Today.

http://bit.ly/3GEMokW
Graph Search.

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Graph Search.

  Consistent Heuristic.

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Graph Search.

- Consistent Heuristic.

Constraint Satisfaction Problems.

Graph Search

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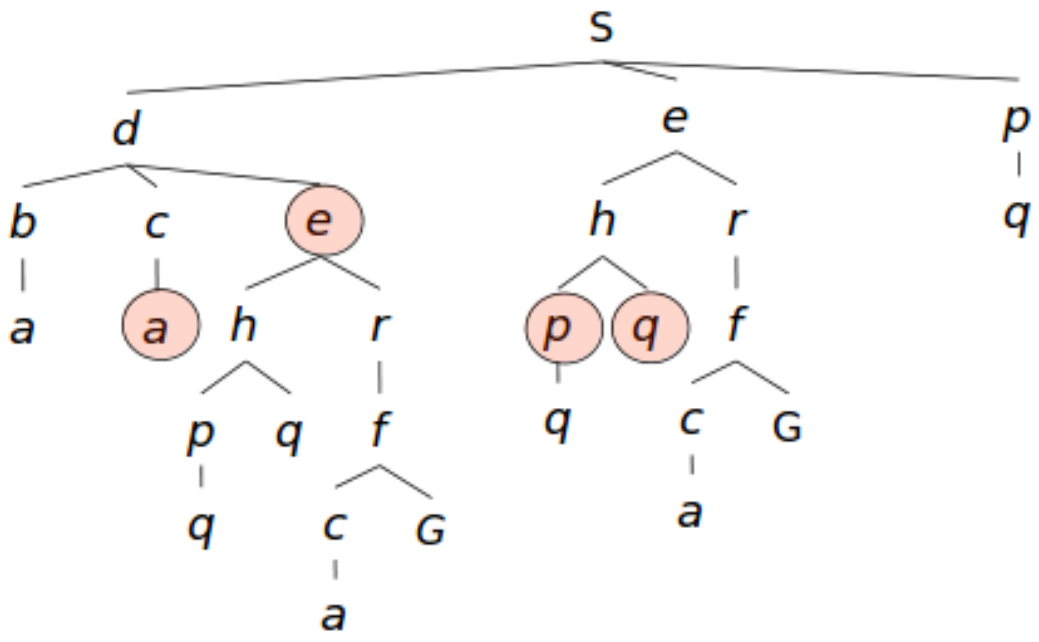

In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

Graph Search

Idea: never expand a state twice

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How to implement:

Graph Search

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Tree search + set of expanded states (“closed set”)

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Expand the search tree node-by-node, but...

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Expand the search tree node-by-node, but...
Before expanding a node,

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- Expand the search tree node-by-node, but...
- Before expanding a node,
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  - If yes skip it, else add to closed set and expand.

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Expand the search tree node-by-node, but...
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If yes skip it, else add to closed set and expand.

Important: store the closed set as a set, not a list

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Can graph search wreck completeness?

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Can graph search wreck completeness? Why/why not?

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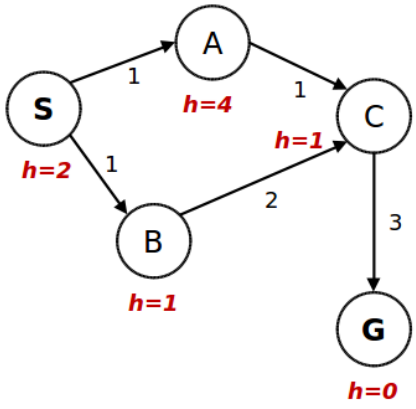
Can graph search wreck completeness? Why/why not?

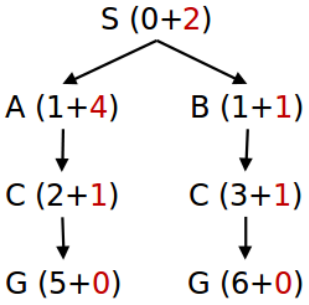
How about optimality?

A* Graph Search Gone Wrong?

State space graph



Search tree



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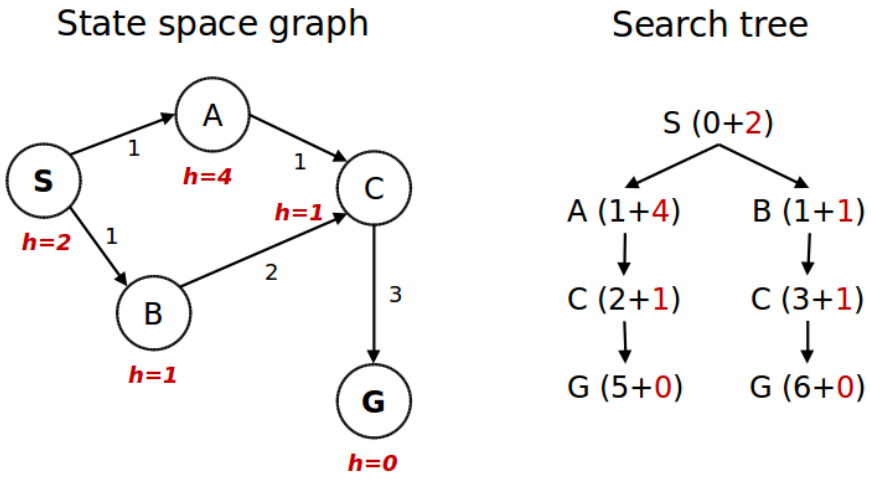

Is $h(\cdot)$ admissible?

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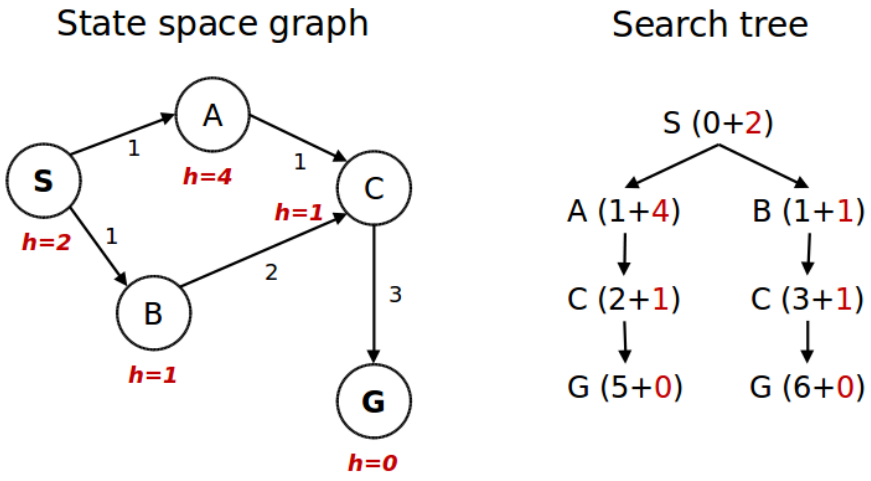

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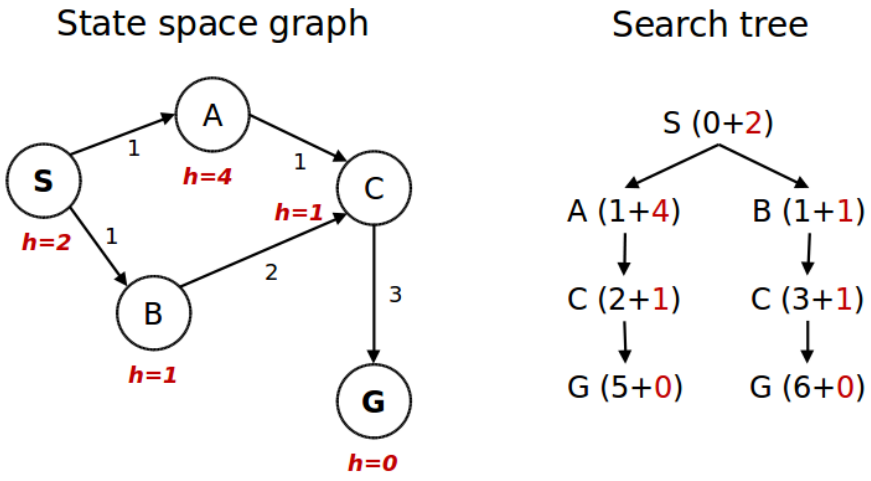

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Will exploring w.r..t $h(B) + g(n)$ be optimal?

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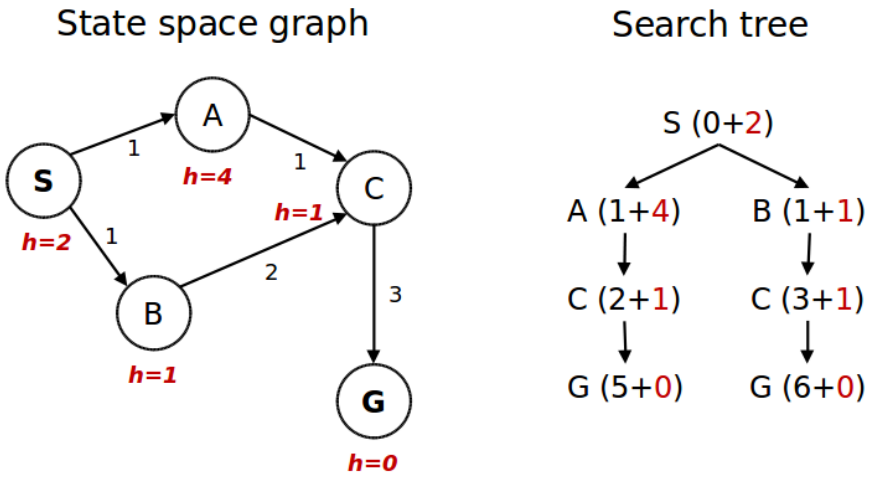

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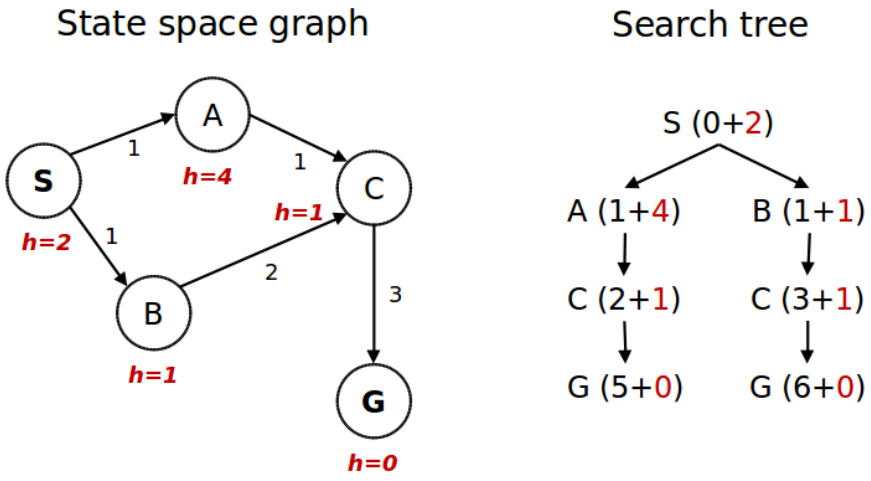

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Expand S.
  A and B in fringe!

