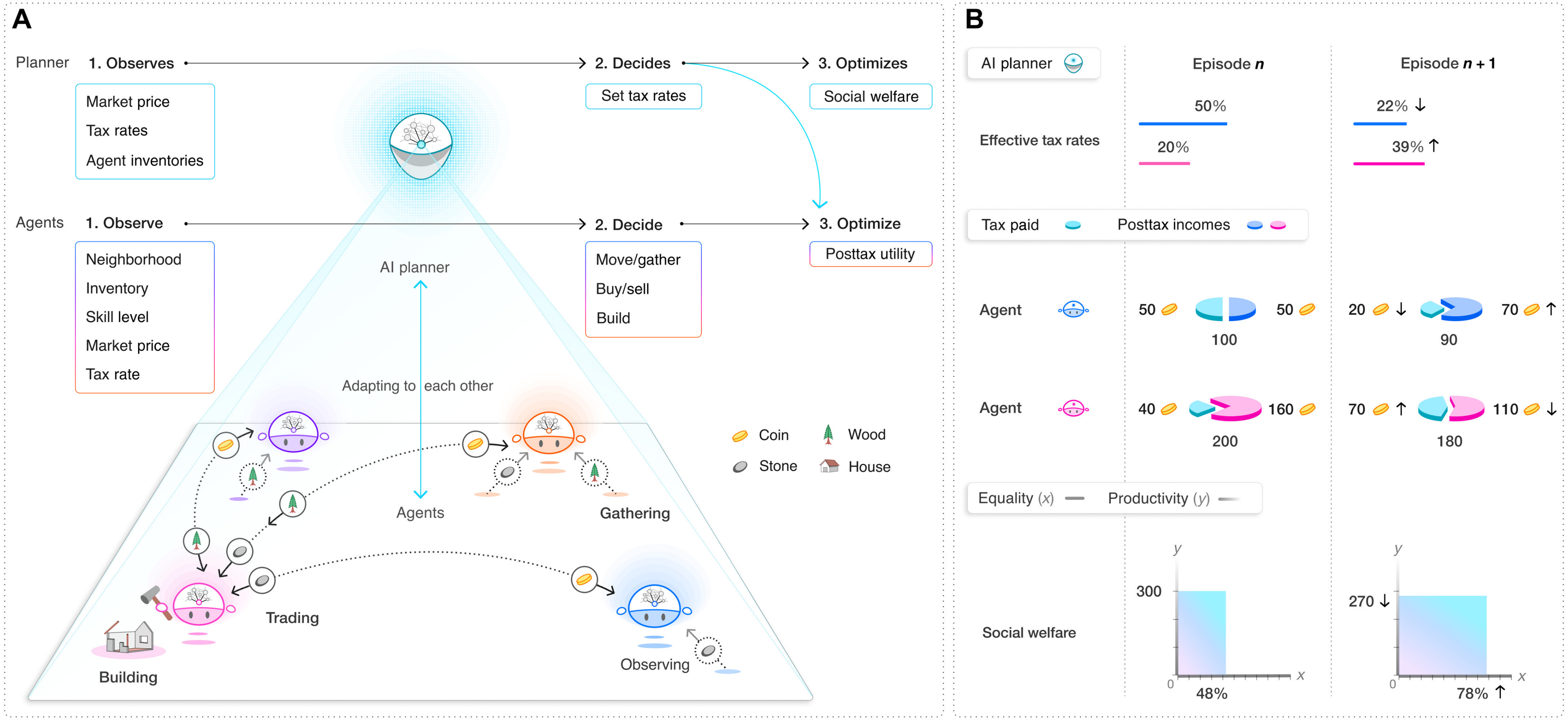
2022: Training Language Models with Human Feedback



[Aligning language models to follow instructions. Ouyang et al. 2022]

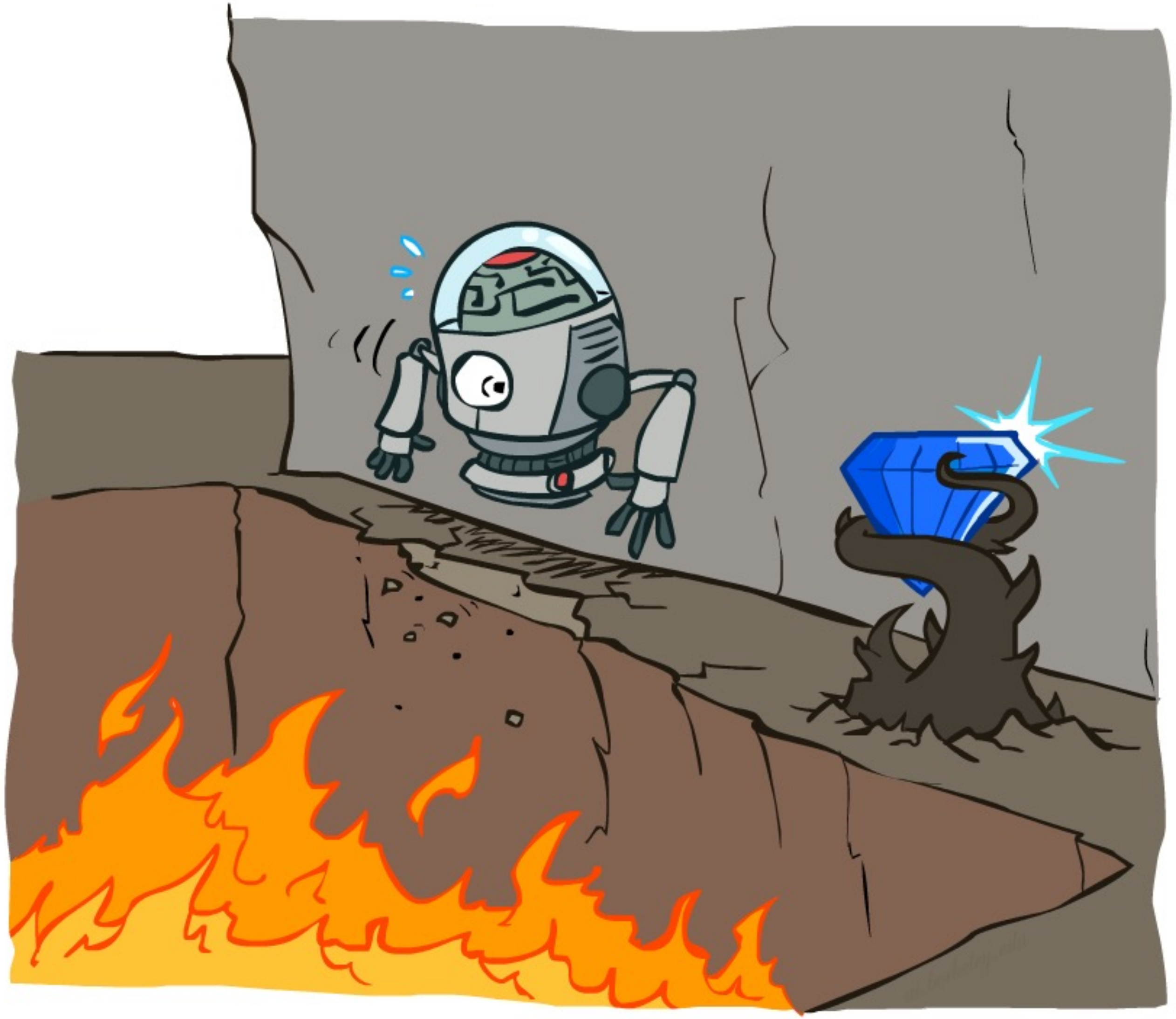
Examples of (Deep) Reinforcement Learning

2022: Economic policy design?



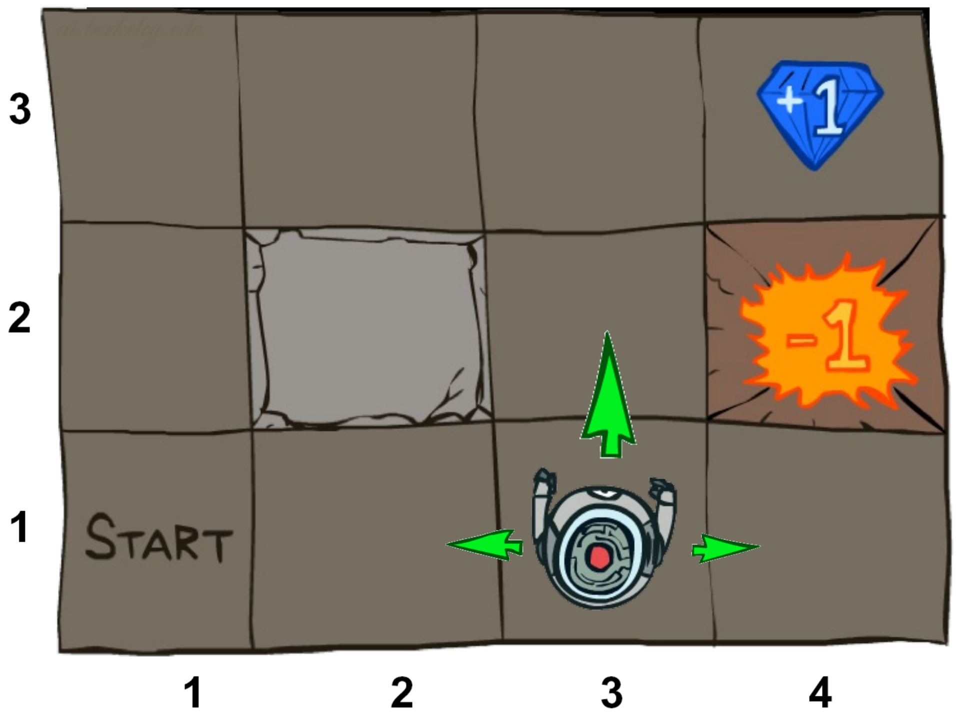
[The AI Economist: Taxation policy design via two-level deep multiagent RL. Zheng et al. Science 2022]

Non-Deterministic Search



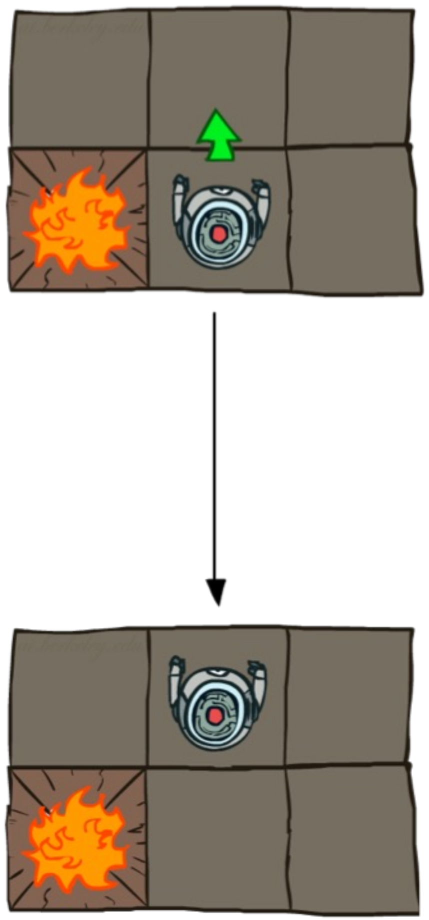
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

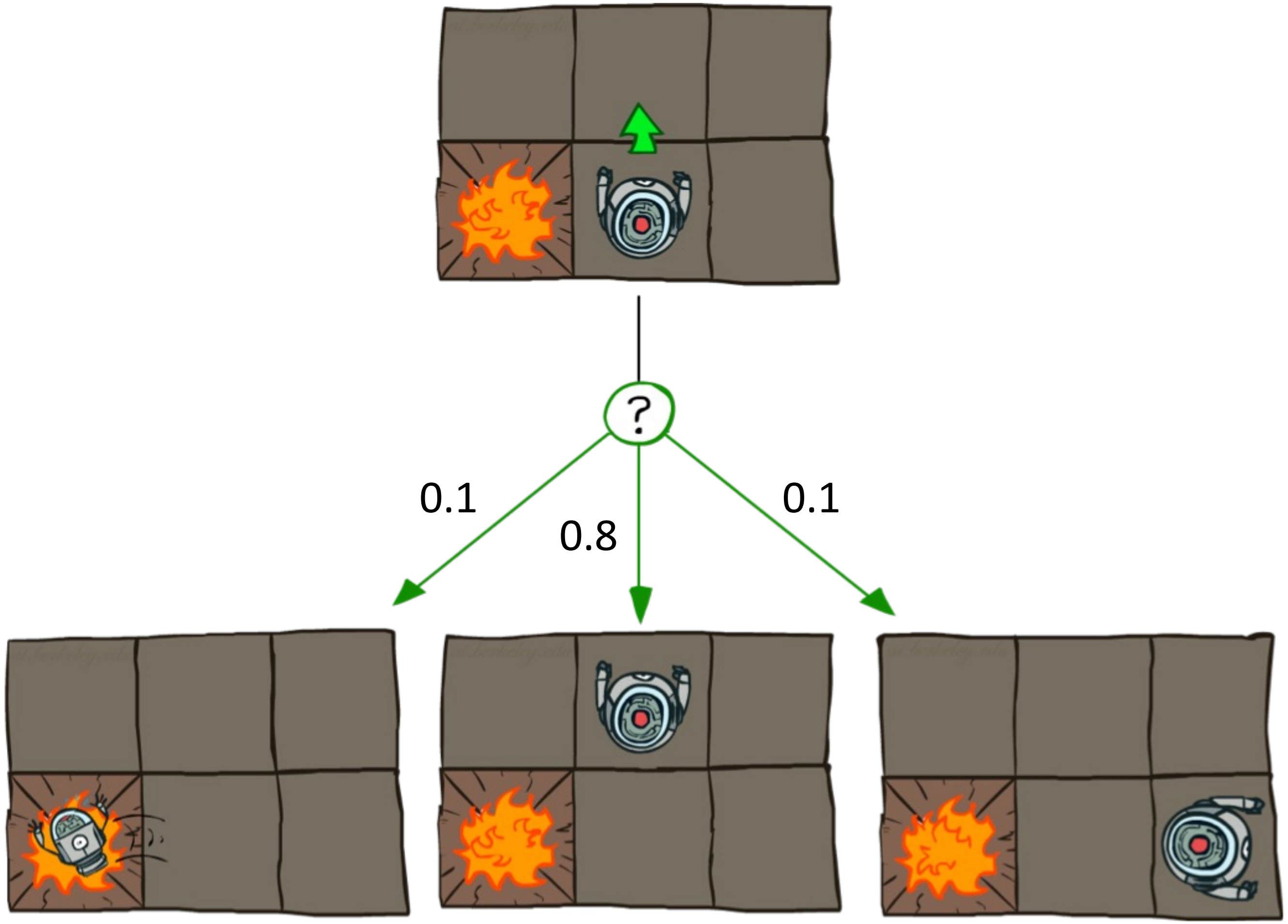


Grid World Actions

Deterministic Grid World

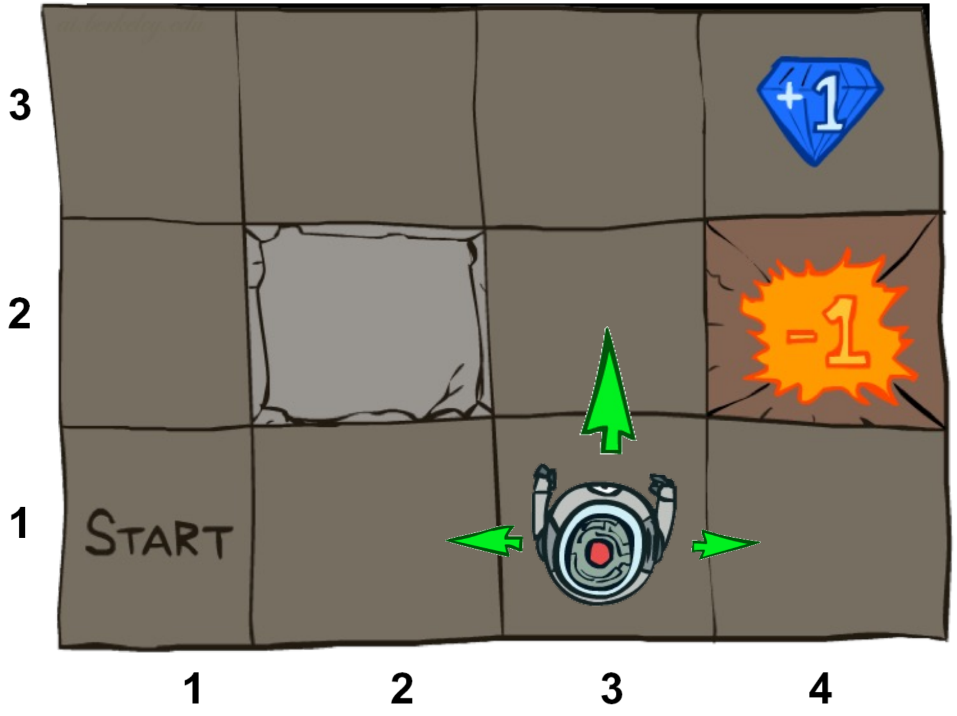


Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
  - A set of states $s \in S$
  - A set of actions $a \in A$
  - A transition function T(s, a, s')
    - Probability that a from s leads to s', i.e., P(s'| s, a)
    - Also called the model or the dynamics
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state
  - Maybe a terminal state



[Demo – gridworld manual intro (L8D1)]

What is Markov about MDPs?

