1 True/False

T/F A rational agent may choose to play a game of chance with negative expected payoff.

True. Money alone does not determine an agent’s utility function. The agent may realize some utility from playing the game.

T/F If a tree search algorithm is complete, the corresponding graph search algorithm will be, too.

True. Graph search does not compromise completeness, only optimality.

T/F For all random variables $X, Y$ and $Z$ and distributions over them,

$$\frac{P(Y|X)P(Z|X)P(X)}{P(Y,Z)} = P(X|Y,Z)$$

False. This would require an independence assumption that $P(Y,Z|X) = P(Y|X)P(Z|X)$

T/F The perceptron algorithm always converges for inseparable data

False. It always converges for separable data.

T/F If a CSP is known to have a tree-structured graph, then forward checking and arc consistency are equivalent operations.

False. Arc consistency still considers all domain values while forward checking only considers those constrained by a newly instantiated variable.

2 Search

Consider the following search problem:

Which path will each search algorithm return, assuming all successor functions work out in such a way that nodes are explored in alphabetical order whenever possible?

(a) Breadth-first search

$S \rightarrow G$
(b) Depth-first search
\[ S \rightarrow A \rightarrow G \]
(c) Uniform-cost search
\[ S \rightarrow B \rightarrow G \]
(d) A* search
\[ S \rightarrow B \rightarrow G \]
(e) Greedy search
\[ S \rightarrow G \]
(f) Name a node that uniform-cost search will expand, but A* will not.
\[ C \]

Describe a reasonably general case in which each of the following will occur, or state that the scenario is impossible. Correct answers should not take more than one or two sentences.

(g) Depth-first search never terminates, despite a finite goal
The leftmost branch of the search tree has infinite depth and contains no goal.
(h) Breadth-first search never terminates, despite a finite goal
impossible
(i) Uniform cost search and breadth-first search expand the same nodes and return the same goal
The cost for every action is 1.

3 CSPs

You must arrange three statues in an exhibit hall: an ice carving of a swan (i), a gold lion (g), and a marble abstract piece (m). There are three tables, 1, 2, and 3, arranged in a row, with 1 closest to the door and 3 farthest into the exhibit hall. It is a hot day and so the ice carving cannot be nearest the door. Your manager also informs you that it will look bad to have to animal sculptures on adjacent tables. Reality tells you that each table must have a different sculpture.

If we formulate this problem as a binary CSP with variables \( X_1, X_2, \) and \( X_3 \), each with domain \( \{i, g, m\} \): 

(a) What are the unary constraint(s) (list them explicitly)
\[ X_1 \neq i \]

(b) What are the binary constraint(s) (list them explicitly)
\[ (X_1, X_2) \in \{(i, m), (m, i), (g, m), (m, g)\} \]
(X_2, X_3) \in \{(i, m), (m, i), (g, m), (m, g)\}
(X_1, X_3) \in \{(i, m), (m, i), (g, m), (m, g), (i, g), (g, i)\}

Assume we enforce the unary constraint(s) in pre-processing for the remaining parts:

(c) Which variable will be assigned first by the MRV heuristic?

\[ X_1 \]

(d) If we assign \( X_3 = i \), show the domains of the remaining variables after forward checking.

\[
\begin{align*}
X_1 & \in \{g, m\} \\
X_2 & \in \{m\} \\
X_3 & = i
\end{align*}
\]

(e) If no variables are assigned, show the initial domains after running arc consistency.

\[ \text{arc consistency has no effect.} \]

\[
\begin{align*}
X_1 & \in \{g, m\} \\
X_2 & \in \{i, g, m\} \\
X_3 & \in \{i, g, m\}
\end{align*}
\]

(f) If it’s a cool day, and we drop the requirement that the ice swan cannot be nearest the door, what are the initial domains after running arc consistency?

\[ \text{arc consistency has no effect.} \]

\[
\begin{align*}
X_1 & \in \{i, g, m\} \\
X_2 & \in \{i, g, m\} \\
X_3 & \in \{i, g, m\}
\end{align*}
\]

4 **Probability**

You are hired by a casino to help detect cheaters who manage to sneak loaded dice into a game which involves rolling a die 4 times in a row. With CS188 under your belt, you’ve decided to use a Naive Bayes model in order to detect cheaters. Assume that the prior probability of a player cheating is \( P(\text{cheat}) = 0.1 \). Non-cheaters all use fair dice, and cheaters all use a single type of loaded die.