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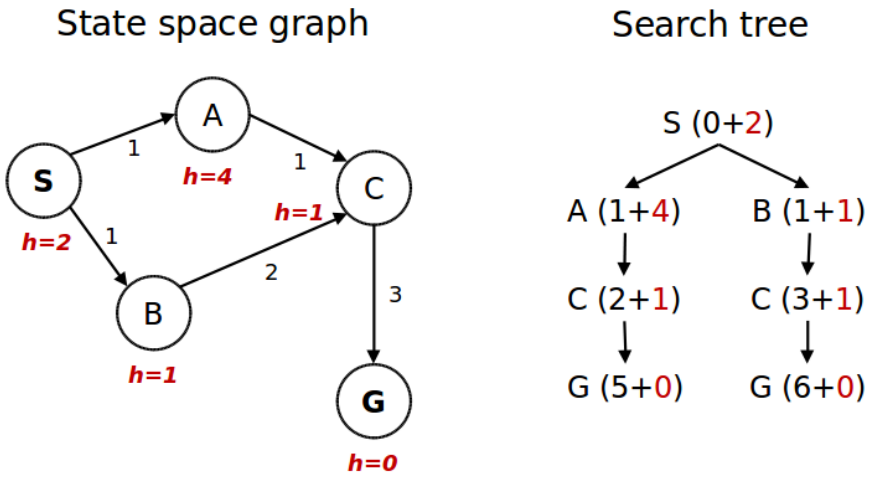

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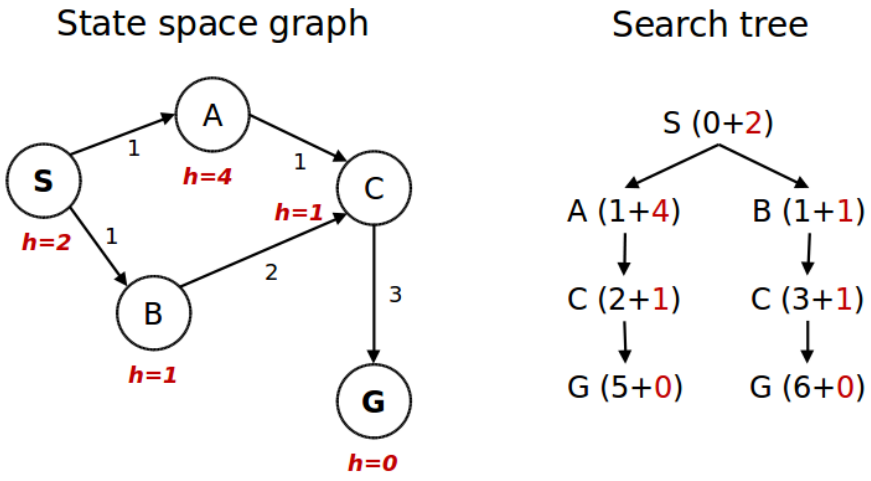

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  C in fringe with key, $3 + h(C) = 4$.

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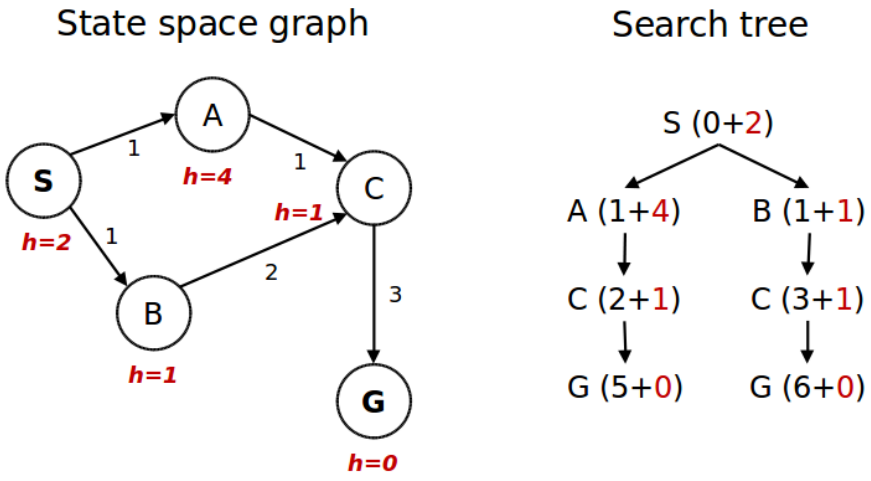

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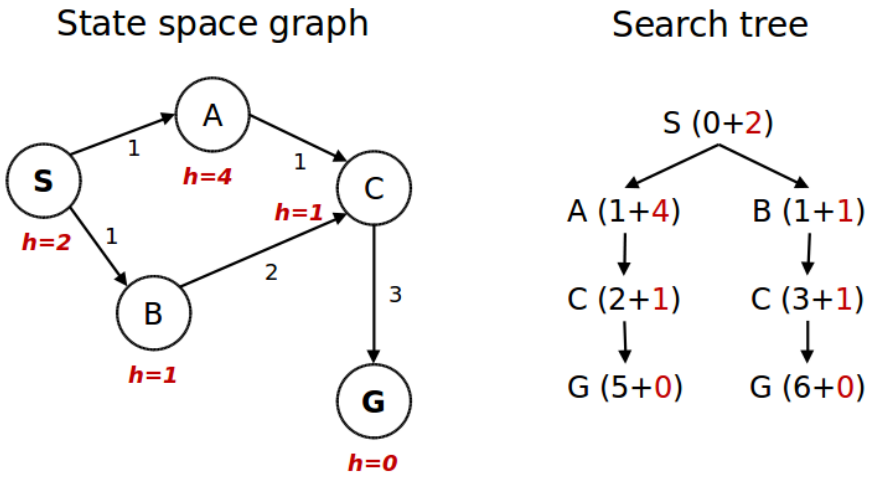

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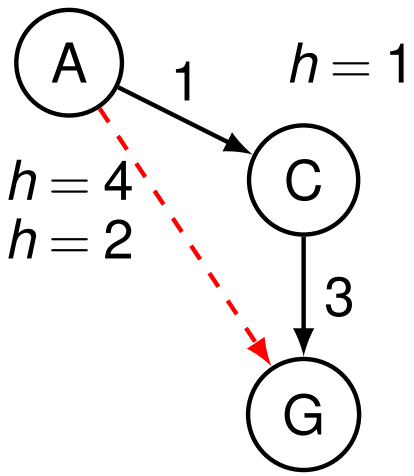
Could have been there in 4.

Consistency of Heuristics

Main idea: est. heuristic costs $\leq$ actual costs



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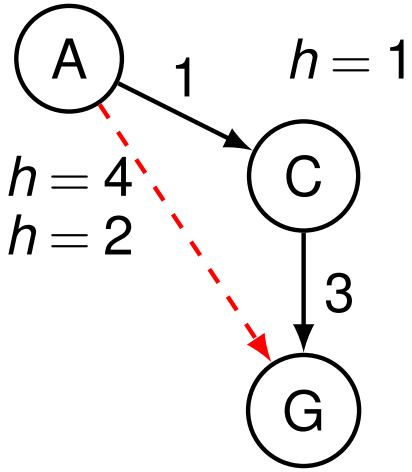

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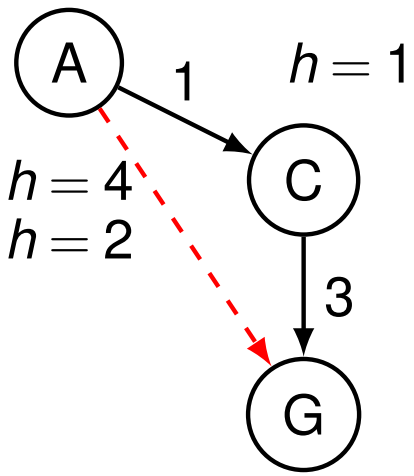
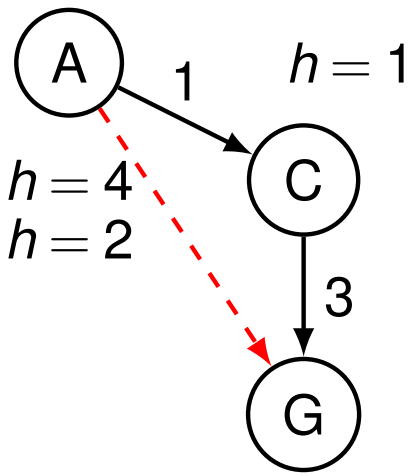


Main idea: est. heuristic costs $\leq$ actual costs

Admissibility: $h(x) \leq$ cost to goal.

Consistency: $h(x) - h(y) \leq cost(x, y)$.

Consistency of Heuristics



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heuristic "arc" cost $\leq$ actual arc cost

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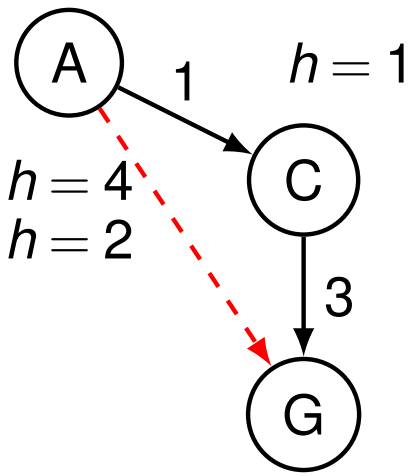

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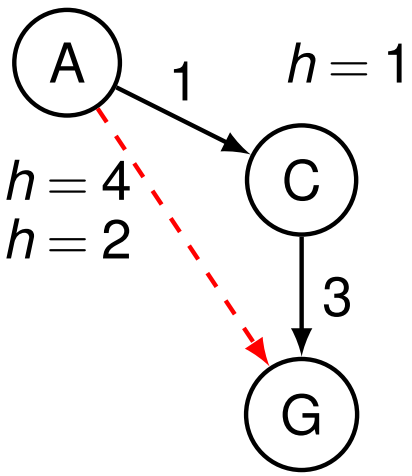

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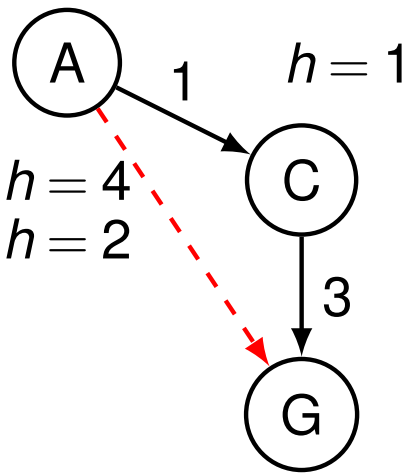
Consistent $\implies$ admissible?

