CS 188: Artificial Intelligence
Spring 2006

Lecture 21: MDPs

4/6/2006

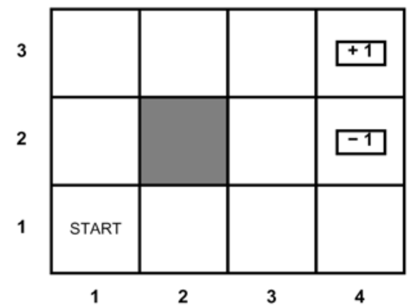
Dan Klein – UC Berkeley

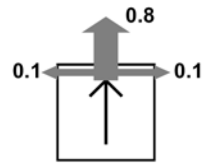
Reinforcement Learning

- [Demos]
- Basic idea:
  - Receive feedback in the form of rewards
  - Must learn to act so as to maximize expected rewards
  - Agent's utility is defined by the reward function
  - Change the rewards, change the behavior!
- Examples:
  - Playing a game, reward at the end for winning / losing
  - Vacuuming a house, reward for each piece of dirt picked up
  - Automated taxi, reward for each passenger delivered

Markov Decision Processes

- Markov decision processes (MDPs)
  - A set of states $s \in S$
  - A model $T(s,a,s') = P(s' \mid s,a)$
    - Probability that action a in state s leads to s'
  - A reward function $R(s)$ (or $R(s,a,s')$)
- MDPs are the simplest case of reinforcement learning
  - In general reinforcement learning, we don't know the model or the reward function

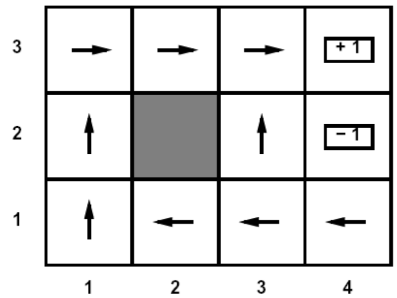




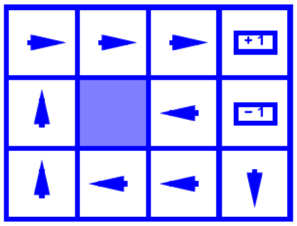
MDP Solutions

- In state-space search, want an optimal sequence of actions from start to a goal
- In an MDP, want an optimal policy $\pi(s)$
  - A policy gives an action for each state
  - Optimal policy is the one which maximizes expected utility (i.e. expected rewards) if followed
  - Gives a reflex agent!

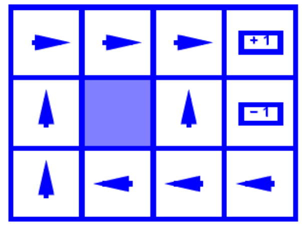
Optimal policy when $R(s) = -0.04$:



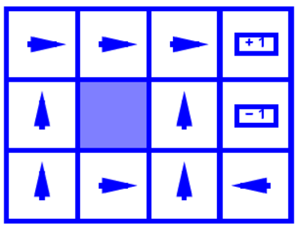
Example Optimal Policies



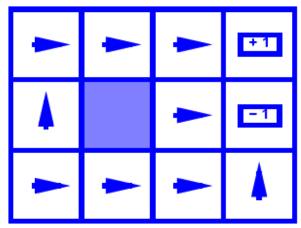
R(s) = -0.01



R(s) = -0.03



R(s) = -0.4



R(s) = -2.0

Stationarity

- In order to formalize optimality of a policy, need to understand utilities of reward sequences
- Typically consider stationary preferences:

$$[r, r_0, r_1, r_2, \ldots] \succ [r, r'_0, r'_1, r'_2, \ldots]$$
$$\Leftrightarrow$$
$$[r_0, r_1, r_2, \ldots] \succ [r'_0, r'_1, r'_2, \ldots]$$

