Announcements

- Homework 3
  - Due 2/18 at 11:59pm

- Project 2
  - Due 2/22 at 4:00pm

- Tutoring: read @260 on Piazza, we now have 1:1 tutoring available
Still assume a Markov decision process (MDP):
- A set of states \( s \in S \)
- A set of actions (per state) \( A \)
- A model \( T(s,a,s') \)
- A reward function \( R(s,a,s') \)
Reinforcement Learning
Example: Prescription Problem

- \( P(\text{cure}) = 0.2 \)
- \( P(\text{cure}) = 0.4 \)
- \( P(\text{cure}) = 0.9 \)
- \( P(\text{cure}) = 0.1 \)
Example: Prescription Problem

\[ P(\text{cure}) = ? \]

\[ P(\text{cure}) = ? \]

\[ P(\text{cure}) = ? \]

\[ P(\text{cure}) = ? \]
Let’s Play!

P(cure) = ?

P(cure) = ?

P(cure) = ?

P(cure) = ?

What Just Happened?

- That wasn’t planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn’t solve it with just computation
  - You needed to actually act to figure it out

- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP
Still assume a Markov decision process (MDP):

- A set of states $s \in S$
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- A reward function $R(s,a,s')$

Still looking for a policy $\pi(s)$

New twist: don’t know $T$ or $R$

- I.e. we don’t know which states are good or what the actions do
- Must actually try actions and states out to learn
Reinforcement Learning

- **Basic idea:**
  - Receive feedback in the form of *rewards*
  - Agent’s utility is defined by the reward function
  - Must (learn to) act so as to **maximize expected rewards**
  - All learning is based on observed samples of outcomes!
Cheetah

episode: 160  return: 4254
Atari
Robots
Robots
The Crawler!

[Demo: Crawler Bot (L10D1)] [You, in Project 3]
Video of Demo Crawler Bot
Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) $A$
  - A model $T(s,a,s')$
  - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$

- New twist: don’t know $T$ or $R$
  - I.e. we don’t know which states are good or what the actions do
  - Must actually try actions and states out to learn
Offline (MDPs) vs. Online (RL)

Offline Solution

Online Learning
Model-Based Learning
Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct

- Step 1: Learn empirical MDP model
  - Count outcomes $s'$ for each $s, a$
  - Normalize to give an estimate of $\hat{T}(s, a, s')$
  - Discover each $\hat{R}(s, a, s')$ when we experience $(s, a, s')$

- Step 2: Solve the learned MDP
  - For example, use value iteration, as before
Example: Model-Based Learning

Assume: $\gamma = 1$

<table>
<thead>
<tr>
<th>Input Policy $\pi$</th>
<th>Observed Episodes (Training)</th>
<th>Learned Model</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\hat{T}(s, a, s')$</td>
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<tr>
<td></td>
<td></td>
<td>$T(B, east, C) = 1.00$</td>
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<td></td>
<td></td>
<td>$T(C, east, D) = 0.75$</td>
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<td>$T(C, east, A) = 0.25$</td>
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<td></td>
<td>$\hat{R}(s, a, s')$</td>
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<td>$R(B, east, C) =$</td>
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<td>$R(C, east, D) =$</td>
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<td>$R(D, exit, x) =$</td>
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<td>$\ldots$</td>
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<td></td>
<td>Episode 1: B, east, C, -1</td>
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<td></td>
<td>C, east, D, -1</td>
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<td>D, exit, x, +10</td>
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<td>Episode 2: B, east, C, -1</td>
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<td>C, east, D, -1</td>
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<td>D, exit, x, +10</td>
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<td></td>
<td>Episode 3: E, north, C, -1</td>
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<td></td>
<td>C, east, D, -1</td>
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<td>D, exit, x, +10</td>
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<td></td>
<td>Episode 4: E, north, C, -1</td>
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<td></td>
<td>C, east, A, -1</td>
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<td></td>
<td>A, exit, x, -10</td>
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</tbody>
</table>
Example: Expected Age

Goal: Compute expected age of cs188 students

**Known P(A):**

\[ E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \ldots \]

**Without P(A), instead collect samples** \([a_1, a_2, \ldots, a_N]\)

**Unknown P(A): “Model Based”**

\[ \hat{P}(a) = \frac{\text{num}(a)}{N} \]

\[ E[A] \approx \sum_a \hat{P}(a) \cdot a \]

**Why does this work?** Because eventually you learn the right model.

**Unknown P(A): “Model Free”**

\[ E[A] \approx \frac{1}{N} \sum_i a_i \]

**Why does this work?** Because samples appear with the right frequencies.
Model-Free Learning
Passive Reinforcement Learning
Passive Reinforcement Learning

- **Simplified task: policy evaluation**
  - Input: a fixed policy \( \pi(s) \)
  - You don’t know the transitions \( T(s,a,s') \)
  - You don’t know the rewards \( R(s,a,s') \)
  - **Goal:** learn the state values

- **In this case:**
  - Learner is “along for the ride”
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world.
Direct Evaluation

- **Goal:** Compute values for each state under $\pi$

- **Idea:** Average together observed sample values
  - Act according to $\pi$
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples

