1 Warm-Up: Expectimax

Consider the zero-sum expectimax game tree from Section 3 shown below. Circles represent chance nodes. Trapezoids that point up represent choices for the player seeking to maximize. Outcome values for the maximizing player are listed for each leaf node.

(a) First, assume that each chance node chooses uniformly between available moves. Assuming optimal play, carry out the expectimax search algorithm and write the value of each node inside the corresponding trapezoid. What is the expected value of this game assuming optimal play? What is the optimal move to make at each node?

(b) Now, assume that each chance node plays the leftmost move with probability 0.5, the middle move with probability 0.25, and the rightmost move with probability 0.25. Assuming optimal play, what is the expected value of this game? What is the optimal move to make?
2 Non-transitive Dice

Consider the following three 4-sided dice A, B, C with the given side values. Assume the dice are all fair, and all rolls are independent.

A: 3,3,3,7
B: 2,2,5,7
C: 1,4,5,6

(a) What is the expected value of each die?

(b) Consider the indicator function $\text{better}(X, Y)$ which has value 1 if $X > Y$, −1 if $X < Y$, and value 0 o.w. What are the expected values of $\text{better}(A, B)$, $\text{better}(B, C)$, and $\text{better}(C, A)$?

A binary relation $R$ is transitive if for all $a, b, c$, $R(a, b)$ and $R(b, c)$ implies $R(a, c)$. For example, the greater-than operator ($>$) on the real numbers is an example of a transitive relation: if $x < y$ and $y < z$ then $x < z$.

(c) Why are dice like those in (a) sometimes called “non-transitive dice”?


3 St. Petersburg Paradox

You are traveling in Russia when a friendly stranger comes up to you and offers you the following game. He will give you $1 and a fair coin. You flip the coin repeatedly until the first tail appears. For every head that appears, you double your money. When the first tail appears, the game is over.

(a) What is the expected value of this game?

(b) What would you pay to play this game? How does this compare to your answer from part (a)?

This game is known as the St. Petersburg Paradox. One way of resolving this paradox is by using the decreasing marginal utility of money.

(c) Assume that you have a utility function for money of the form $U(x) = \log_2(x)$ where $x$ is your total winnings from this game. What is the expected utility of this game to you? What is the expected value of this game in dollars? You may find the following identities helpful:

$$r + r^2 + r^3 + ... = \sum_{i=0}^{\infty} r^{i+1} = \frac{r}{1-r}, \quad r \in [0, 1)$$

$$r(0) + r^2(1) + r^3(2) + r^4(3) + ... = \sum_{i=0}^{\infty} ir^{i+1} = \left(\frac{r}{1-r}\right)^2, \quad r \in [0, 1)$$
4  Suicidal Pacman

Pacman is sometimes suicidal when doing a minimax search because of its worst case analysis. We will build here a small expectimax tree to see the difference in behavior.

Consider the following rules (slightly simplified from assignment):

- Ghosts cannot change direction unless they are facing a wall. The possible actions are east, west, south, and north (not stop). Initially, they have no direction and can move to any adjacent square.
- We use random ghosts which choose uniformly between all their legal moves.
- Assume that Pacman cannot stop
- If Pacman runs into a space with a ghost, it dies before having the chance to eat any food which was there.
- The game is scored as follows:
  - -1 for each action Pacman takes
  - 10 for each food dot eaten
  - -500 for losing (if Pacman is eaten)
  - 500 for winning (all food dots eaten)

Given the following “trapped” maze, build the expectimax tree with max and chance nodes clearly identified. Use the game score as the evaluation function at the leaves. If you don’t want to make little drawings, all possible states of the game have been labeled for you on the next page: use them to identify the states of the game. Pacman moves first, followed by the lower left ghost, then the top right ghost.
(a) Build the expectimax tree. What is Pacman’s optimal move?

(b) What would pacman do if it was using minimax instead?
(c) By changing the probabilities of action for the ghosts, can you get expectimax to make the same decision as minimax?

(d) Now say you are using the following alternate game score components:

- -1 for Pacman making a move
- -1.5 for losing
- 0 for eating food
- 0.3 for winning

Use this new game score as your evaluation function at the leaves. Note this yields a monotonic transformation of the original utilities: a function which preserves the ordering of the state according to their utility. Could this change the decision of Pacman using expectimax?