

# Announcements

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- **HW 4** due **today** (Oct 3) at 11:59pm PT
- **Project 3** due **Friday** (Oct 6) at 11:59pm PT
- **HW 5** released soon, due **next Tuesday** (Oct 10) at 11:59pm PT
- **Midterm** is on **Monday** (Oct 16) 7-9pm PT
  - See [exam logistics page for details](#)
  - Fill out [exam requests form](#) for by **this Friday** (Oct 6) 11:59pm PT

# CS188 Outline

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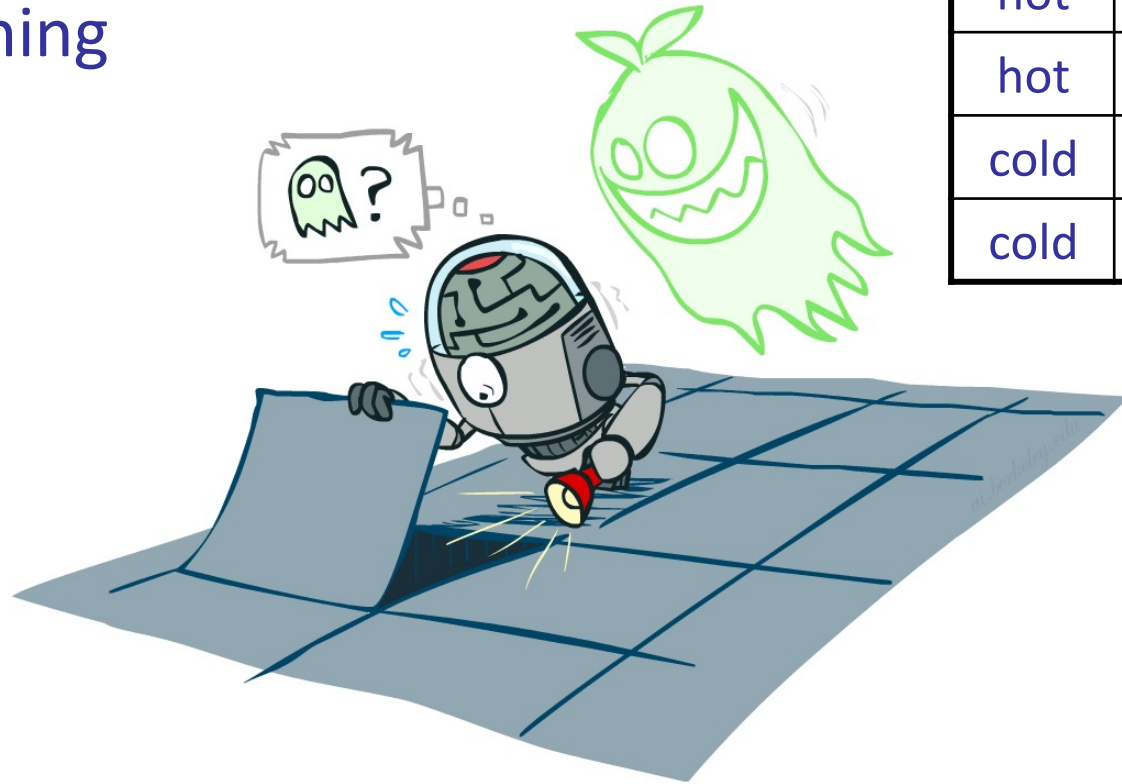
- We're done with Part I: Search and Planning!
- Part II: Probabilistic Reasoning

Why should we care about probability, randomness, uncertainty in AI?

- **To better model natural environment:** the world has unpredictable events
  - **To better model natural cognition:** the agent may be uncertain about the world state or which actions to take
  - **To develop more efficient algorithms:** approximate solutions via random sampling
- Part III: Machine Learning

# CS188 Outline

- We're done with Part I: Search and Planning!
- Part II: Probabilistic Reasoning
  - Form and update beliefs:
    - Diagnosis
    - Speech recognition
    - Tracking objects
    - Robot mapping
    - Genetics
    - Error correcting codes
    - Explain human cognition
    - ... lots more!
- Part III: Machine Learning



