CS 188: Artificial Intelligence
Spring 2006

Lecture 5: Robot Motion Planning

1/31/2006

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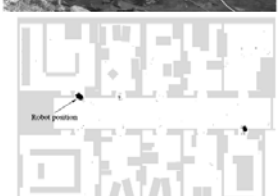
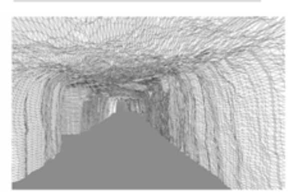
Many slides from either Stuart Russell or Andrew Moore

Robotics Tasks

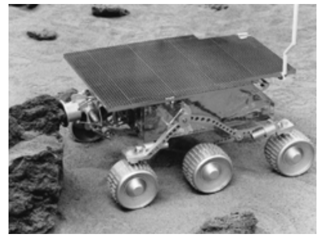
- Motion planning (today)
  - How to move from A to B
  - Known obstacles
  - Offline planning
- Localization (later)
  - Where exactly am I?
  - Known map
  - Ongoing localization (why?)
- Mapping (much later)
  - What's the world like?
  - Exploration / discovery
  - SLAM: simultaneous localization and mapping

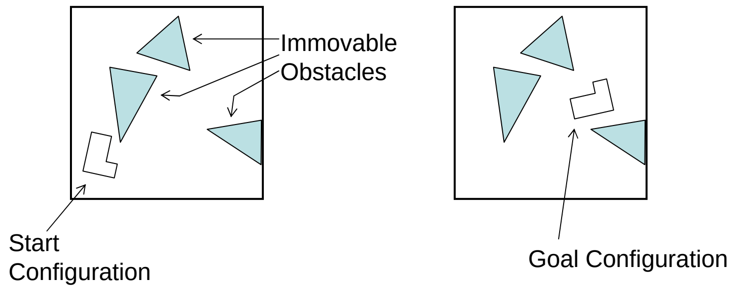




Mobile Robots

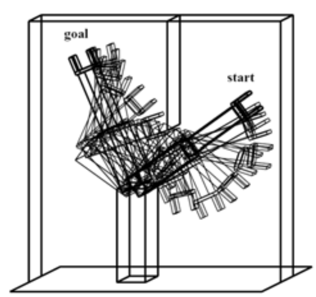
- High-level objectives: move robots around obstacles
- Low-level: fine motor control to achieve motion
- Why is this hard?





Manipulator Robots

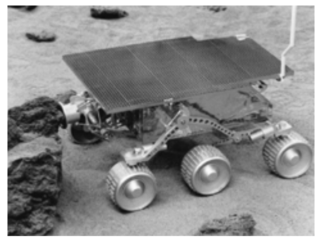
- High-level goals: reconfigure environment
- Low-level: move from configuration A to B (point-to-point motion)
  - Why is this already hard?
- Also: compliant motion



Sensors and Effectors

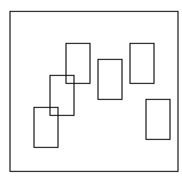
- Sensors vs. Percepts
  - Agent programs receive percepts
  - Agent bodies have sensors
  - Includes *proprioceptive* sensors
  - Real world: sensors break, give noisy answers, miscalibrate, etc.

- Effectors vs. Actuators
  - Agent programs have actuators (control lines)
  - Agent bodies have effectors (gears and motors)
  - Real-world: wheels slip, motors fail, etc.

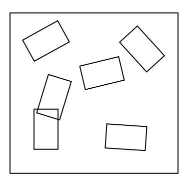


Degrees of Freedom

- The degrees of freedom are the numbers required to specify a robot's configuration
- Positional DOFs:
  - (x, y, z) of free-flying robot
  - direction robot is facing
- Effector DOFs
  - Arm angle
  - Wing position
- Static state: robot shape and position
- Dynamic state: derivatives of static DOFs (why have these?)