- “Markov” generally means that given the present state, the future and the past are independent

- For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

$$=$$

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

- This is just like search, where the successor function could only depend on the current state (not the history)

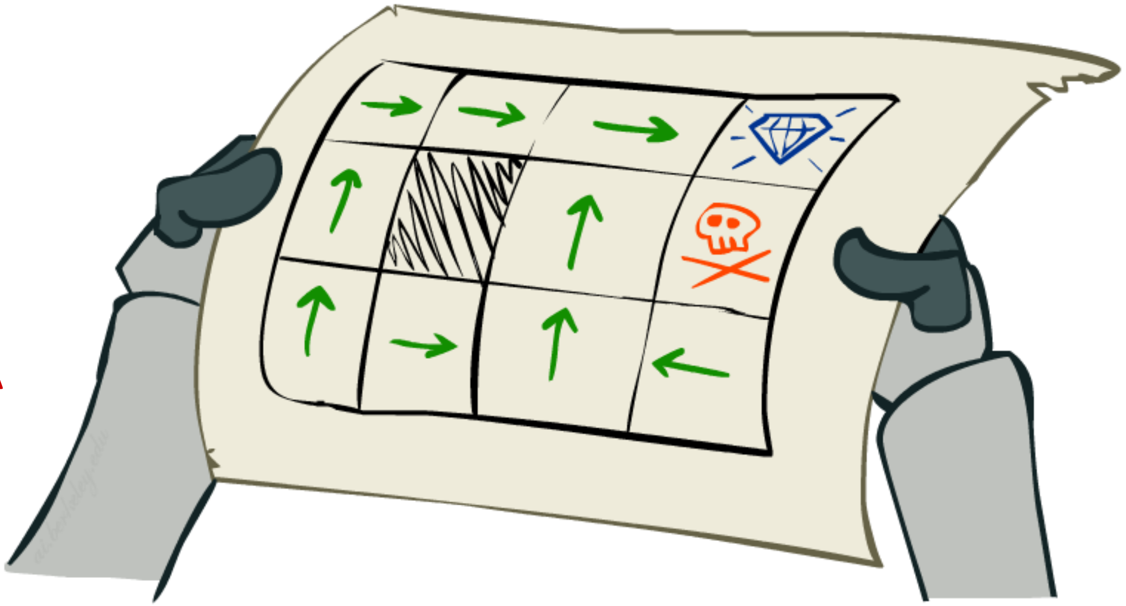
Andrey Markov
(1856-1922)

Policies

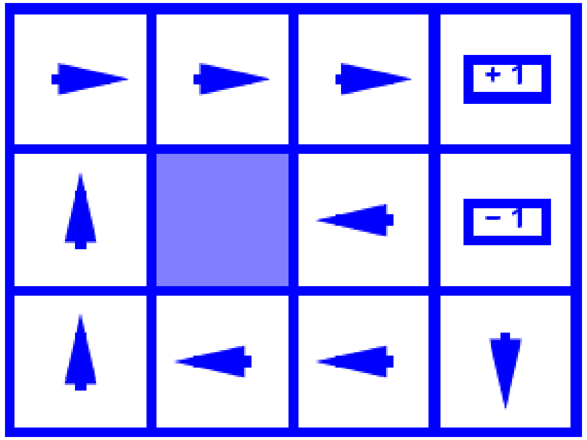
- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

- For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$
  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed
  - An explicit policy defines a reflex agent

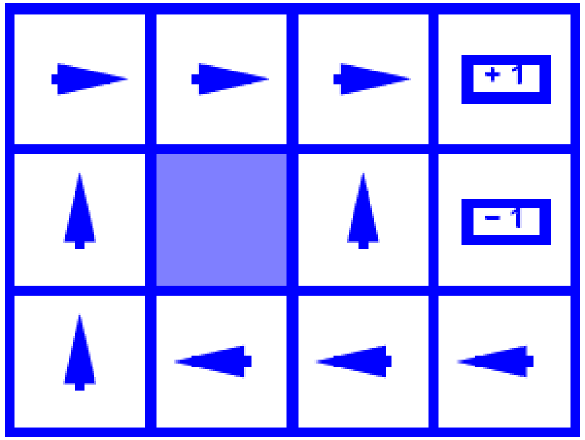
- Expectimax didn't compute entire policies
  - It computed the action for a single state only



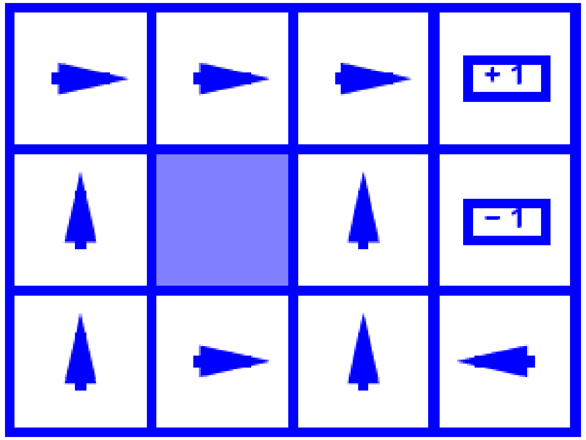
Optimal Policies



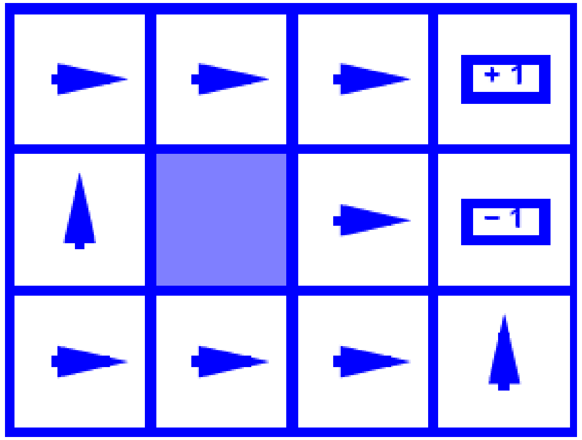
R(s) = -0.01



R(s) = -0.03

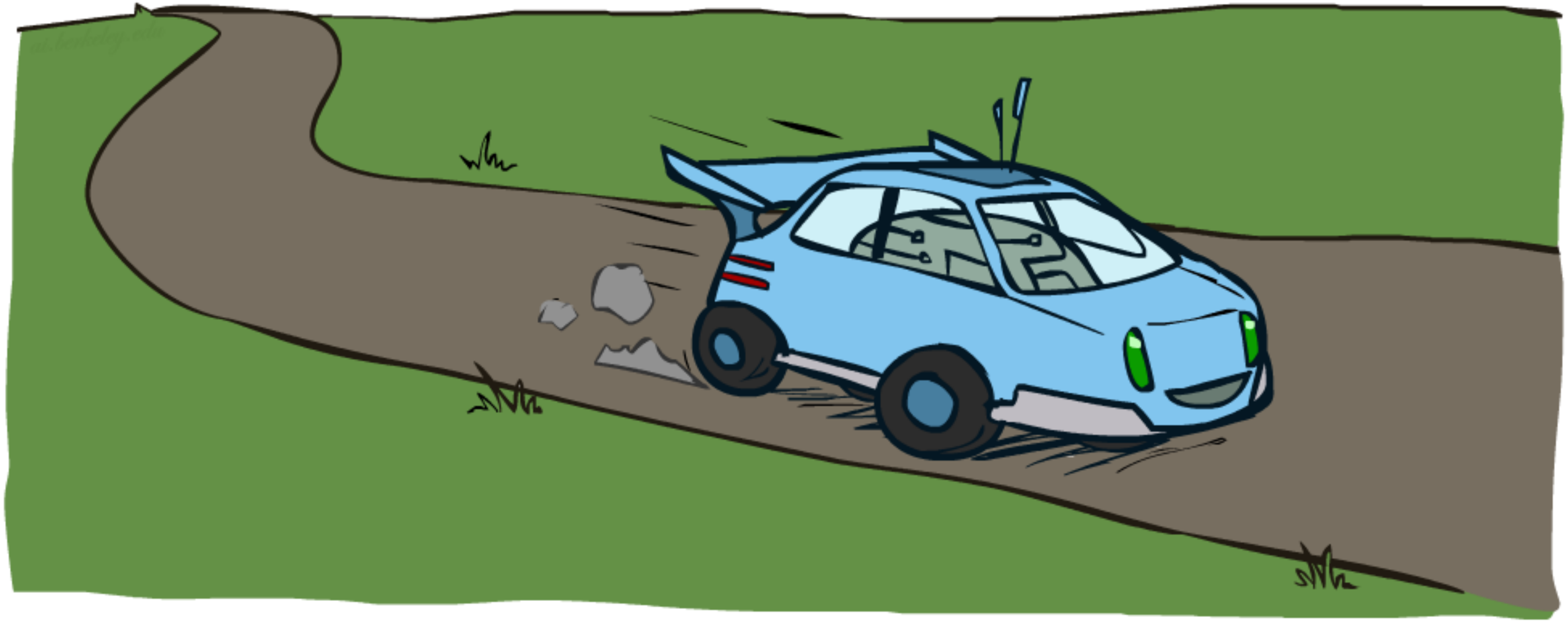


R(s) = -0.4



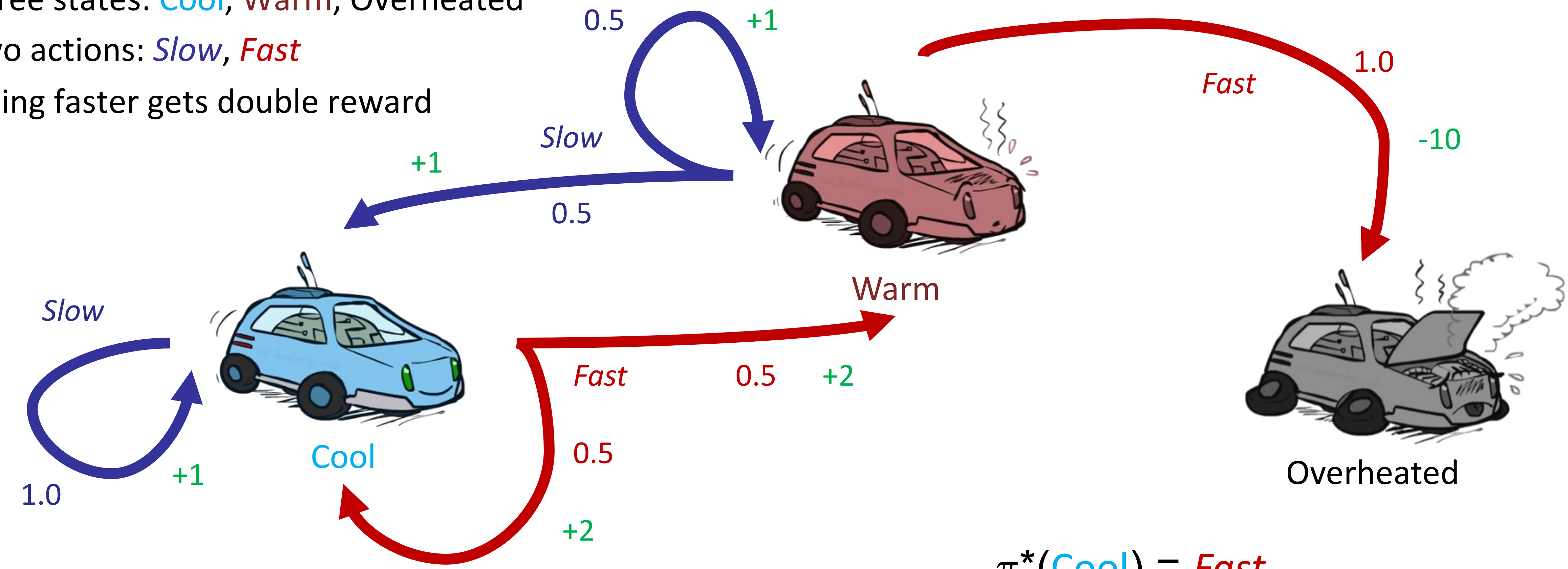
R(s) = -2.0

Example: Racing



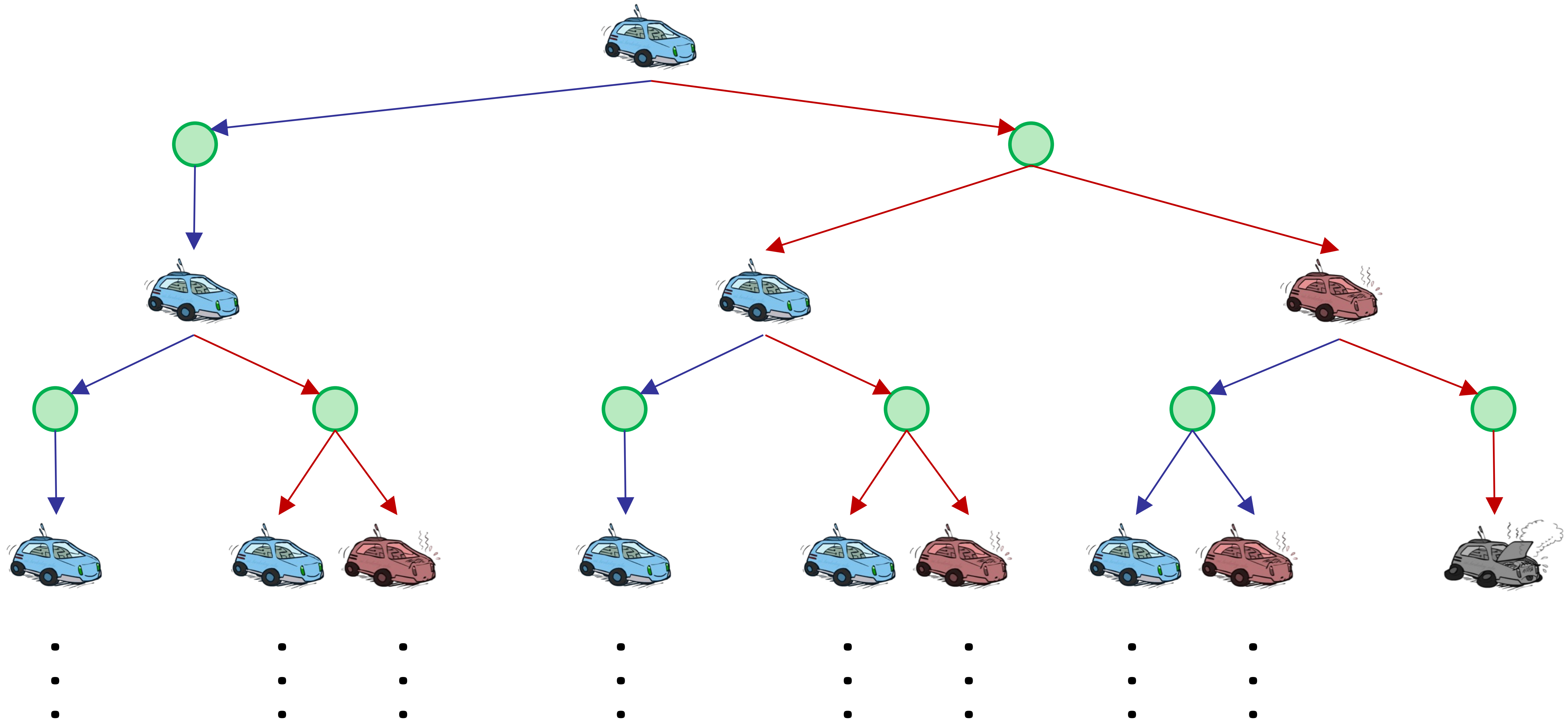
Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
- Going faster gets double reward



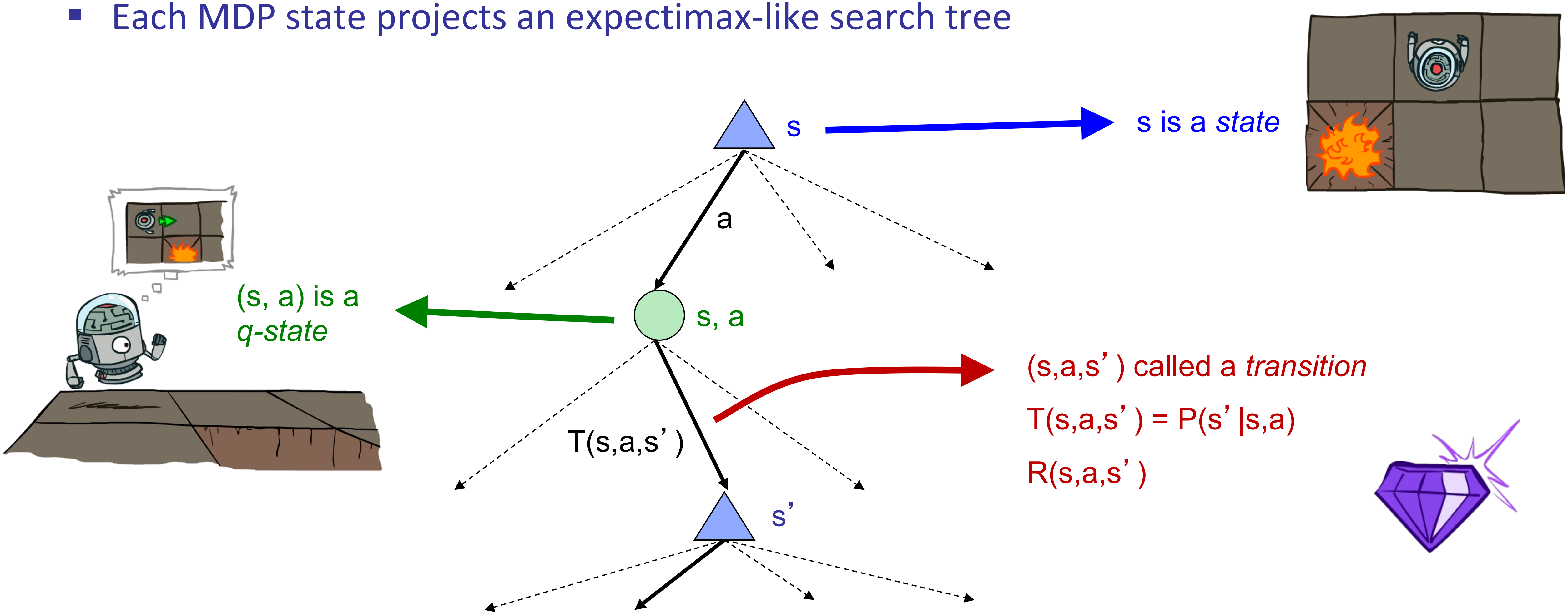
Optimal Policy: π^*(Cool) = *Fast*
π^*(Warm) = *Slow*
π^*(Overheated) = end

Racing Search Tree

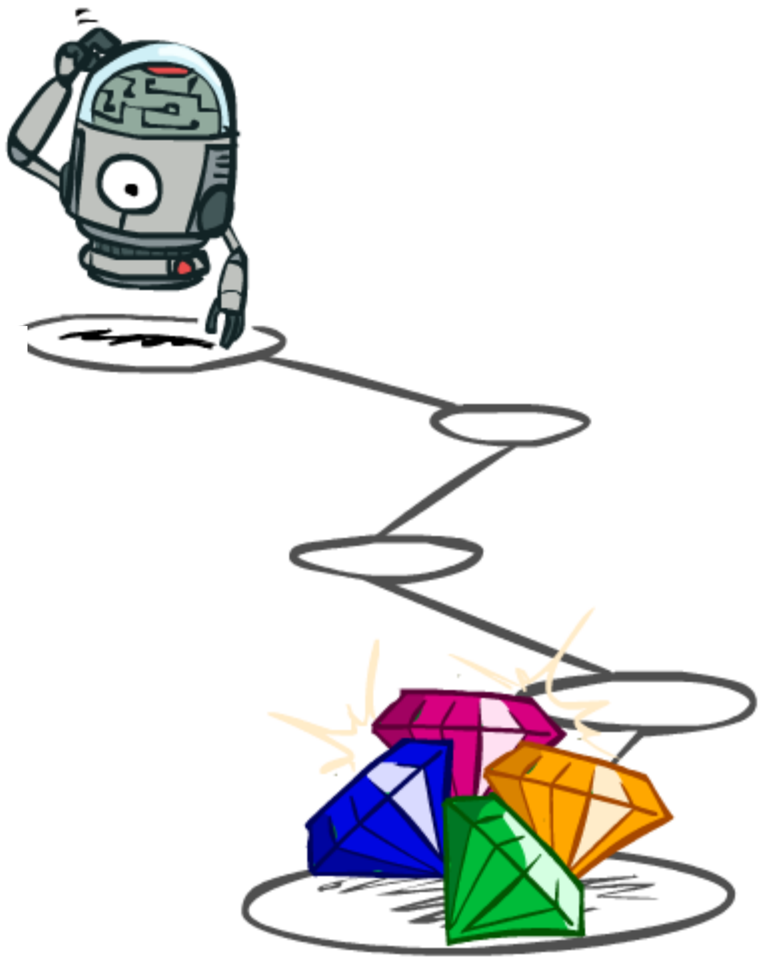


MDP Search Trees

- Each MDP state projects an expectimax-like search tree



Utilities of Sequences

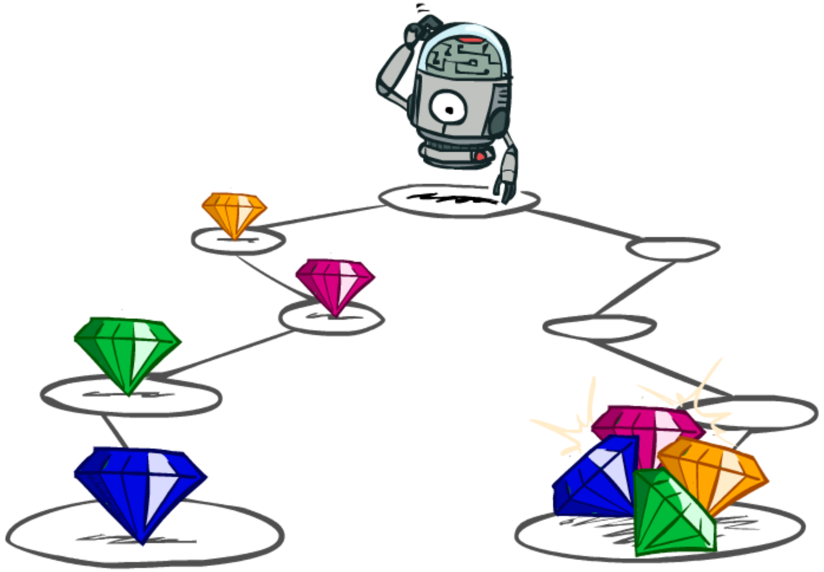


Utilities of Sequences

- What preferences should an agent have over reward sequences?

