CS188 Spring 2012 Section 9: VPI

1 Used Car Purchase

[Adapted from problem 16.11 in Russell & Norvig]

A used car buyer can decide to carry out various investigations with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the investigations, decide which car to buy. We will assume that the buyer is deciding whether to buy car c and that there is time to carry out at most one investigation which costs \$50 and which can help to figure out the quality of the car. A car can be in good shape (of good quality $Q = +q$) or in bad shape (of bad quality Q=$-q$), and the investigation might help to indicate what shape the car is in. There are only two outcomes for the investigation I: pass (I=pass) or fail (I=fail). Car c costs \$1,500, and its market value is \$2,000 if it is in good shape; if not, \$700 in repairs will be needed to make it in good shape. The buyers estimate is that c has 70% chance of being in good shape.

(a) Draw the decision network that represents this problem. (The only action is whether to buy or not the car).

(b) Calculate the expected net gain from buying car c, given no investigation.

Investigations can be described by the probability that the car will pass or fail the investigation given that the car is in good or bad shape. We have the following information:

$$P(I = \text{pass}|Q = +q) = 0.9$$
$$P(I = \text{pass}|Q = -q) = 0.2$$

From this we can determine the following about whether the car will pass (or fail) its investigation, and the probability that it is in good (or bad) shape given each possible investigation outcome:

$$P(I = \text{pass}) = \sum_q P(I = \text{pass}, Q = q) = 0.69$$
$$P(I = fail) = 1 - P(I = \text{pass}) = 0.31$$
$$P(Q = +q|I = \text{pass}) = \frac{P(I = \text{pass}|Q = +q)P(Q = +q)}{P(I = \text{pass})} \approx 0.91$$
$$P(Q = +q|I = \text{fail}) = \frac{P(I = \text{fail}|Q = +q)P(Q = +q)}{P(I = \text{fail})} \approx 0.22$$

(c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

(d) Calculate the value of (perfect) information of the investigation. Should the buyer pay for an investigation?