$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

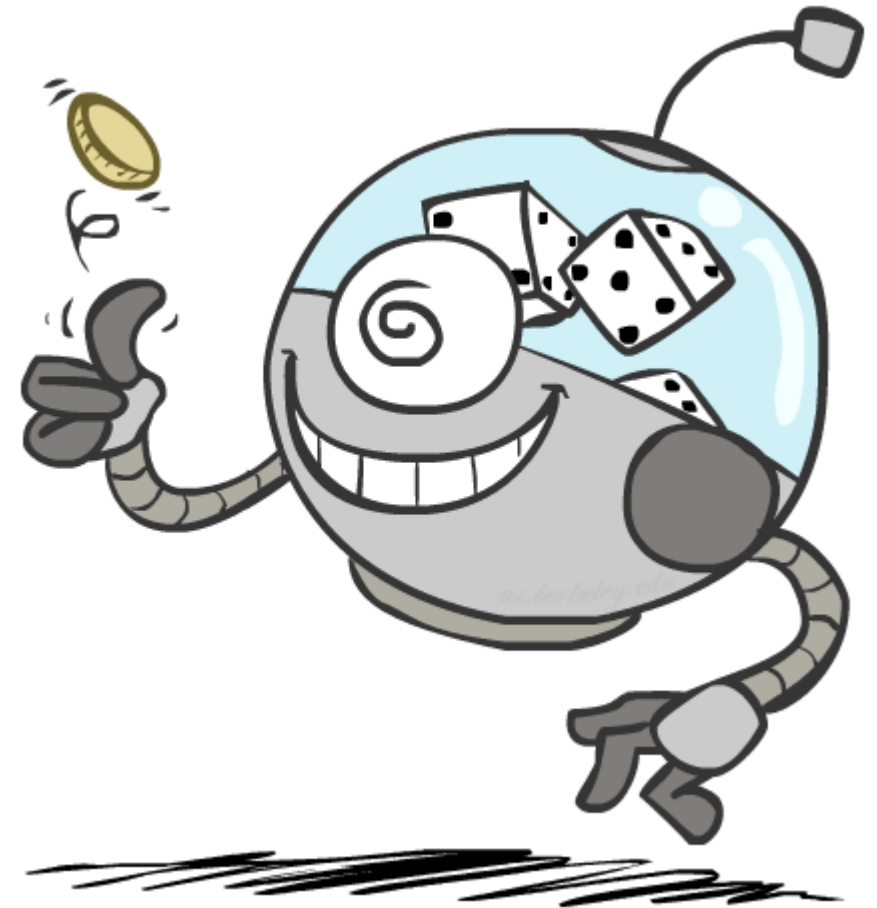
# CS 188: Artificial Intelligence

## Probability



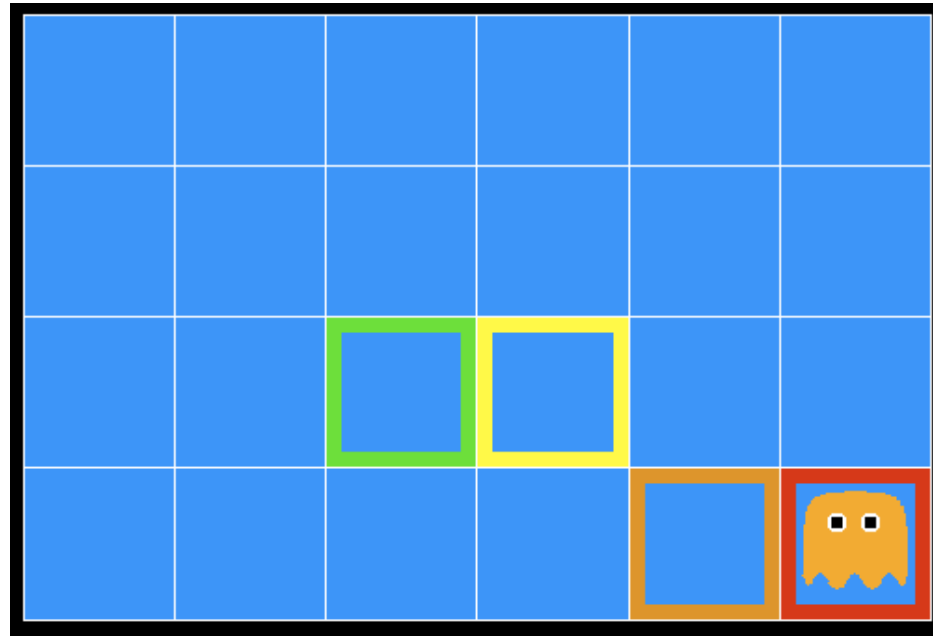
# Today

- Probability
  - Random Variables
  - Joint and Marginal Distributions
  - Conditional Distribution
  - Product Rule, Chain Rule, Bayes' Rule
  - Inference by Enumeration
- You'll need all this stuff A LOT for the next few weeks, so make sure you go over it now!



# Inference in Ghostbusters

- A ghost is in the grid somewhere
- Sensor readings tell how close a square is to the ghost
  - On the ghost: red
  - 1 or 2 away: orange
  - 3 or 4 away: yellow
  - 5+ away: green
- **Sensors are noisy**, but we know  $P(\text{Color} \mid \text{Distance})$



$P(\text{red} \mid 3)$	$P(\text{orange} \mid 3)$	$P(\text{yellow} \mid 3)$	$P(\text{green} \mid 3)$
0.05	0.15	0.5	0.3

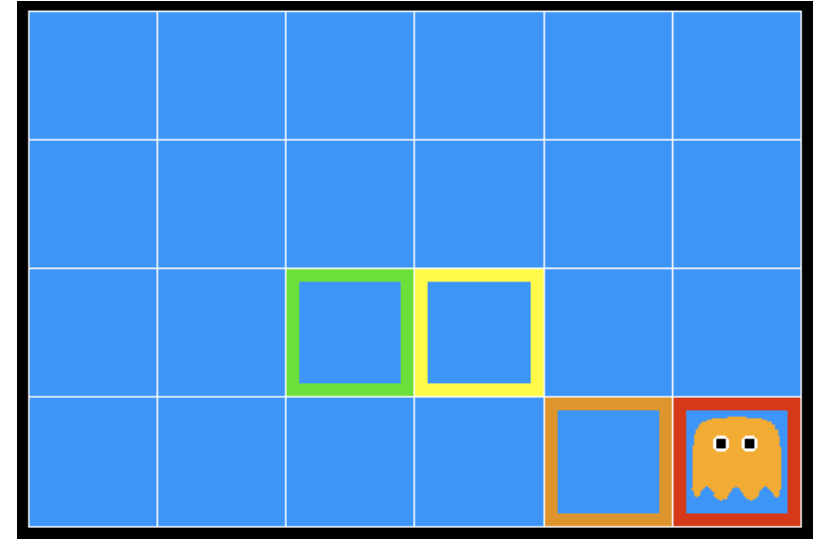
# Video of Demo Ghostbuster – No probability

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# Uncertainty

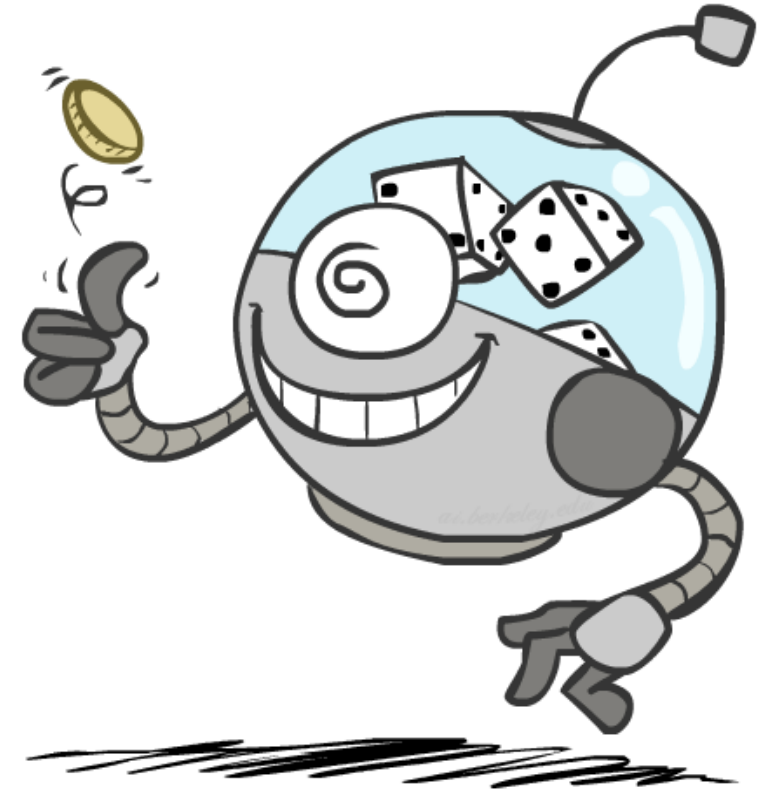
- General situation:
  - **Observed variables (evidence):** Agent knows certain things about the state of the world (e.g., sensor readings or symptoms)
  - **Unobserved variables:** Agent needs to reason about other aspects (e.g. where an object is or what disease is present)
  - **Model:** Agent knows something about how the known variables relate to the unknown variables
- Probabilistic reasoning and inference gives us a framework for managing our beliefs and knowledge