Consistency of Heuristics



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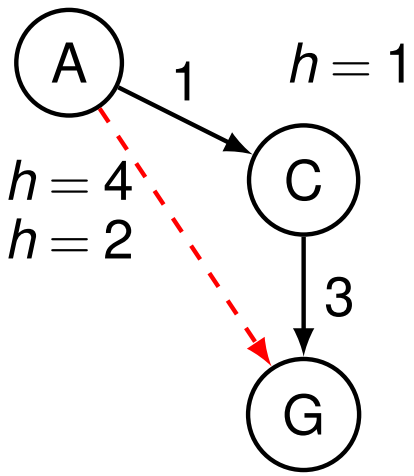

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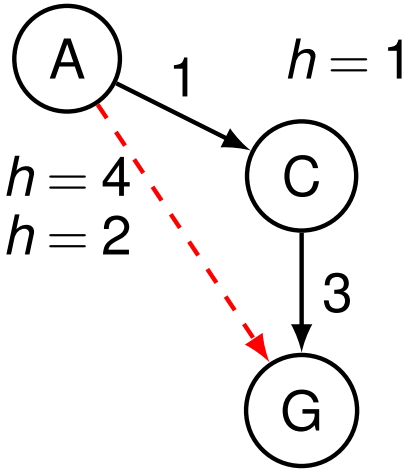

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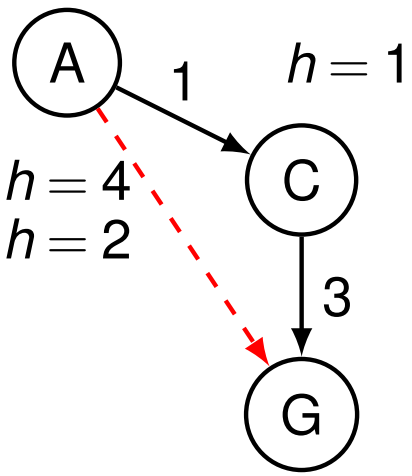

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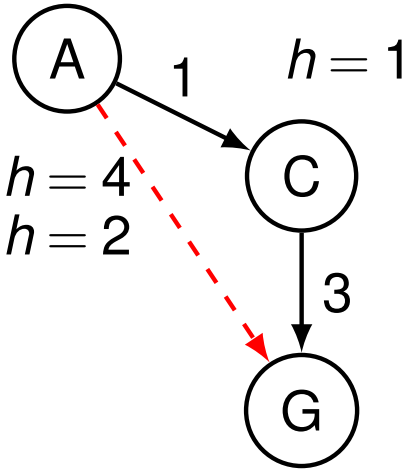
Consistent: f value along a path never decreases

Consistency of Heuristics



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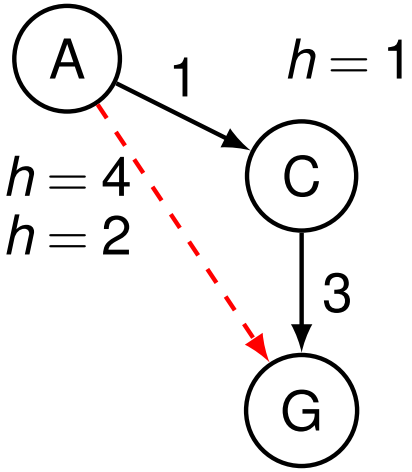

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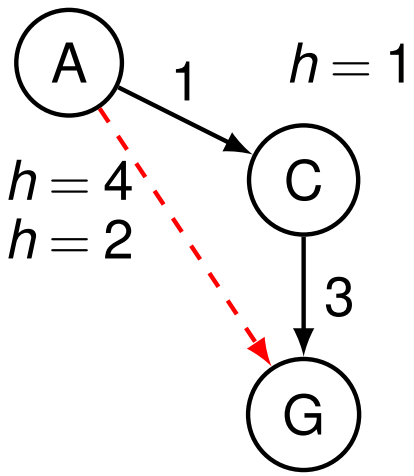
Consistent $\implies$ admissible? Yes? No?

Consistent: f value along a path never decreases

Admissible:

$f(C) = h(C) + 1 = 3$.

Consistency of Heuristics



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heuristic "arc" cost $\leq$ actual arc cost

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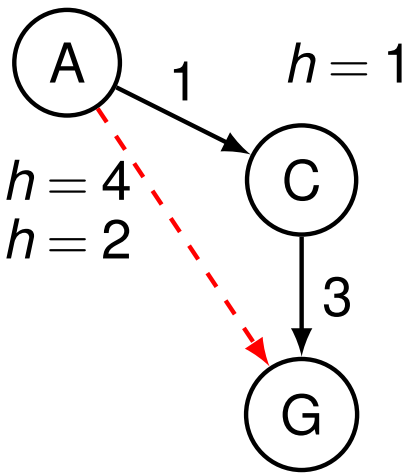
Consistency of Heuristics



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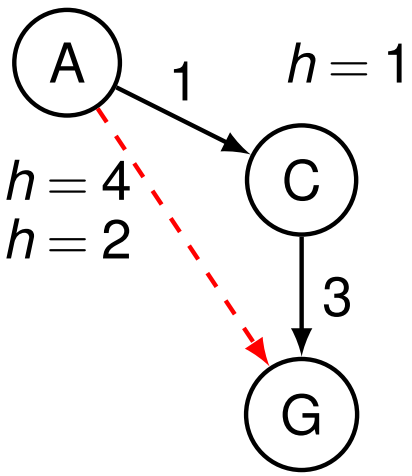

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Admissible:

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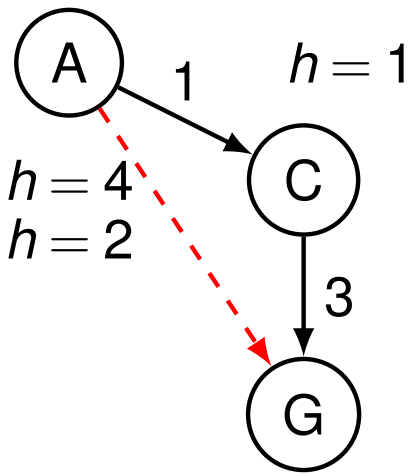
Consistent: $f(A) = 2$

Consistency of Heuristics



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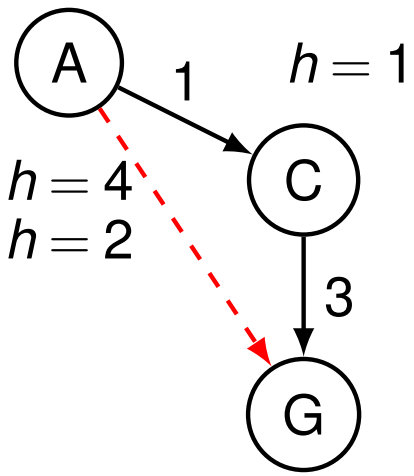
Consistency of Heuristics



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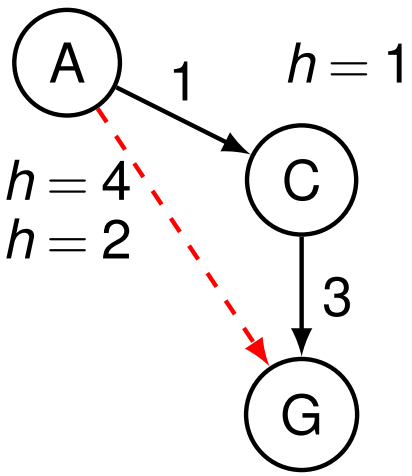
Claim: If y is expanded due to x, $f(y) \geq f(x)$.

Consistency of Heuristics



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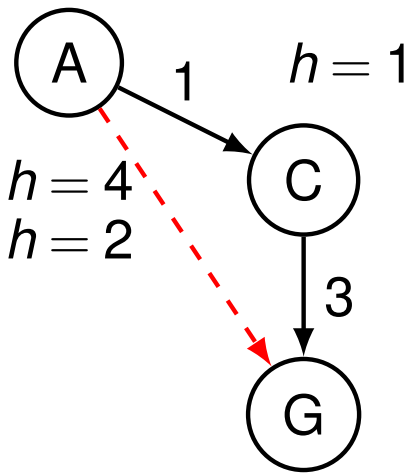
Proof:

$f(y) = g(x) + cost(x, y) + h(y)$

Consistency of Heuristics



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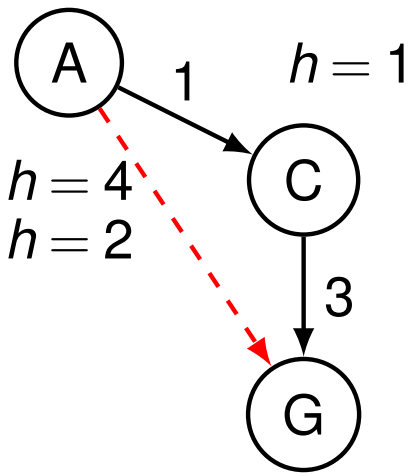

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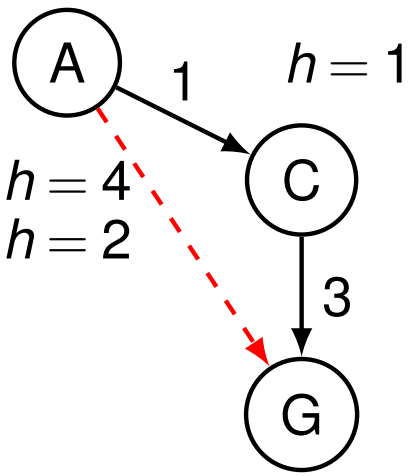
Consistency of Heuristics



Main idea: est. heuristic costs $\leq$ actual costs

Admissibility: $h(x) \leq$ cost to goal.

Consistency: $h(x) - h(y) \leq cost(x, y)$.

heuristic "arc" cost $\leq$ actual arc cost

Consistent $\implies$ admissible? Yes? No?

Consistent: f value along a path never decreases

Admissible:

$f(C) = h(C) + 1 = 3$.

$f(A) = h(A) = 4$.

Consistent: $f(A) = 2 < 3 = f(C)$.

Claim: If y is expanded due to x, $f(y) \geq f(x)$.

