CS 188: Artificial Intelligence
Spring 2006

Lecture 22: Reinforcement Learning
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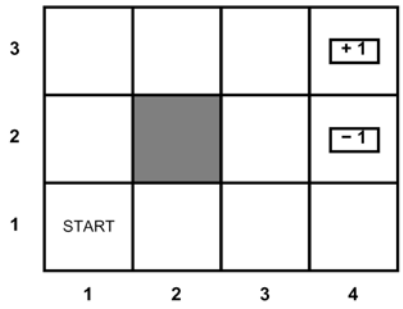
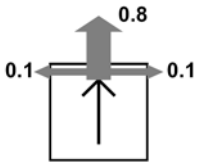
Today

- More MDPs: policy iteration
- Reinforcement learning
  - Passive learning
  - Active learning

Recap: MDPs

- Markov decision processes (MDPs)
  - A set of states s ∈ S
  - A model T(s,a,s')
    - Probability that the outcome of action a in state s is s'
  - A reward function R(s)
- Solutions to an MDP
  - A policy π(s)
  - Specifies an action for each state
  - We want to find a policy which maximizes total expected utility = expected (discounted) rewards





Bellman Equations

- The value of a state according to π

$$U^{\pi}(s) = R(s) + \gamma \sum_{s'} U^{\pi}(s')T(s, \pi(s), s')$$

- The policy according to a value U

$$\pi^{U}(s) = \arg\max_{a} \sum_{s'} U(s')T(s, a, s')$$

- The optimal value of a state

$$U^{*}(s) = R(s) + \gamma \max_{a} \sum_{s'} U^{*}(s')T(s, a, s')$$

Recap: Value Iteration

- Idea:
  - Start with (bad) value estimates (e.g. $U_0(s) = 0$)
  - Start with corresponding (bad) policy $\pi_0(s)$
  - Update values using the Bellman relations (once)

$$U_{i+1}(s) = R(s) + \gamma \sum_{s'} U_i(s') T(s, \pi_i(a), s')$$

  - Update policy based on new values

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} U_{i+1}(s') T(s, a, s')$$

  - Repeat until convergence

Policy Iteration

- Alternate approach:
  - Policy evaluation: calculate exact utility values for a fixed policy
  - Policy improvement: update policy based on values
  - Repeat until convergence
- This is policy iteration
  - Can converge faster under some conditions

Policy Evaluation

- If we have a fixed policy π, use a simplified Bellman update to calculate utilities:

$$U^{\pi}(s) = R(s) + \gamma \sum_{s'} U^{\pi}(s') T(s, \pi(s), s')$$

- Unlike in value iteration, policy does not change during update process
- Converges to the expected utility values for this π
- Can also solve for U with linear algebra methods instead of iteration

Policy Improvement

- Once values are correct for current policy, update the policy

$$\pi_{i+1}(s) = \arg\max_{a} \sum_{s'} U(s') T(s, a, s')$$

- Note:
  - Value iteration: update U, π, U, π U, π…
  - Policy iteration: U, U, U, U… π, U, U, U, U… π
  - Otherwise, basically the same!

Reinforcement Learning

- Reinforcement learning:
  - Still have an MDP:
    - A set of states $s \in S$
    - A model T(s,a,s')
    - A reward function R(s)
  - Still looking for a policy $\pi(s)$
  - New twist: don't know T or R
    - I.e. don't know which states are good or what the actions do
    - Must actually try actions and states out to learn

Example: Animal Learning

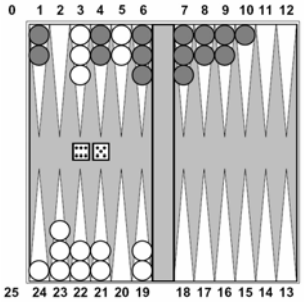
- RL studied experimentally for more than 60 years in psychology
  - Rewards: food, pain, hunger, drugs, etc.
  - Mechanisms and sophistication debated
- Example: foraging
  - Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
  - Bees have a direct neural connection from nectar intake measurement to motor planning area

Example: Autonomous Helicopter

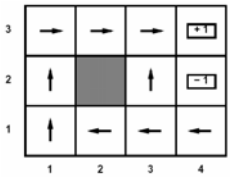
Example: Autonomous Helicopter

Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to U(s) using a neural network
- Combined with depth 3 search, one of the top 3 players in the world
- (We'll cover game playing in a few weeks)



Passive Learning

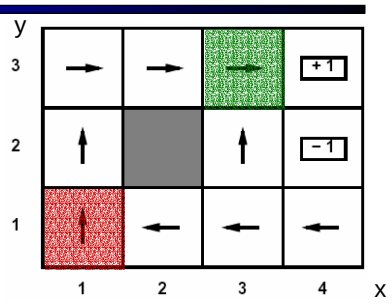


- Simplified task
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s)
  - You DO know the policy $\pi(s)$
  - Goal: learn the state values (and maybe the model)
- In this case:
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - We'll get to the general case soon

Example: Direct Estimation

- Episodes:

(1,1) -1 up	(1,1) -1 up
(1,2) -1 up	(1,2) -1 up
(1,2) -1 up	(1,3) -1 right
(1,3) -1 right	(2,3) -1 right
(2,3) -1 right	(3,3) -1 right
(3,3) -1 right	(3,2) -1 up
(3,2) -1 up	(4,2) -100
(3,3) -1 right	
(4,3) +100	