- More or less? [1, 2, 2] or [2, 3, 4]

- Now or later? [0, 0, 1] or [1, 0, 0]



Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

1

Worth Now

γ

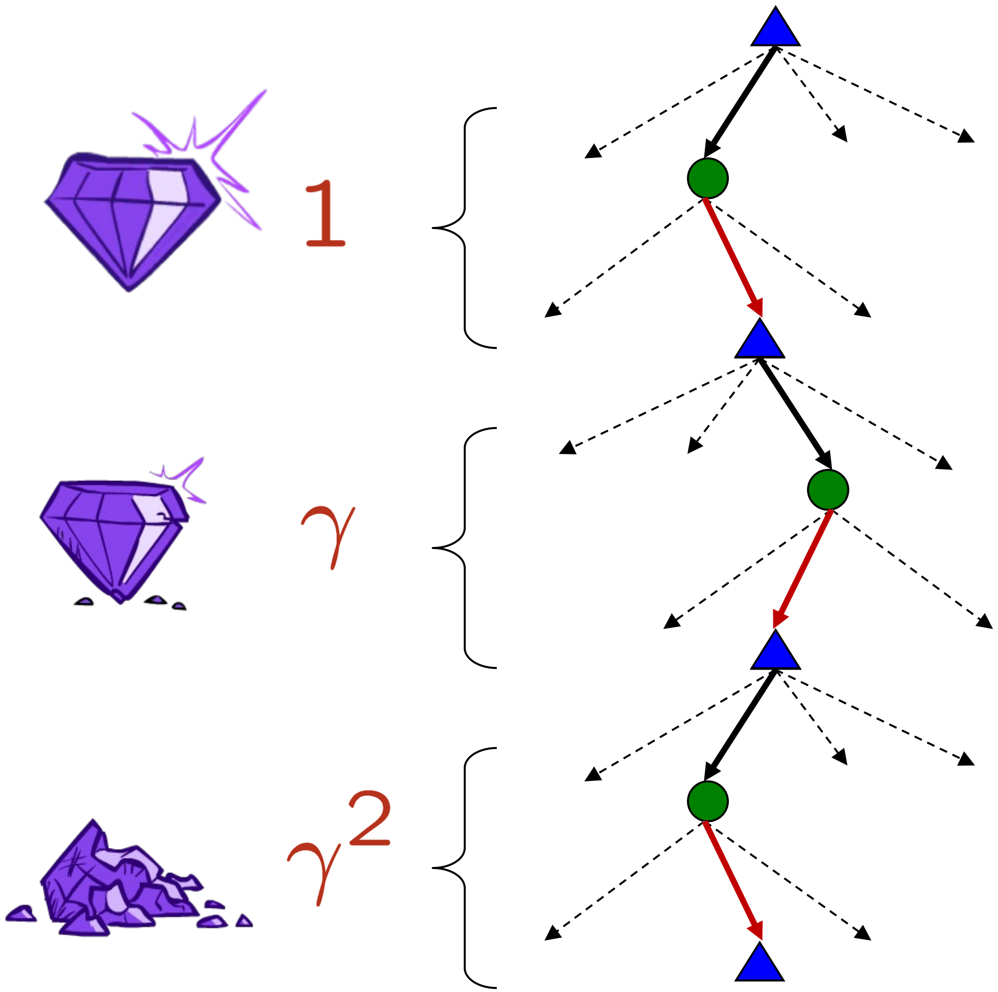
Worth Next Step

γ^2

Worth In Two Steps

Discounting

- How to discount?
  - Each time we descend a level, we multiply in the discount once
- Why discount?
  - Sooner rewards probably do have higher utility than later rewards
  - Also helps our algorithms converge
- Example: discount of 0.5
  - U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3 = 2.75
  - U([3,2,1]) = 1*3 + 0.5*2 + 0.25*1 = 5.25
  - U([1,2,3]) < U([3,2,1])



Discounting in Public Policy

COUNCIL OF ECONOMIC ADVISERS ISSUE BRIEF
JANUARY 2017

DISCOUNTING FOR PUBLIC POLICY: THEORY AND RECENT EVIDENCE ON THE MERITS OF UPDATING THE DISCOUNT RATE

Weighing benefits and costs that take place over time requires discounting those amounts to present value equivalents. This necessitates selecting a discount rate which can adjust for the fact that resources are more valuable today than in the future if consumers prefer to consume today rather than wait, or if firms could be earning a positive return on invested resources. Current guidance from the office of management and budget requires using both a 7 percent and 3 percent real discount rate in regulatory benefit-cost analyses. This issue brief reassesses the current choice of discount rates and methodologies for selecting the 3 percent and 7 percent rates. Empirical evidence suggests that real interest rates around the world have come down since the last evaluation of the rates, and new theoretical advances considering future uncertainty likely suggest lower long term rates, as well. In general the evidence supports lowering these discount rates, with a plausible best guess based on the available information being that the lower discount rate should be at most 2 percent while the upper discount rate should also likely be reduced.

γ = 0.93 or 0.97

obamawhitehouse.archives.gov/sites/default/files/page/files/201701_cea_discounting_issue_brief.pdf

Quiz: Discounting

- Given:

10				1
a	b	c	d	e

  - Actions: East, West, and Exit (only available in exit states a, e)
  - Transitions: deterministic

- Quiz 1: For $\gamma = 1$, what is the optimal policy?

10				1

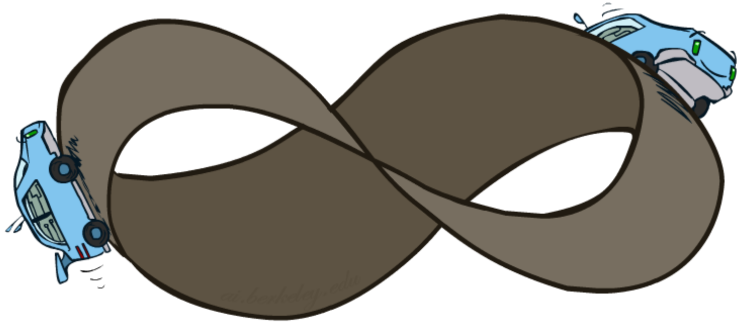
- Quiz 2: For $\gamma = 0.1$, what is the optimal policy?

10				1

- Quiz 3: For which γ are West and East equally good when in state d?

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?
- Solutions:
  - Finite horizon: (similar to depth-limited search)
    - Terminate episodes after a fixed T steps (e.g. life)
    - Gives nonstationary policies (π depends on time left)



Shaunae Miller wins 400m gold medal in 2016 Olympics [Photo credit: NBC]

Infinite Utilities?!

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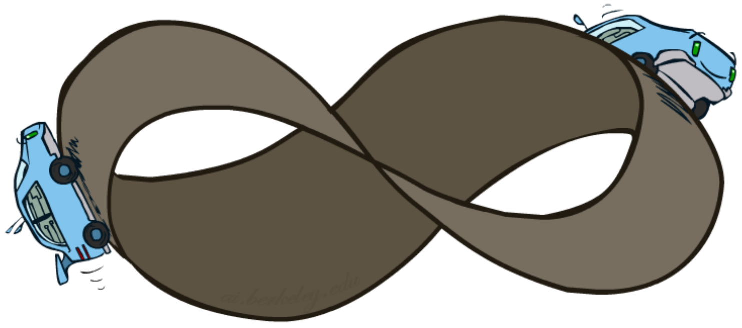


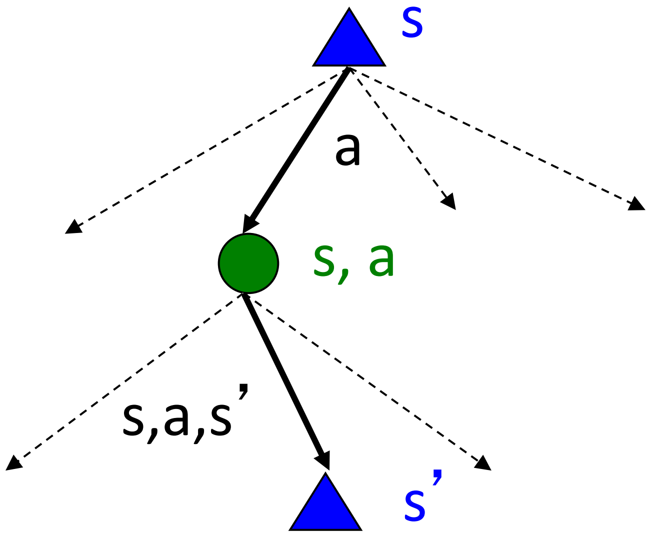
  - Discounting: use $0 < \gamma < 1$

$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max}/(1-\gamma)$$

    - Smaller γ means smaller "horizon" – shorter term focus
  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