Proof:

$$f(y) = g(x) + cost(x, y) + h(y)$$
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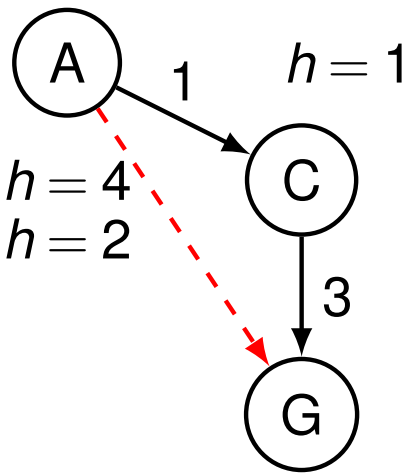

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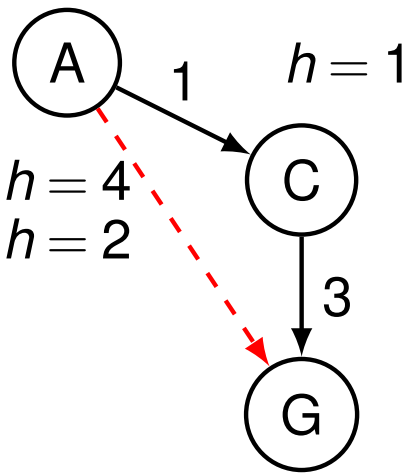

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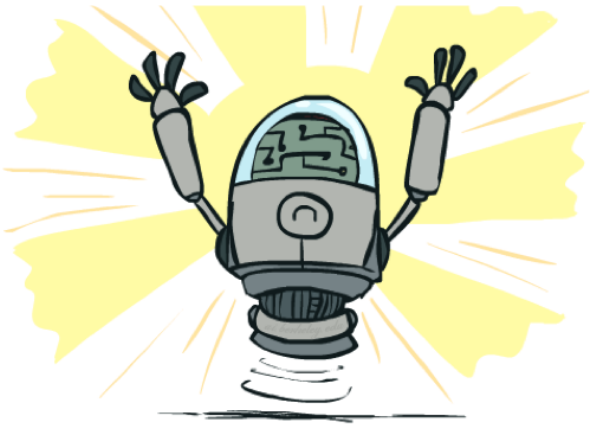
The "estimate" of plan cost keeps rising as you progress.

Optimality of A* Graph Search



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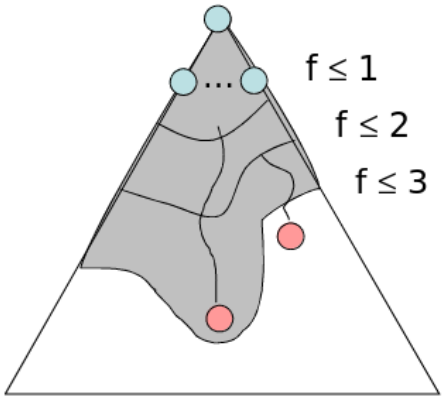
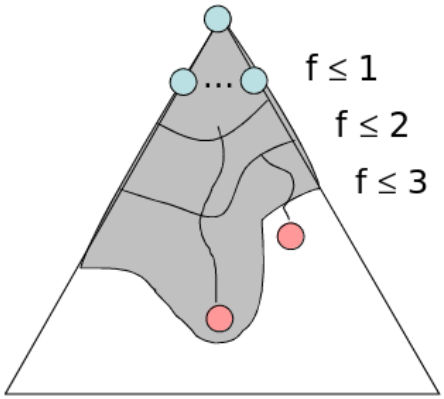


Sketch: consider what A* does with a consistent heuristic:

Optimality of A* Graph Search



Sketch: consider what A* does with a consistent heuristic:

Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)

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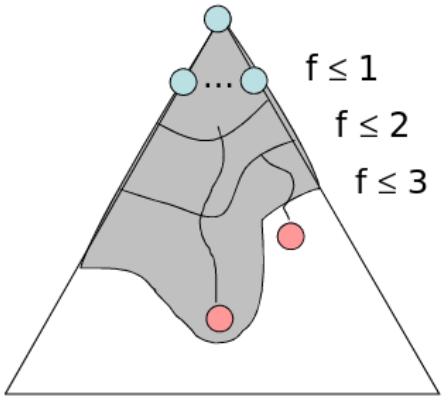

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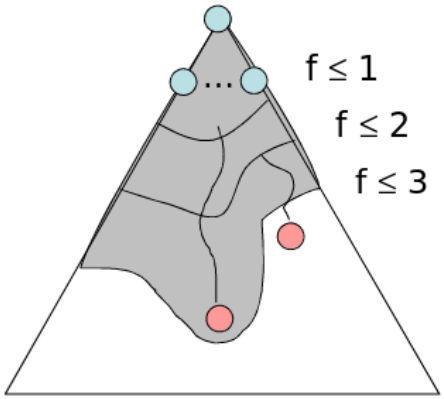

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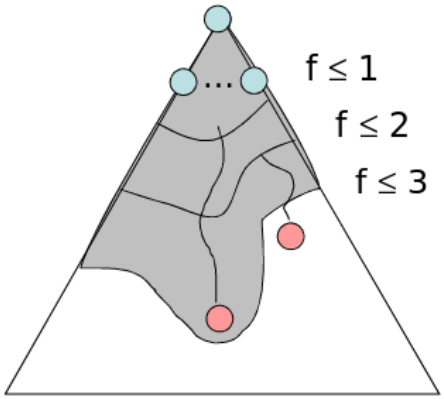

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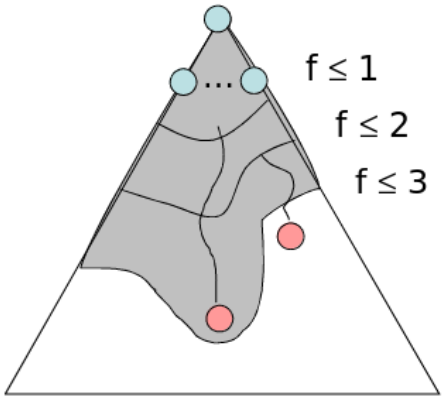

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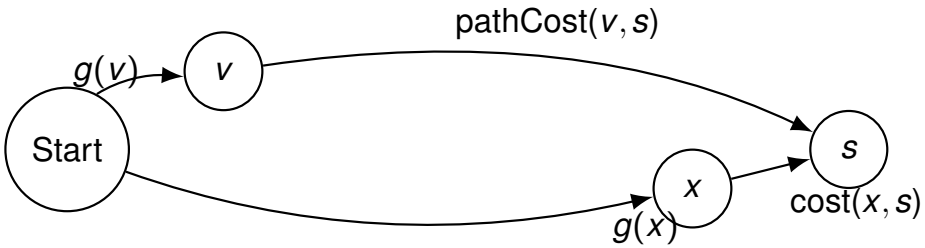

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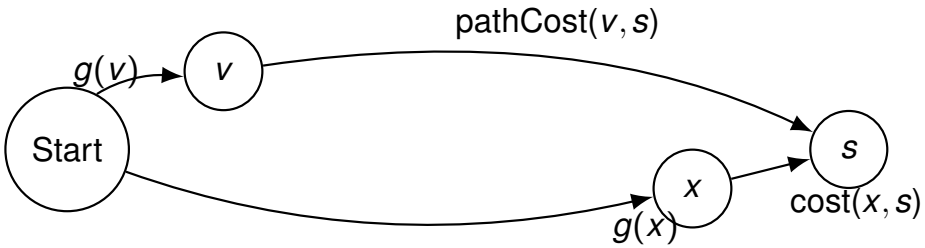

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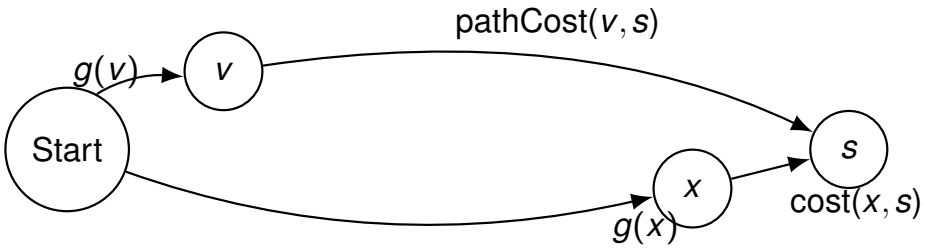

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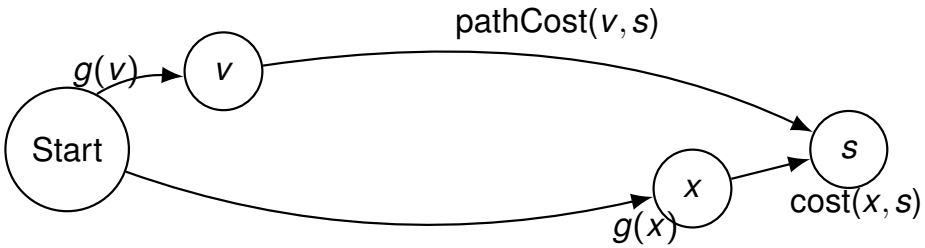

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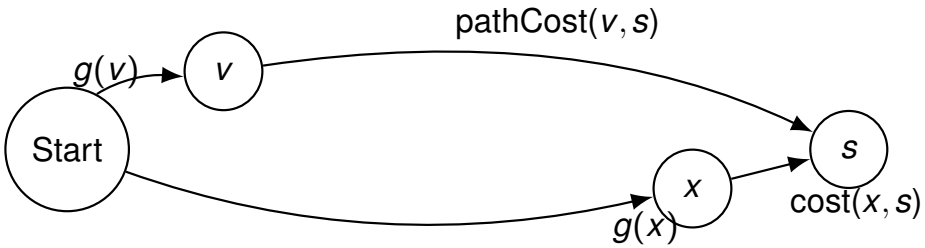

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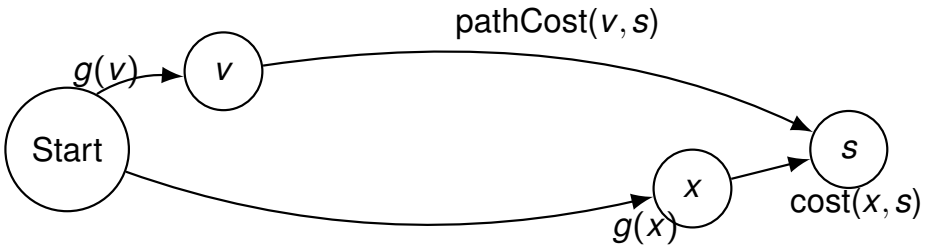

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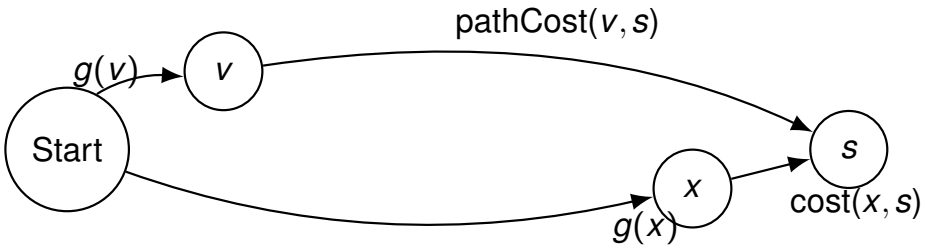