- **This is called direct evaluation**
Example: Direct Evaluation

**Input Policy** $\pi$

```
A
B
C
D
E
```

*Assume: $\gamma = 1$*

**Observed Episodes (Training)**

- **Episode 1**
  - $B$, east, $C$, -1
  - $C$, east, $D$, -1
  - $D$, exit, $x$, +10

- **Episode 2**
  - $B$, east, $C$, -1
  - $C$, east, $D$, -1
  - $D$, exit, $x$, +10

- **Episode 3**
  - $E$, north, $C$, -1
  - $C$, east, $D$, -1
  - $D$, exit, $x$, +10

- **Episode 4**
  - $E$, north, $C$, -1
  - $C$, east, $A$, -1
  - $A$, exit, $x$, -10

**Output Values**

```
A     -10
B     +8
C     +4
D     +10
E     -2
```
Problems with Direct Evaluation

- What’s good about direct evaluation?
  - It’s easy to understand
  - It doesn’t require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions

- What bad about it?
  - It wastes information about state connections
  - Each state must be learned separately
  - So, it takes a long time to learn

Output Values

If B and E both go to C under this policy, how can their values be different?
Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate $V$ for a fixed policy:
  - Each round, replace $V$ with a one-step-look-ahead layer over $V$
    
    $V_{0}^{\pi}(s) = 0$

    $$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_{k}^{\pi}(s')]$$

  - This approach fully exploited the connections between the states
  - Unfortunately, we need $T$ and $R$ to do it!

- Key question: how can we do this update to $V$ without knowing $T$ and $R$?
  - In other words, how to we take a weighted average without knowing the weights?
Sample-Based Policy Evaluation?

- We want to improve our estimate of $V$ by computing these averages:

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

- Idea: Take samples of outcomes $s'$ (by doing the action!) and average

$$\text{sample}_1 = R(s, \pi(s), s'_1) + \gamma V_k^\pi(s'_1)$$
$$\text{sample}_2 = R(s, \pi(s), s'_2) + \gamma V_k^\pi(s'_2)$$
$$\quad \ldots$$
$$\text{sample}_n = R(s, \pi(s), s'_n) + \gamma V_k^\pi(s'_n)$$

$$V_{k+1}^\pi(s) \leftarrow \frac{1}{n} \sum_{i} \text{sample}_i$$
Temporal Difference Learning

- Big idea: learn from every experience!
  - Update $V(s)$ each time we experience a transition $(s, a, s', r)$
  - Likely outcomes $s'$ will contribute updates more often

- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average

Sample of $V(s)$:

$$sample = R(s, \pi(s), s') + \gamma V^\pi(s')$$

Update to $V(s)$:

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$$

Same update:

$$V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$$
Exponential Moving Average

- Exponential moving average
  - The running interpolation update: \( \bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n \)
  - Makes recent samples more important:
    \[
    \bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \ldots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \ldots}
    \]
  - Forgets about the past (distant past values were wrong anyway)

- Decreasing learning rate (alpha) can give converging averages
Example: Temporal Difference Learning

Assume: $\gamma = 1, \alpha = 1/2$

$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha \left[R(s, \pi(s), s') + \gamma V^\pi(s') \right]$
Active Reinforcement Learning
Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages.
- However, if we want to turn values into a (new) policy, we’re sunk:

\[ \pi(s) = \arg \max_a Q(s, a) \]

\[ Q(s, a) = \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V(s') \right] \]

- Idea: learn Q-values, not values.
- Makes action selection model-free too!
Detour: Q-Value Iteration

- **Value iteration**: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given $V_k$, calculate the depth $k+1$ values for all states:
    \[
    V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V_k(s') \right]
    \]

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s, a) = 0$, which we know is right
  - Given $Q_k$, calculate the depth $k+1$ q-values for all q-states:
    \[
    Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]
    \]
Q-Learning

- **Q-Learning**: sample-based Q-value iteration
  \[ Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right] \]

- Learn \(Q(s,a)\) values as you go
  - Receive a sample \((s, a, s', r)\)
  - Consider your old estimate: \(Q(s, a)\)
  - Consider your new sample estimate:
    \[ sample = R(s, a, s') + \gamma \max_{a'} Q(s', a') \]
  - Incorporate the new estimate into a running average:
    \[ Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha)[sample] \]

[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]
Video of Demo Q-Learning -- Gridworld
Video of Demo Q-Learning -- Crawler
Q-Learning:
act according to current policy (and also explore...)

- Full reinforcement learning: optimal policies (like value iteration)
  - You don’t know the transitions $T(s,a,s')$
  - You don’t know the rewards $R(s,a,s')$
  - You choose the actions now
  - Goal: learn the optimal policy / values

- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens...
Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you’re acting suboptimally!

- This is called **off-policy learning**

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn’t matter how you select actions (!)
Exploration vs. Exploitation
How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions (\(\varepsilon\)-greedy)
    - Every time step, flip a coin
    - With (small) probability \(\varepsilon\), act randomly
    - With (large) probability \(1-\varepsilon\), act on current policy
  
- Problems with random actions?
  - You do eventually explore the space, but keep thrashing around once learning is done
  - One solution: lower \(\varepsilon\) over time