CS 188: Artificial Intelligence  
Spring 2009

Lecture 20: Decision Networks  
4/2/2009

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Slides adapted from Dan Klein

Announcements

- Written 3 released tonight, due April 14

Decision Networks

- MEU: choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision networks:
  - Bayes nets with nodes for utility and actions
  - Let's us calculate the expected utility for each action
- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, must be parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)

Decision Networks

- Action selection:
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action

Example: Decision Networks

Weather Forecast Umbrella

Weather

Forecast

Umbrella

Optimal decision – leave
MEU(leave) = \sum_w P(w)U(leave, w)
= 0.7 \cdot 100 + 0.3 \cdot 0 = 70

Umbrella – take
MEU(take) = \sum_w P(w)U(take, w)
= 0.7 \cdot 20 + 0.3 \cdot 70 = 35

Umbrella = leave

Umbrella = take

Find P(W|F=bad)
- Select for evidence
- First we join P(W) and P(bad(W))
- Then we normalize

Evidence in Decision Networks

Weather

Forecast

Weather
Example: Decision Networks

Umbrella = leave

\[ EU(\text{leave}|\text{bad}) = \sum_{w} P(w|\text{bad}) U(\text{bad}, w) \]
\[ = 0.34 \cdot 100 + 0.66 \cdot 0 = 34 \]

Umbrella = take

\[ EU(\text{take}|\text{bad}) = \sum_{w} P(w|\text{bad}) U(\text{take}, w) \]
\[ = 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \]

Optimal decision = take

\[ \text{MEU}(F = \text{bad}) = \max_{a} EU(a|\text{bad}) = 53 \]

Conditioning on Action Nodes

- An action node can be a parent of a chance node
- Chance node conditions on the outcome of the action
- Action nodes are like observed variables in a Bayes' net, except we max over their values

Value of Information

- Idea: compute value of acquiring each possible piece of evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has MEU = k/2
  - Fair price of drilling rights: k/2
- Question: what’s the value of information
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say “oil in a” or “oil in b,” prob 0.5 each
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

Value of Perfect Information

- Current evidence E=e, utility depends on S=s
  \[ \text{MEU}(a) = \max_{a} \sum_{e} P(e|a) U(s, a) \]
- Potential new evidence E’: suppose we knew E’ = e’
  \[ \text{MEU}(e, e’) = \max_{a} \sum_{e} P(e|a) U(s, a) \]
- BUT E’ is a random variable whose value is currently unknown, so:
  - Must compute expected gain over all possible values
  \[ \text{VPI}(E’) = \sum_{e} P(e|a) (\text{MEU}(e, e’) – \text{MEU}(e)) \]
  - (VPI = value of perfect information)

VPI Example: Weather

MEU with no evidence

\[ \text{MEU}(a) = \max_{a} EU(a) = 70 \]

MEU if forecast is bad

\[ \text{MEU}(F = \text{bad}) = \max_{a} EU(a|\text{bad}) = 53 \]

MEU if forecast is good

\[ \text{MEU}(F = \text{good}) = \max_{a} EU(a|\text{good}) = 95 \]

Forecast distribution

\[ P(\text{bad}) = 0.34, \ P(\text{good}) = 0.66 \]

\[ \text{VPI}(E’) = \sum_{e} P(e|a) (\text{MEU}(e, e’) – \text{MEU}(e)) \]

VPI Example: Ghostbusters

- Reminder: ghost his hidden, sensors are noisy
- T: Top square is red
- B: Bottom square is red
- G: Ghost is in the top

Sensor model:

\[ P(\text{t} | \text{g}) = 0.8 \]
\[ P(\text{t} | \neg \text{g}) = 0.4 \]
\[ P(\text{b} | \text{g}) = 0.4 \]
\[ P(\text{b} | \neg \text{g}) = 0.8 \]

Joint Distribution

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VPI Example: Ghostbusters

Utility of bust is 2, no bust is 0

- Q1: What’s the value of knowing T if I know nothing?
- Q1': \( \mathbb{E}_{P(T)}[\text{MEU}(t) - \text{MEU}(\overline{t})] \)
- Q2: What’s the value of knowing B if I already know that T is true (red)?
- Q2': \( \mathbb{E}_{P(B|t)}[\text{MEU}(t, b) - \text{MEU}(t)] \)
- How low can the value of information ever be?

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VPI Properties

- Nonnegative in expectation
  \( \forall E', e : \text{VPI}_e(E') \geq 0 \)
- Nonadditive --consider, e.g., obtaining \( E_i \) twice
  \[ \text{VPI}_e(E_j, E_k) \neq \text{VPI}_e(E_j) + \text{VPI}_e(E_k) \]
- Order-independent
  \[ \text{VPI}_e(E_{j_i}, E_{k_i}) = \text{VPI}_e(E_j) + \text{VPI}_{E_{j_i}, E_{k_i}}(E_{k_i}) \]
  \[ = \text{VPI}_e(E_k) + \text{VPI}_{E_{j_i}, E_{k_i}}(E_{j_i}) \]

Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn’t order either one. What’s the value of knowing which it is?

- If you have $10 to bet and odds are 3 to 1 that Berkeley will beat Stanford, what’s the value of knowing the outcome in advance, assuming you can make a fair bet for either Cal or Stanford?

- What if you are morally obligated not to bet against Cal, but you can refrain from betting?