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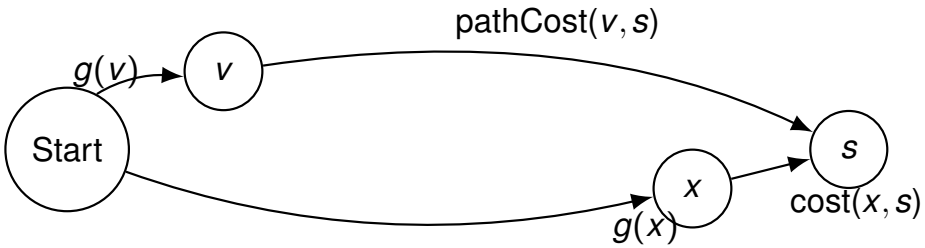

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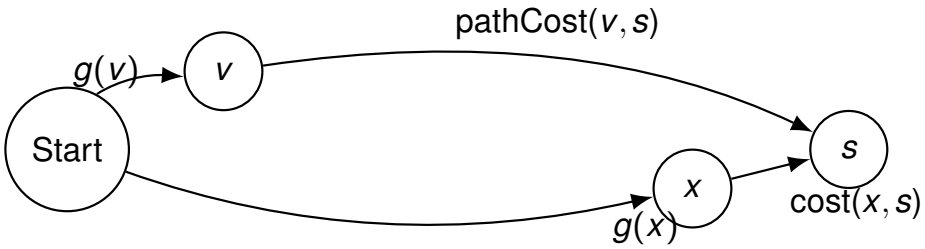

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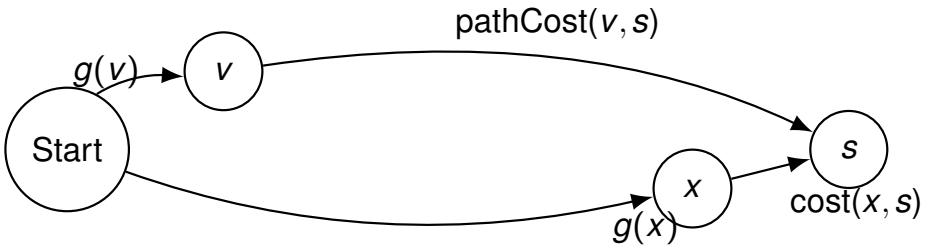

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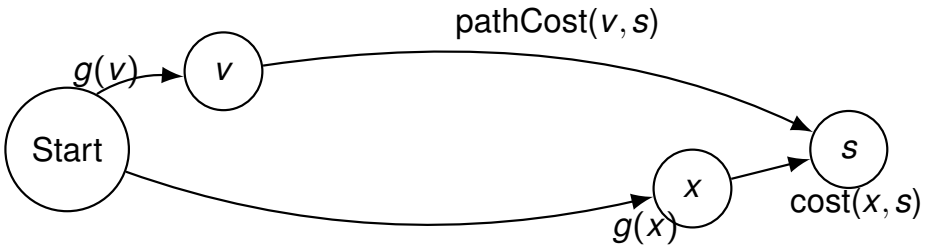

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But then v would have been expanded before s!

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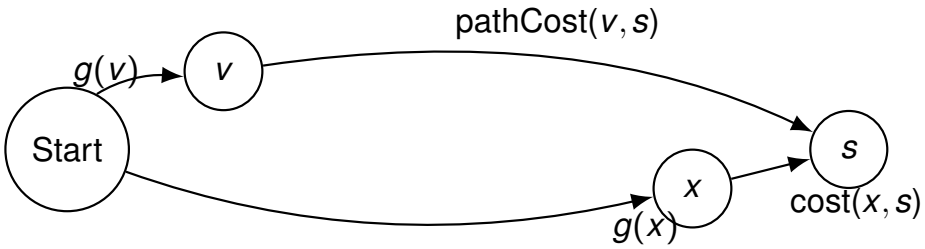

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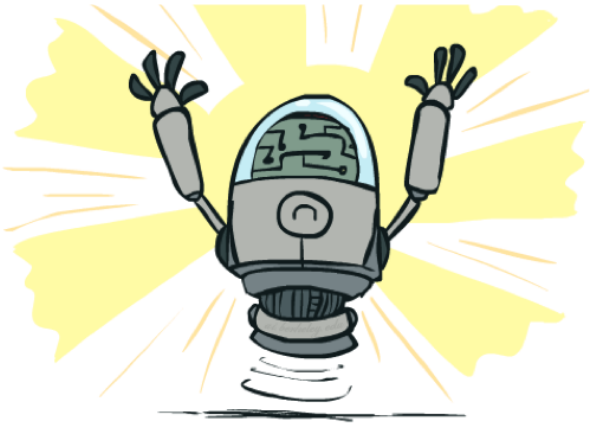
Optimality

Tree search:

A* is optimal if heuristic is admissible.



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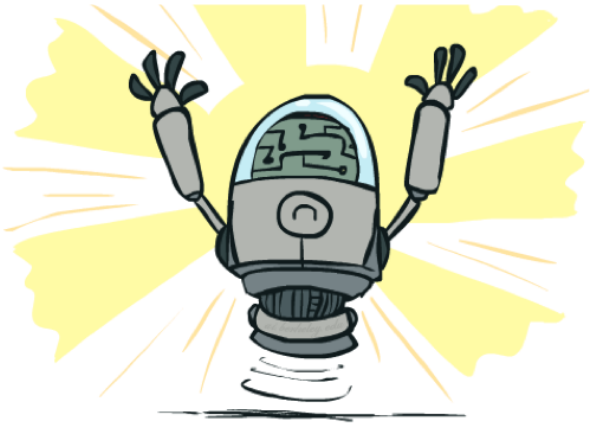

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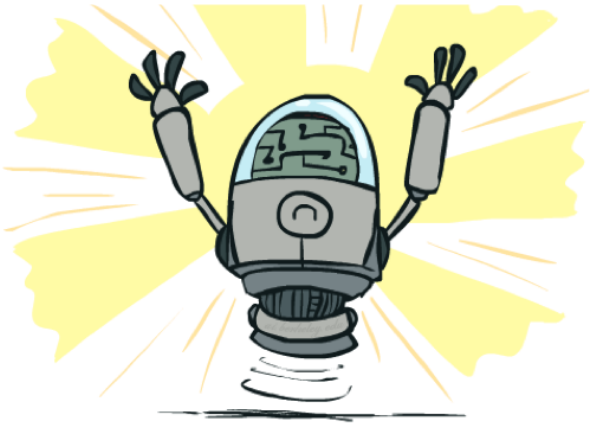
Optimality



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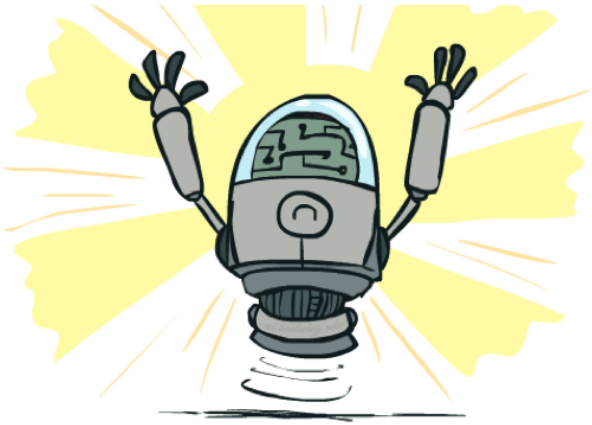

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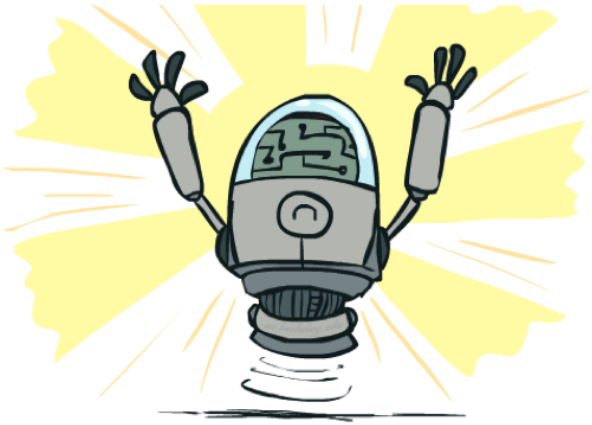
Graph search:

A* optimal if heuristic is consistent

Optimality



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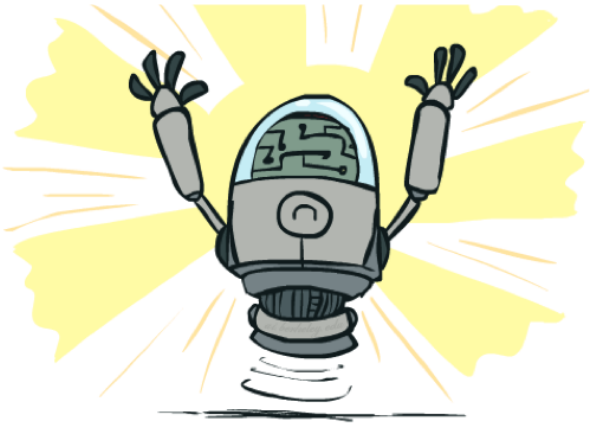

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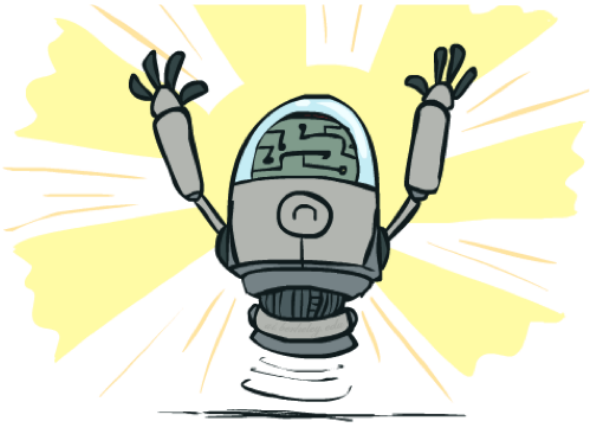

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- A* optimal if heuristic is consistent
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Consistency implies admissibility

In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

CS 188: Artificial Intelligence

Next:

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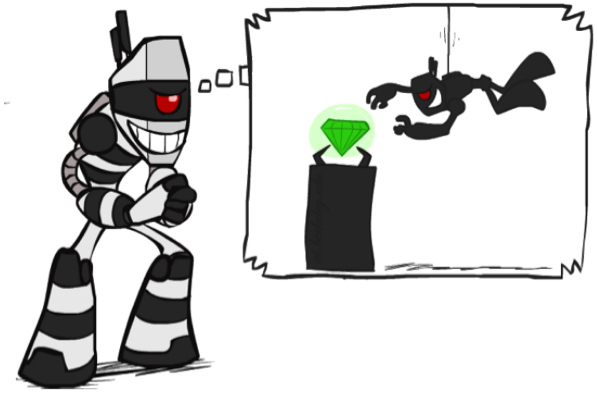
Next:

Constraint Satisfaction Problems.