# Random Variables

- A **random variable** is some aspect of the world about which we (may) have uncertainty
  - **R** = Is it raining?
  - **T** = Is it hot or cold?
  - **D** = How long will it take to drive to work?
  - **L** = Where is the ghost?
- We denote random variables with capital letters
- Like variables in a CSP, random variables have **domains**
  - **R** in  $\{\text{true, false}\}$  (often write as  $\{+r, -r\}$ )
  - **T** in  $\{\text{hot, cold}\}$
  - **D** in  $[0, \infty)$
  - **L** in possible locations, maybe  $\{(0,0), (0,1), \dots\}$

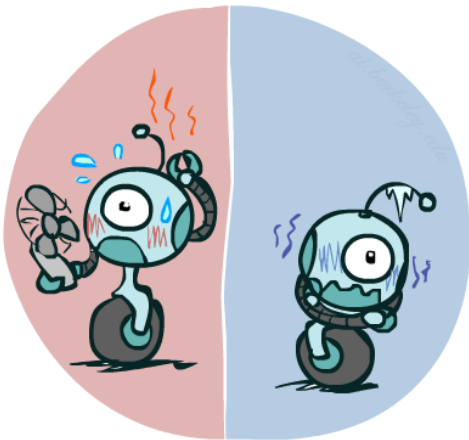


# Probability Distributions

- Associate a probability with each **value** of that **random variable**

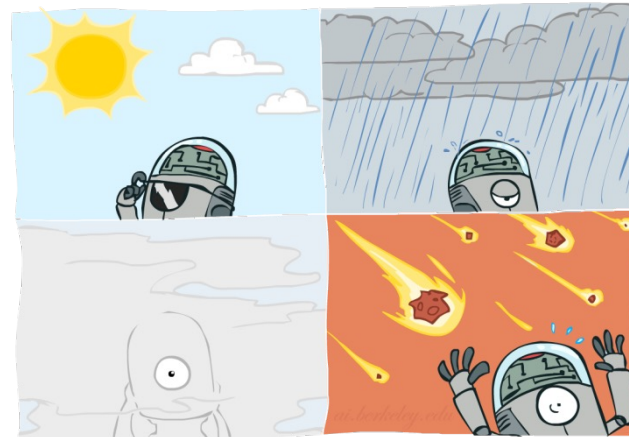
- Temperature:

- Weather:



$P(T)$

T	P
hot	0.5
cold	0.5



$P(W)$

W	P
sun	0.6
rain	0.1
fog	0.3
meteor	0.0

# Probability Distributions

- Unobserved **random variables** have distributions

T	P
hot	0.5
cold	0.5
W	P
sun	0.6
rain	0.1
fog	0.3
meteor	0.0

Shorthand notation:

$P(\textit{hot})$  same as  $P(T = \textit{hot})$

$P(\textit{cold})$  same as  $P(T = \textit{cold})$

$P(\textit{rain})$  same as  $P(W = \textit{rain})$

...

OK if all domain entries are unique

- A distribution is a TABLE of probabilities of values

- A probability (of a **lower case value**) is a single number:

$$P(W = \textit{rain}) = 0.1$$

- Must have:  $\forall x \ P(X = x) \geq 0$  and  $\sum_x P(X = x) = 1$

# Joint Distributions

- A *joint distribution* over a set of **random variables**:  $X_1, X_2, \dots, X_N$  specifies a real number for each assignment (or *outcome*):

$$P(X_1 = x_1, X_2 = x_2, \dots, X_N = x_N)$$

$$P(x_1, x_2, \dots, x_N)$$

- Must obey:  $P(x_1, x_2, \dots, x_n) \geq 0$

$$\sum_{(x_1, x_2, \dots, x_n)} P(x_1, x_2, \dots, x_n) = 1$$

- Size of distribution if  $n$  variables with domain sizes  $d$ ?
  - For all but the smallest distributions, impractical to write out!

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

# Probabilistic Models

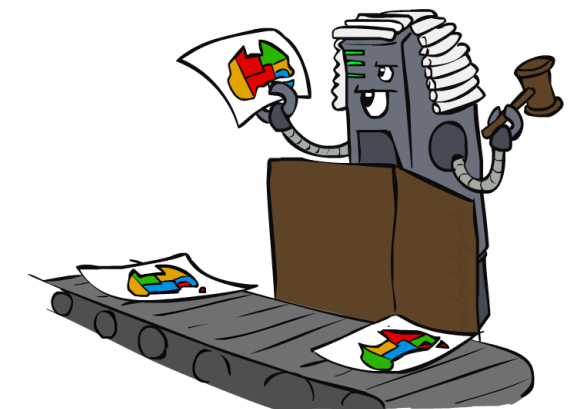
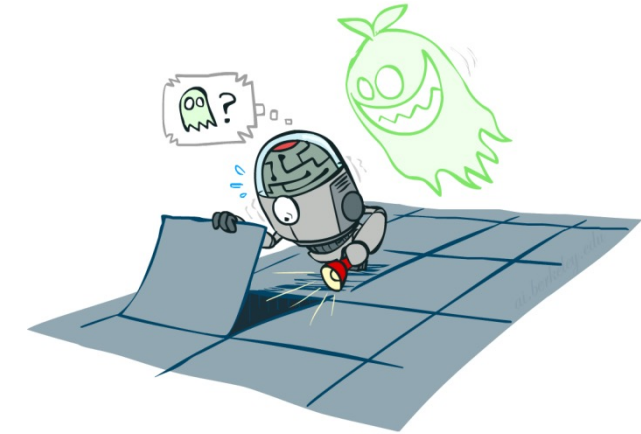
- A probabilistic model is a joint distribution over a set of random variables
- Probabilistic models:
  - (Random) variables with domains
  - Assignments are called *outcomes*
  - Joint distributions: say whether assignments (outcomes) are likely
  - *Normalized*: sum to 1.0
  - Ideally: only certain variables directly interact
- Constraint satisfaction problems:
  - Variables with domains
  - Constraints: state whether assignments are possible
  - Ideally: only certain variables directly interact

