Announcements

- Projects:
  - The Python tutorial (Project 0) is due tomorrow
  - Project 1 (Search) is out, due next Wednesday
  - You don’t need to submit answers the project’s discussion questions
  - Use Python 2.5 (on EECS instructional machines)
  - 5 slip days for projects; up to two per deadline
  - Try pair programming, not divide-and-conquer
Today

- A* Search
- Heuristic Design
- Local Search

Recap: Search

- Search problem:
  - States (configurations of the world)
  - Successor function: a function from states to lists of (state, action, cost) triples; drawn as a graph
  - Start state and goal test

- Search tree:
  - Nodes: represent paths / plans
  - Paths have costs (sum of action costs)

- Search Algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
Example: Pancake Problem

BOUND FOR SORTING BY PREFIX REVERSAL

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For a permutation $\sigma$ of the integers from 1 to $n$, let $f(\sigma)$ be the smallest number of prefix reversals that will transform $\sigma$ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all $\sigma$ in (the symmetric group) $S_n$. We show that $f(n) \leq (5n + 5)/3$, and that $f(n) \geq 17n/16$ for $n$ a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n + 3$.
General Tree Search

function TREE-SEARCH( problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
  if there are no candidates for expansion then return failure
  choose a leaf node for expansion according to strategy
  if the node contains a goal state then return the corresponding solution
  else expand the node and add the resulting nodes to the search tree
end

Action: flip top two
Cost: 2

Action: flip all four
Cost: 4

Path to reach goal:
Flip four, flip three
Total cost: 7

Uniform Cost Search

- Strategy: expand lowest path cost

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location
Best First (Greedy)

- **Strategy:** expand a node that you think is closest to a goal state
  - **Heuristic:** estimate of distance to nearest goal for each state

- **A common case:**
  - Best-first takes you straight to the (wrong) goal

- **Worst-case:** like a badly-guided DFS

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Example: Heuristic Function

Heuristic: the largest pancake that is still out of place
Combining UCS and Greedy

- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Best-first** orders by goal proximity, or *forward cost* $h(n)$

- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

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When should A* terminate?

- **Should we stop when we enqueue a goal?**

  - No: only stop when we dequeue a goal
Is A* Optimal?

- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

Admissible Heuristics

- A heuristic $h$ is **admissible** (optimistic) if:

  $$h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

- E.g. Euclidean distance in a Pacman maze

- Coming up with admissible heuristics is most of what’s involved in using A* in practice.
Optimality of A*: Blocking

Notation:
- \( g(n) = \) cost to node \( n \)
- \( h(n) = \) estimated cost from \( n \) to the nearest goal (heuristic)
- \( f(n) = g(n) + h(n) = \) estimated total cost via \( n \)
- \( G^* \): the lowest cost goal node
- \( G \): another goal node

Proof:
- What could go wrong?
- We’d have to have to pop a suboptimal goal \( G \) off the fringe before \( G^* \)
- This can’t happen:
  - Imagine a suboptimal goal \( G \) is on the queue
  - Some node \( n \) which is a subpath of \( G^* \) must be on the fringe (why?)
  - \( n \) will be popped before \( G \)
Properties of A*

Uniform-Cost

A*

UCS vs A* Contours

- Uniform-cost expanded in all directions
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

[demo: position search UCS / A*]
Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics.

- Often, admissible heuristics are solutions to relaxed problems (examples coming up).

- Inadmissible heuristics are often useful too (why?).

- Very common hack: use $\alpha \times h(n)$ for admissible $h$, $\alpha > 1$ to generate a faster but less optimal, inadmissible $h'$ from admissible $h$.

Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- What states can I reach from the start state?
- What should the costs be?

![8 Puzzle](image)
8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
  - $h(\text{start}) = 8$
- This is a relaxed-problem heuristic

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8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total Manhattan distance
- Why admissible?
  - $h(\text{start}) = 3 + 1 + 2 + \ldots = 18$

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8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What’s wrong with it?

- With A*: a trade-off between quality of estimate and work per node!

Trivial Heuristics, Dominance

- Dominance: $h_a \geq h_c$ if
  \[ \forall n : h_a(n) \geq h_c(n) \]

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible
    \[ h(n) = \max(h_a(n), h_b(n)) \]

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic
Graph Search, Reconsidered

- Idea: never expand a state twice
- How to implement:
  - Tree search + list of expanded states (closed list)
  - Expand the search tree node-by-node, but...
    - Before expanding a node, check to make sure its state is new
- Python trick: store the closed list as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?
Optimality of A* Graph Search

- Consider what A* does:
  - Expands nodes in increasing total f value (f-contours)
    Reminder: $f(n) = g(n) + h(n) = \text{cost to } n + \text{heuristic}$
  - Proof idea: the optimal goal(s) has the lowest f value, so it must get expanded first

There's a problem with this argument. What are we assuming is true?

Consistency

- Wait, how do we know we expand in increasing f value?
- Couldn't we pop some node $n$, and find its child $n'$ to have lower f value?
- YES:

What can we require to prevent these inversions?
- Consistency: $c(n, a, n') \geq h(n) - h(n')$
- Real cost must always exceed reduction in heuristic