U(1,1) ~ (92 + -106) / 2 = -7

U(3,3) ~ (99 + 97 + -102) / 3 = -31.3

Model-Based Learning

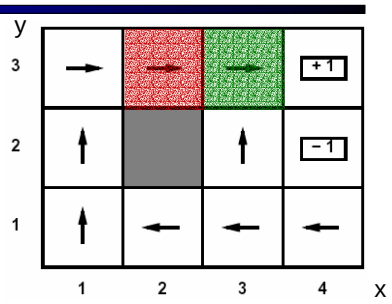
- Idea:
  - Learn the model empirically (rather than values)
  - Solve the MDP as if the learned model were correct
- Empirical model learning
  - Simplest case:
    - Count outcomes for each s,a
    - Normalize to give estimate of T(s,a,s')
    - Discover R(s) the first time we enter s
  - More complex learners are possible (e.g. if we know that all squares have related action outcomes "stationary noise")

Example: Model-Based Learning

- Episodes:

(1,1) -1 up
(1,2) -1 up
(1,2) -1 up
(1,3) -1 right
(2,3) -1 right
(3,3) -1 right
(3,2) -1 up
(3,3) -1 right
(4,3) +100

(1,1) -1 up
(1,2) -1 up
(1,3) -1 right
(2,3) -1 right
(3,3) -1 right
(3,2) -1 up
(4,2) -100



T(<3,3>, right, <4,3>) = 1 / 3

T(<2,3>, right, <3,3>) = 2 / 2

R(3,3) = -1, R(4,1) = 0?

Model-Free Learning

- Big idea: why bother learning T?
  - Update each time we experience a transition
  - Frequent outcomes will contribute more updates (over time)
- Temporal difference learning (TD)
  - Policy still fixed!
  - Move values toward value of whatever successor occurs

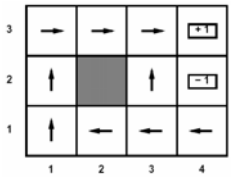
$$U^{\pi}(s) = R(s) + \gamma \sum_{s'} U^{\pi}(s') T(s, \pi(s), s')$$

$$U^{\pi}(s) \leftarrow U^{\pi}(s) + \alpha \left(R(s) + \gamma U^{\pi}(s') - U^{\pi}(s) \right)$$

- [DEMO]

Example: Passive TD

$$U^{\pi}(s) \leftarrow U^{\pi}(s) + \alpha\left(R(s) + \gamma U^{\pi}(s') - U^{\pi}(s)\right)$$



(1,1) -1 up
(1,2) -1 up
(1,2) -1 up
(1,3) -1 right
(2,3) -1 right
(3,3) -1 right
(3,2) -1 up
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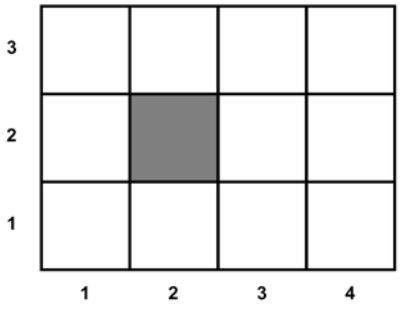


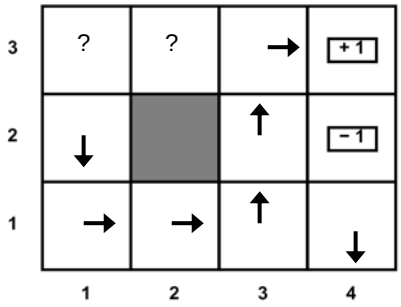
Take $\gamma = 1$, $\alpha = 0.1$

(Greedy) Active Learning

- In general, want to learn the optimal policy
- Idea:
  - Learn an initial model of the environment:
  - Solve for the optimal policy for this model (value or policy iteration)
  - Refine model through experience and repeat

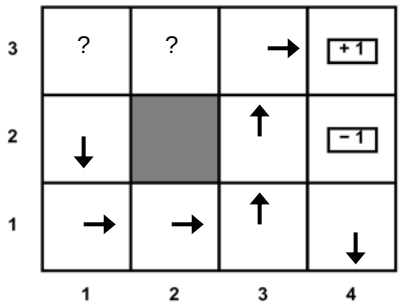
Example: Greedy Active Learning

- Imagine we find the lower path to the good exit first
- Some states will never be visited following this policy from (1,1)
- We'll keep re-using this policy because following it never collects the regions of the model we need to learn the optimal policy



What Went Wrong?

- Problem with following optimal policy for current model:
  - Never learn about better regions of the space
- Fundamental tradeoff: exploration vs. exploitation
  - Exploration: must take actions with suboptimal estimates to discover new rewards and increase eventual utility
  - Exploitation: once the true optimal policy is learned, exploration reduces utility
  - Systems must explore in the beginning and exploit in the limit



Next Time

- Active reinforcement learning
  - Q-learning
  - Balancing exploration / exploitation
- Function approximation
  - Generalization for reinforcement learning
  - Modeling utilities for complex spaces