Section 9 Handout

Problem 1: Value of Information

An oil wildcatter can decide to carry out the seismic resonance test at a given site, and then based on the result of the test he will decide whether to drill or not on the site. The seismic resonance test results can be "closed pattern" (more likely when the hold holds the oil) or "diffuse pattern" (more likely when empty). The seismic resonance test cost \$20K.

The site might contain an oil deposit (O=o) or might not (O=¬o). The oil wildcatter's estimate is that the chance of hitting an oil deposit is 40%. The cost of drilling is \$100K and the payoff for hitting an oil deposit is \$400K (including the cost of drilling).

a) Draw the decision network that represents this problem (the only action is to drill or not drill on the site)

b) Calculate the expected net gain from drilling on the site, given no test

c) Suppose that the seismic resonance test tells you that

$P(T = \text{closed} \mid O = o) = 0.8$

$P(T = \text{closed} \mid O = \neg o) = 0.1$

Calculate the probability that the test reveals closed pattern and the probability that the site contains an oil deposit given each possible test outcome

d) Calculate the optimal decisions given either the test outcome (closed or diffuse), and their expected utilities.

e) Calculate the value of information of the test. Should the oil wildcatter carry out the test?

Problem 2: HMM

You sometimes get colds, which make you sneeze. You also get allergies, which make you sneeze. Sometimes you are well, which doesn't make you sneeze. You decide to model the process using the following HMM, with hidden states $X \in$ {*well, allergy, cold*} and observations $E \in$ {*quiet, sneeze*}:

Transitions

$P(X_1)$

well	1
allergy	0
cold	0

$P(X_t \mid X_{t-1} = well)$

well	0.7
allergy	0.2
cold	0.1

$P(X_t \mid X_{t-1} = allergy)$

well	0.6
allergy	0.3
cold	0.1

$P(X_t \mid X_{t-1} = cold)$

well	0.2
allergy	0.2
cold	0.6

Emissions

$P(E_t \mid X_t = well)$

quiet	1.0
sneeze	0.0

$P(E_t \mid X_t = allergy)$

quiet	0.0
sneeze	1.0

$P(E_t \mid X_t = cold)$

quiet	0.0
sneeze	1.0

Note that colds are "stickier" in that you tend to have them for multiple days, while allergies come and go on a quicker time scale. However, allergies are more frequent. Assume that on the first day, you are well.

a) Imagine that you observe the sequence quiet, sneeze, sneeze. What is the probability that you were well all three days and observed these effects?

b) What is the posterior distribution over your state on day 2 (X_2) if
E_1 = quiet, E_2 = sneeze?

c) What is the posterior distribution over your state on day 3 (X_3) if
E_1 = quiet, E_2 = sneeze, E_3 = sneeze?