CS 188: Artificial Intelligence
Spring 2006

Lecture 4: A* Search (and Friends)
1/26/2006

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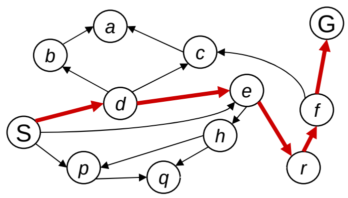
Many slides from either Stuart Russell or Andrew Moore

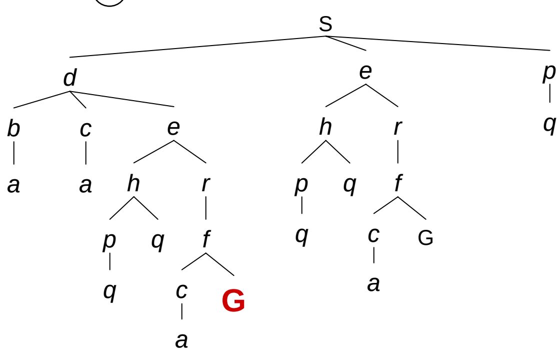
Today

- A* Search
- Heuristic Design
- Local Search

Problem Graphs vs Search Trees



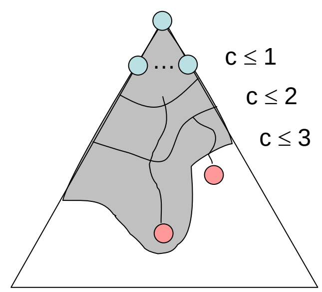
Each NODE in in the search tree is an entire PATH in the problem graph.

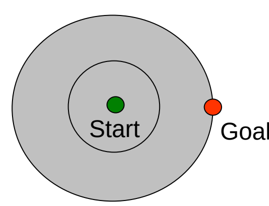


We almost always construct both on demand – and we construct as little as possible.

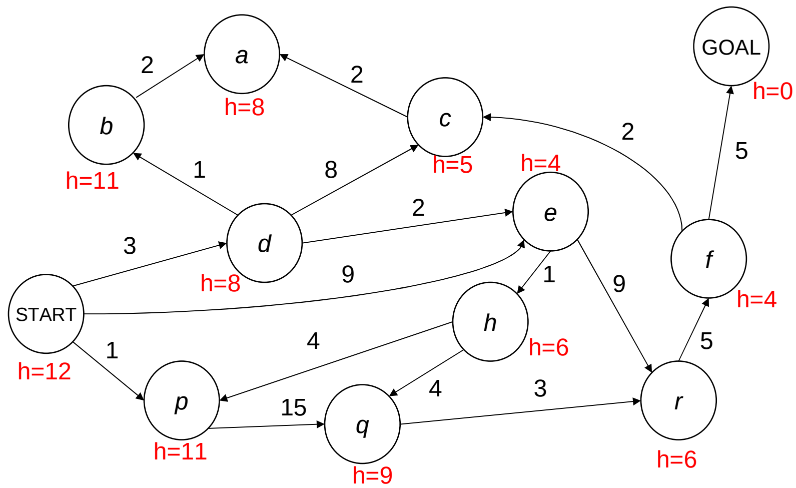
Uniform Cost Problems

- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location



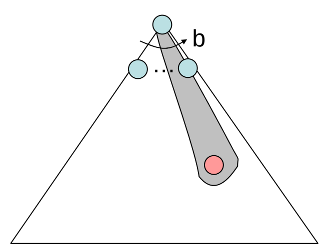


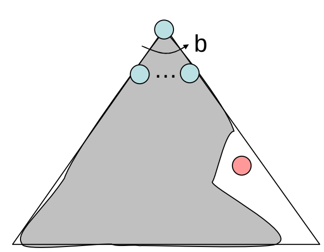
Best-First Search



Best-First Search

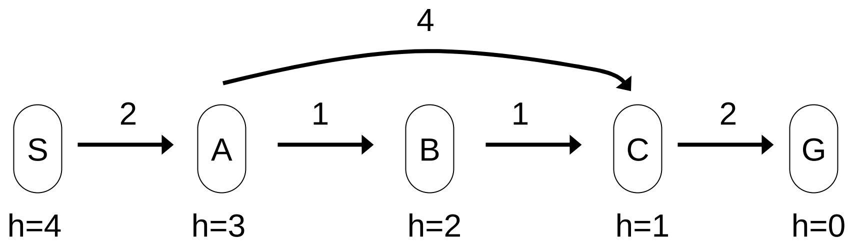
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS in the worst case
  - Can explore everything
  - Can get stuck in loops if no cycle checking
- Like DFS in completeness (finite states w/ cycle checking)