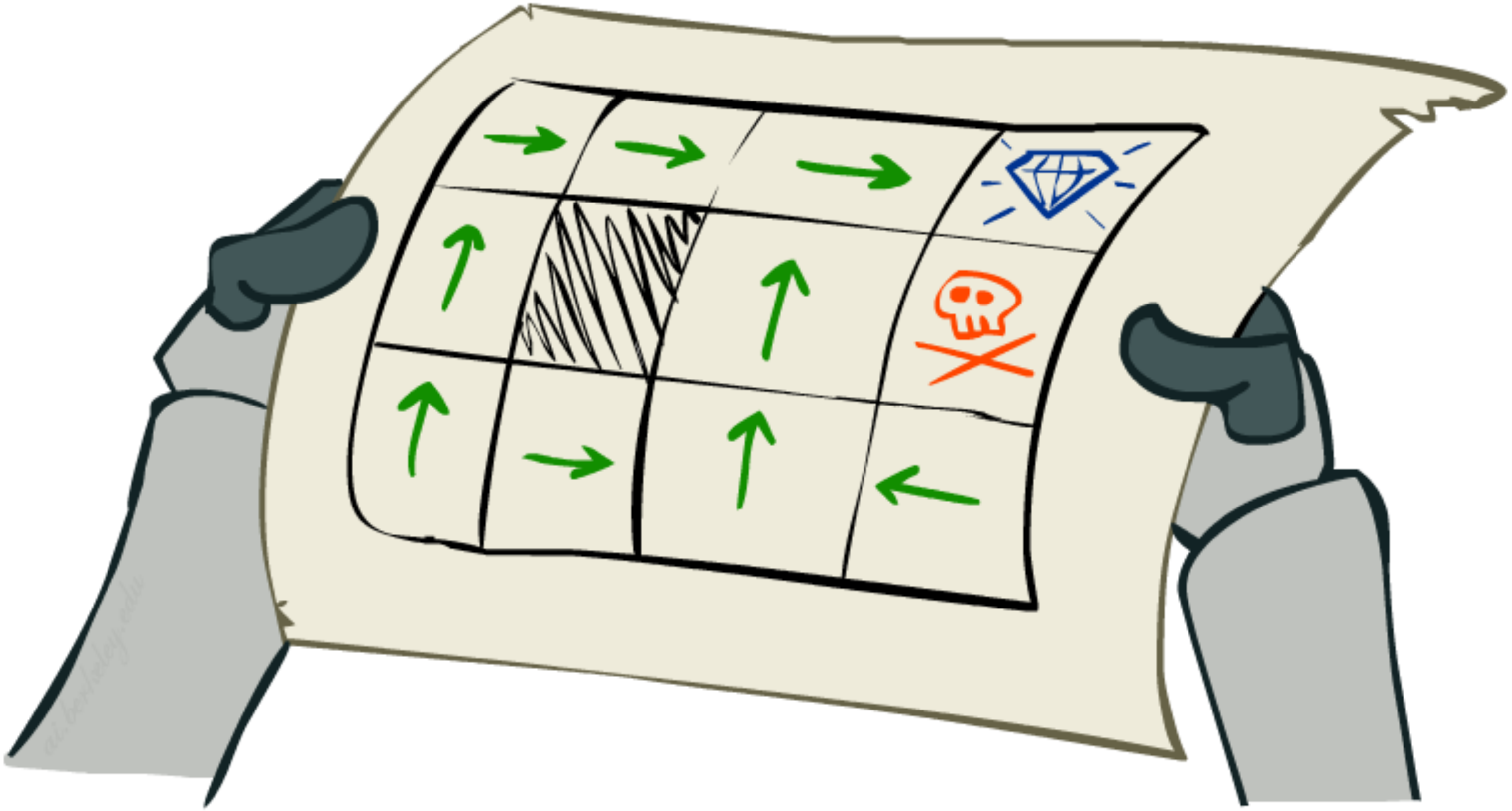
Recap: Defining MDPs

- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards

Solving MDPs



Recall: Racing

- A robot car wants to travel far, quickly
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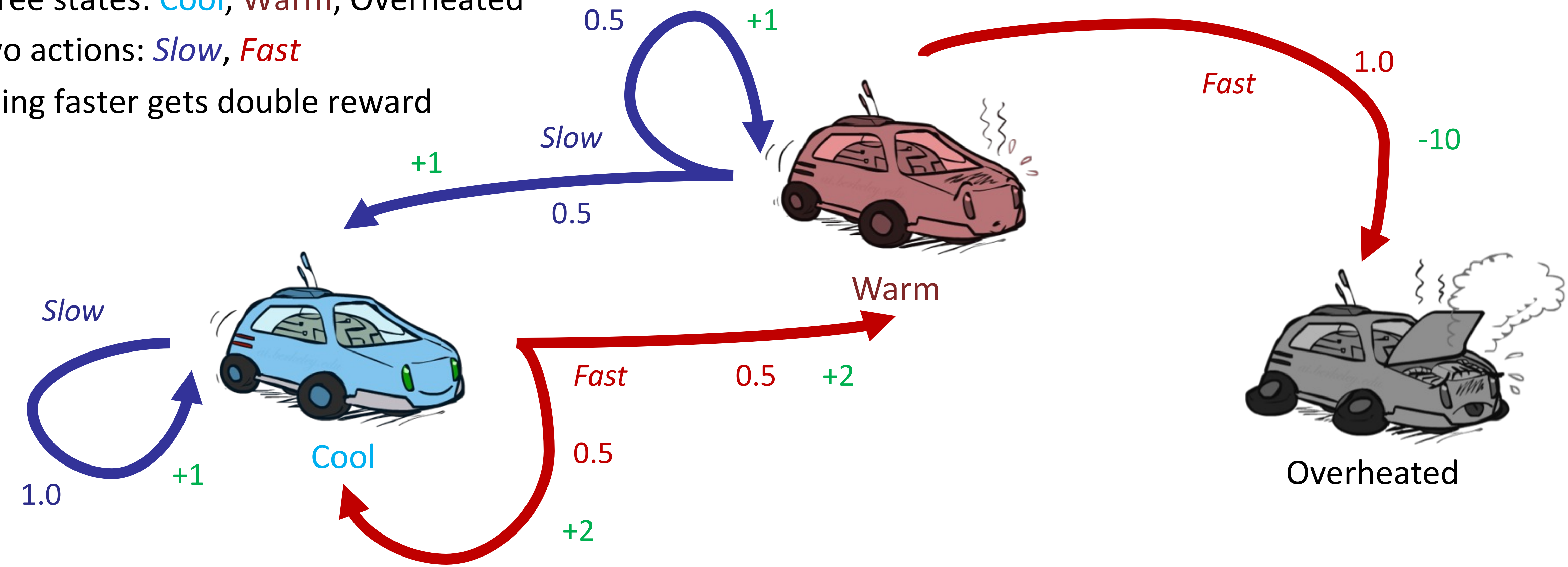
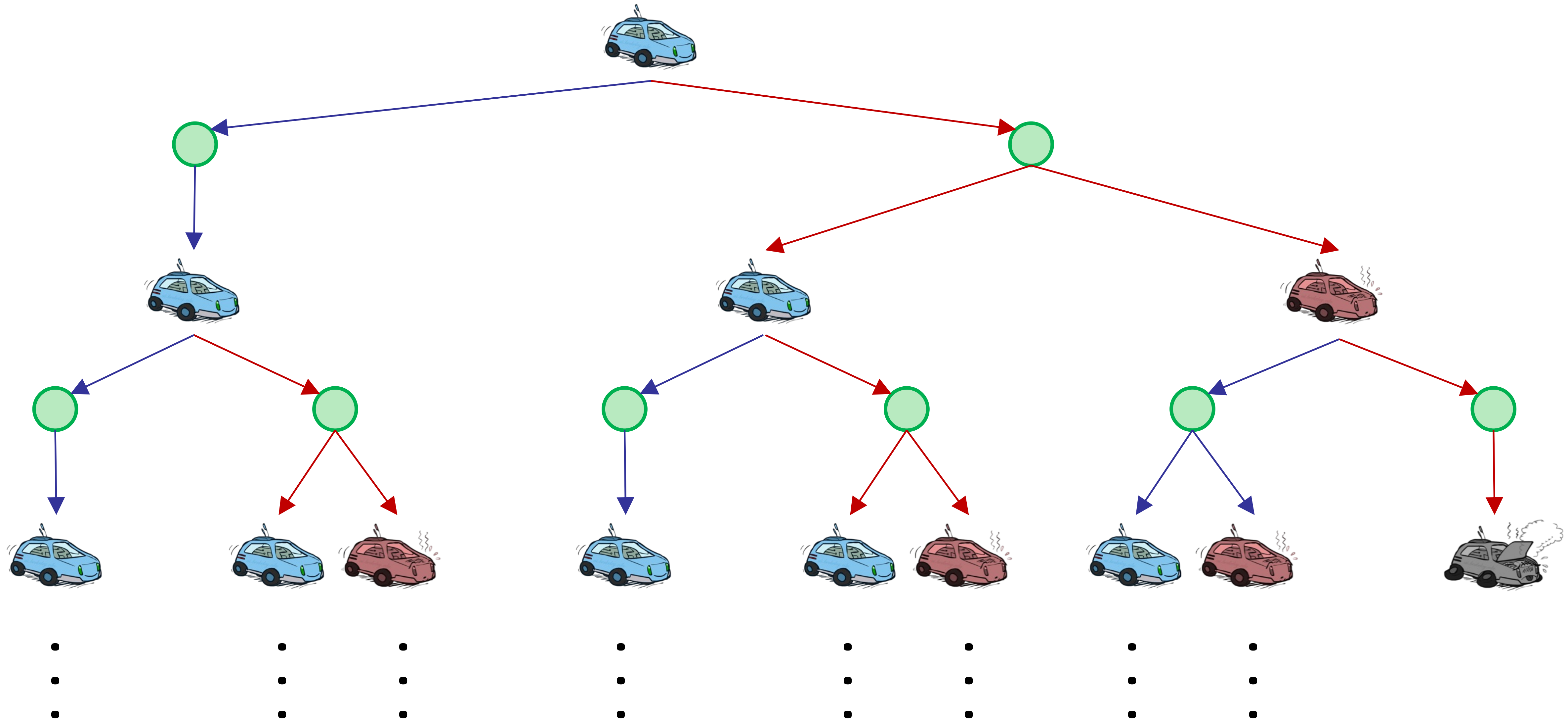
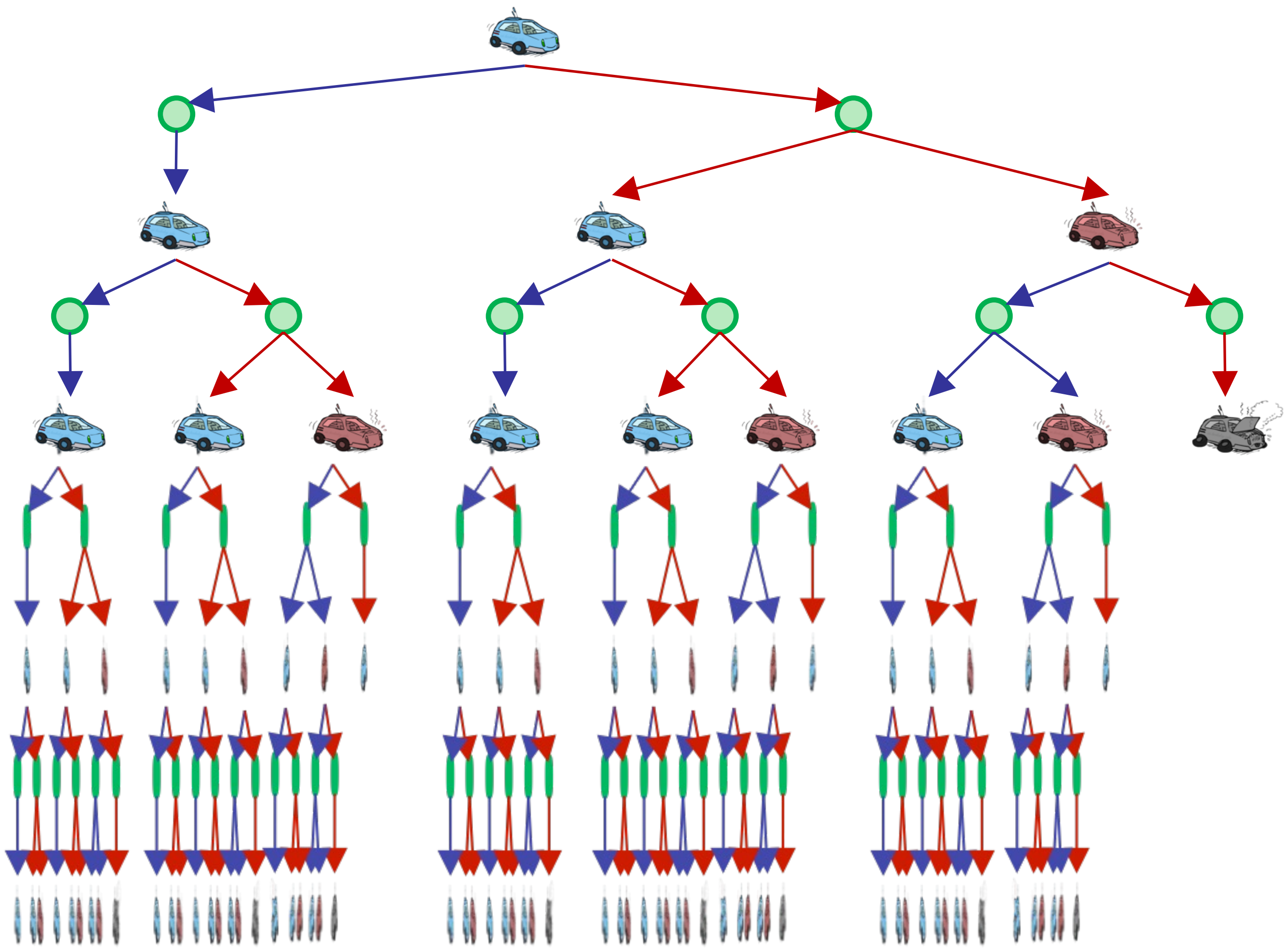


Racing Search Tree



Racing Search Tree

- We're doing way too much work with expectimax!
- Problem: States are repeated
  - Idea: Only compute needed quantities once, cache the rest in a lookup table
- Problem: Tree goes on forever
  - Idea: Do a depth-limited computation, but with increasing depths until change is small
  - Note: deep parts of the tree eventually don't matter if $\gamma < 1$



Optimal Quantities

- The value (utility) of a state s:

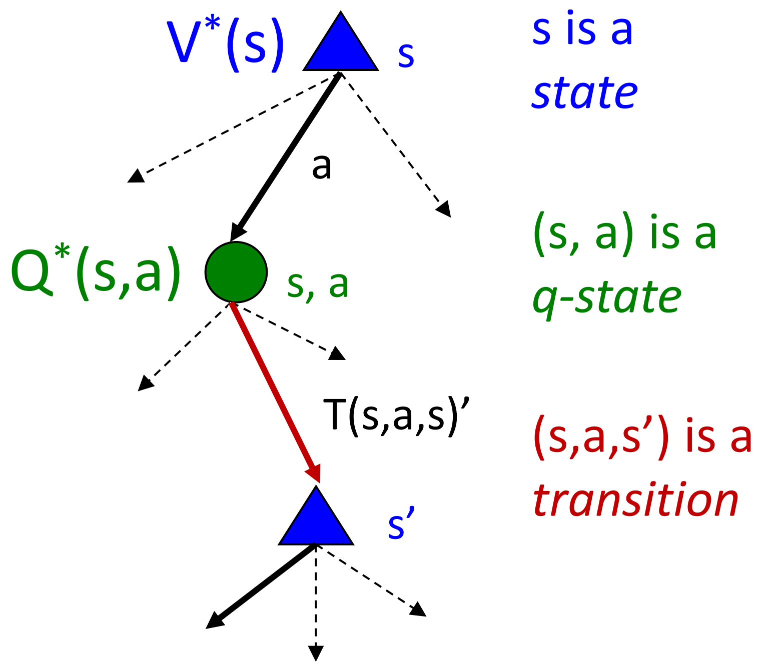
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

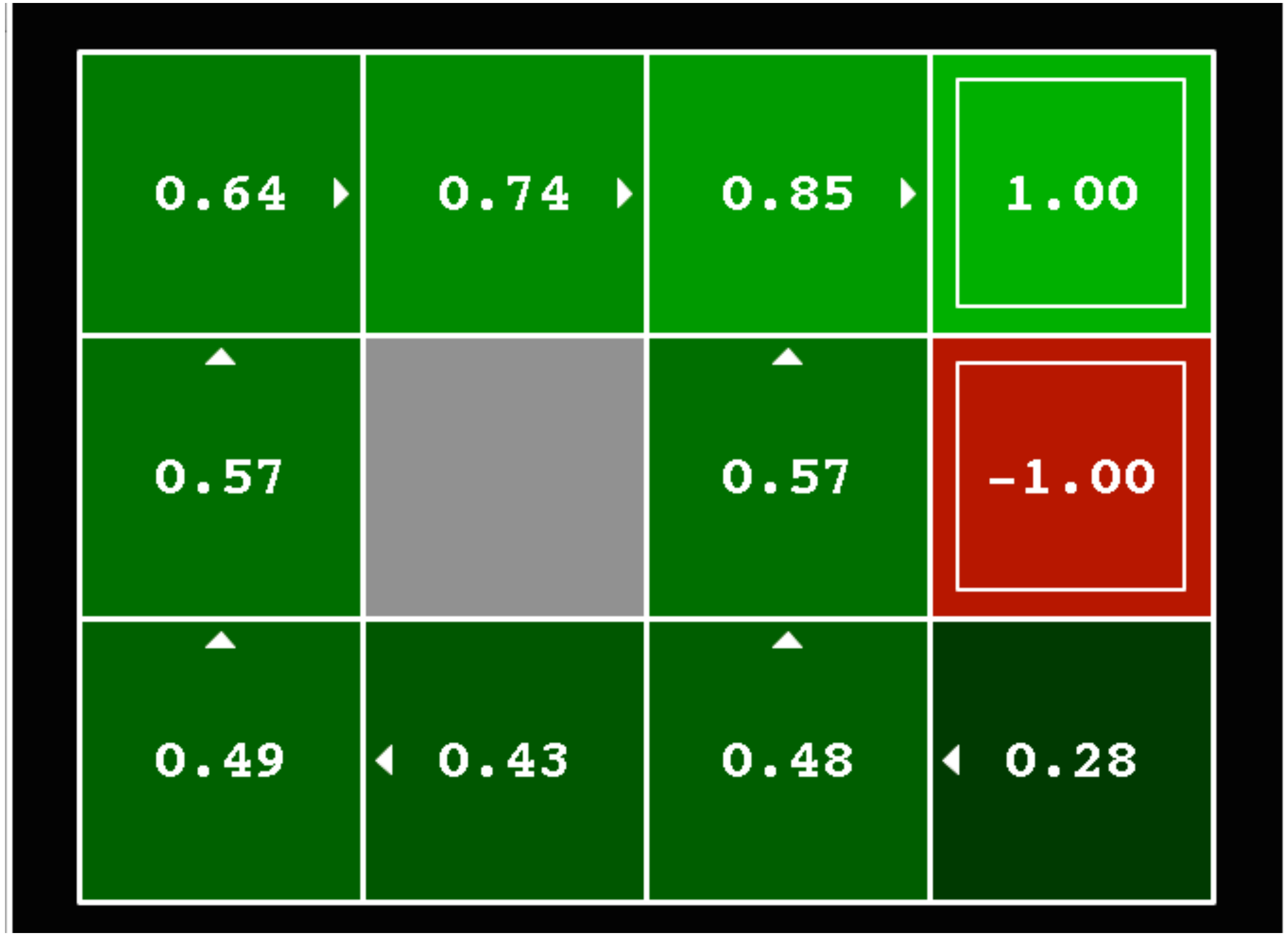
- The optimal policy:

  $\pi^*(s)$ = optimal action from state s



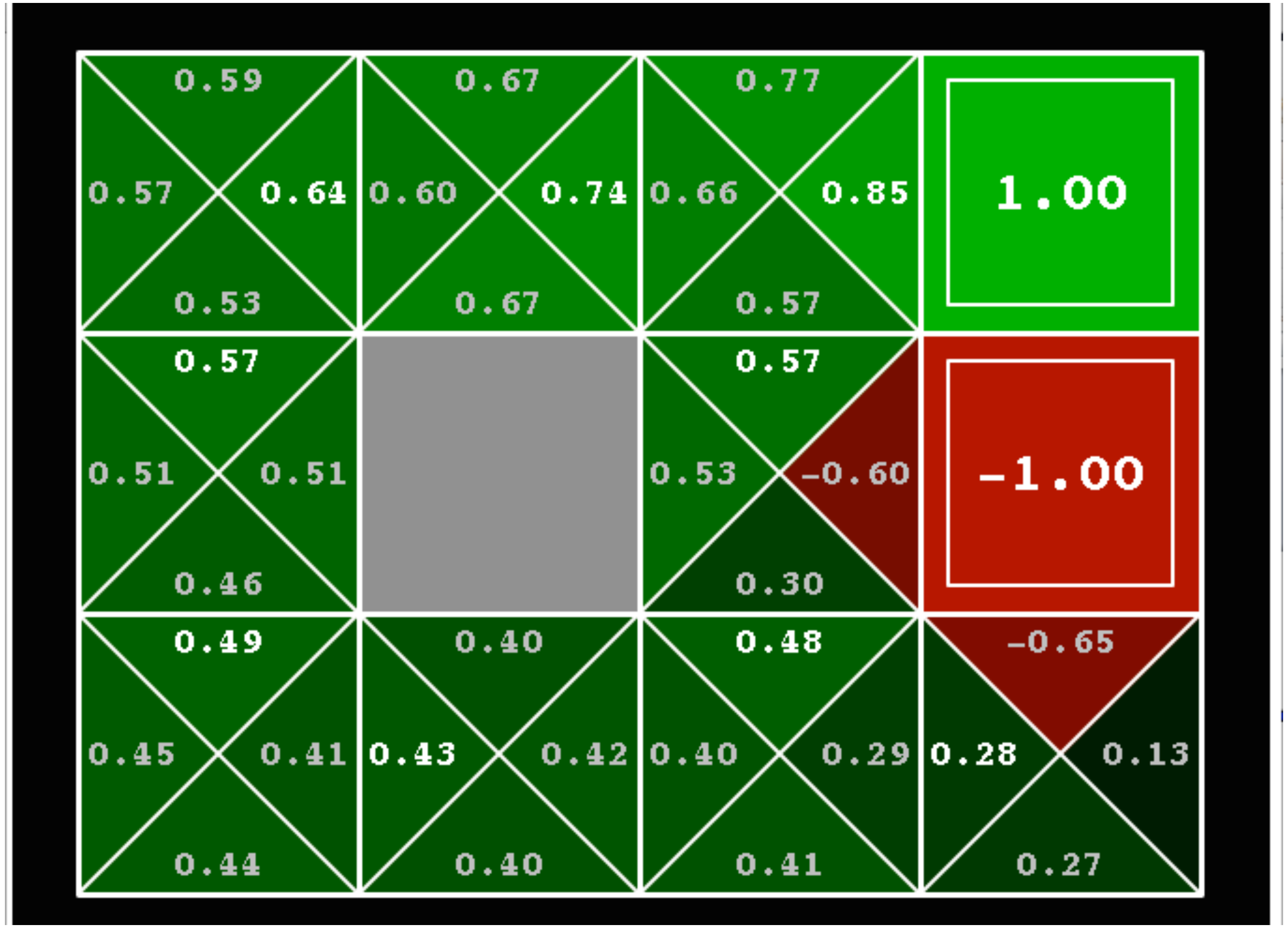
[Demo – gridworld values (L8D4)]

Snapshot of Demo – Gridworld V* Values



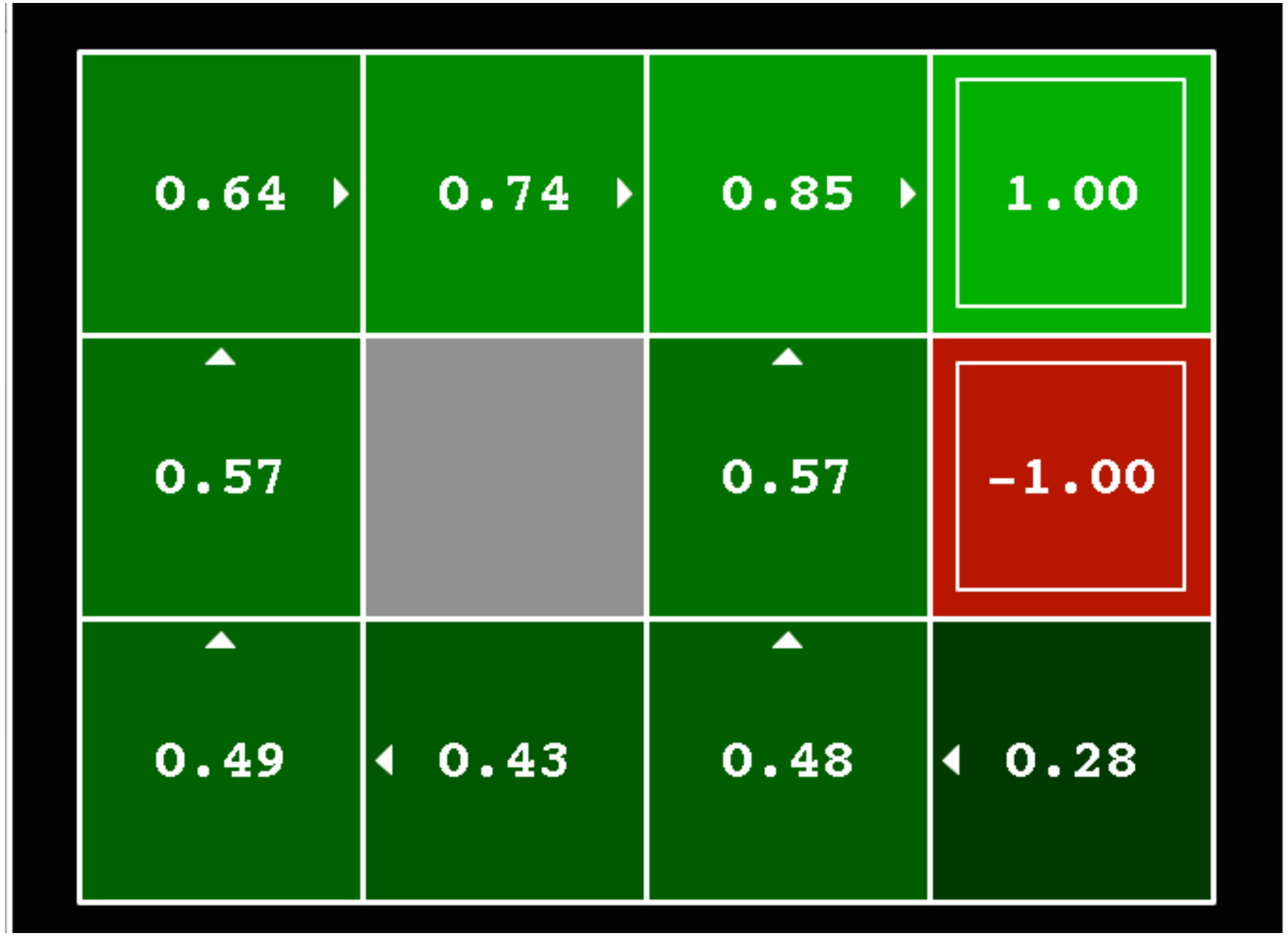
Noise = 0.2
Discount = 0.9
Living reward = 0

