CS 188: Artificial Intelligence
Spring 2006

Lecture 10: Perceptrons
2/16/2006

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Many slides from either Stuart Russell or Andrew Moore

Today

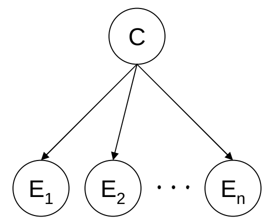
- Naïve Bayes models
  - Smoothing
  - Real world issues
- Perceptrons
  - Mistake-driven learning
  - Data separation, margins, and convergence

General Naïve Bayes

- This is an example of a *naive Bayes* model:

$$P(\text{Cause}, \text{Effect}_1 \ldots \text{Effect}_n) =$$

$$P(\text{Cause}) \prod_i P(\text{Effect}_i | \text{Cause})$$



- Total number of parameters is *linear* in n!

Example: Spam Filtering

- Model: $P(C, W_1 \ldots W_n) = P(C) \prod_i P(W_i | C)$
- Parameters:

$P(C)$

```
ham : 0.66
spam: 0.33
```

$P(W|\text{spam})$

```
the     : 0.016
to      : 0.015
and     : 0.012
...
free    : 0.001
click   : 0.001
...
morally : 0.001
nicely  : 0.001
...
```

$P(W|\text{ham})$

```
the     : 0.021
to      : 0.013
and     : 0.011
...
free    : 0.005
click   : 0.004
...
screens : 0.000
minute  : 0.000
...
```

Estimation: Laplace Smoothing

- Laplace's estimate:
  - Pretend you saw every outcome once more than you actually did

H H T

$$P_{LAP}(x) = \frac{c(x) + 1}{\sum_x [c(x) + 1]}$$

$$= \frac{c(x) + 1}{N + |X|}$$

$$P_{ML}(X) =$$

$$P_{LAP}(X) =$$

  - Can derive this as a *maximum a posteriori* estimate using *Dirichlet priors* (see cs281a)

Estimation: Laplace Smoothing

- Laplace's estimate (extended):
  - Pretend you saw every outcome k extra times

H H T

$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

  - What's Laplace with k = 0?
  - k is the strength of the prior

$$P_{LAP,0}(X) =$$

$$P_{LAP,1}(X) =$$

$$P_{LAP,100}(X) =$$

- Laplace for conditionals:
  - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y) + k}{c(y) + k|X|}$$

Estimation: Linear Interpolation

- In practice, Laplace often performs poorly for P(X|Y):
  - When |X| is very large
  - When |Y| is very large
- Another option: linear interpolation
  - Get unconditional P(X) from the data
  - Make sure the estimate of P(X|Y) isn't too different from P(X)

$$P_{LIN}(x|y) = \alpha \hat{P}(x|y) + (1.0 - \alpha)\hat{P}(x)$$

  - What if α is 0? 1?
- For even better ways to estimate parameters, as well as details of the math see cs281a, cs294-5

Real NB: Smoothing

- For real classification problems, smoothing is critical
  - … and usually done badly, even in big commercial systems
- New odds ratios:

$$\frac{P(W|\text{ham})}{P(W|\text{spam})} \qquad \frac{P(W|\text{spam})}{P(W|\text{ham})}$$

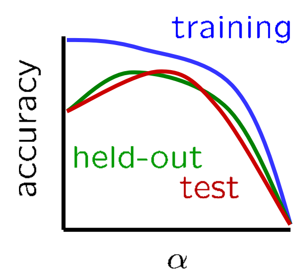
```
helvetica : 11.4
seems     : 10.8
group     : 10.2
ago       :  8.4
areas     :  8.3
...
```

```
verdana : 28.8
credit  : 28.4
order   : 27.2
<font>  : 26.9
money   : 26.5
...
```

Do these make more sense?

Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: the probabilities P(Y|X), P(Y)
  - Hyper-parameters, like the amount of smoothing to do: k, α
- Where to learn?
  - Learn parameters from training data
  - Must tune hyper-parameters on different data
    - Why?
  - For each value of the hyper-parameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data



Spam Example

Word	P(w|spam)	P(w|ham)	Tot Spam	Tot Ham
(prior)	0.33333	0.66666	-1.1	-0.4

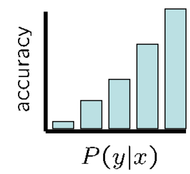
P(spam | w) = 0.989

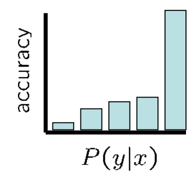
Confidences from a Classifier

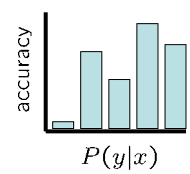
- The confidence of a probabilistic classifier:
  - Posterior over the top label

$$\text{confidence}(x) = \arg\max_y P(y|x)$$

  - Represents how sure the classifier is of the classification
  - Any probabilistic model will have confidences
  - No guarantee confidence is correct
- Calibration
  - Weak calibration: higher confidences mean higher accuracy
  - Strong calibration: confidence predicts accuracy rate
  - What's the value of calibration?

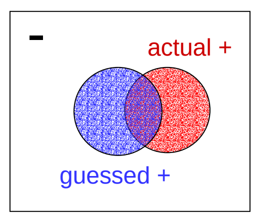






Precision vs. Recall

- Let's say we want to classify web pages as homepages or not
  - In a test set of 1K pages, there are 3 homepages
  - Our classifier says they are all non-homepages
  - 99.7 accuracy!
  - Need new measures for rare positive events

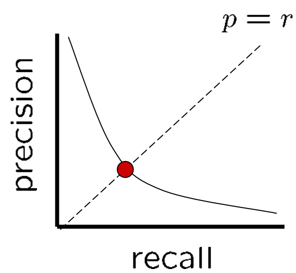


- Precision: fraction of guessed positives which were actually positive
- Recall: fraction of actual positives which were guessed as positive
- Say we guess 5 homepages, of which 2 were actually homepages
  - Precision: 2 correct / 5 guessed = 0.4
  - Recall: 2 correct / 3 true = 0.67
- Which is more important in customer support email automation?
- Which is more important in airport face recognition?

Precision vs. Recall

- Precision/recall tradeoff
  - Often, you can trade off precision and recall
  - Only works well with weakly calibrated classifiers
- To summarize the tradeoff:
  - Break-even point: precision value when p = r
  - F-measure: harmonic mean of p and r:

$$F_1 = \frac{2}{1/p + 1/r}$$



Errors, and What to Do

- Examples of errors

```
Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the
latest version of OmniPage Pro, for just $99.99* - the
regular list price is $499! The most common question we've
received about this offer is - Is this genuine? We would like
to assure you that this offer is authorized by ScanSoft, is
genuine and valid. You can get the . . .
```

```
. . . To receive your $30 Amazon.com promotional certificate,
click through to

  http://www.amazon.com/apparel

and see the prominent link for the $30 offer. All details are
there. We hope you enjoyed receiving this message. However,
if you'd rather not receive future e-mails announcing new
store launches, please click . . .
```

What to Do About Errors?

- Need more features– words aren't enough!
  - Have you emailed the sender before?
  - Have 1K other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?
- Naïve Bayes models can incorporate a variety of features, but tend to do best in homogeneous cases (e.g. all features are word occurrences)

Features

- A feature is a function which signals a property of the input
- Examples:
  - ALL_CAPS: value is 1 iff email in all caps
  - HAS_URL: value is 1 iff email has a URL
  - NUM_URLS: number of URLs in email
  - VERY_LONG: 1 iff email is longer than 1K
  - SUSPICIOUS_SENDER: 1 iff reply-to domain doesn't match originating server
- Features are anything you can think of code to evaluate on an input
  - Some cheap, some very very expensive to calculate
  - Can even be the output of another classifier
  - Domain knowledge goes here!
- In naïve Bayes, how did we encode features?