2 DOFs

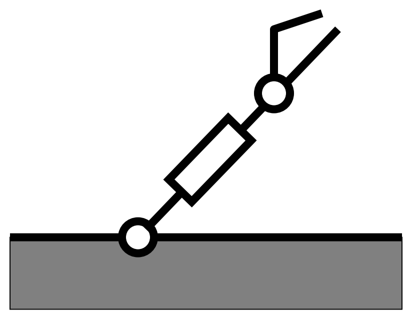


3 DOFs

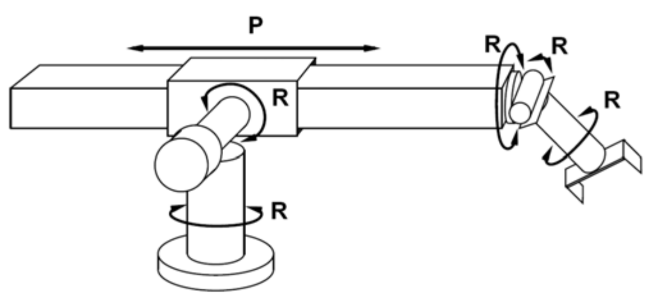
Question: How many DOFs for a polyhedron free-flying in 3D space?

Example

- How many DOFs?
  - What are the natural coordinates for specifying the robot's configuration?
  - These are the *configuration space* coordinates
  - What are the natural coordinates for specifying the effector tip's position?
  - These are the *work space* coordinates



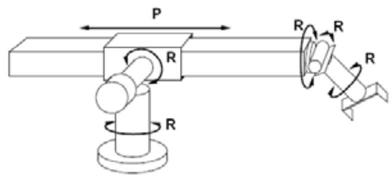
Example

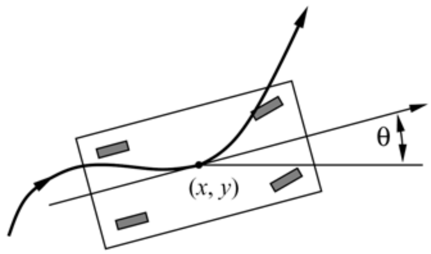


- How many DOFs?
  - How does this compare to your arm?
  - How many are required for arbitrary positioning of end-effector?

Holonomicity

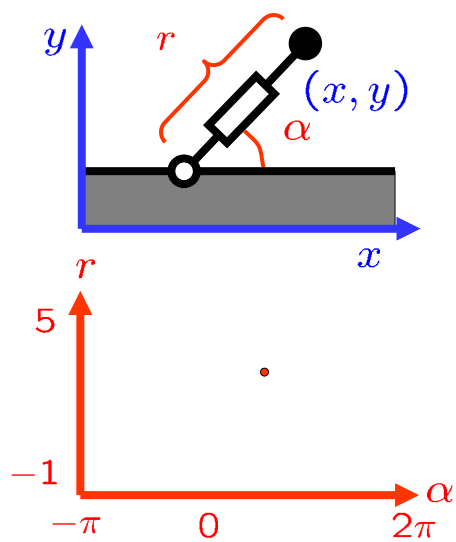
- *Holonomic robots* control all their DOFs (e.g. manipulator arms)
  - Easier to control
  - Harder to build
- *Non-holonomic* robots do not directly control all DOFs (e.g. a car)





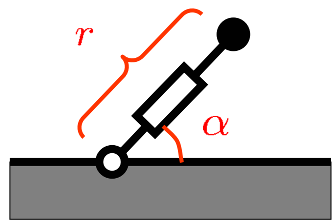
Configuration Space

- Workspace:
  - The world's (x, y) system
  - Obstacles specified here
- Configuration space
  - The robot's state
  - Planning happens here



Kinematics

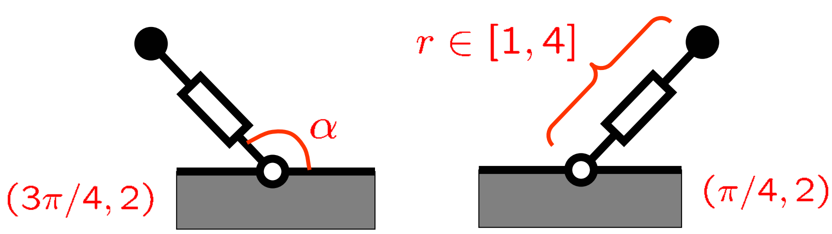
- Kinematics
  - The mapping from configurations to workspace coordinates
  - Generally involves some trigonometry
  - Usually pretty easy
- Inverse Kinematics
  - The inverse: effector positions to configurations
  - Usually non-unique (why?)