Snapshot of Demo – Gridworld Q* Values



Noise = 0.2
Discount = 0.9
Living reward = 0

Snapshot of Demo – Gridworld π^* Values



Noise = 0.2
Discount = 0.9
Living reward = 0

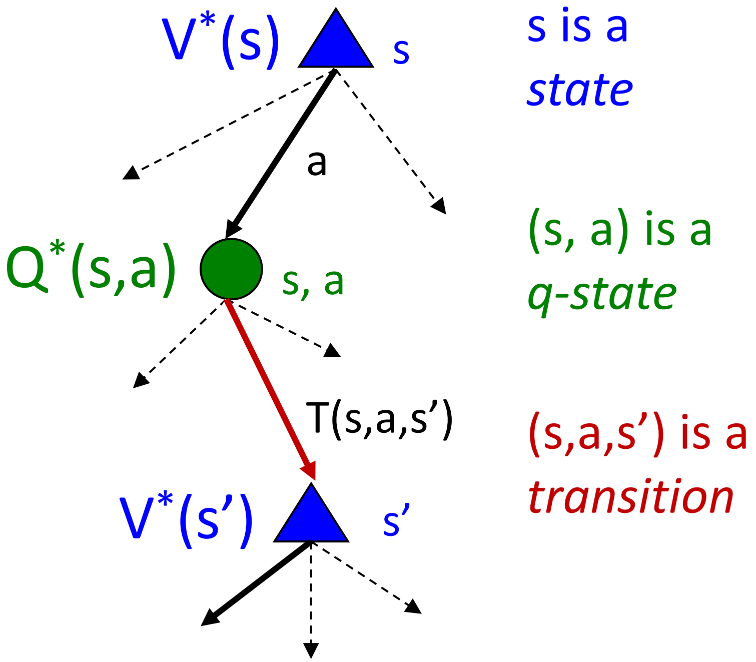
Values of States

- Recursive definition of value (similar to expectimax):

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

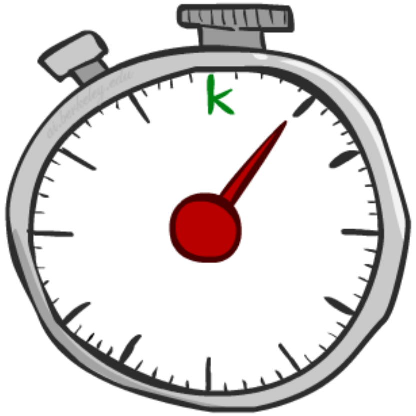
$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

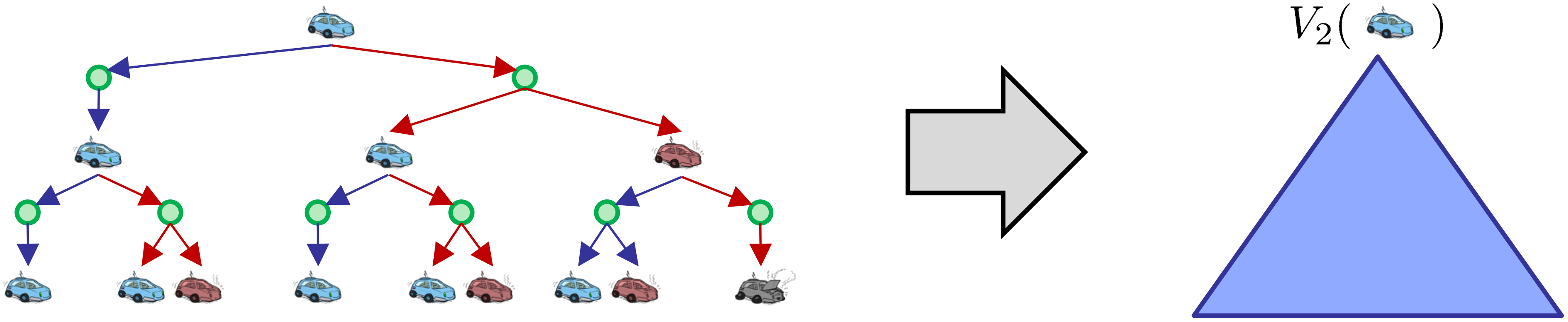


- But how do we solve these equations?

Time-Limited Values

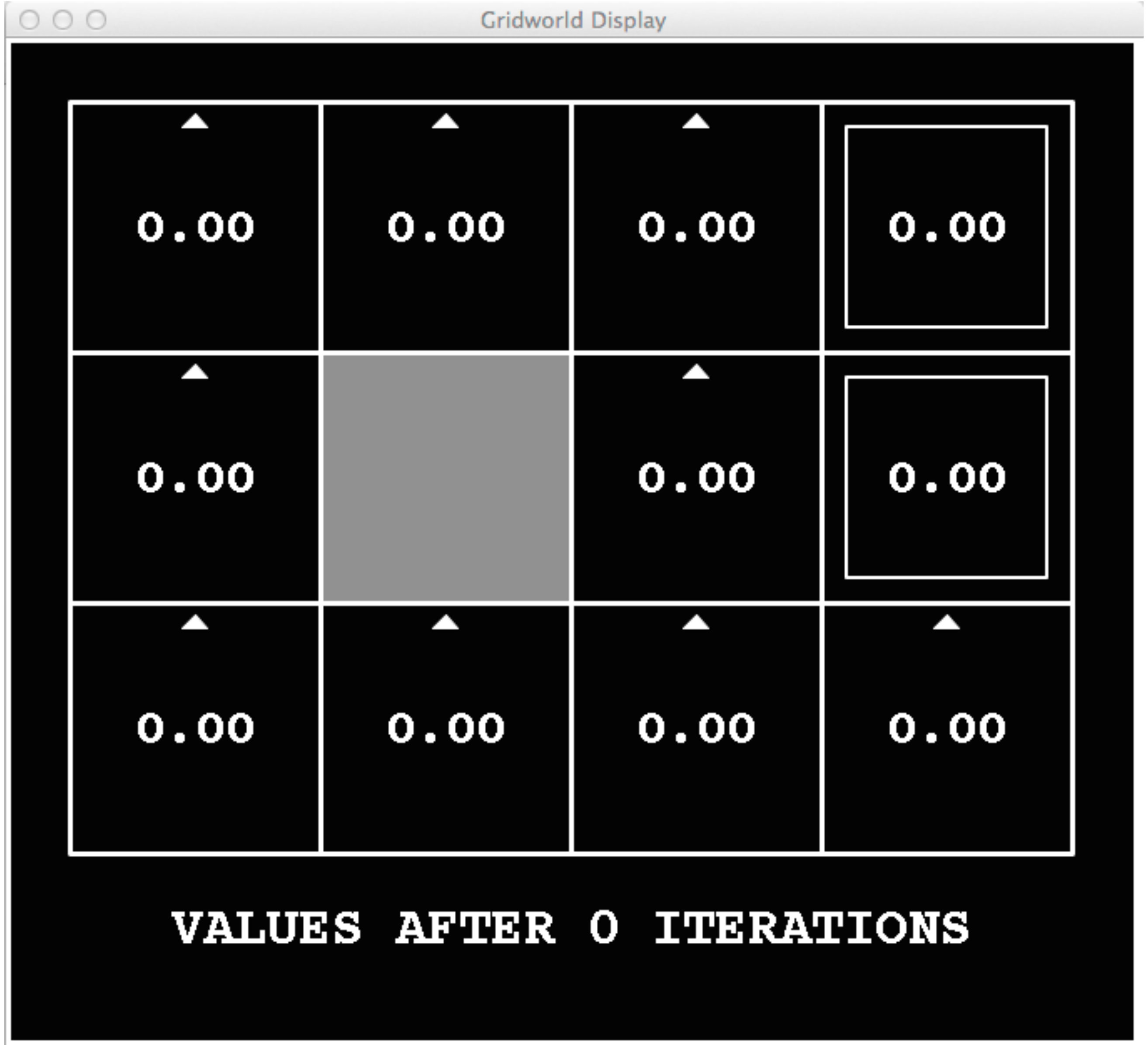
- Key idea: time-limited values
- Define $V_k(s)$ to be the optimal value of s if the game ends in k more time steps
  - Equivalently, it’s what a depth-k expectimax would give from s





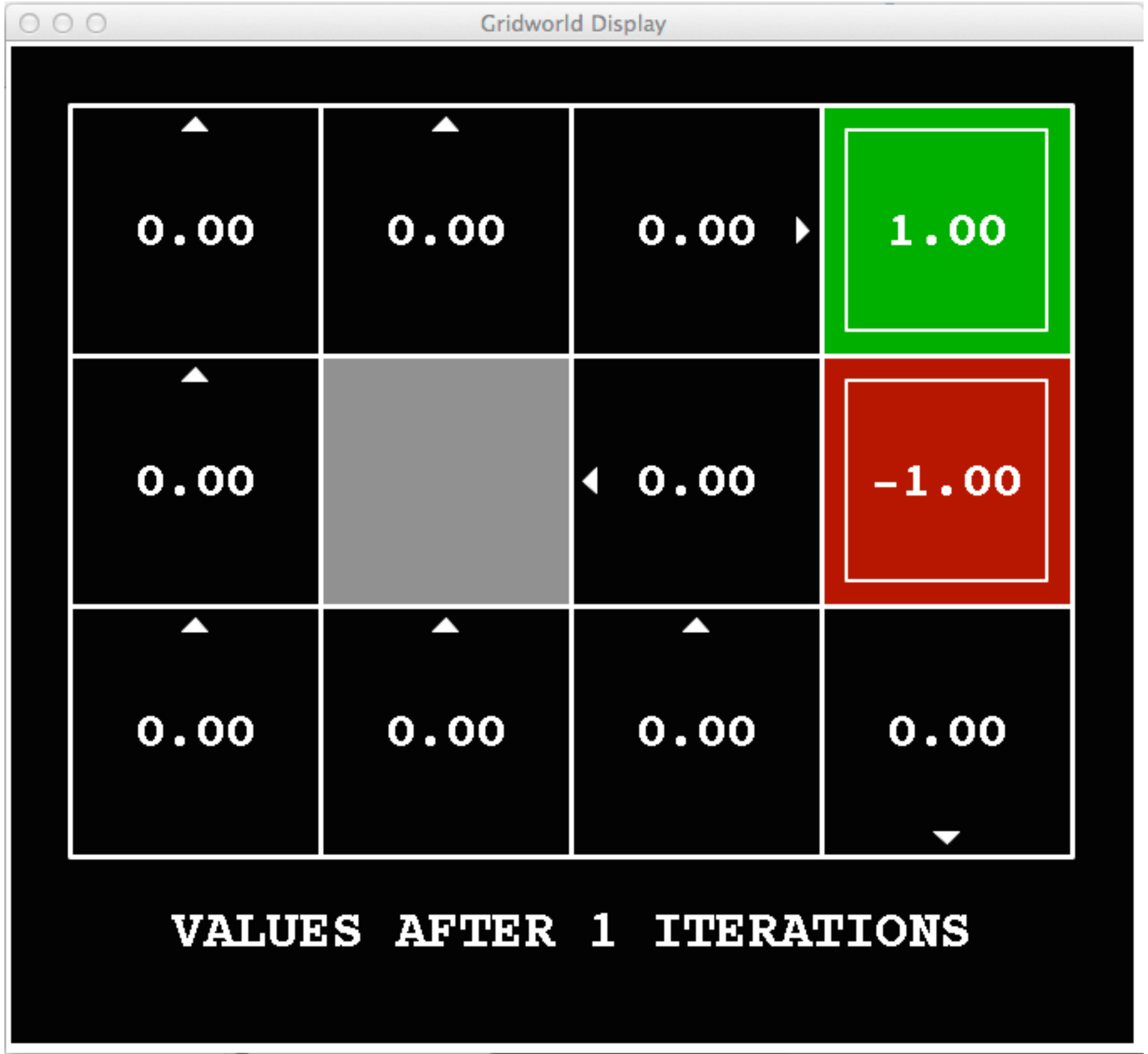
[Demo – time-limited values (L8D6)]

k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

k=1



Noise = 0.2
Discount = 0.9
Living reward = 0

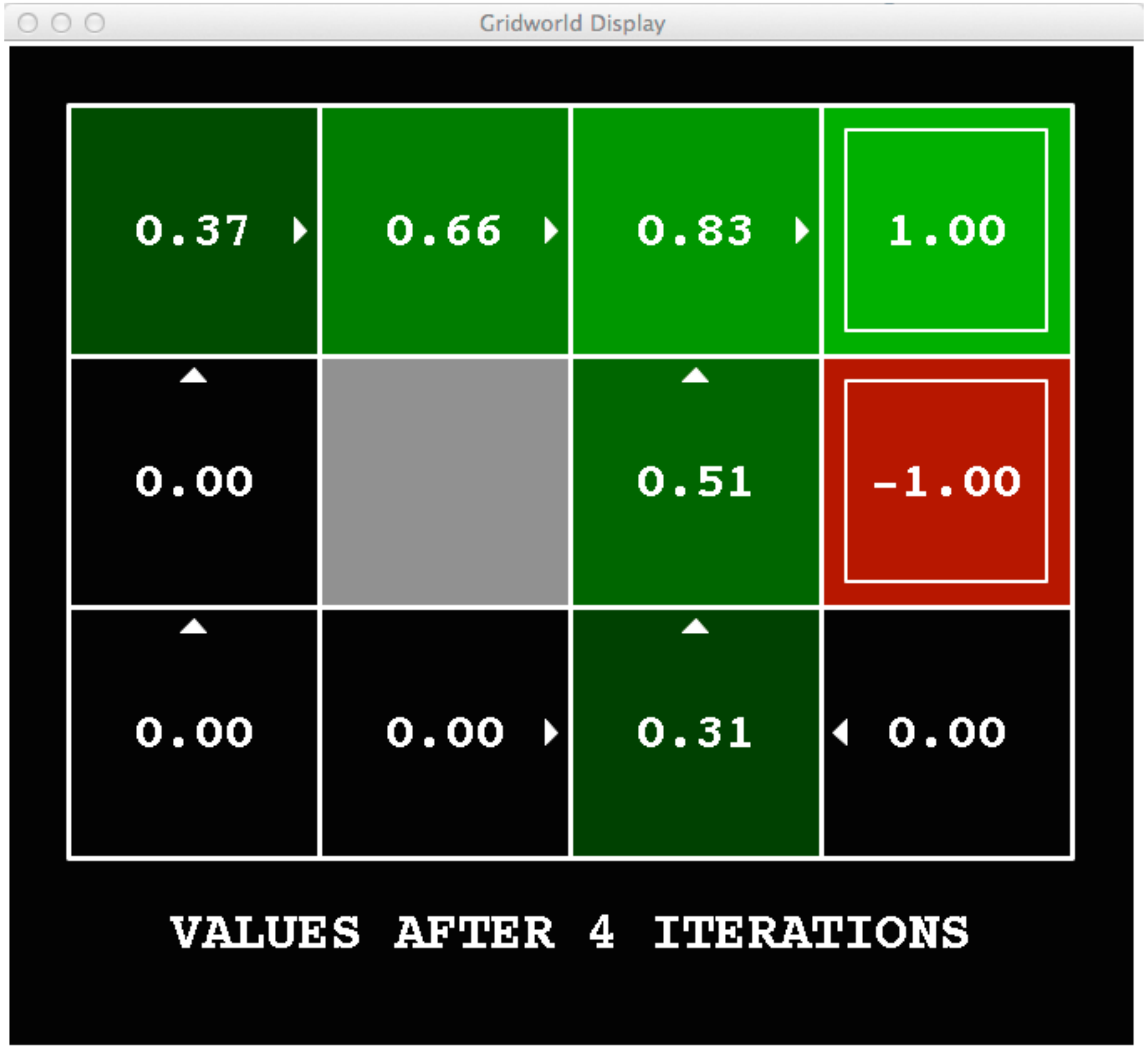
k=2

Noise = 0.2
Discount = 0.9
Living reward = 0

k=3

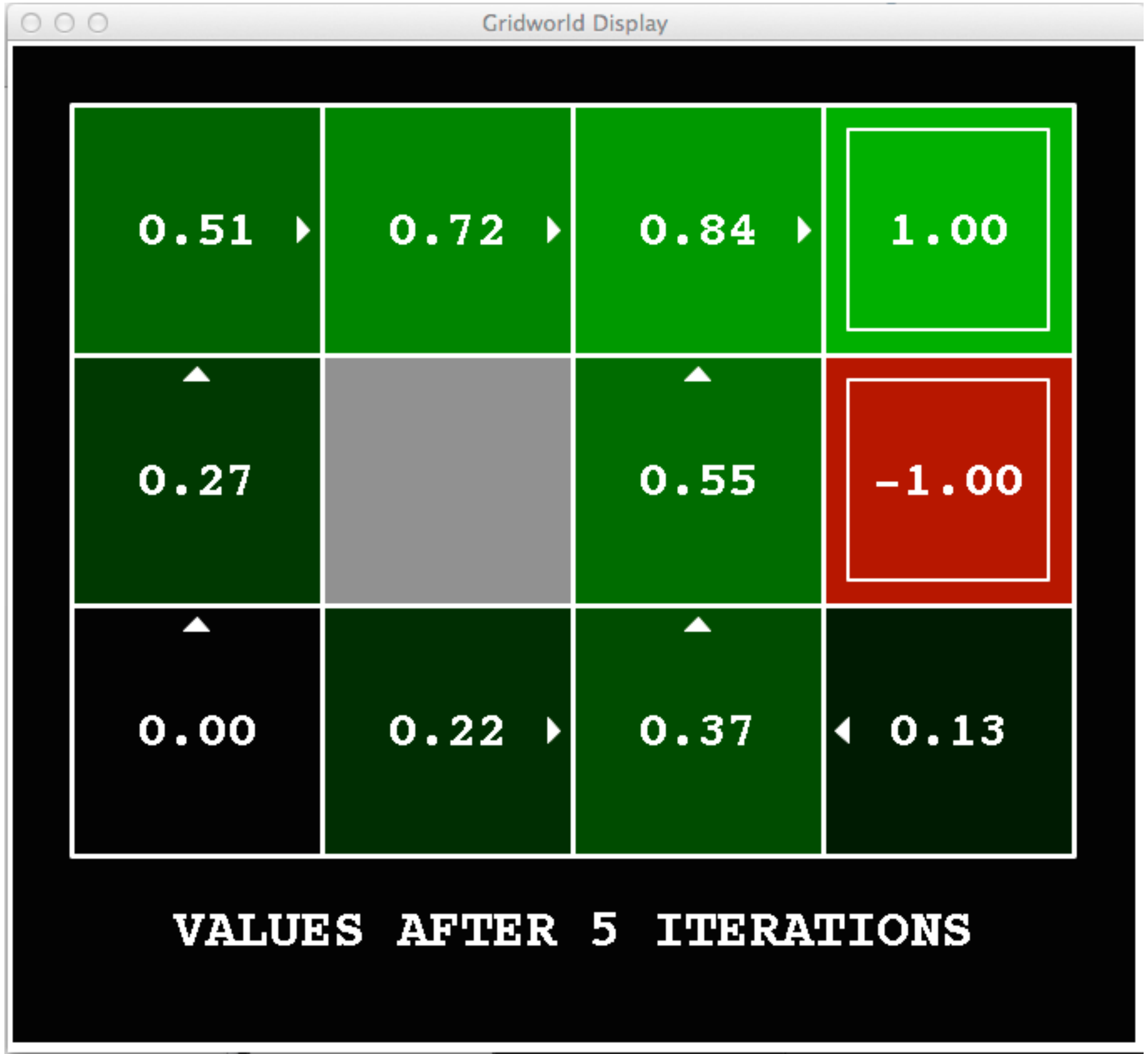
Noise = 0.2
Discount = 0.9
Living reward = 0