What is Search For?

Assumptions about the world:



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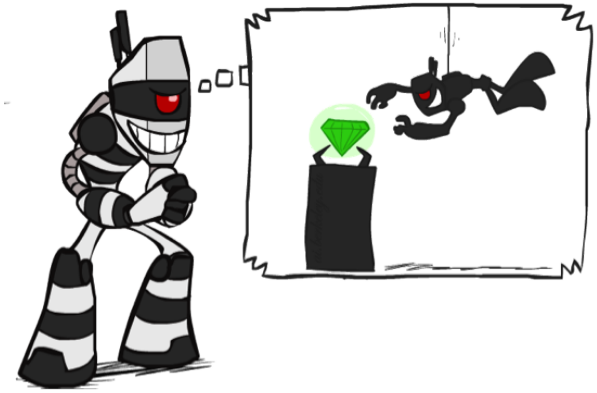

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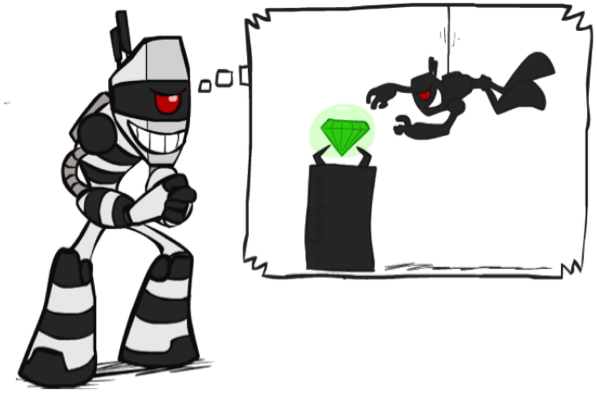

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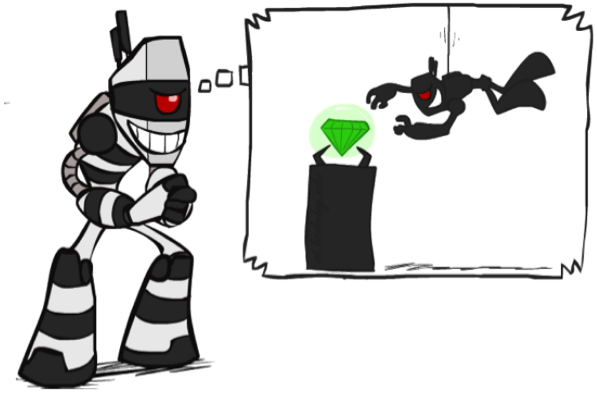

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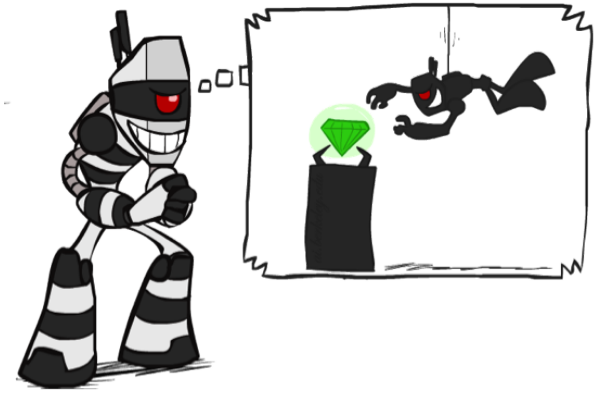

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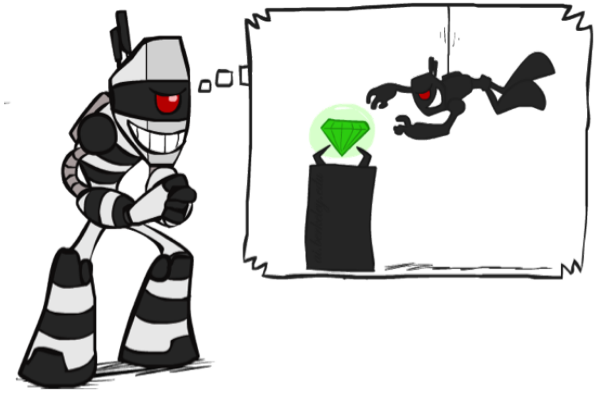

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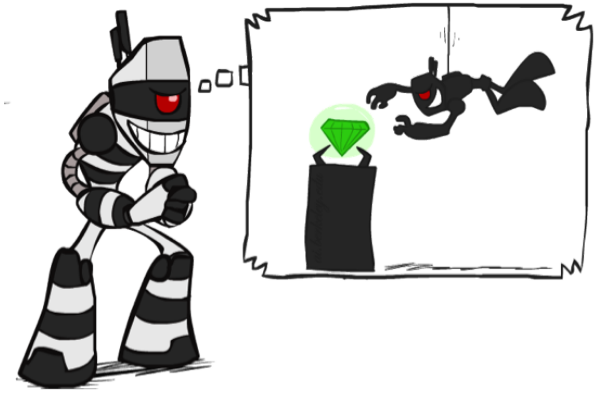

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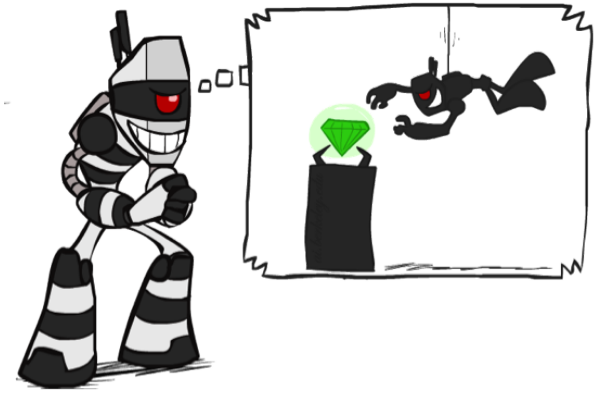

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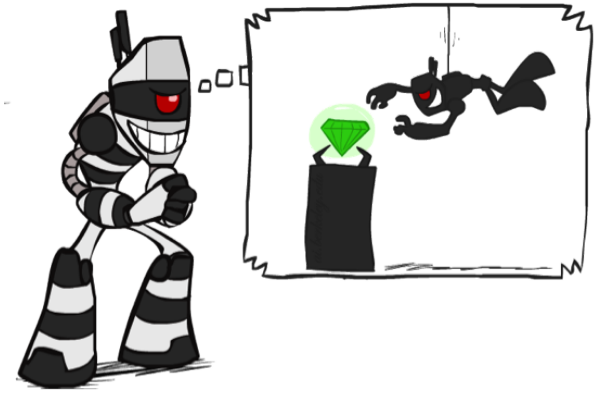

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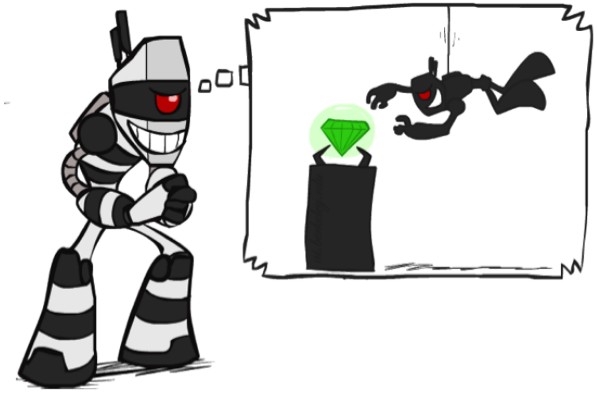

Identification: assignments to variables

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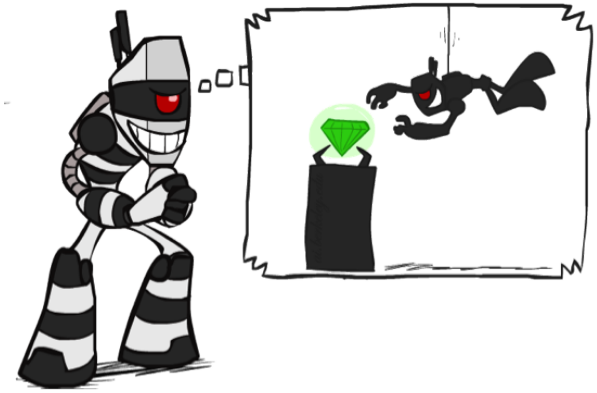

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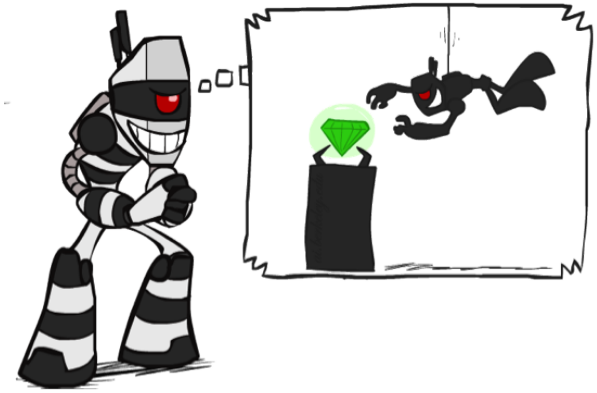

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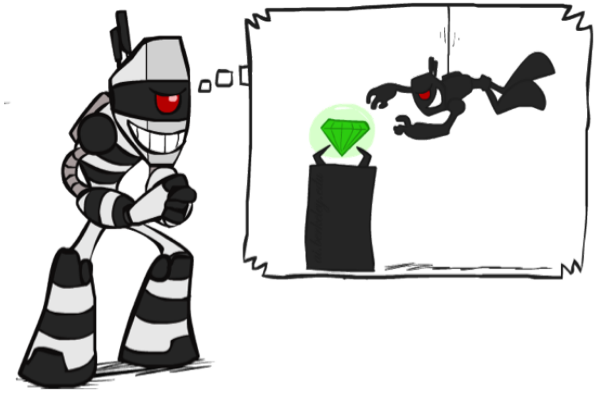

