Assignment 3, Due March 6

1. Problem 15.1 from Russell and Norvig.

2. On Page 447 of Russell and Norvig, various conditional probabilities are stated for the belief network e.g. Probability of Burglary conditioned on JohnCalls is 0.016. There are seven such numbers stated. Please derive these. The simplest way may be to first explicitly compute the 32 entries of the joint probability distribution, but you can use any technique you want. This is a pencil-and-paper problem.

3. Implement an HMM to model the weather problem in the handout by Eric Fosler. Assume prior probabilities of \( P(Sunny) = 0.5, P(Rainy) = 0.25, \) and \( P(Foggy) = 0.25. \)
   - Develop a function to return \( P(q_t | y) \) where \( y \) is a collection of observations of \( T = 10 \) states. In other words, given 10 days of observations of umbrella vs. no-umbrella, find the conditional probability of each type of weather for each day. Assume the following sequence of observations: \( \{T, T, F, F, T, F, T, T, F, F\} \).
   - Compare \( P(q_5 | y_1, \ldots, y_{10}) \) with \( P(q_5 | y_1, \ldots, y_5) \), i.e., the posterior on the weather on the fifth day given all 10 days’ observations vs. that for only the first five days. Are they different? If so, why?
   - Compare the result you get for \( P(q_t | y) \) using the forward-backward method with the result you get using the brute force method. (The brute force method consists of explicitly finding the joint probability distribution and then using Bayes’ rule to get the posterior.) Count the number of flops needed for the forward-backward algorithm vs. the brute force method; this will give you an idea of the complexity. Also this is a good way to debug your code for the forward-backward method.
   - Compute the most probable sequence of states \( q_t \) given the observations using the Viterbi method (dynamic programming).

4. Solve Problem 24.10 (pg 772) from Russell and Norvig. You can do this using pencil and paper, or you can modify the HMM code you implemented for the weather problem and have it crank out the answer for you. Your choice.

You may work on this assignment in groups of up to 3 people. While any programming language could be used for this or other assignments, you may find that MATLAB is the most convenient for this assignment. The TAs will be available to provide MATLAB help in the sections.