k=4



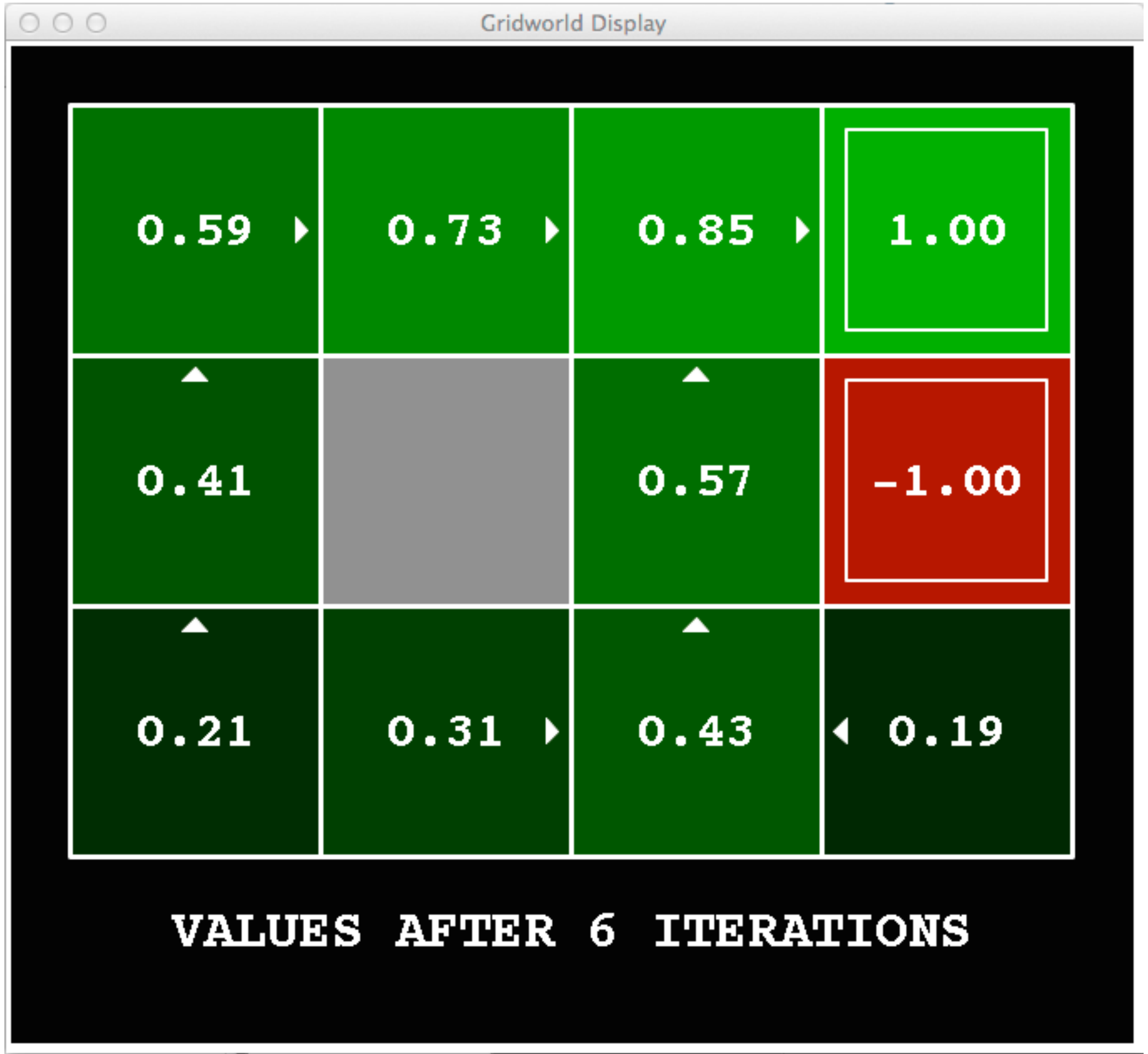
Noise = 0.2
Discount = 0.9
Living reward = 0

k=5



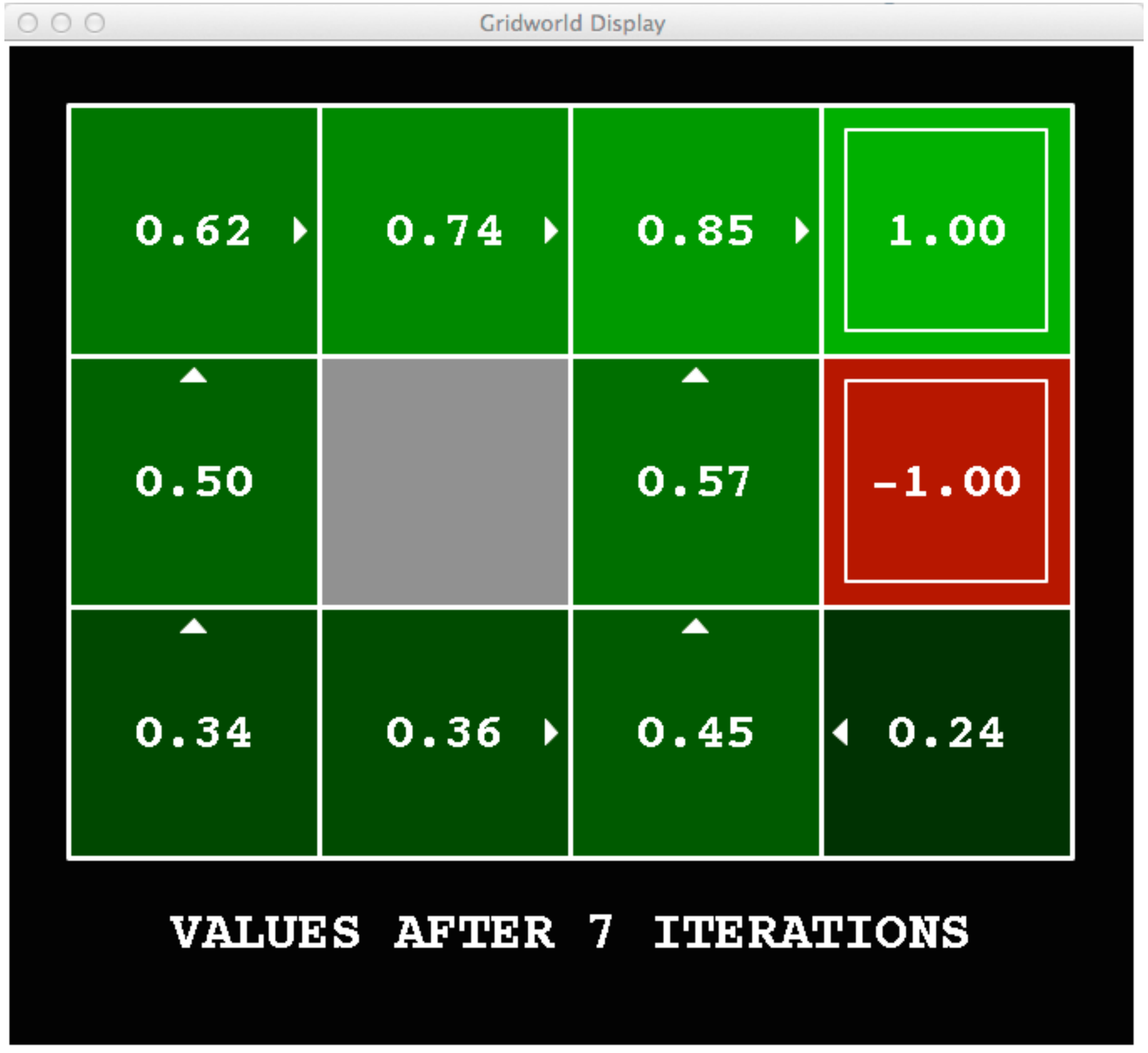
Noise = 0.2
Discount = 0.9
Living reward = 0

k=6



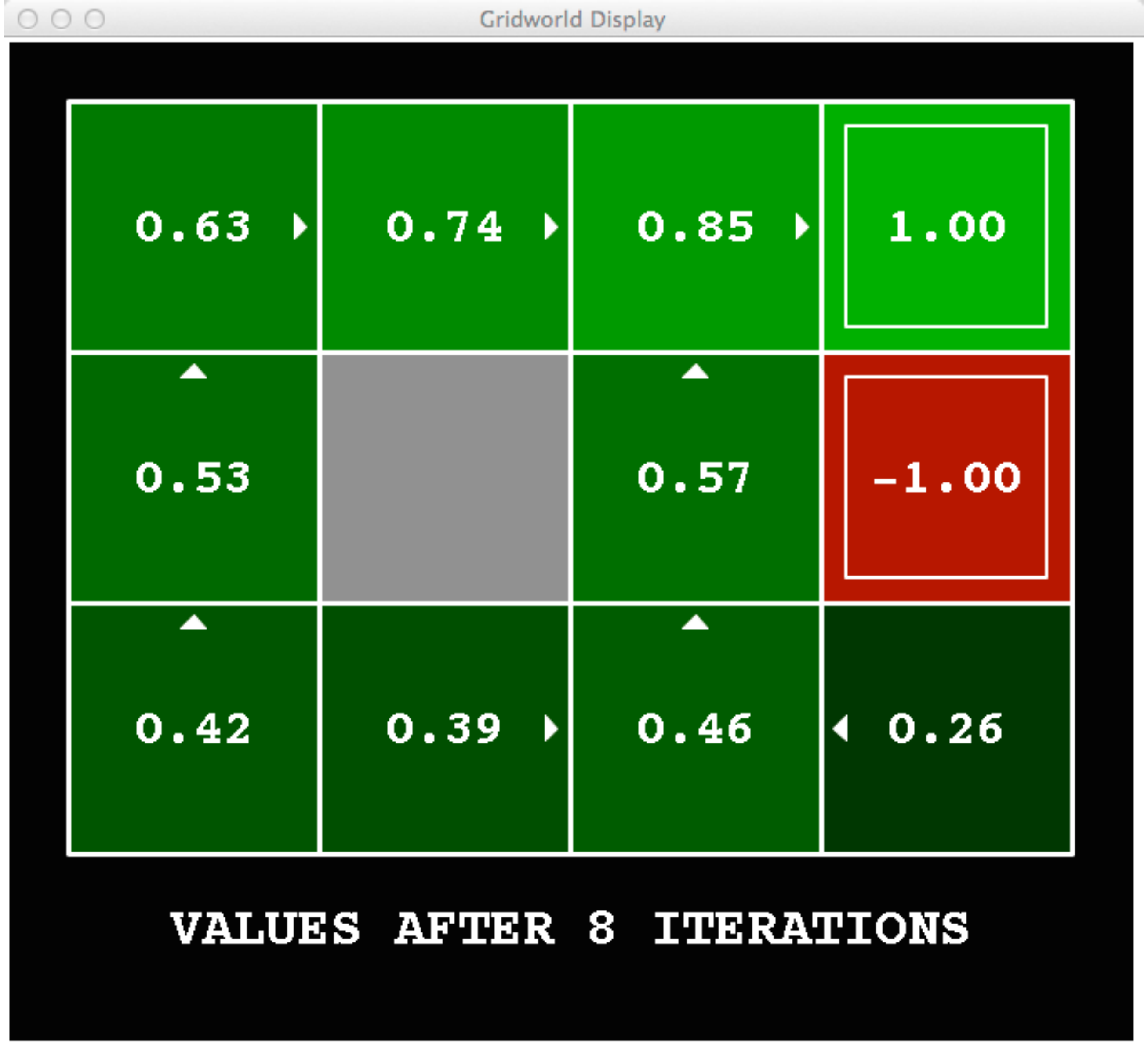
Noise = 0.2
Discount = 0.9
Living reward = 0

k=7



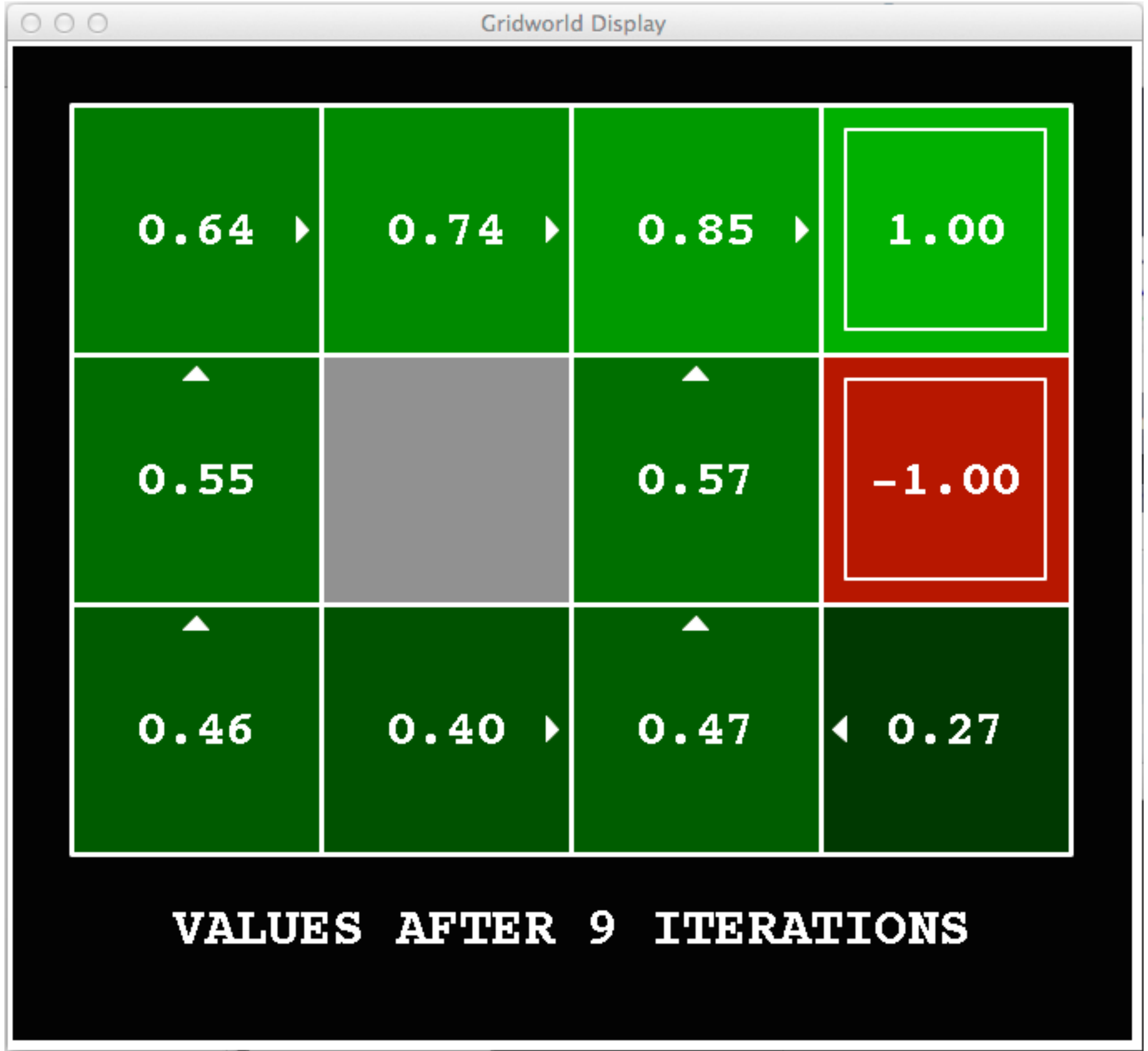
Noise = 0.2
Discount = 0.9
Living reward = 0

k=8



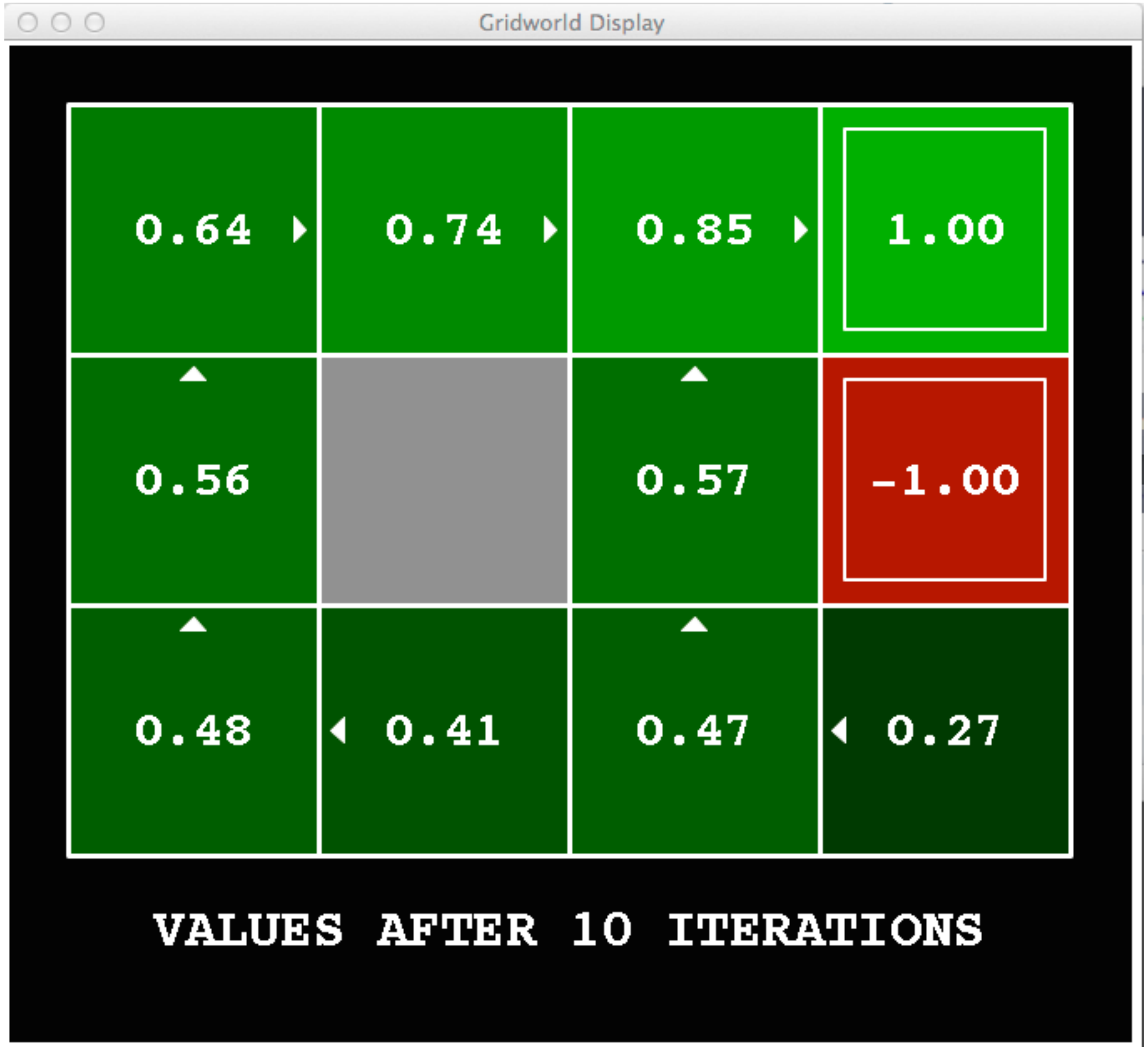
Noise = 0.2
Discount = 0.9
Living reward = 0