Feature Extractors

- A feature extractor maps inputs to feature vectors

```
Dear Sir.

First, I must
solicit your
confidence in
this
transaction,
this is by
virtue of its
nature as being
utterly
confidencial and
top secret. …
```

```
W=dear     :  1
W=sir      :  1
W=this     :  2
...
W=wish     :  0
...
MISSPELLED :  2
NAMELESS   :  1
ALL_CAPS   :  0
NUM_URLS   :  0
...
```

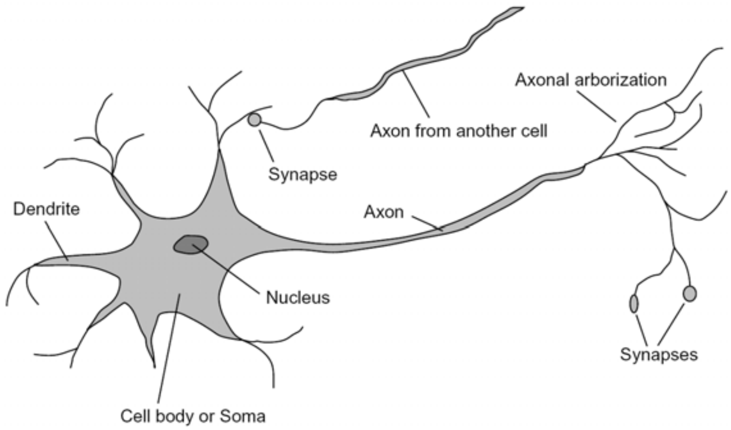
- Many classifiers take feature vectors as inputs
- Feature vectors usually very sparse, use sparse encodings (i.e. only represent non-zero keys)

Generative vs. Discriminative

- Generative classifiers:
  - E.g. naïve Bayes
  - We build a causal model of the variables
  - We then query that model for causes, given evidence
- Discriminative classifiers:
  - E.g. perceptron (next)
  - No causal model, no Bayes rule, often no probabilities
  - Try to predict output directly
  - Loosely: mistake driven rather than model driven

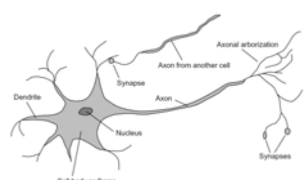
Some (Vague) Biology

- Very loose inspiration: human neurons



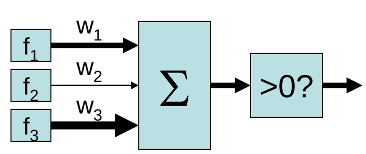
The Binary Perceptron

- Inputs are features
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x)$$

- If the activation is:
  - Positive, output 1
  - Negative, output 0



Example: Spam

- Imagine 4 features:
  - Free (number of occurrences of “free”)
  - Money (occurrences of “money”)
  - BIAS (always has value 1)

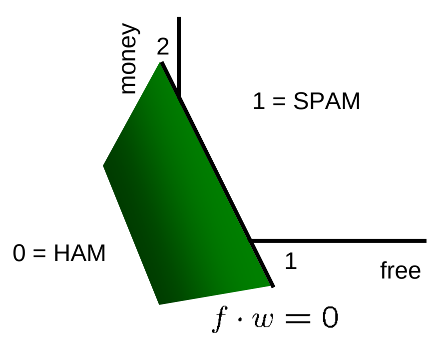
x	$f(x)$	w	$\sum_i w_i \cdot f_i(x)$
“free money”	BIAS : 1 free : 1 money : 1 the : 0 ...	BIAS : -3 free : 4 money : 2 the : 0 ...	$(1)(-3)\ +$ $(1)(4)\ +$ $(1)(2)\ +$ $(0)(0)\ +$ ... $= 3$

Binary Decision Rule

- In the space of feature vectors
  - Any weight vector is a hyperplane
  - One side will be class 1
  - Other will be class 0