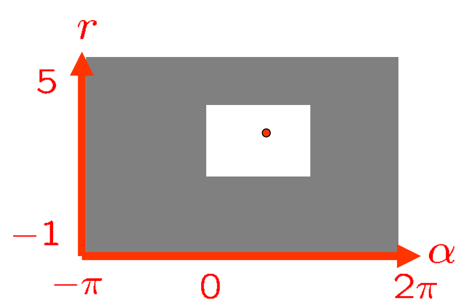
$$x = r\cos(\alpha)$$
$$y = r\sin(\alpha)$$

Forward kinematics

Configuration Space

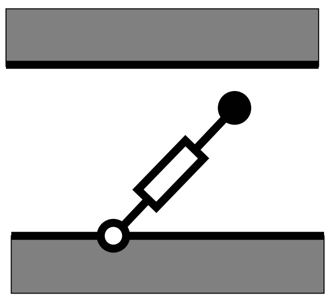


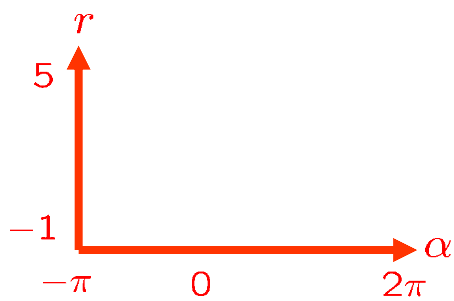
- Configuration space
  - Just a coordinate system
  - Not all points are reachable / legal
- Legal configurations:
  - No collisions
  - No self-intersection



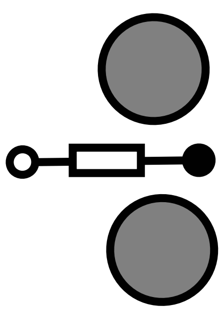
Obstacles in C-Space

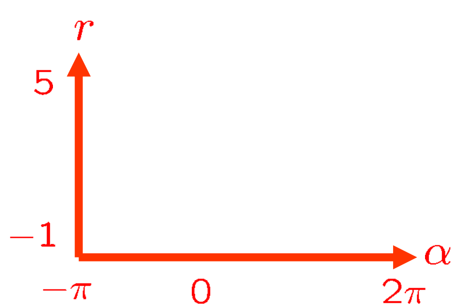
- What / where are the obstacles?
- Remaining space is *free space*





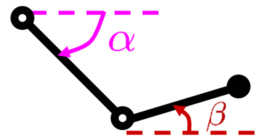
More Obstacles

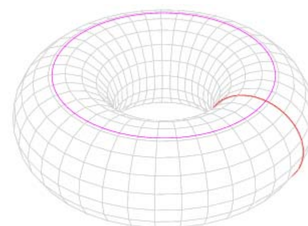




Topology

- You very quickly get into issues of topology:
  - Point robot in 3D: R^3
  - Directional robot with fixed position in 3D: SO(3)
  - Two rotational-jointed robot in 2D: $S_1 x S_1$
- For the present purposes, we'll basically ignore these issues
- In practice, you have to deal with it properly