Distribution over T,W

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Constraint over T,W

T	W	P
hot	sun	T
hot	rain	F
cold	sun	F
cold	rain	T



# Events

- An *event* is a set  $E$  of outcomes

$$P(E) = \sum_{(x_1 \dots x_n) \in E} P(x_1 \dots x_n)$$

- From a joint distribution, we can calculate the probability of any event
  - Probability that it's hot AND sunny?
  - Probability that it's hot?
  - Probability that it's hot OR sunny?
- Typically, the events we care about are *partial assignments*, like  $P(T=\text{hot})$

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

# Quiz: Events

- $P(+x, +y)$  ?
- $P(+x)$  ?
- $P(-y \text{ OR } +x)$  ?

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

# Quiz: Events

- $P(+x, +y) ?$                       0.2
- $P(+x) ?$                                $0.2 + 0.3 = 0.5$
- $P(-y \text{ OR } +x) ?$                        $0.1 + 0.3 + 0.2 = 0.6$

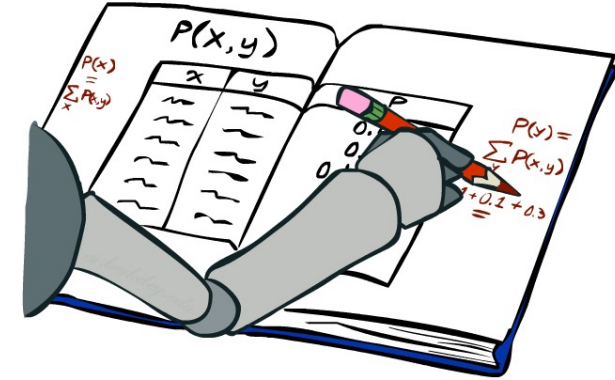
$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1



# Marginal Distributions

- Marginal distributions are sub-tables which eliminate **random variables**
- Marginalization (summing out): Combine collapsed rows by adding



$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(t) = \sum_w P(t, w)$$



$P(T)$

T	P
hot	0.5
cold	0.5

$$P(w) = \sum_t P(t, w)$$



$P(W)$

W	P
sun	0.6
rain	0.4

$$P(X_1 = x_1) = \sum_{x_2} P(X_1 = x_1, X_2 = x_2)$$

$x_2$

hidden (unobserved) variables

# Quiz: Marginal Distributions

$P(X, Y)$

X	Y	P
+X	+Y	0.2
+X	-Y	0.3
-X	+Y	0.4
-X	-Y	0.1

$$P(x) = \sum_y P(x, y)$$



$$P(y) = \sum_x P(x, y)$$

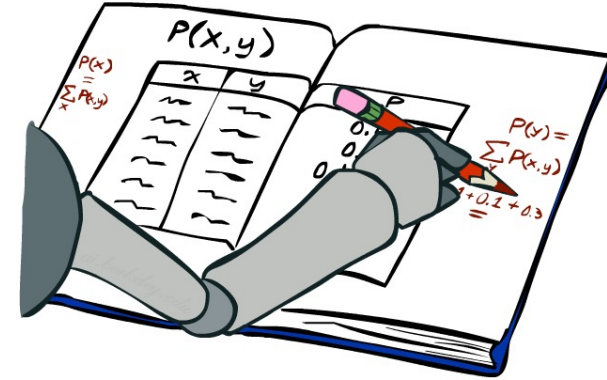


$P(X)$

X	P
+X	
-X	

$P(Y)$

Y	P
+Y	
-Y	



# Quiz: Marginal Distributions

$P(X, Y)$

X	Y	P
+X	+Y	0.2
+X	-Y	0.3
-X	+Y	0.4
-X	-Y	0.1

$$P(x) = \sum_y P(x, y)$$



$$P(y) = \sum_x P(x, y)$$

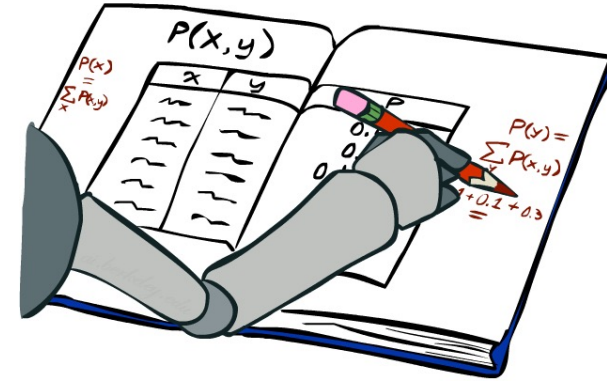


$P(X)$

X	P
+X	0.5
-X	0.5

$P(Y)$

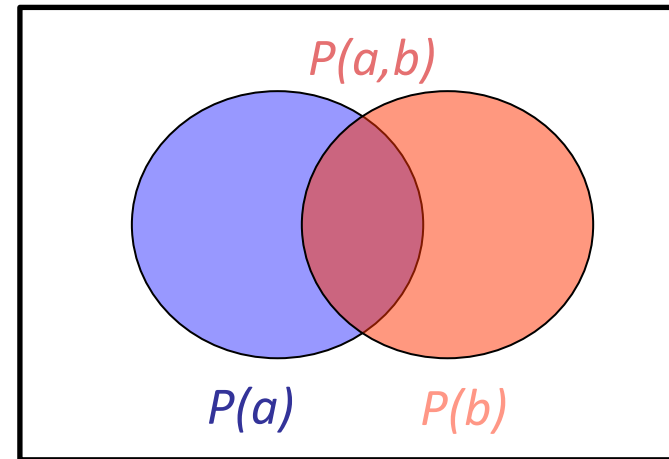
Y	P
+Y	0.6
-Y	0.4



# Conditional Probabilities

- A simple relation between joint and conditional probabilities
  - In fact, this is taken as the *definition* of a conditional probability

$$\begin{array}{c} \text{evidence} \\ \downarrow \\ P(a|b) = \frac{P(a,b)}{P(b)} \\ \uparrow \\ \text{query} = (\text{proportion of } b \text{ where } a \text{ holds}) \end{array}$$

