Last name:_________________ First name:_________________ SID:___________ Class account login:___________

Collaborators: ____

CS188 Spring 2011 Written 3: Probability

Due: Friday, 3/11/11 at 5:29pm in 283 Soda Drop Box (no slip days).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually. Recall to make a photo-copy of your solutions to allow you to resubmit for partial credit recovery. See course webpage for details.

1 [9 pts] Of Weddings and Bears

You are the ruler of a distant kingdom, have just gotten engaged to your beloved, have set a date for the wedding, and now need to send a message to your soon-to-be in-laws. Hopefully (?), they will attend the wedding ($A = +a$), but they may not ($A = \neg a$). You dispatch your fastest messenger across the barren wastes. Unfortunately, a number of things could go wrong. Your messenger could be beset by a bevy of wild bears ($B = +b$). Your messenger may be captured ($C = +c$) by a cohort of cave trolls. It is possible that both ills befall the messenger. Using your knowledge about the dangers of bears, of cave trolls, and of your in-laws, you have constructed the following joint probability table:

A	B	C	$P(A, B, C)$
$+a$	$+b$	$+c$	0.0000
$+a$	$+b$	$\neg c$	0.0008
$+a$	$\neg b$	$+c$	0.0396
$+a$	$\neg b$	$\neg c$	0.6336
$\neg a$	$+b$	$+c$	0.0020
$\neg a$	$+b$	$\neg c$	0.0072
$\neg a$	$\neg b$	$+c$	0.1584
$\neg a$	$\neg b$	$\neg c$	0.1584

[1pt] (a) What is the joint distribution $P(B, C)$? Your answer should be in the form of a table.

[1pt] (b) Are B and C independent? Justify your answer using the actual probabilities computed in part (a).

[1pt] (c) You are also a naturalist, so you are curious about the probability of a bear attack in your model. What is $P(B)$, the marginal over B given no evidence? (This is called the *prior* distribution, here the prior probability of bear attack.)

[1pt] (d) If your in-laws do not attend the wedding ($A = \neg a$), what is the conditional distribution over bear attack, $P(B|A = \neg a)$? (This is often called a *posterior* distribution.) Does it make intuitive sense how the answers have shifted from part (c)?

[1pt] (e) Suppose you learn that your messenger was captured by cave trolls. What is the new posterior distribution $P(B|A = \neg a, C = +c)$? Is the conditional probability of bear attack higher or lower than from part (d)? Does this make intuitive sense? The phenomenon where the discovery that one possible cause is true decreases belief in other possible causes is called *explaining-away*.

Now, suppose you are making your own decision (D) about whether ($+d$) or not ($\neg d$) to host the wedding at all. You have no evidence about B or C, but your decision crucially depends on A. If both you and your in-laws attend the wedding, then everyone will be happy. If only you attend, you will be greatly embarrassed, perhaps to the point that you will have to declare war on your in-laws just to save face! However, if only your in-laws show up, they will be extraordinarily embarrassed—as will you—and there will certainly be a large and costly war. Finally, if neither of you attend the wedding, then there will only be slight embarrassment of having to reschedule.

Based on all these considerations, you create the utility function (U) below, which relates the amount of embarrassment suffered under the possible outcomes.

A	D	U
$+a$	$+d$	-1
$+a$	$\neg d$	-10000
$\neg a$	$+d$	-5000
$\neg a$	$\neg d$	-10

[2pt] (f) Given this chart and the probabilities above, what is the expected utility $\mathbb{E}[U|D = d]$ if you decide to attend and the expected utility $\mathbb{E}[U|D = \neg d]$ if you decide not to attend? Note that A is independent of your decision to attend.

$$\mathbb{E}[U \mid D = d] =$$

$$\mathbb{E}[U \mid D = \neg d] =$$

[2pt] (g) Now suppose that, before you must make your decision, you learn that your messenger was beset by bears ($B = +b$). Calculate the expected utility for each decision under this condition.

$$\mathbb{E}[U \mid D = d, B = b] =$$

$$\mathbb{E}[U \mid D = \neg d, B = b] =$$