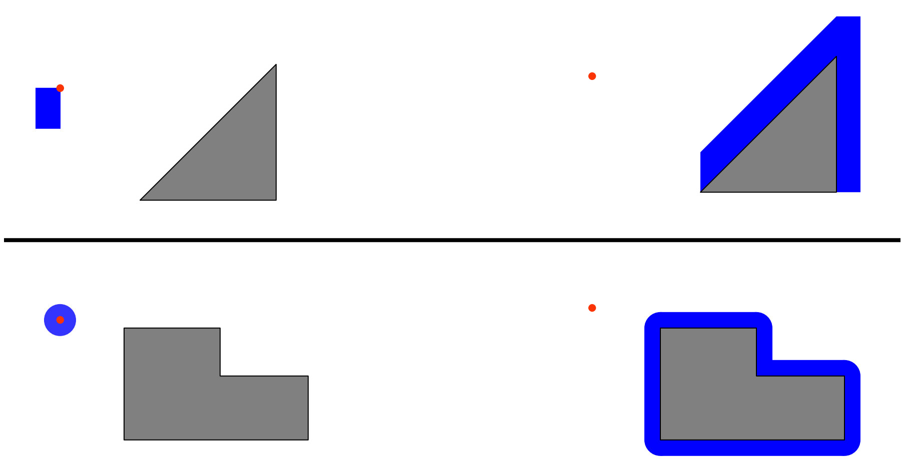




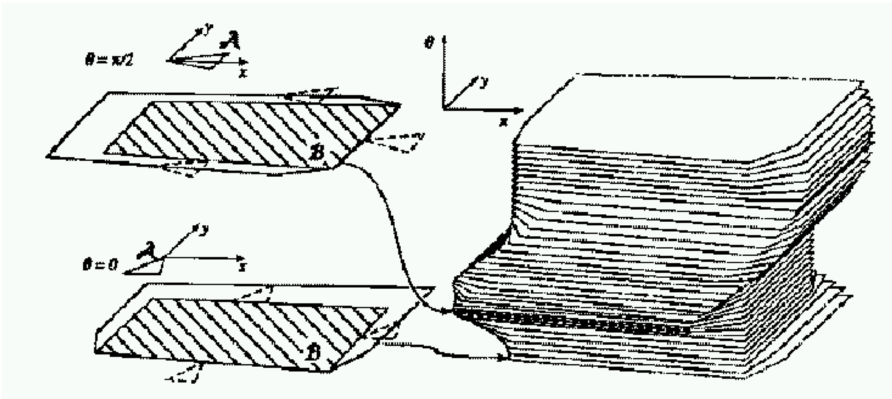
Example: 2D Polygons

Workspace

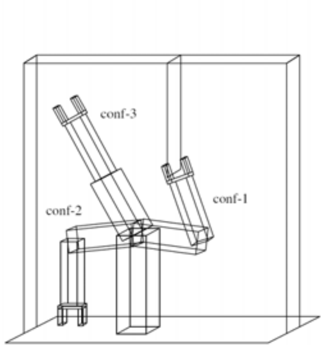
Configuration Space

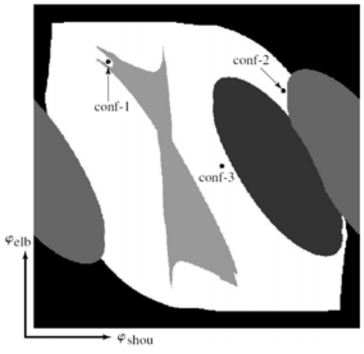


Example: Rotation



Example: A Less Simple Arm



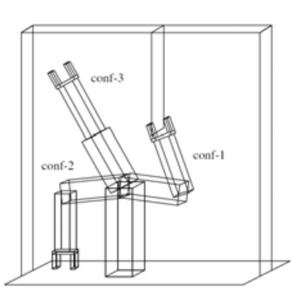


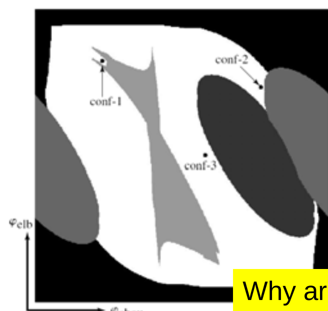
Summary

- Degrees of freedom
- Legal robot configurations form configuration space
- Obstacles have complex images in c-space

Motion as Search

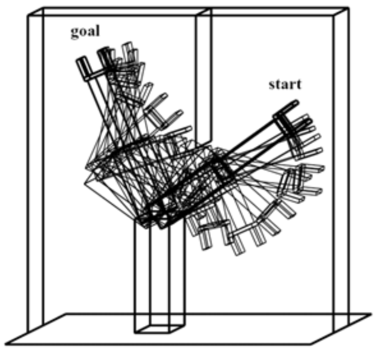
- Motion planning as path-finding problem
  - Problem: configuration space is continuous
  - Problem: under-constrained motion
  - Problem: configuration space can be complex

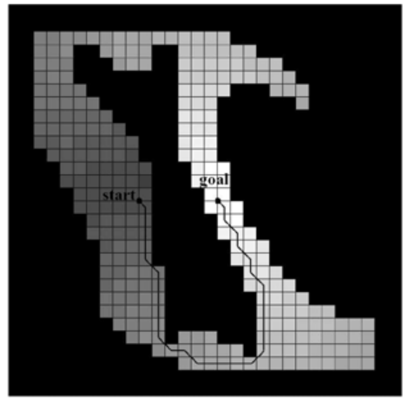




Why are there two paths from 1 to 2?