Combining Best-First and UCS

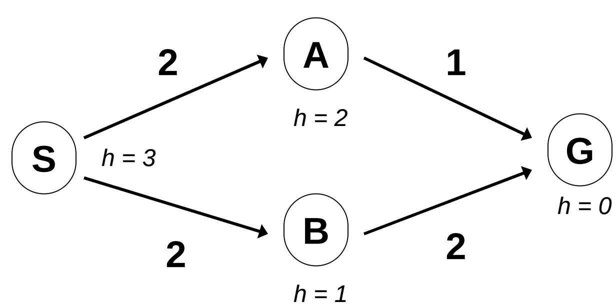
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Best-first orders by goal proximity, or *forward cost* h(n)



- What happens with each ordering?
- A* orders by the sum: f(n) = g(n) + h(n)

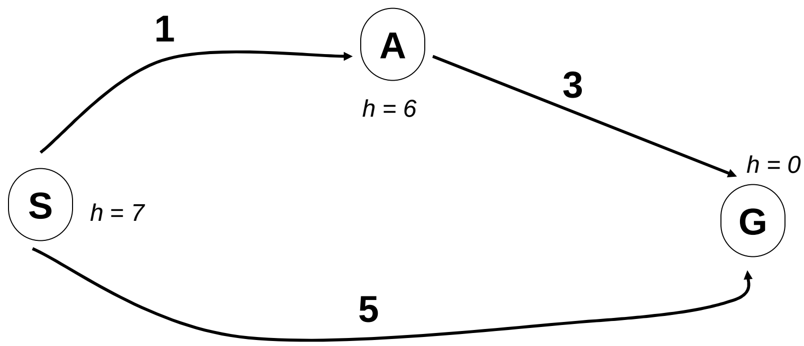
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost > estimated good goal cost
- We need estimates to be less than actual costs!

Admissible Heuristics

- A heuristic is *admissible* (optimistic) if:

$$h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

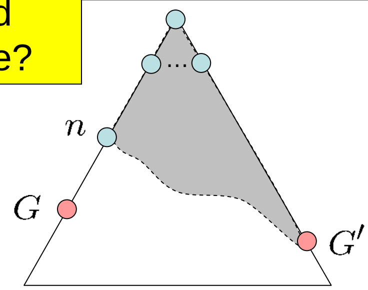
- E.g. Euclidean distance on a map problem

- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A*: Blocking

This proof assumed tree search! Where?

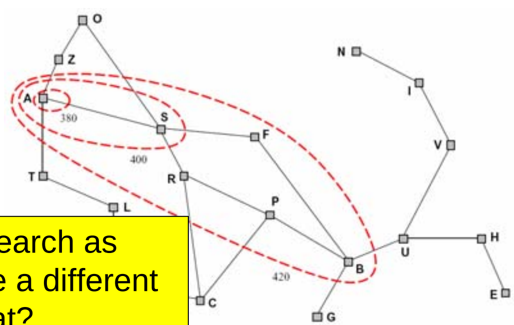
- Proof:
  - What can go wrong?
  - We'd have to have to pop a suboptimal goal off the fringe queue
  - Imagine a suboptimal goal G' is on the queue
  - Consider any unexpanded (fringe) node *n* on a shortest path to optimal G
  - *n* will be popped before G



$$f(n) \leq g(G)$$
$$g(G) < g(G')$$
$$g(G') = f(G')$$
$$f(n) < f(G')$$

Optimality of A*: Contours

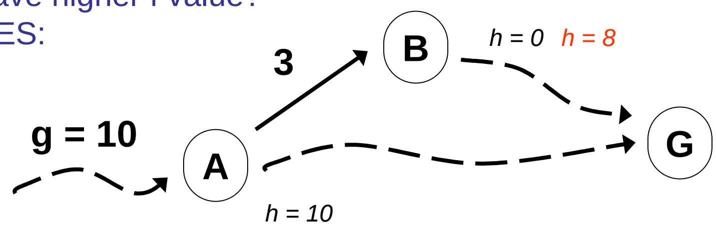
- Consider what A* does:
  - Expands nodes in increasing total f value (f-contours)
  - Optimal goals have lower f value, so get expanded first



Holds for graph search as well, but we made a different assumption. What?