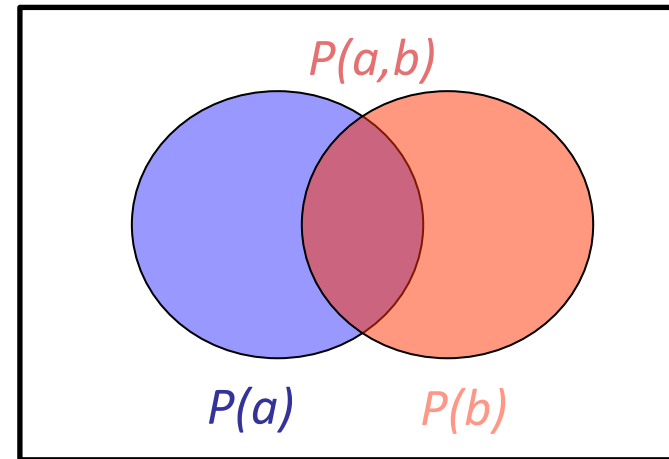


# Conditional Probabilities

- A simple relation between joint and conditional probabilities
  - In fact, this is taken as the *definition* of a conditional probability

$$P(a|b) = \frac{P(a, b)}{P(b)}$$

= (proportion of  $b$  where  $a$  holds)



$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(W = s|T = c) = \frac{P(W = s, T = c)}{P(T = c)} = \frac{0.2}{0.5} = 0.4$$

$$\begin{aligned} &= P(W = s, T = c) + P(W = r, T = c) \\ &= 0.2 + 0.3 = 0.5 \end{aligned}$$

# Quiz: Conditional Probabilities

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

■  $P(+x \mid +y)$  ?

■  $P(-x \mid +y)$  ?

■  $P(-y \mid +x)$  ?

$$P(a|b) = \frac{P(a, b)}{P(b)}$$

# Quiz: Conditional Probabilities

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

■  $P(+x \mid +y) ?$        $0.2 / 0.6 = 1/3$

■  $P(-x \mid +y) ?$        $0.4 / 0.6 = 2/3$

■  $P(-y \mid +x) ?$        $0.3 / 0.5 = 3/5$

$$P(a|b) = \frac{P(a, b)}{P(b)}$$

# Conditional Distributions

- Conditional distributions are probability distributions over some variables given fixed values of others

$$P(a|b) = \frac{P(a, b)}{P(b)}$$

Conditional Distributions

$P(W|T)$

$P(W T = hot)$	
W	P
sun	0.8
rain	0.2

$P(W T = cold)$	
W	P
sun	0.4
rain	0.6

Joint Distribution

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3



# Normalization Trick

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$\begin{aligned}P(W = s|T = c) &= \frac{P(W = s, T = c)}{P(T = c)} \\ &= \frac{P(W = s, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.2}{0.2 + 0.3} = 0.4\end{aligned}$$



$$\begin{aligned}P(W = r|T = c) &= \frac{P(W = r, T = c)}{P(T = c)} \\ &= \frac{P(W = r, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.3}{0.2 + 0.3} = 0.6\end{aligned}$$

$P(W|T = c)$

W	P
sun	0.4
rain	0.6

# Normalization Trick

$$\begin{aligned}
 P(W = s|T = c) &= \frac{P(W = s, T = c)}{P(T = c)} \\
 &= \frac{P(W = s, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\
 &= \frac{0.2}{0.2 + 0.3} = 0.4
 \end{aligned}$$

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

**SELECT** the joint probabilities matching the evidence



$P(c, W)$

T	W	P
cold	sun	0.2
cold	rain	0.3

**NORMALIZE** the selection (make it sum to one)



$P(W|T = c)$

W	P
sun	0.4
rain	0.6

$$\begin{aligned}
 P(W = r|T = c) &= \frac{P(W = r, T = c)}{P(T = c)} \\
 &= \frac{P(W = r, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\
 &= \frac{0.3}{0.2 + 0.3} = 0.6
 \end{aligned}$$

# Normalization Trick

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

**SELECT** the joint probabilities matching the evidence



$P(c, W)$

T	W	P
cold	sun	0.2
cold	rain	0.3

**NORMALIZE** the selection (make it sum to one)



$P(W|T = c)$

W	P
sun	0.4
rain	0.6

- Why does this work? Sum of selection is  $P(\text{evidence})!$  ( $P(T=c)$ , here)

$$P(x_1|x_2) = \frac{P(x_1, x_2)}{P(x_2)} = \frac{P(x_1, x_2)}{\sum_{x_1} P(x_1, x_2)}$$

# Quiz: Normalization Trick

- $P(X | Y=-y)$  ?

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

**SELECT** the joint probabilities matching the evidence



**NORMALIZE** the selection (make it sum to one)



# Quiz: Normalization Trick

- $P(X \mid Y=-y)$  ?