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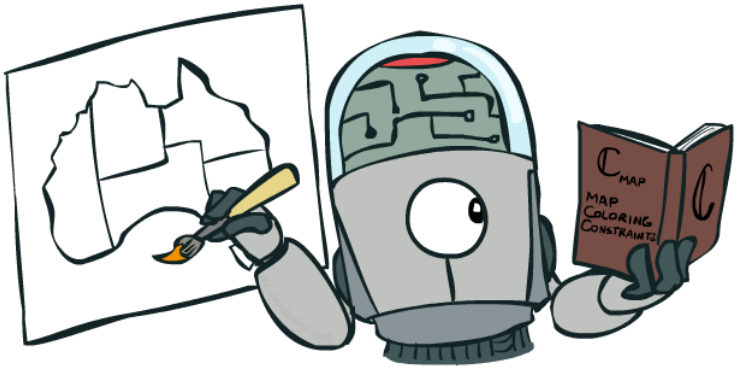
CSPs are specialized identification problems

Constraint Satisfaction Problems



Constraint Satisfaction Problems

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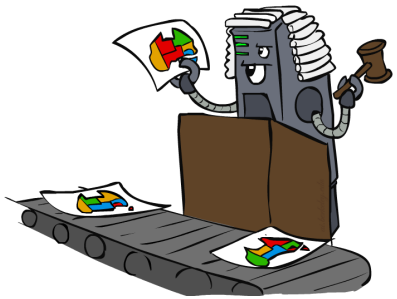

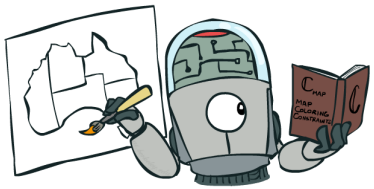

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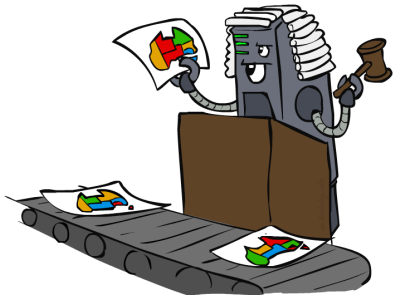

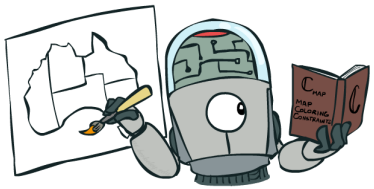

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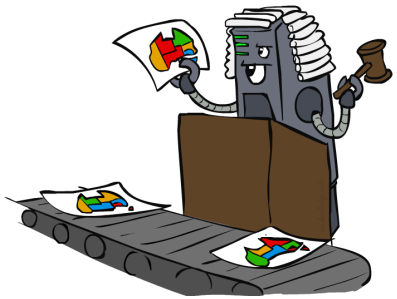

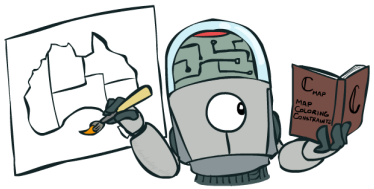

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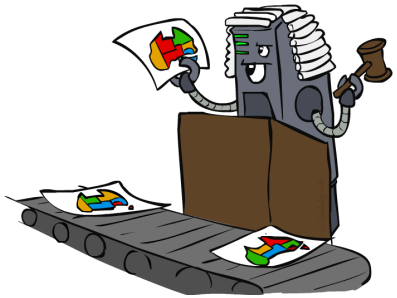

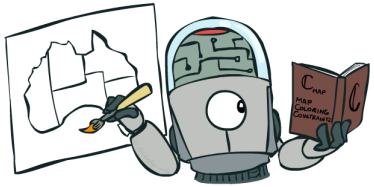

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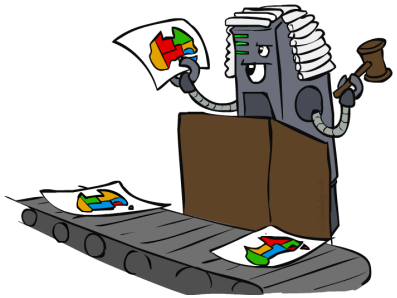

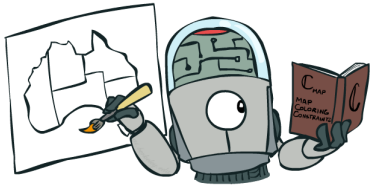

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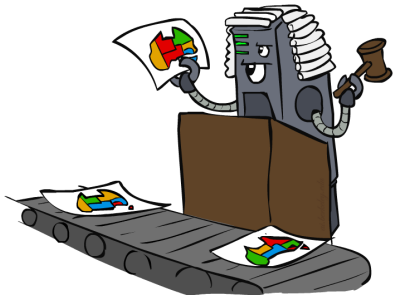

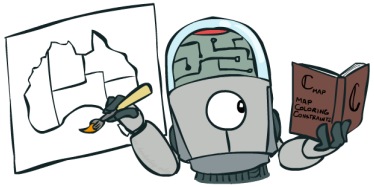

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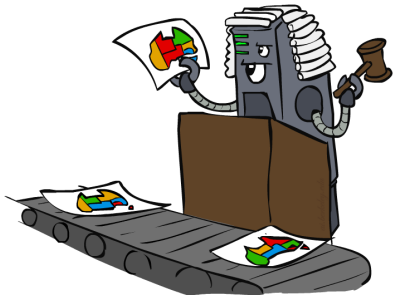

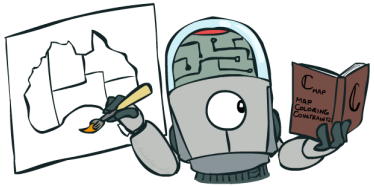

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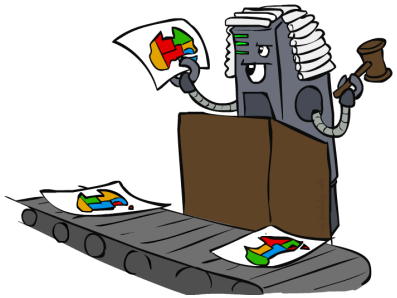

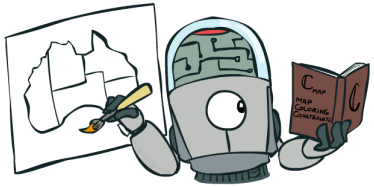

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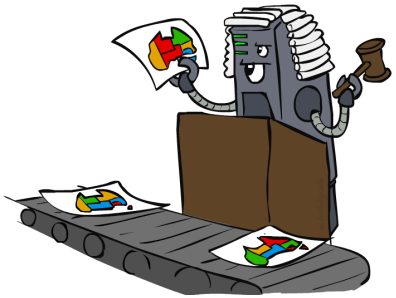

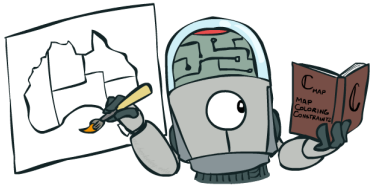

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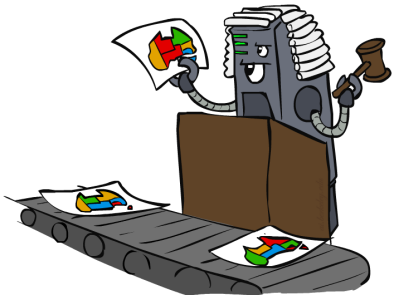

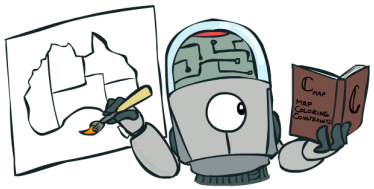

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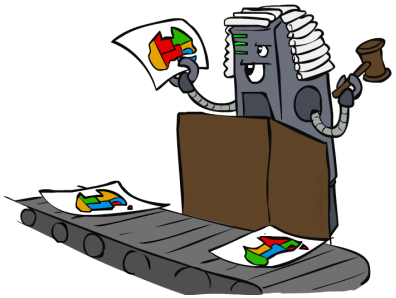

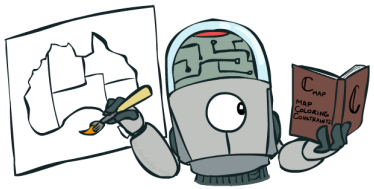

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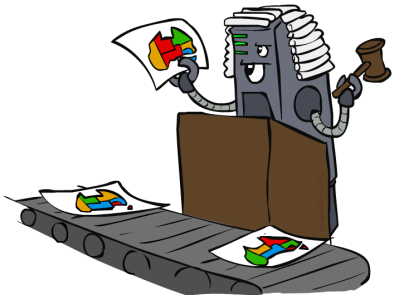

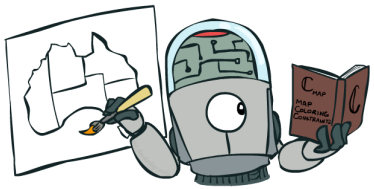

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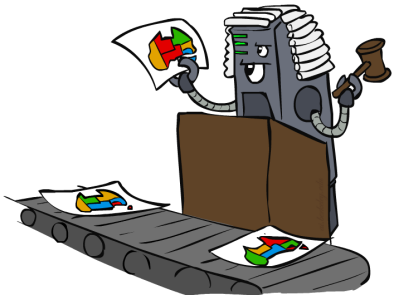

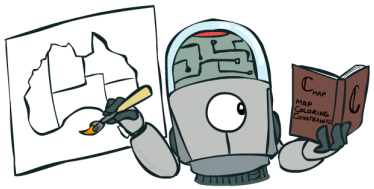

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Simple example of a formal representation language.