Consistency

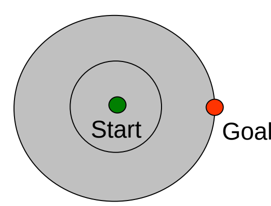
- Wait, how do we know we expand in increasing f value?
- Couldn't we pop some node *n*, and find its child *n'* to have higher f value?
- YES:



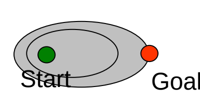
- What do we need to do to fix this?
- Consistency: $h(n) \leq c(n, a, n') + h(n')$
- Real cost always exceeds reduction in heuristic

UCS vs A* Contours

- Uniform-cost expanded in all directions



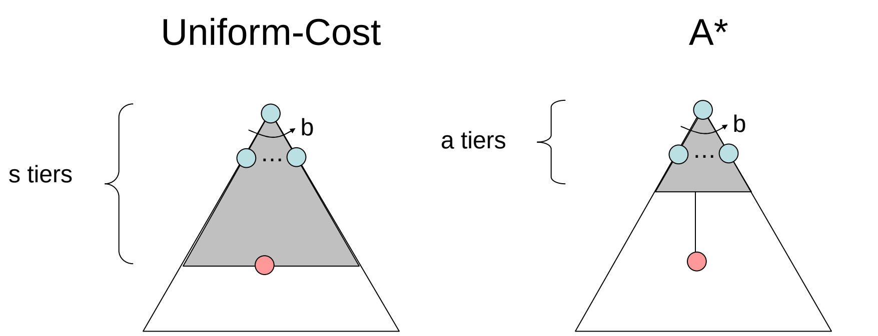
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



Properties of A*

Algorithm	Complete	Optimal	Time	Space
UCS = BFS*	Y	Y	$O(s\, b^s)$*	$O(b^s)$*
A*	Y	Y	$O(a\, b^a)$	$O(b^a)$

**Assume all costs are 1* *Assume one goal, non-goals have* $h(n) = g^*(G) - a$

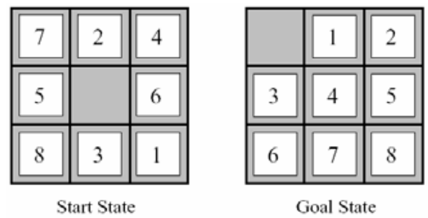


Admissible Heuristics

- Most of the work is in coming up with admissible heuristics
- Good news: usually admissible heuristics are also consistent
- More good news: inadmissible heuristics are often quite effective (especially when you have no choice)

8-Puzzle I

- Number of tiles misplaced?
- Why is it admissible?
- h(start) = 8
- This is a relaxed-problem heuristic



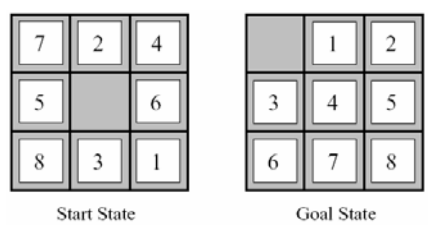
	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
ID	112	6,300	3.6×10^6
TILES	13	39	227

8-Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any one direction at any time?
- Total *Manhattan* distance
- Why admissible?
- h(start) =

  3 + 1 + 2 + …

  = 18



	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MAN-HATTAN	12	25	73

8-Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes?
  - What's wrong with it?

- With A*, trade-off between quality of estimate and work per node!