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

**SELECT** the joint probabilities matching the evidence



X	Y	P
+x	-y	0.3
-x	-y	0.1

**NORMALIZE** the selection  
(make it sum to one)



X	P
+x	0.75
-x	0.25

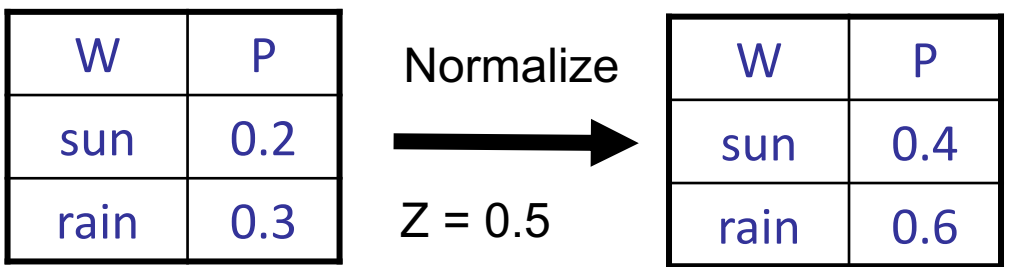
# To Normalize

- (Dictionary) To bring or restore to a normal condition

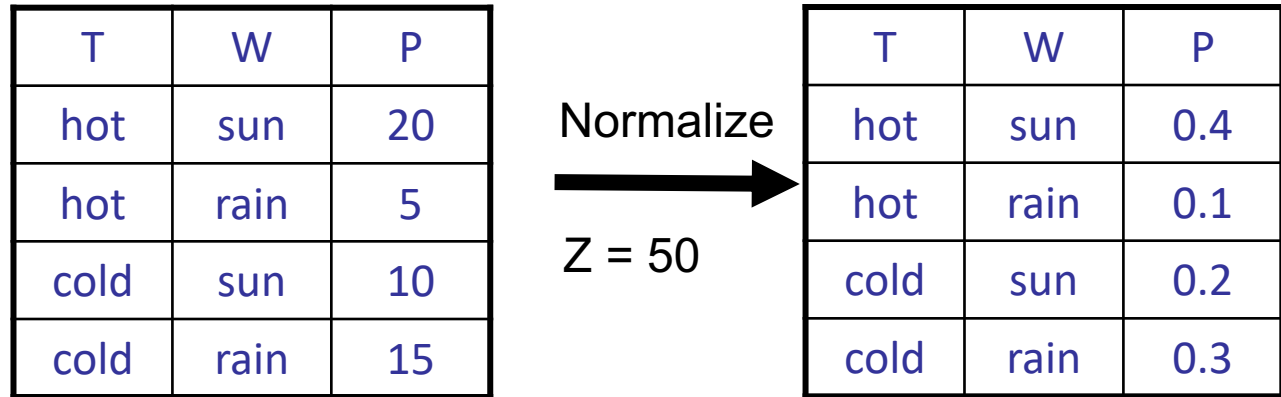
All entries sum to ONE

- Procedure:
  - Step 1: Compute  $Z = \text{sum over all entries}$
  - Step 2: Divide every entry by  $Z$

- Example 1



- Example 2



# Probabilistic Inference

- *Probabilistic inference*: compute a desired probability from other known probabilities (e.g. conditional from joint)
- We generally compute conditional probabilities
  - $P(\text{on time} \mid \text{no reported accidents}) = 0.90$
  - These represent the agent's *beliefs* given the evidence
- Probabilities change with new evidence:
  - $P(\text{on time} \mid \text{no accidents, 5 a.m.}) = 0.95$
  - $P(\text{on time} \mid \text{no accidents, 5 a.m., raining}) = 0.80$
  - Observing new evidence causes *beliefs to be updated*



# Inference by Enumeration

- $P(W)$ ?

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20



# Inference by Enumeration

- $P(W)?$   
↑  
query

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

- $P(W)$ ?

$$P(\text{sun}) = 0.3 + 0.1 + 0.1 + 0.15 = 0.65$$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

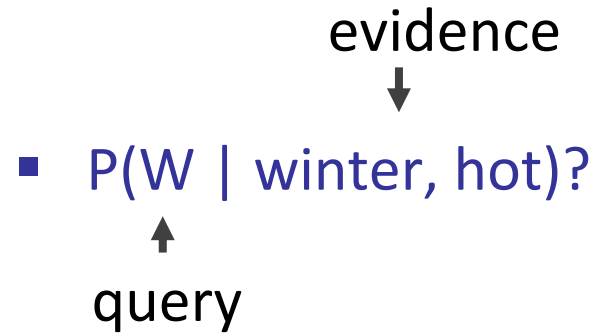
- $P(W)$ ?

$$P(\text{sun}) = 0.3 + 0.1 + 0.1 + 0.15 = 0.65$$

$$P(\text{rain}) = 0.05 + 0.05 + 0.05 + 0.20 = 0.35$$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration



S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

- $P(W \mid \text{winter, hot})?$

unnormalized  $P(\text{sun} \mid \text{winter, hot}) = 0.10$

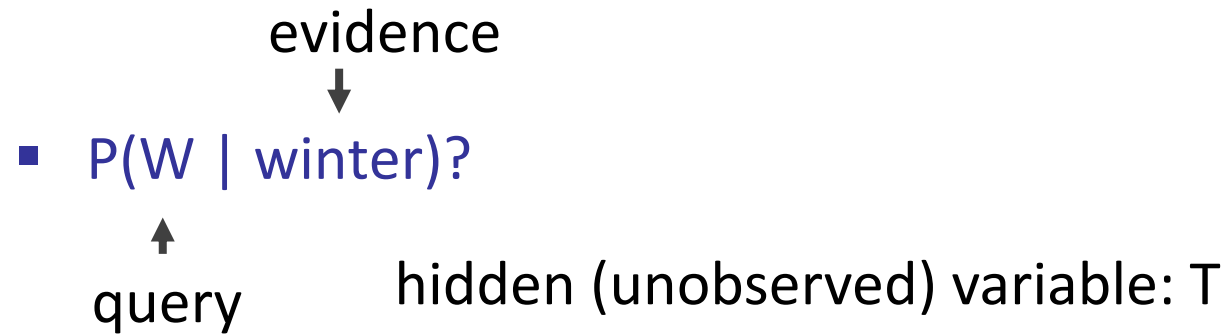
unnormalized  $P(\text{rain} \mid \text{winter, hot}) = 0.05$

$P(\text{sun} \mid \text{winter, hot}) = 0.10 / 0.15 = 2/3$

$P(\text{rain} \mid \text{winter, hot}) = 0.05 / 0.15 = 1/3$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration



S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

- $P(W \mid \text{winter})?$

unnormalized  $P(\text{sun} \mid \text{winter}) = 0.1 + 0.15 = 0.25$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

- $P(W \mid \text{winter})?$

unnormalized  $P(\text{sun} \mid \text{winter}) = 0.1 + 0.15 = 0.25$

unnormalized  $P(\text{rain} \mid \text{winter}) = 0.05 + 0.20 = 0.25$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20



# Inference by Enumeration

- $P(W \mid \text{winter})?$

unnormalized  $P(\text{sun} \mid \text{winter}) = 0.1 + 0.15 = 0.25$

unnormalized  $P(\text{rain} \mid \text{winter}) = 0.05 + 0.20 = 0.25$

$P(\text{sun} \mid \text{winter}) = 0.25 / 0.50 = 0.5$

$P(\text{rain} \mid \text{winter}) = 0.25 / 0.50 = 0.5$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