k=9



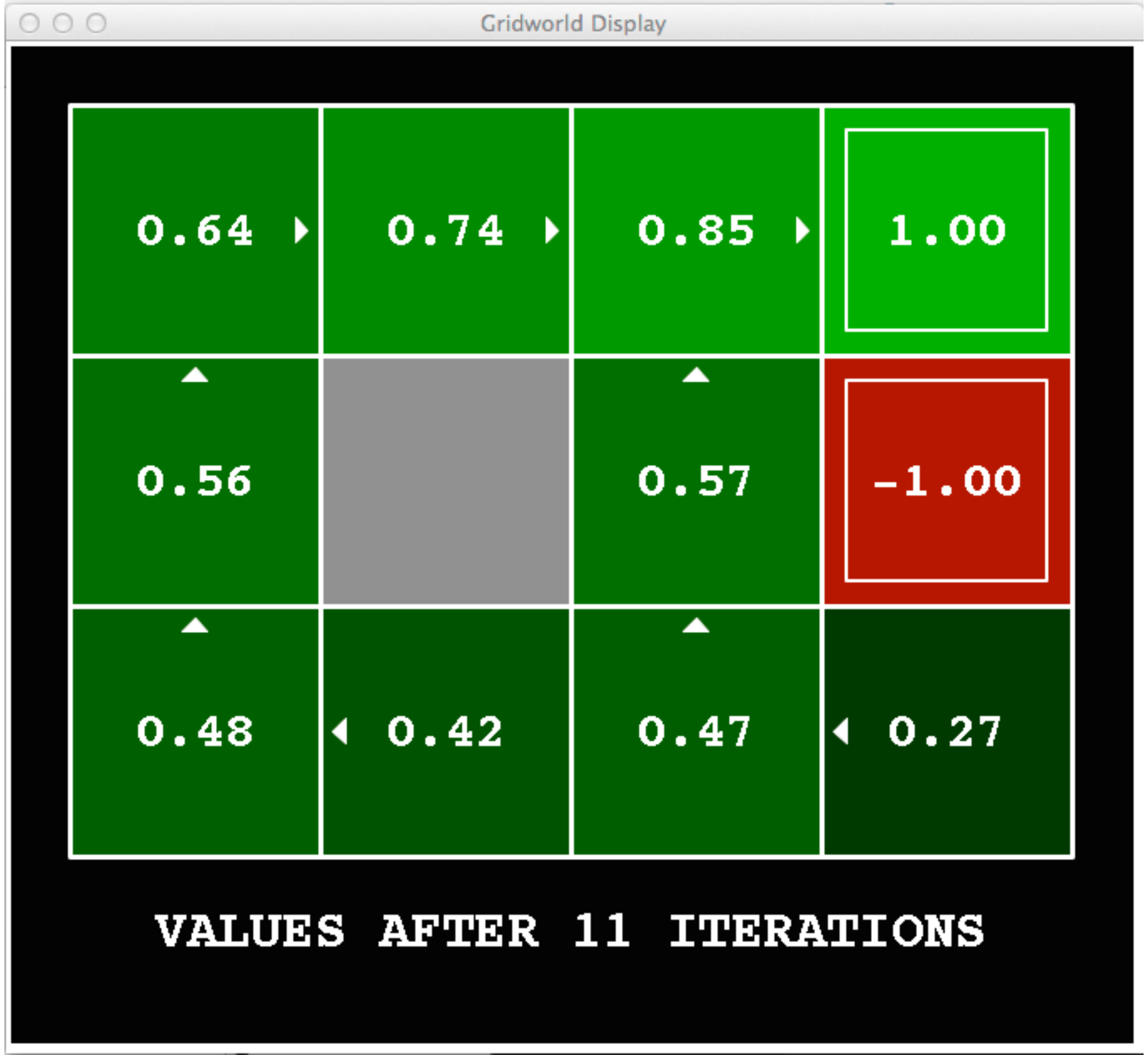
Noise = 0.2
Discount = 0.9
Living reward = 0

k=10



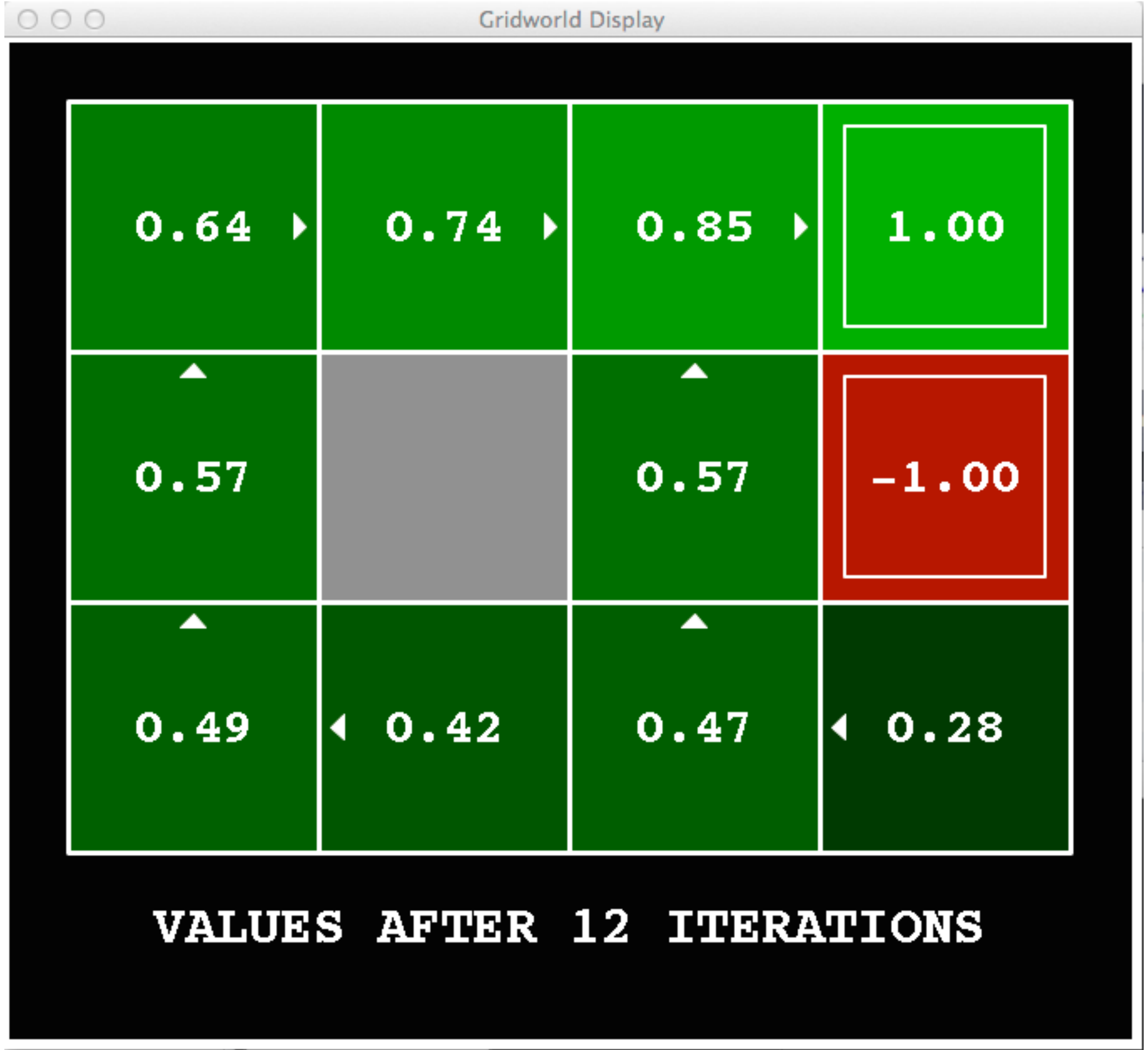
Noise = 0.2
Discount = 0.9
Living reward = 0

k=11



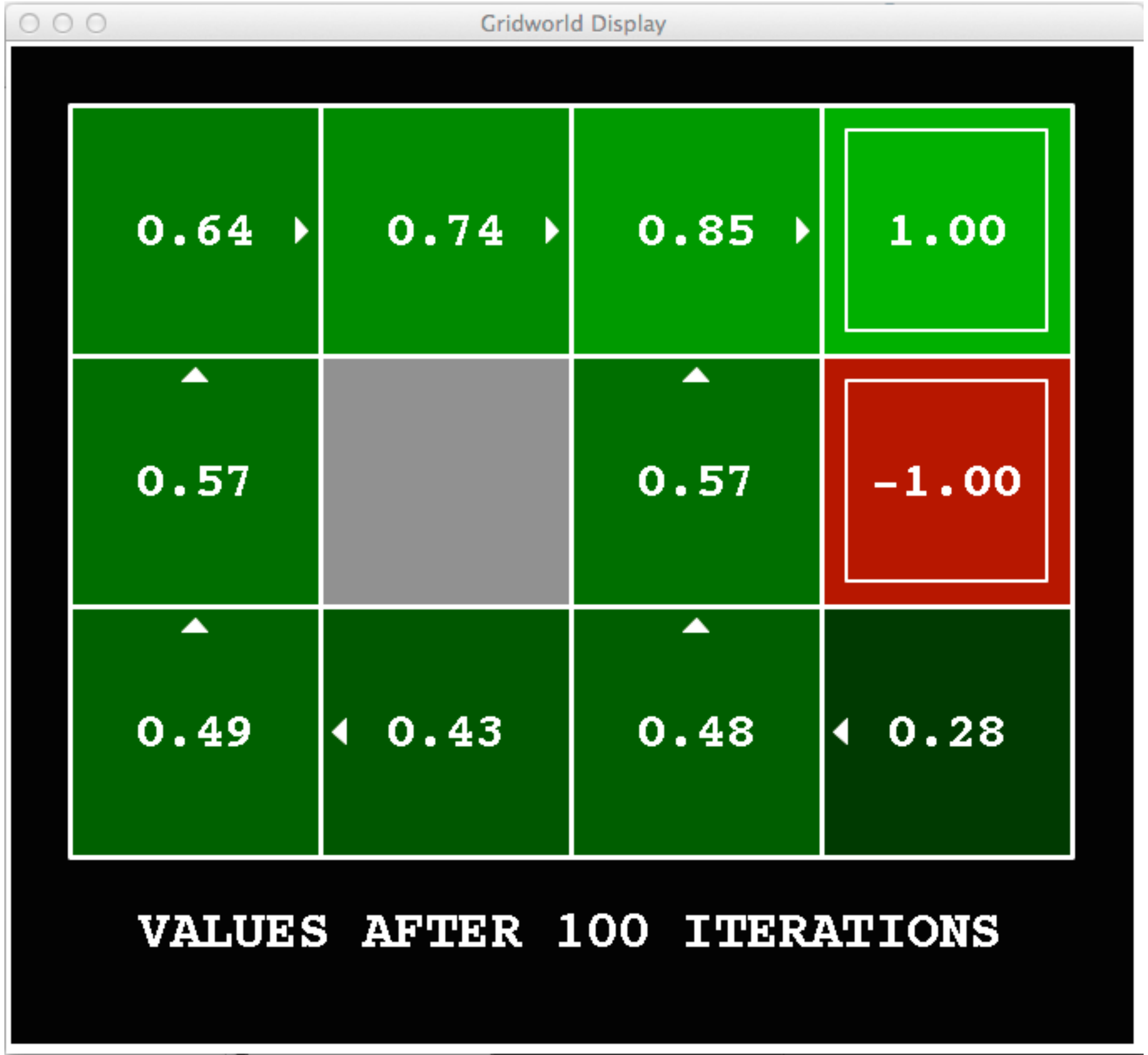
Noise = 0.2
Discount = 0.9
Living reward = 0

k=12



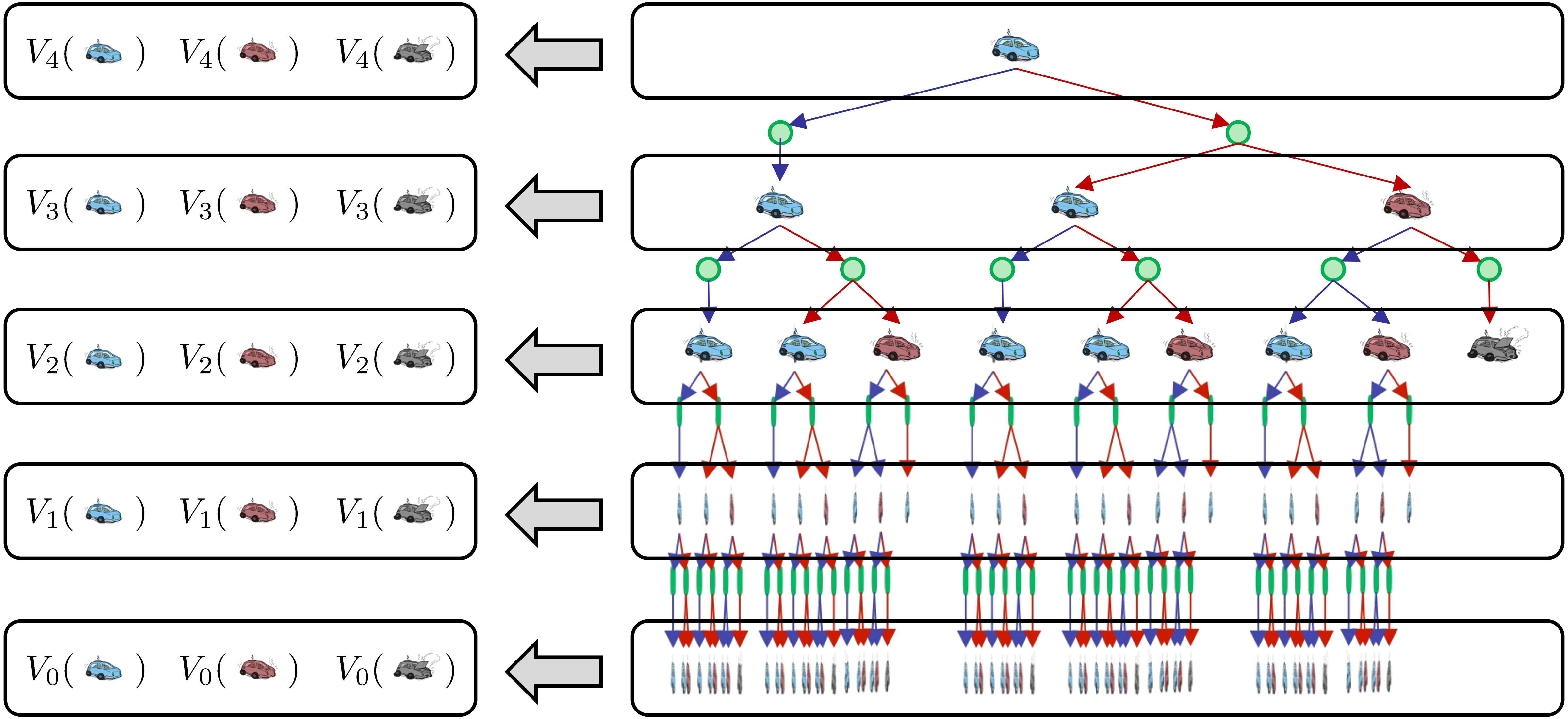
Noise = 0.2
Discount = 0.9
Living reward = 0

k=100

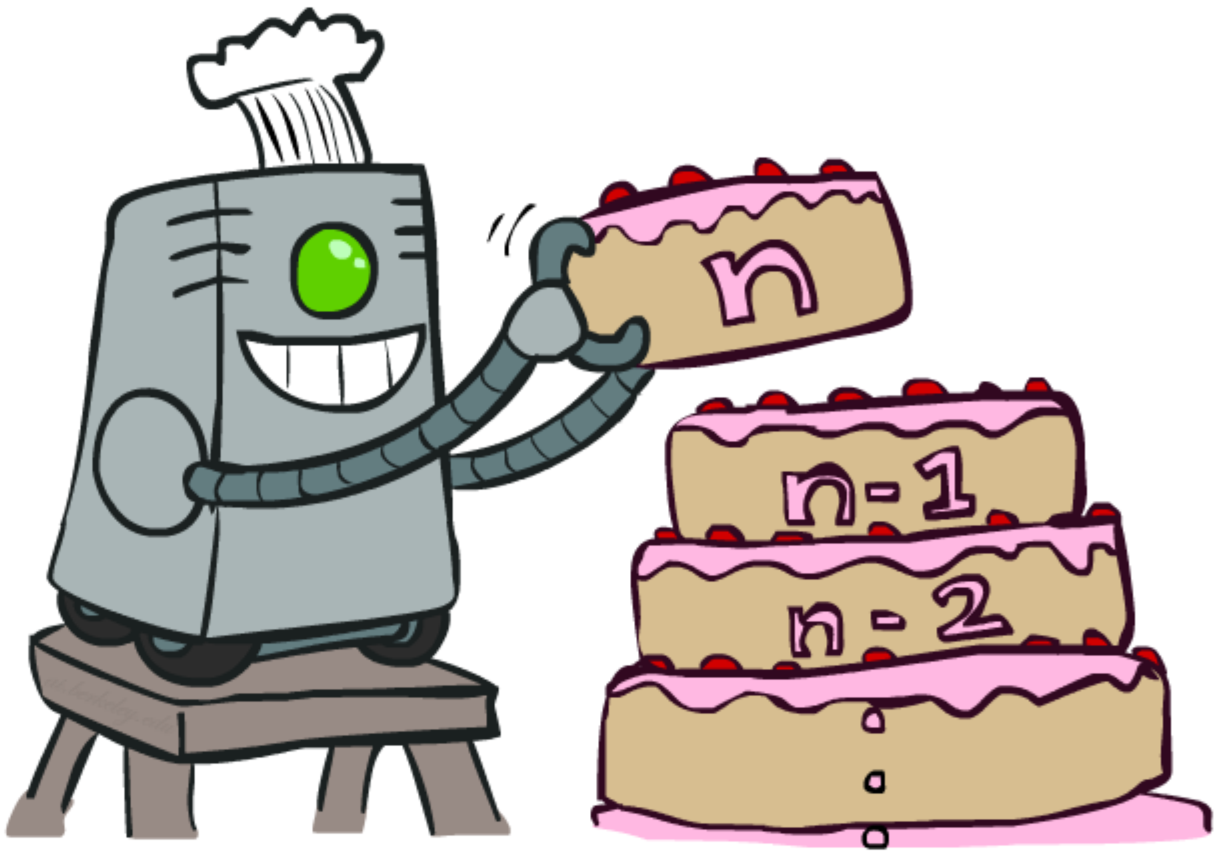


Noise = 0.2
Discount = 0.9
Living reward = 0

Computing Time-Limited Values



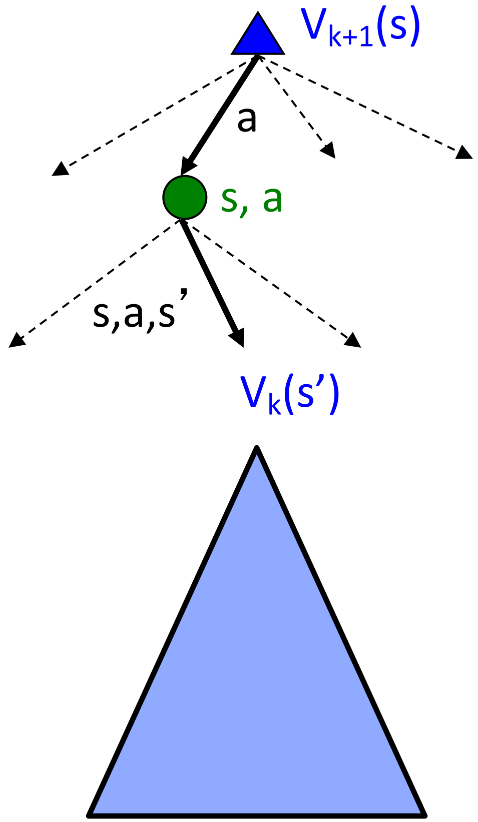
Value Iteration



Value Iteration

- Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero
- Given vector of $V_k(s)$ values, do one step of expectimax from each state:

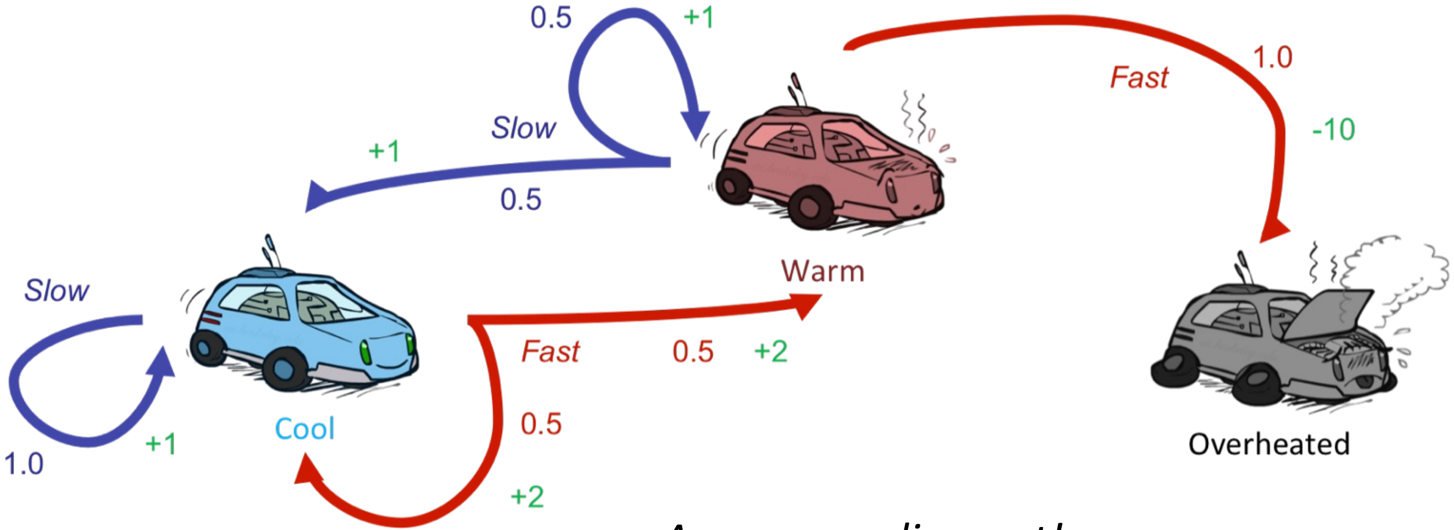
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- Repeat until convergence, which yields V*
- Complexity of each iteration: $O(S^2A)$
- Theorem: will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do

Example: Value Iteration

V_2			
V_1			
V_0	0	0	0

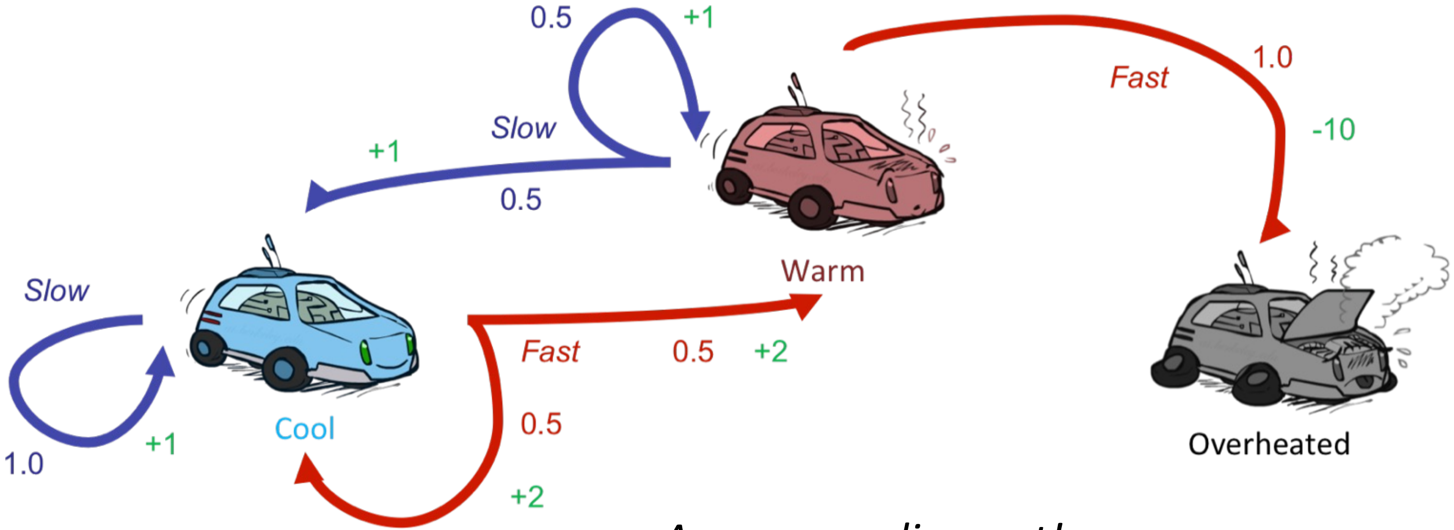


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Example: Value Iteration

V_2			
V_1	?		
V_0	0	0	0



Assume no discount!

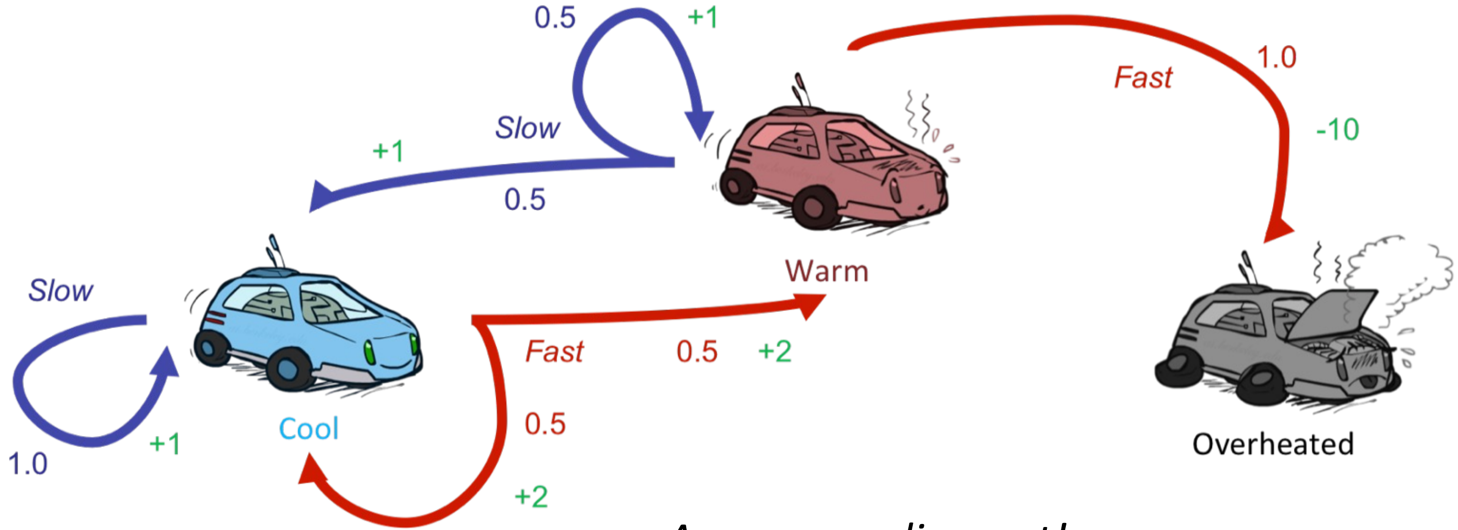
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

a=slow: $1(1 + 0) = 1$

a=fast: $0.5(2 + 0) + 0.5(2 + 0) = 2$

Example: Value Iteration

V_2			
V_1	2	?	
V_0	0	0	0



Assume no discount!

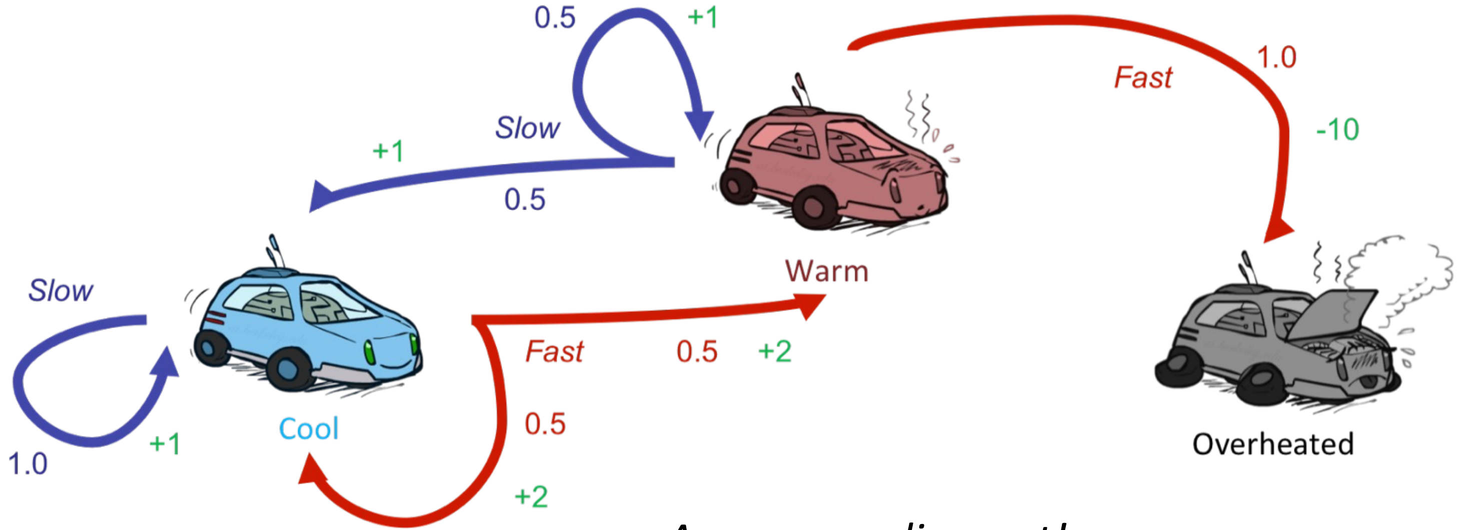
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

a=slow: 0.5(1 + 0) + 0.5(1 + 0) = 1

a=fast: 1(-10 + 0) = -10

Example: Value Iteration

V_2	?		
V_1	2	1	0
V_0	0	0	0



Assume no discount!

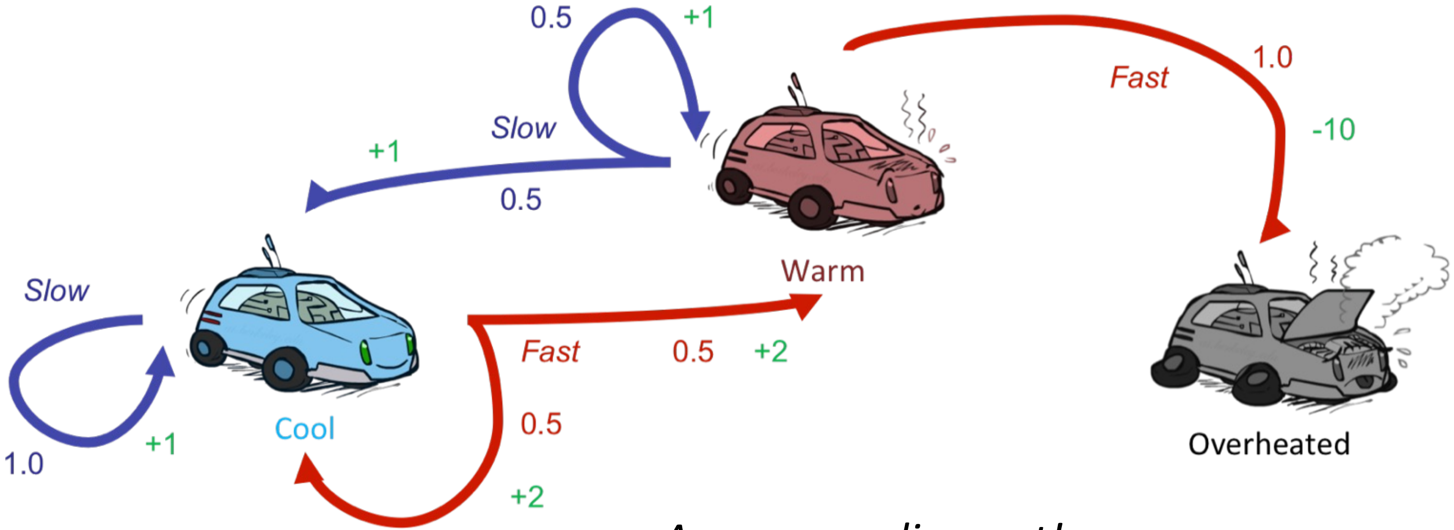
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

a=slow: $1(1 + 2) = 3$

a=fast: $0.5(2 + 2) + 0.5(2 + 1) = 3.5$

Example: Value Iteration

V_2	3.5	?	0
V_1	2	1	0
V_0	0	0	0



Assume no discount!

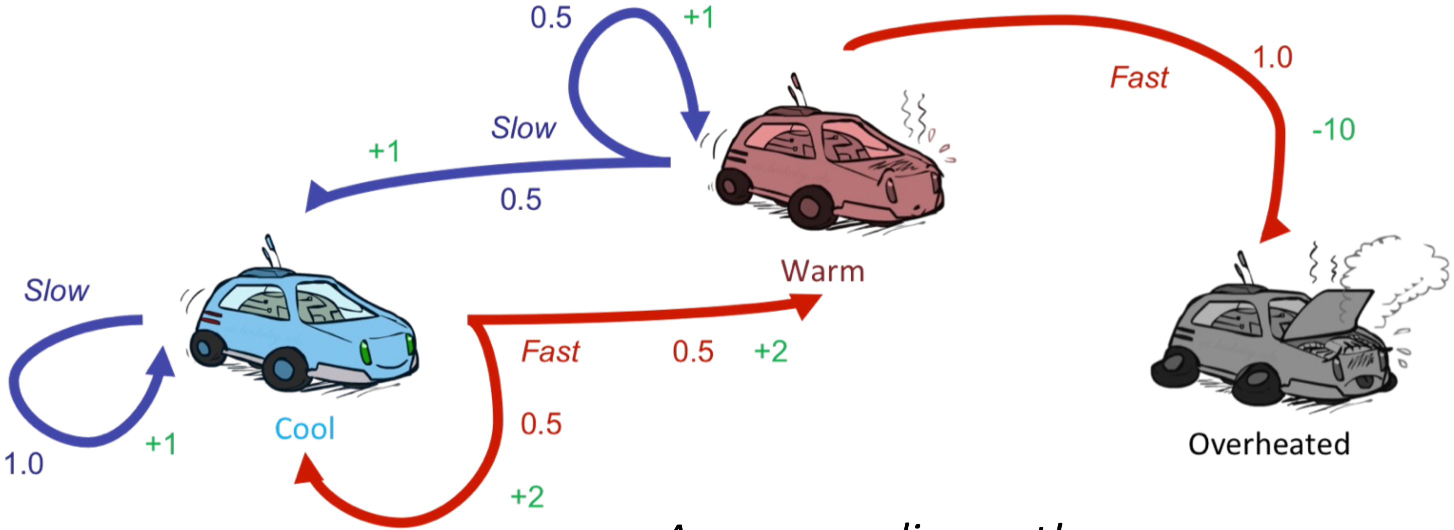
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

a=slow: 0.5(1 + 2) + 0.5(1 + 1) = 2.5

a=fast: 1(-10 + 0) = -10

Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

What we did today

- Working with stochastic environments (but world model known)
- Introduced *MDPs* (describe problem) and *policies* (solution)
  - MDPs look similar to expectimax search trees
- Discussed how to solve MDPs
  - Optimal *state value V*(s)*, and *q-state value Q*(s,a)* are key quantities
  - *Bellman equation* characterizes the optimal value function:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

    - A key equation in RL and this class!
  - *Value iteration* is an algorithm to solve the Bellman equation

Next Time: Policy-Based Methods