Decomposition Methods

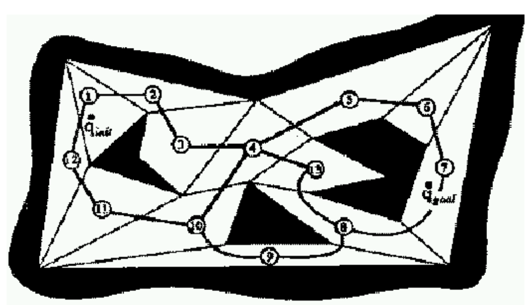




- Break c-space into discrete regions
- Solve as a discrete problem

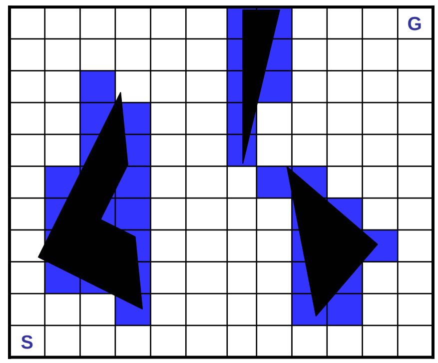
Exact Decomposition?

- With polygon obstacles: decompose exactly
- Problems?
  - Doesn't scale at all
  - Doesn't work with complex, curved obstacles



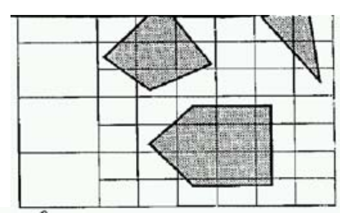
Approximate Decomposition

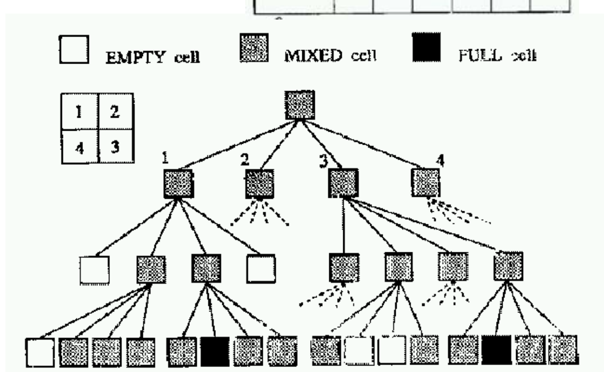
- Break c-space into a grid
  - Search (A*, etc)
  - What can go wrong?
  - If no path found, can subdivide and repeat
- Problems?
  - Still scales poorly
  - Incomplete*
  - Wiggly paths



Hierarchical Decomposition

- Actually used in practical systems
- But:
  - Not optimal
  - Not complete
  - Still hopeless above a small number of dimensions





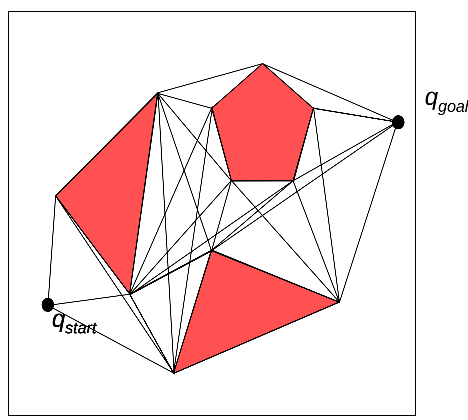
Skeletonization Methods

- Decomposition methods turn configuration space into a grid

- Skeletonization methods turn it into a set of points, with preset linear path between them