- Theorem: only two ways to define stationary utilities
  - Additive utility:

$$U([s_0, s_1, s_2, \ldots]) = R(s_0) + R(s_1) + R(s_2) + \cdots$$

  - Discounted utility:

$$U([s_0, s_1, s_2, \ldots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) \cdots$$

How (Not) to Solve an MDP

- The inefficient way:
  - Enumerate policies
  - Calculate the expected utility (discounted rewards) starting from the start state
    - E.g. by simulating a bunch of runs
  - Choose the best policy

- We'll return to a (better) idea like this later

Utilities of States

- Idea: calculate the utility (value) of each state

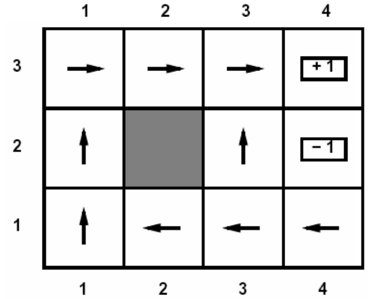
  U(s) = expected (discounted) sum of rewards assuming optimal actions

- Given the utilities of states, MEU tells us the optimal policy

$$\pi^U(s) = \arg\max_a E_{P(s'|a,s)} U(s')$$

$$= \arg\max_a U(s') T(s,a,s')$$

3	0.812	0.868	0.912	+1
2	0.762		0.660	− 1
1	0.705	0.655	0.611	0.388
	1	2	3	4



Infinite Utilities?!

- Problem: infinite state sequences with infinite rewards
- Solutions:
  - Finite horizon:
    - Terminate after a fixed T steps
    - Gives nonstationary policy (π depends on time left)
  - Absorbing state(s): guarantee that for every policy, agent will eventually “die”
  - Discounting: for $0 < \gamma < 1$

$$U([s_0, \ldots s_\infty]) = \sum_{t=0}^{\infty} \gamma^t R(s_t) \leq R_{\max}/(1-\gamma)$$

    - Smaller γ means smaller horizon

The Bellman Equation

- Definition of state utility leads to a simple relationship amongst utility values:

  Expected rewards = current reward +
  γ x expected sum of rewards after taking best action

- Formally:

$$\begin{aligned} U(s) &= R(s) + \gamma \max_a E_{P(s'|a,s)} U(s') \\ &= R(s) + \gamma \max_a \sum_{s'} U(s') T(s,a,s') \\ &= R(s) + \gamma \sum_{s'} U(s') T(s, \pi^U(a), s') \end{aligned}$$

Example: Bellman Equations

3	0.812	0.868	0.912	+ 1
2	0.762		0.660	– 1
1	0.705	0.655	0.611	0.388
	1	2	3	4

$$
\begin{aligned}
U(1,1) = -0.04 & & \\
\quad + \gamma \max\{ & 0.8U(1,2)+0.1U(2,1)+0.1U(1,1), & \textit{up} \\
& 0.9U(1,1)+0.1U(1,2) & \textit{left} \\
& 0.9U(1,1)+0.1U(2,1) & \textit{down} \\
& 0.8U(2,1)+0.1U(1,2)+0.1U(1,1)\} & \textit{right}
\end{aligned}
$$

Value Iteration

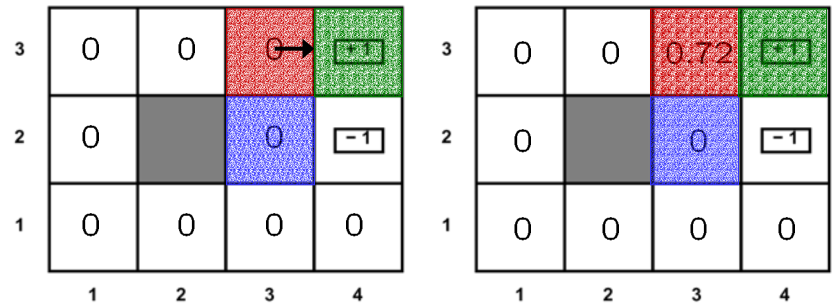
- Idea:
  - Start with bad guesses at utility values (e.g. $U_0(s) = 0$)
  - Update using the Bellman equation (called a value update or Bellman update):

$$
\begin{aligned}
U_{i+1}(s) &= R(s) + \gamma \max_a E_{P(s'|a,s)} U_i(s') \\
&= R(s) + \gamma \max_a \sum_{s'} U_i(s') T(s,a,s')
\end{aligned}
$$