Trivial Heuristics, Dominance

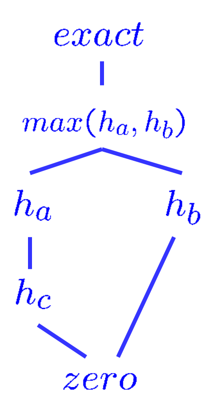
- Dominance:

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



Course Scheduling

- From the university's perspective:
  - Set of courses $\{c_1, c_2, \ldots c_n\}$
  - Set of room / times $\{r_1, r_2, \ldots r_n\}$
  - Each pairing (c_k, r_m) has a cost w_{km}
  - What's the best assignment of courses to rooms?
- States: list of pairings
- Actions: add a legal pairing
- Costs: cost of the new pairing

- Admissible heuristics?

- (Who can think of a cs170 answer to this problem?)

Other A* Applications

- Machine translation
- Statistical parsing
- Speech recognition
- Robot motion planning (next class)
- Routing problems (see homework!)
- Planning problems (see homework!)
- …

Summary: A*

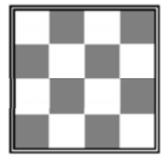
- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible heuristics
- Heuristic design is key: often use relaxed problems

Local Search Methods

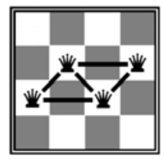
- Queue-based algorithms keep fallback options (backtracking)
- Local search: improve what you have until you can't make it better
- Generally much more efficient (but incomplete)

Types of Problems

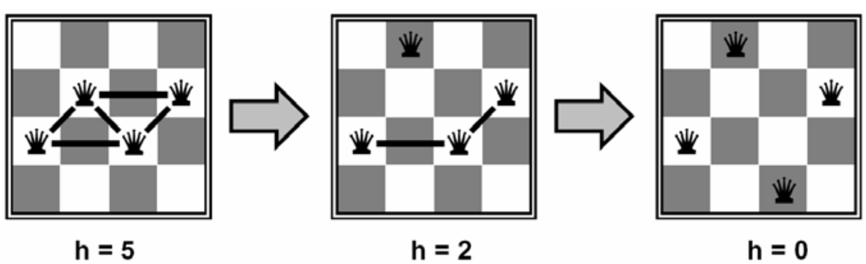
- Planning problems:
  - We want a path to a solution (examples?)
  - Usually want an optimal path
  - Incremental formulations



- Identification problems:
  - We actually just want to know what the goal is (examples?)
  - Usually want an optimal goal
  - Complete-state formulations
  - *Iterative improvement algorithms*



Example: N-Queens

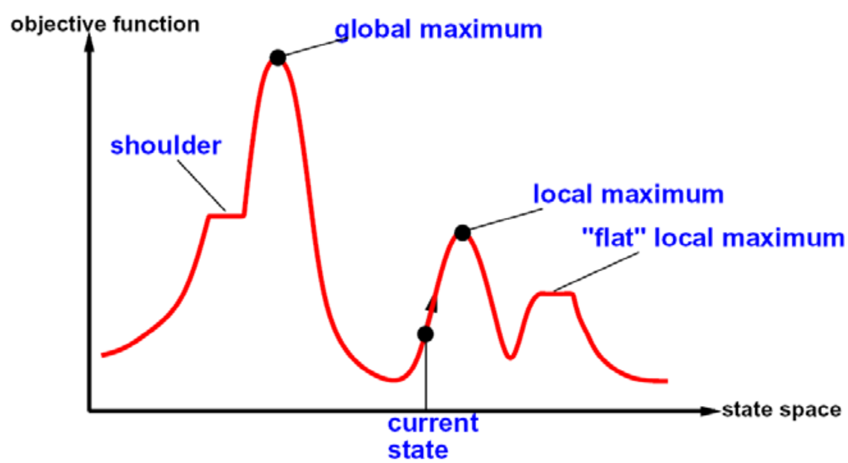


- Start wherever, move queens to reduce conflicts
- Almost always solves large n-queens nearly instantly
- How is this different from best-first search?