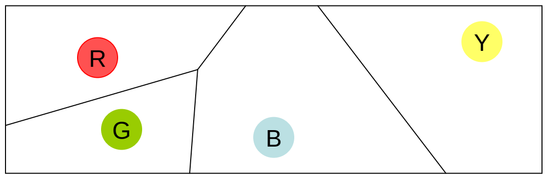
Visibility Graphs

- Shortest paths:
  - No obstacles: straight line
  - Otherwise: will go from vertex to vertex
  - Fairly obvious, but somewhat awkward to prove
- Visibility methods:
  - All free vertex-to-vertex lines (visibility graph)
  - Search using, e.g. A*
  - Can be done in $O(n^3)$ easily, $O(n^2 \log(n))$ less easily
- Problems?
  - Bang, screech!
  - Not robust to control errors
  - Wrong kind of optimality?



Voronoi Decomposition

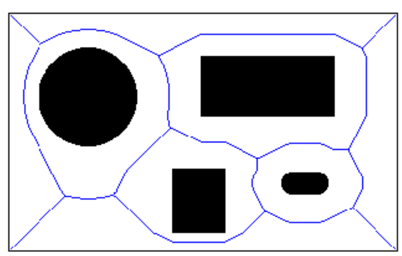
- Voronoi regions: points colored by closest obstacle

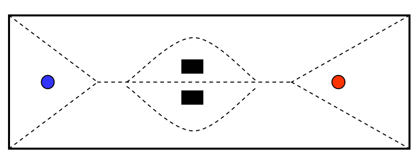


- Voronoi diagram: borders between regions
  - Can be calculated efficiently for points (and polygons) in 2D
  - In higher dimensions, some approximation methods

Voronoi Decomposition

- Algorithm:
  - Compute the Voronoi diagram of the configuration space
  - Compute shortest path (line) from start to closest point on Voronoi diagram
  - Compute shortest path (line) from goal to closest point on Voronoi diagram.
  - Compute shortest path from start to goal along Voronoi diagram
- Problems:
  - Hard over 2D, hard with complex obstacles
  - Can do weird things:

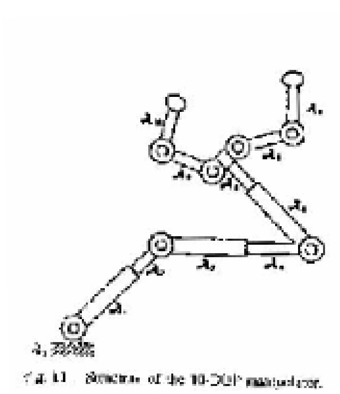


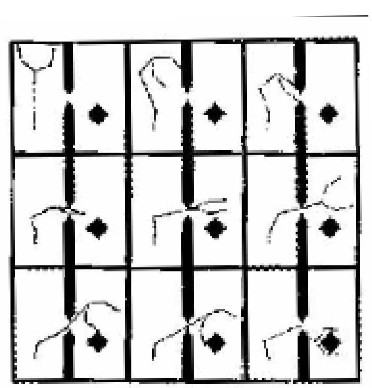


Probabilistic Roadmaps

- Idea: just pick random points as nodes in a visibility graph
- This gives *probabilistic roadmaps*
  - Very successful in practice
  - Lets you add points where you need them
  - If insufficient points, incomplete, or weird paths