(a) The casino has kept a record of the rolls of the last three known cheaters: they were [1, 2, 6, 6], [6, 3, 5, 4], and [6, 6, 6, 6]. Using this data, estimate the distribution of the loaded die.

\[
\begin{align*}
P(1|\text{cheat}) = P(2|\text{cheat}) = P(3|\text{cheat}) = P(4|\text{cheat}) = P(5|\text{cheat}) = \frac{1}{12} \\
P(6|\text{cheat}) = \frac{7}{12}
\end{align*}
\]
(b) What is the posterior probability that a gambler is cheating given a roll sequence of [6, 2, 6, 6] according to your estimate of the distribution from (a)?

\[
P(\text{cheat}|6, 2, 6, 6) \propto P(6|\text{cheat})^3 P(2|\text{cheat}) P(\text{cheat}) = \frac{7^3}{124 \cdot 10}
\]

\[
P(\neg \text{cheat}|6, 2, 6, 6) \propto P(6|\neg \text{cheat})^3 P(2|\neg \text{cheat}) P(\neg \text{cheat}) = \frac{9 \cdot 2^4}{6^4 \cdot 10} = \frac{9 \cdot 2^4}{124 \cdot 10}
\]

Normalizing, we have \( P(\text{cheat}|6, 2, 6, 6) = \frac{7^3}{7^3 + 9 \cdot 2^4} \) and \( P(\neg \text{cheat}|6, 2, 6, 6) = \frac{9 \cdot 2^4}{7^3 + 9 \cdot 2^4} \).

(c) If the utility of falsely accusing a non-cheater is -10, what is the minimum utility for catching a cheater which will make accusing the gambler in (b) the rational action?

We must solve \( 0 = -10 \cdot \frac{9 \cdot 2^4}{7^3 + 9 \cdot 2^4} + x \cdot \frac{7^3}{7^3 + 9 \cdot 2^4} \). The solution is \( \frac{10 \cdot 9 \cdot 2^4}{7^3} \).

5 Classification

Imagine we have features \( f_1, f_2, f_3, f_4 \) and three classes, \( \{x, y, z\} \). Assume we are training a multi-class perceptron and a given point in the training, it has the following weight vectors:

\[
\begin{align*}
    w_x &= (0, 0, 0, 0) \\
    w_y &= (0, 2, 0, 0) \\
    w_z &= (2, 0, 1, 0)
\end{align*}
\]

(a) If we next encounter the instance \( (1, 0, 0, 1) \) with true label \( x \), write the resulting weights after processing this new instance:

The classifier will guess \( z \), which is an error. After the update, we have:

\[
\begin{align*}
    w_x &= (1, 0, 0, 1) \\
    w_y &= (0, 2, 0, 0) \\
    w_z &= (1, 0, 1, -1)
\end{align*}
\]

(b) If we next encounter the instance \( (0, 1, 1, 1) \) with true label \( y \), write the resulting weights after processing this new instance:

No update is made because this datum is classified correctly as \( y \).

Consider a domain with three Boolean attributes, \( \{X, Y, Z\} \) and the target function \( f(x, y, z) = x \text{ xor } z \). Let \( H \) be the space of decision trees over these attributes.

(c) Is \( f \) realizable? If so, draw a decision tree which proves it; if not, argue why it is not. \( f \) is realizable.
Consider the following data set:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>f</th>
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<tbody>
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<td>0</td>
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</table>

(d) Draw the decision tree which would be learned from this data using the recursive splitting algorithm presented in class. Assume that splits are chosen using information gain, and gain ties are broken to prefer splits by alphabetical order.