  - Repeat until convergence

- Theorem: will converge to unique optimal values
  - Basic idea: bad guesses get refined towards optimal values
  - Policy may converge before values do

Example: Bellman Updates

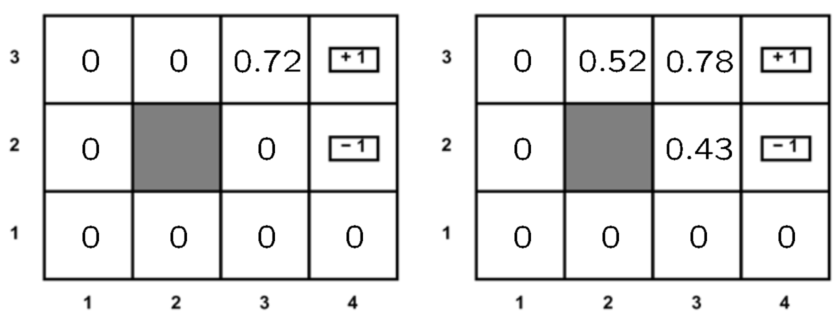


$$U_{i+1}(s) = R(s) + \gamma \max_a \sum_{s'} U_i(s')T(s,a,s')$$

$$= 0 + 0.9 \sum_{s'} U_i(s')T(\langle 3,3\rangle, \text{right}, s')$$

$$= 0 + 0.9\,[0.8 \cdot 1 + 0.1 \cdot 0 + 0.1 \cdot 0]$$

Example: Value Iteration



- Information propagates outward from terminal states and eventually all states have correct value estimates
- [DEMO]

Convergence*

- Define the max-norm: $||U|| = \max_s |U(s)|$
- Theorem: For any two approximations U and V

$$||U^{t+1} - V^{t+1}|| \leq \gamma \, ||U^t - V^t||$$

  - I.e. any distinct approximations must get closer to each other, so, in particular, any approximation must get closer to the true U and value iteration converges to a unique, stable, optimal solution
- Theorem:

$$||U^{t+1} - U^t|| < \epsilon, \Rightarrow ||U^{t+1} - U|| < 2\epsilon\gamma/(1-\gamma)$$

  - I.e. one the change in our approximation is small, it must also be close to correct

Policy Iteration

- Alternate approach:
  - Policy evaluation: calculate utilities for a fixed policy
  - Policy improvement: update policy based on resulting utilities
  - Repeat until convergence
- This is policy iteration
  - Can converge faster under some conditions

Policy Evaluation

- If we have a fixed policy π, use simplified Bellman equation to calculate utilities:

$$U_{i+1}^{\pi}(s) = R(s) + \gamma \sum_{s'} U_i(s')T(s, \pi(s), s')$$

Policy Improvement

- For fixed utilities, easy to find the best action according to one-step lookahead

$$\pi_{i+1}^{U}(s) = \arg\max_{a} \sum_{s'} U(s')T(s, a, s')$$

Comparison

- In value iteration:
  - Every pass (or “backup”) updates both policy (based on current utilities) and utilities (based on current policy
- In policy iteration:
  - Several passes to update utilities
  - Occasional passes to update policies
- Hybrid approaches (asynchronous policy iteration):
  - Any sequences of partial updates to either policy entries or utilities will converge if every state is visited infinitely often

Next Class

- In real reinforcement learning:
  - Don’t know the reward function R(s)
  - Don’t know the model T(s,a,s’)
  - So can’t do Bellman updates!
- Need new techniques:
  - Q-learning
  - Model learning
  - Agents actually have to interact with the environment rather than simulate it!