# Inference by Enumeration

- General case:

- Evidence variables:  $E_1 \dots E_k = e_1 \dots e_k$
  - Query\* variable:  $Q$
  - Hidden variables:  $H_1 \dots H_r$
- }  $X_1, X_2, \dots, X_n$   
All variables

- We want:

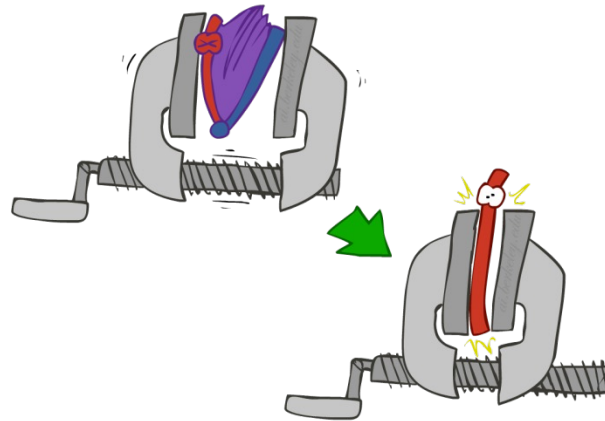
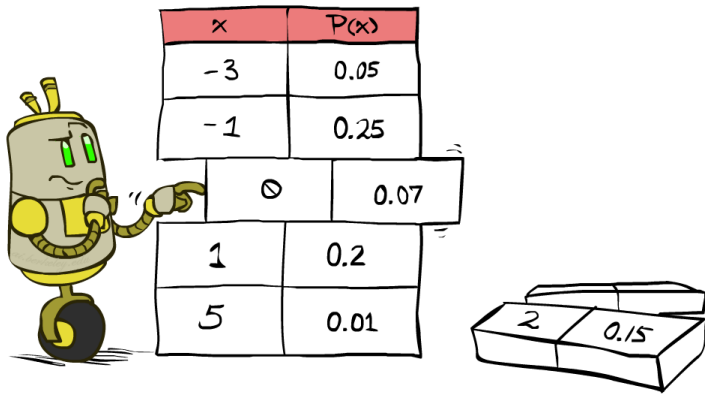
$$P(Q|e_1 \dots e_k)$$

*\* Works fine with multiple query variables, too*

- Step 1: **Select** the entries consistent with the evidence

- Step 2: **Sum out** H to get joint of Query and evidence

- Step 3: **Normalize**



$$\times \frac{1}{Z}$$

$$P(Q, e_1 \dots e_k) = \sum_{h_1 \dots h_r} P(Q, \underbrace{h_1 \dots h_r, e_1 \dots e_k}_{X_1, X_2, \dots, X_n})$$

$$Z = \sum_q P(Q, e_1 \dots e_k)$$

$$P(Q|e_1 \dots e_k) = \frac{1}{Z} P(Q, e_1 \dots e_k)$$

# Inference by Enumeration

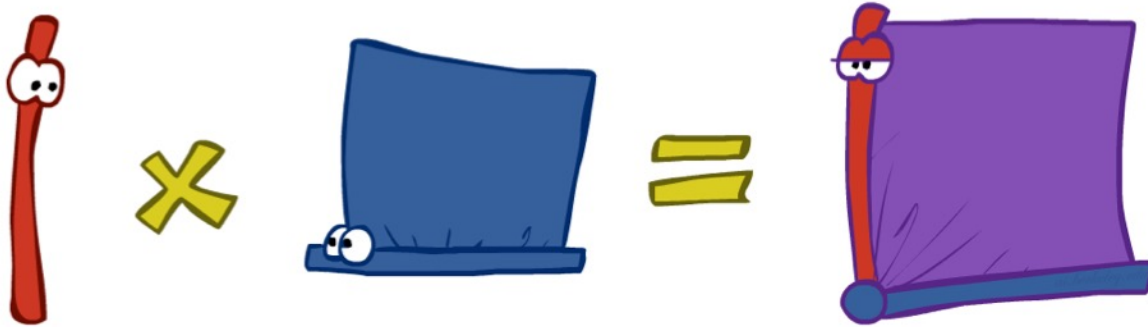
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- Obvious problems:
  - Worst-case time complexity  $O(d^n)$
  - Space complexity  $O(d^n)$  to store the joint distribution

# The Product Rule

- Sometimes have conditional distributions but want the joint

$$P(y)P(x|y) = P(x, y) \quad \longleftrightarrow \quad P(x|y) = \frac{P(x, y)}{P(y)}$$



# The Product Rule

$$P(y)P(x|y) = P(x, y)$$

- Example:

$P(W)$

R	P
sun	0.8
rain	0.2

$P(D|W)$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3



$P(D, W)$

D	W	P
wet	sun	
dry	sun	
wet	rain	
dry	rain	

# The Chain Rule

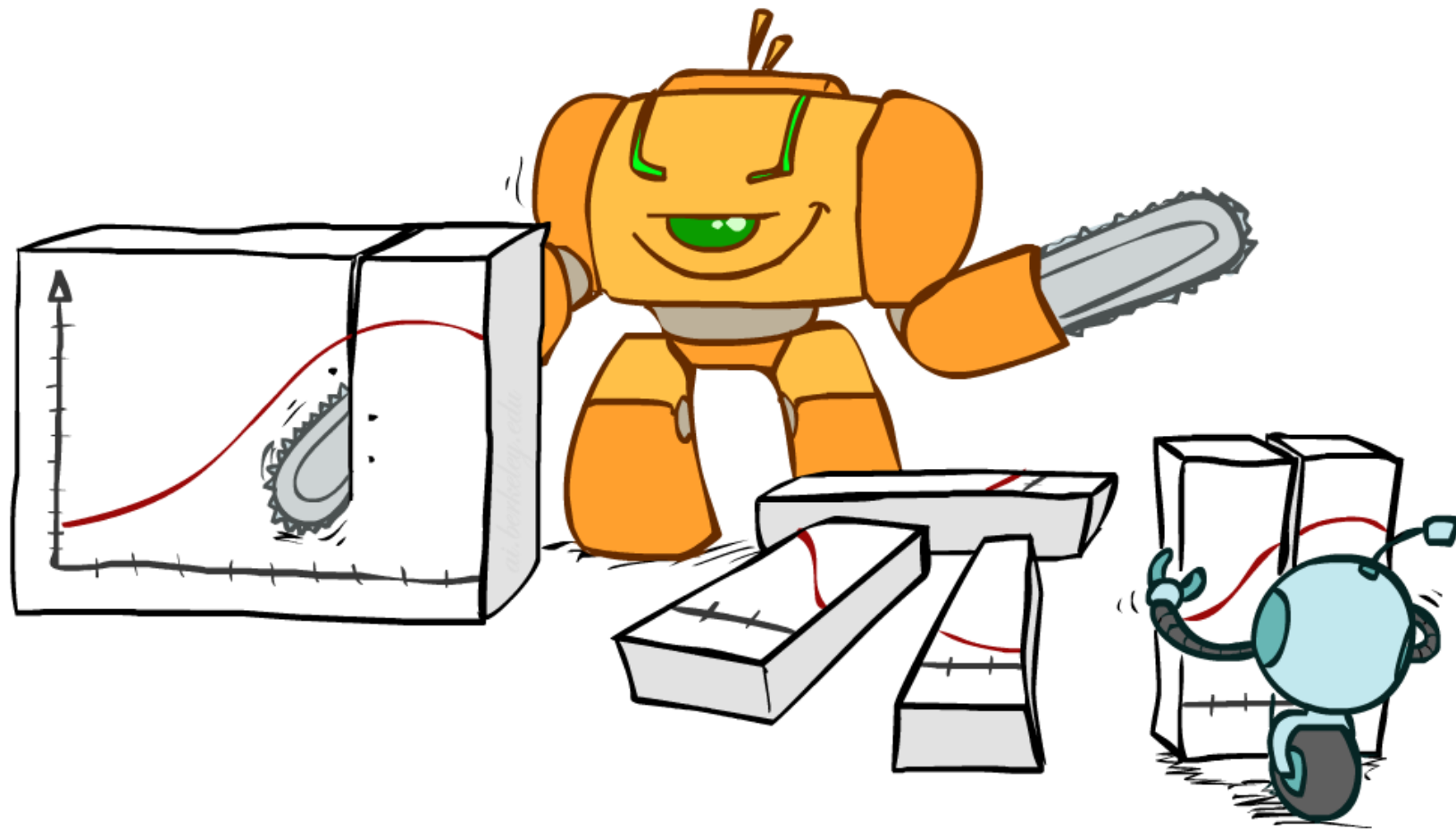
- More generally, can always write any joint distribution as an incremental product of conditional distributions

$$P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)$$

$$P(x_1, x_2, \dots, x_n) = \prod_i P(x_i|x_1 \dots x_{i-1})$$

- Why is this always true?

# Bayes Rule



# Bayes' Rule

- Two ways to factor a joint distribution over two variables:

$$P(x, y) = P(x|y)P(y) = P(y|x)P(x)$$

- Dividing, we get:

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

- Why is this at all helpful?
  - Lets us build one conditional from its reverse
  - Often one conditional is tricky but the other one is simple
  - Foundation of many systems we'll see later (e.g. ASR, MT)
- In the running for most important AI equation!

That's my rule!





# Bayes' Rule

- Two ways to factor a joint distribution over two variables:

$$P(x, y) = P(x|y)P(y) = P(y|x)P(x)$$

- Dividing, we get:

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)} \qquad P(\text{cause}|\text{effect}) = \frac{P(\text{effect}|\text{cause})P(\text{cause})}{P(\text{effect})}$$

- Why is this at all helpful?
  - Lets us build one conditional from its reverse
  - Often one conditional is tricky but the other one is simple
  - Foundation of many systems we'll see later (e.g. ASR, MT)
- In the running for most important AI equation!

# Inference with Bayes' Rule

- Example: Diagnostic probability from causal probability:

$$P(\text{cause}|\text{effect}) = \frac{P(\text{effect}|\text{cause})P(\text{cause})}{P(\text{effect})}$$

- Example:

- M: meningitis, S: stiff neck

$$\left. \begin{aligned} P(+m) &= 0.0001 \\ P(+s|+m) &= 0.8 \\ P(+s|-m) &= 0.01 \end{aligned} \right\} \text{Example givens}$$

$$P(+m|+s) = \frac{P(+s|+m)P(+m)}{P(+s)} = \frac{P(+s|+m)P(+m)}{P(+s|+m)P(+m) + P(+s|-m)P(-m)} = \frac{0.8 \times 0.0001}{0.8 \times 0.0001 + 0.01 \times 0.999}$$

# Quiz: Bayes' Rule

- Given:

$$P(W)$$

R	P
sun	0.8
rain	0.2

$$P(D|W)$$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

- What is  $P(W \mid \text{dry})$  ?

# Quiz: Bayes' Rule

- Given:

$$P(W)$$

R	P
sun	0.8
rain	0.2

$$P(D|W)$$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

- What is  $P(W | \text{dry})$  ?

$$\text{unnormalized } P(\text{sun} | \text{dry}) = P(\text{dry} | \text{sun}) * P(\text{sun}) = 0.9 * 0.8 = 0.72$$

$$\text{unnormalized } P(\text{rain} | \text{dry}) = P(\text{dry} | \text{rain}) * P(\text{rain}) = 0.3 * 0.2 = 0.06$$

$$P(\text{sun} | \text{dry}) = 0.72 / 0.78 = 12/13$$

$$P(\text{rain} | \text{dry}) = 0.06 / 0.78 = 1/13$$

# Ghostbusters, Revisited

- Let's say we have two distributions:
  - Prior distribution** over ghost location:  $P(G)$ 
    - Let's say this is uniform
  - Sensor reading model:  $P(R | G)$ 
    - Given: we know what our sensors do
    - $R$  = reading color measured at (1,1)
    - E.g.  $P(R = \text{yellow} | G=(1,1)) = 0.1$

- We can calculate the **posterior distribution**  $P(G|r)$  over ghost locations given a reading using Bayes' rule:

$$\text{unnormalized } P(g|r) = P(r|g)P(g)$$

$P(G)$

0.11	0.11	0.11
0.11	0.11	0.11
0.11	0.11	0.11

$P(G|r)$

0.17	0.10	0.10
0.09	0.17	0.10
<0.01	0.09	0.17

# Video of Demo Ghostbusters with Probability

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# Next Time: Bayes' Nets

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