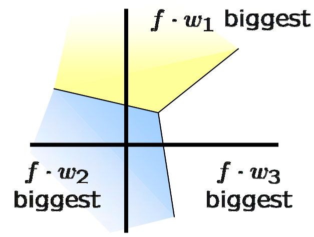
w

```
BIAS  : -3
free  :  4
money :  2
the   :  0
...
```



The Multiclass Perceptron

- If we have more than two classes:
  - Have a weight vector for each class
  - Calculate an activation for each class



$$\text{activation}_w(x, c) = \sum_i w_{c,i} \cdot f_i(x)$$

  - Highest activation wins

$$c = \arg\max_c \ (\text{activation}_w(x, c))$$

Example

"win the vote"

BIAS	:	1
win	:	1
game	:	0
vote	:	1
the	:	1
...		

w_{SPORTS}

BIAS	:	-2
win	:	4
game	:	4
vote	:	0
the	:	0
...		

$w_{POLITICS}$

BIAS	:	1
win	:	2
game	:	0
vote	:	4
the	:	0
...		

w_{TECH}

BIAS	:	2
win	:	0
game	:	2
vote	:	0
the	:	0
...		

The Perceptron Update Rule

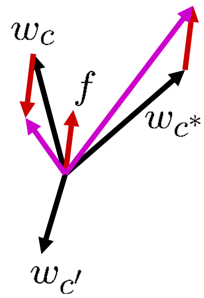
- Start with zero weights
- Pick up training instances one by one
- Try to classify

$$c = \arg\max_c \; w_c \cdot f(x)$$
$$= \arg\max_c \; \textstyle\sum_i w_{c,i} \cdot f_i(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_c = w_c - f(x)$$

$$w_{c^*} = w_{c^*} + f(x)$$



Example

"win the vote"

"win the election"

"win the game"

w_{SPORTS}

```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

$w_{POLITICS}$

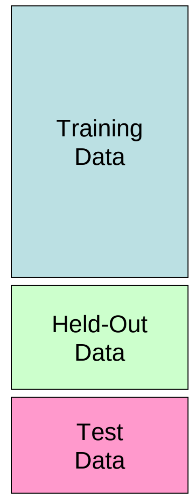
```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

w_{TECH}

```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

Mistake-Driven Classification

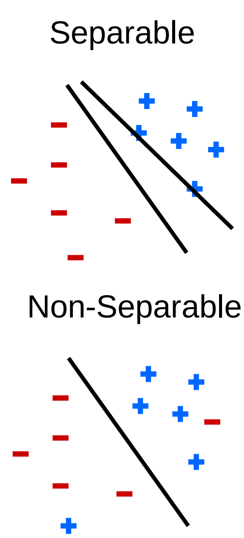
- In naïve Bayes, parameters:
  - From data statistics
  - Have a causal interpretation
  - One pass through the data
- For the perceptron parameters:
  - From reactions to mistakes
  - Have a discriminative interpretation
  - Go through the data until held-out accuracy maxes out



Properties of Perceptrons

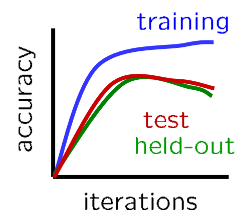
- Separability: some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{1}{\delta^2}$$

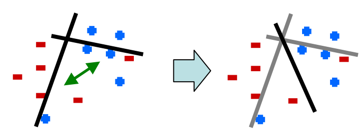


Issues with Perceptrons

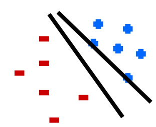
- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining isn't quite as bad as overfitting, but is similar



- Regularization: if the data isn't separable, weights might thrash around
  - Averaging weight vectors over time can help (averaged perceptron)



- Mediocre generalization: finds a "barely" separating solution



Summary

- Naïve Bayes
  - Build classifiers using model of training data
  - Smoothing estimates is important in real systems
  - Classifier confidences are useful, when you can get them
- Perceptrons:
  - Make less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data