Roadmap Example



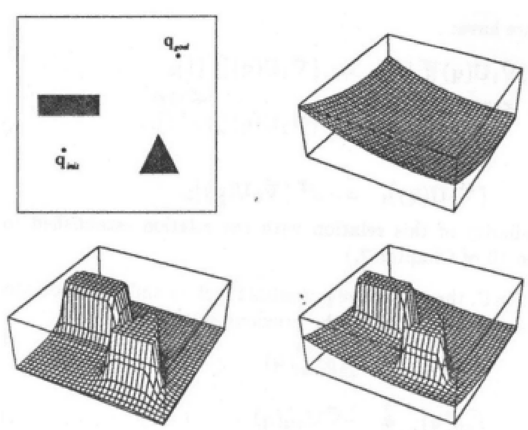


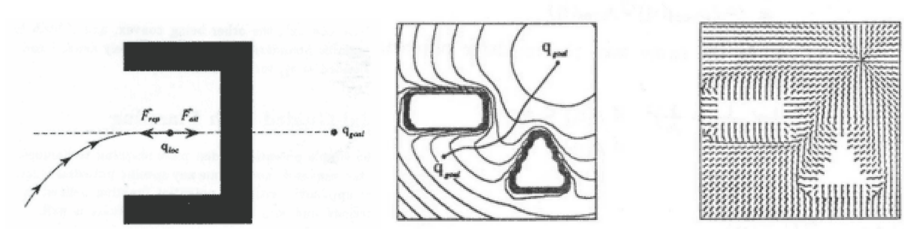
Potential Field Methods

- So far: implicit preference for short paths
- Rational agent should balance distance with risk!
- Idea: introduce cost for being close to an obstacle
- Can do this with discrete methods (how?)
- Usually most natural with continuous methods

Potential Fields

- Cost for:
  - Being far from goal
  - Being near an obstacle
- Go downhill
- What could go wrong?





Potential Field Methods

Define a function $u\left(\underset{\sim}{q}\right)$

u : Configurations $\rightarrow \Re$

Such that

$u \rightarrow$ huge as you move towards an obstacle

$u \rightarrow$ small as you move towards the goal

Write $d_g\left(\underset{\sim}{q}\right) =$ distance from $\underset{\sim}{q}$ to $\underset{\sim}{q}$ goal

$d_i\left(\underset{\sim}{q}\right) =$ distance from $\underset{\sim}{q}$ to nearest obstacle

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One definition of u: $u(q) = d_i(q) - d_g(q)$

Preferred definition: $u(q) = \frac{1}{2}\sum\left(d_g(q)\right)^2 + \frac{1}{2}\eta\frac{1}{d_i(q)^2}$

SIMPLE MOTION PLANNER:

Gradient descent on *u*

Local Search Methods

- Queue-based algorithms keep fallback options (backtracking)
- Local search: improve what you have until you can't make it better
- Generally much more efficient (but incomplete)

Gradient Methods

- How to deal with continous (therefore infinite) state spaces?
- Discretization: bucket ranges of values
  - E.g. force integral coordinates
- Continuous optimization
  - E.g. gradient ascent (or descent)

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$$

$$x \leftarrow x + \alpha \nabla f(x)$$

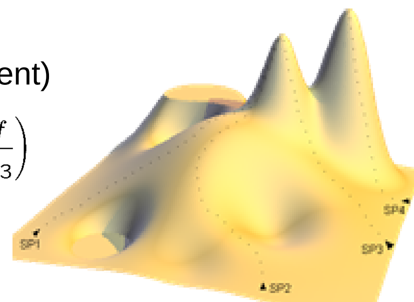
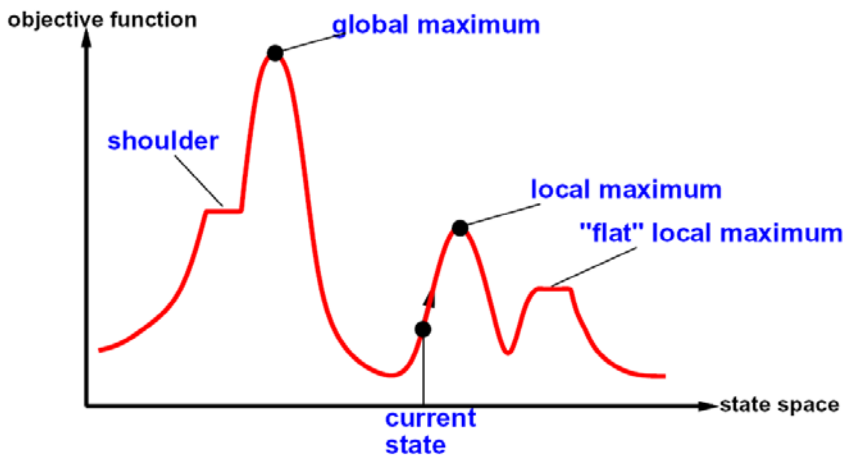


Image from vias.org

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

Hill Climbing Diagram



- Random restarts?
- Random sideways steps?