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

Hill Climbing Diagram



- Random restarts?
- Random sideways steps?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

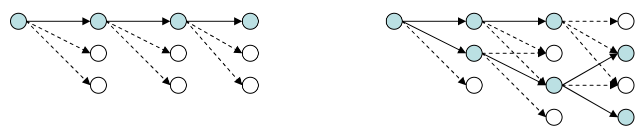
```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape, the less likely you are to every make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

Beam Search

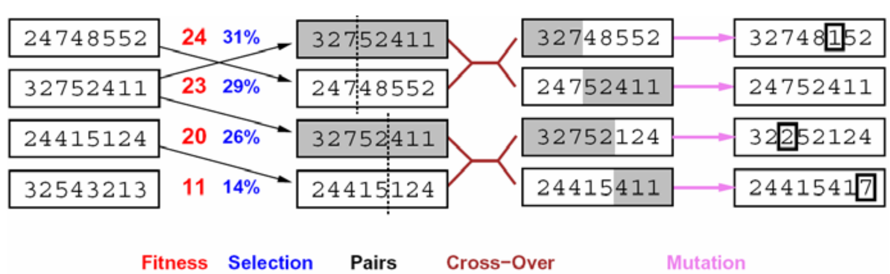
- Like greedy search, but keep K states at all times:



Greedy Search Beam Search

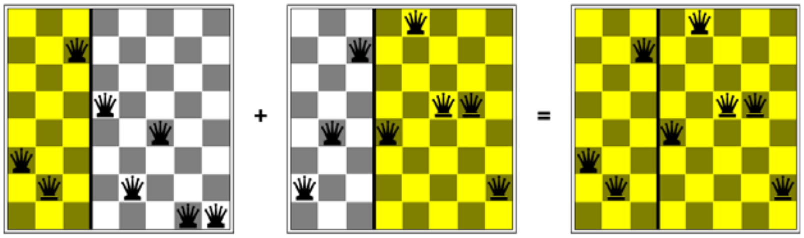
- Variables: beam size, encourage diversity?
- The best choice in MANY practical settings
- Complete? Optimal?
- Why do we still need optimal methods?

Genetic Algorithms



- Genetic algorithms use a natural selection metaphor
- Like beam search (selection), but also have pairwise crossover operators, with optional mutation
- Probably the most misunderstood, misapplied (and even maligned) technique around!

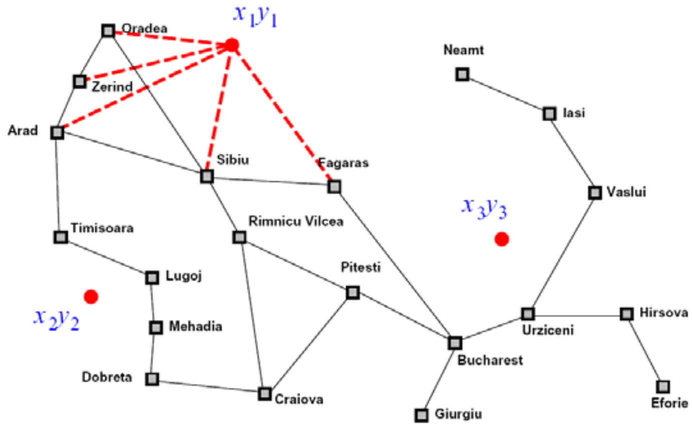
Example: N-Queens



- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

Continuous Problems

- Placing airports in Romania
  - States: $(x_1,y_1,x_2,y_2,x_3,y_3)$
  - Cost: sum of squared distances to closest city

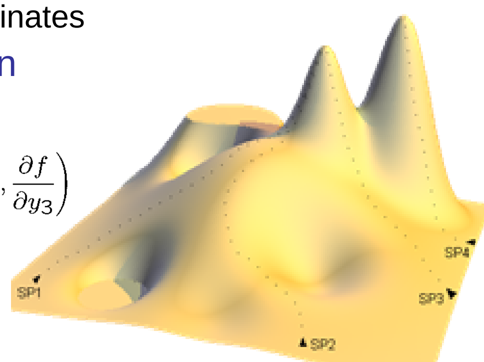


Gradient Methods

- How to deal with continous (therefore infinite) state spaces?
- Discretization: bucket ranges of values
  - E.g. force integral coordinates
- Continuous optimization
  - E.g. gradient ascent

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

$$x \leftarrow x + \alpha \nabla f(x)$$



  - More on this next class…

Image from vias.org