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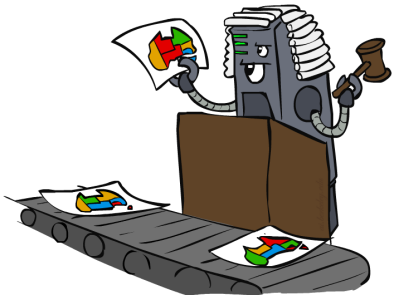

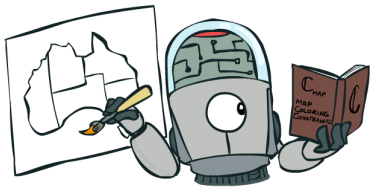

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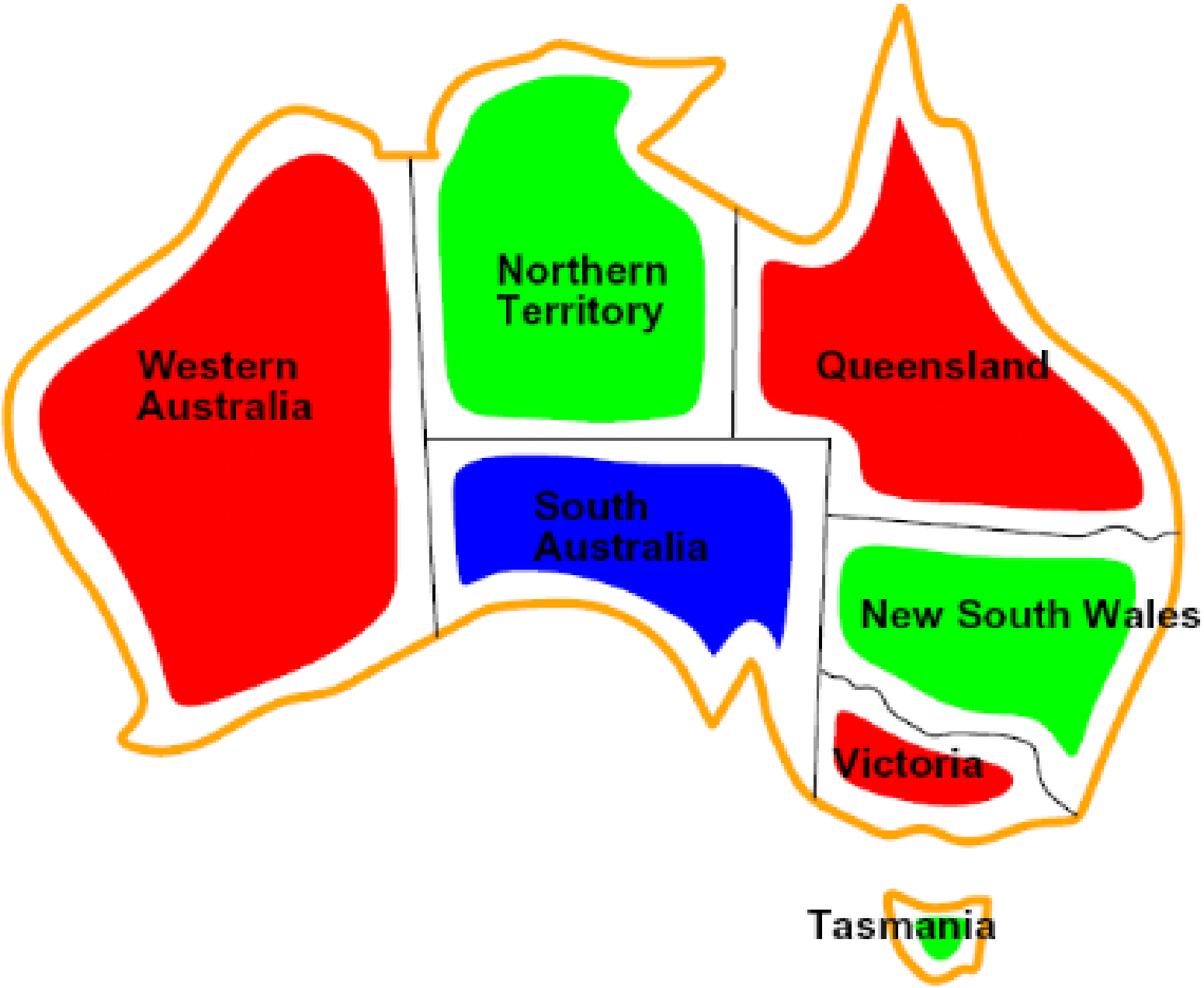
Simple example of a formal representation language.

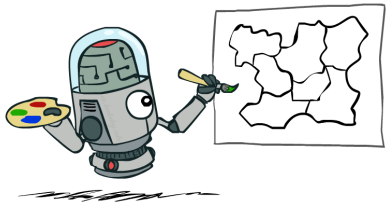
Allows useful still general-purpose algorithms with more power than standard search algorithms

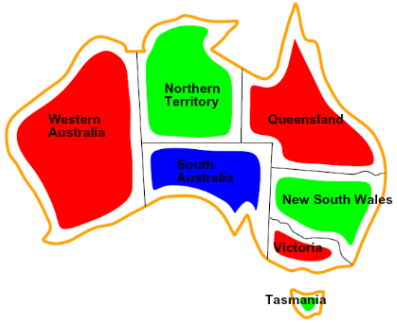
CSP Examples



Example: Map Coloring

Variables: **WA, NA, Q, NSW, V, SA, T**





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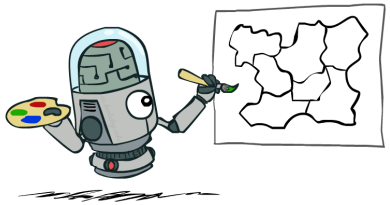
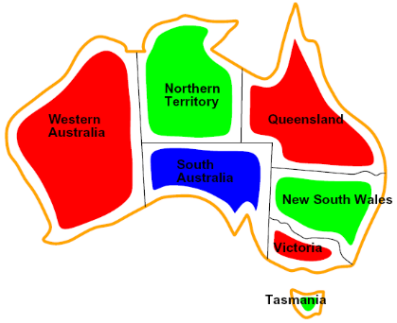


Variables: **WA, NA, Q, NSW, V, SA, T**

Domains: D=red,green,blue



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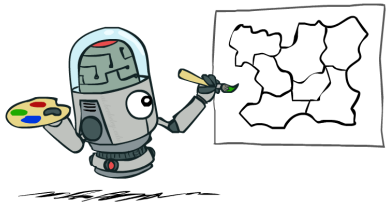

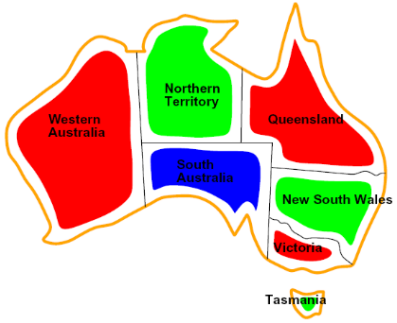

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Constraints: adjacent regions must have different colors.

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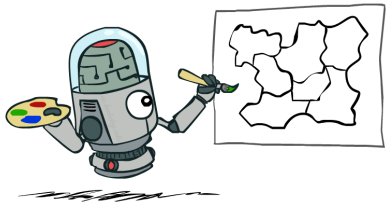

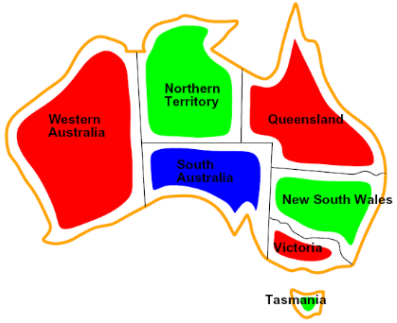

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Implicit: WA $\neq$ NT.

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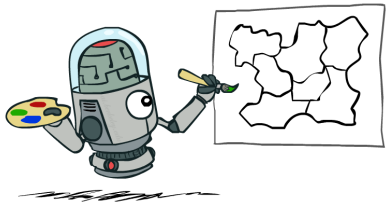

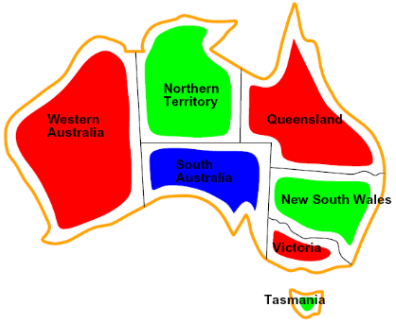

Variables: **WA, NA, Q, NSW, V, SA, T**

Domains: D=red,green,blue

Constraints: adjacent regions must have different colors.

Implicit: WA $\neq$ NT.

Explicit: (WA,NT) $\in \{$ (red,green),(red,blue),...,$\}$.

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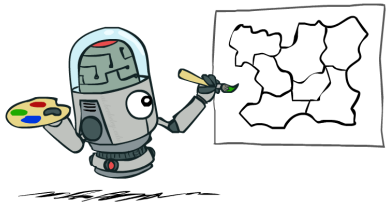
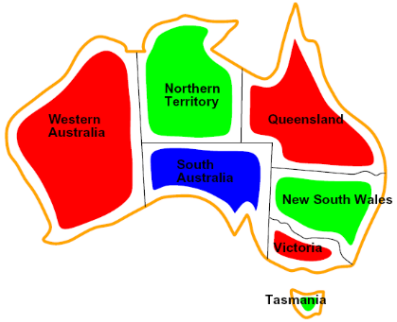




Variables: **WA, NA, Q, NSW, V, SA, T**

Domains: D=red,green,blue

Constraints: adjacent regions must have different colors.

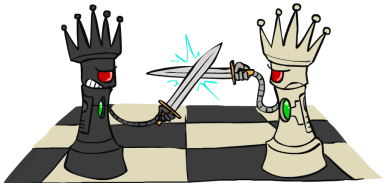
Implicit: WA $\neq$ NT.

Explicit: (WA,NT) $\in$ { (red,green),(red,blue),...,}.

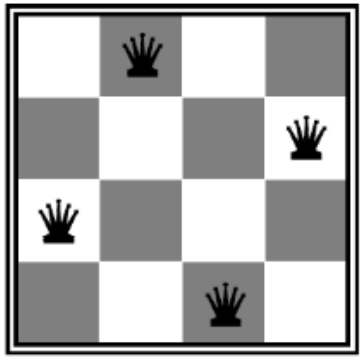
Goal Test: do assignments satisfy all constraints?

{ **WA = red, NT= green, Q = red, NSW=green, V=red, SA=blue, T=green** }

Example: N-Queens



Formulation 1:



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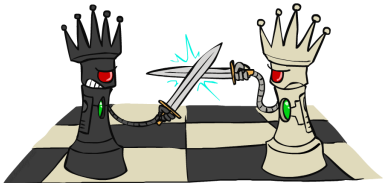
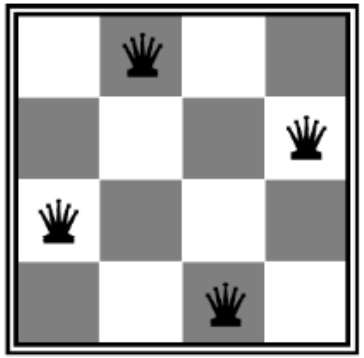
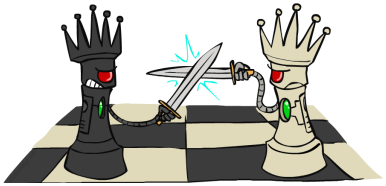




Formulation 1:

Variables: X_{ij}

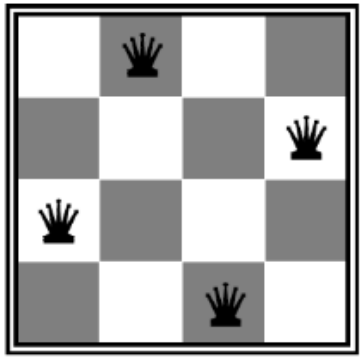
Example: N-Queens



Formulation 1:

Variables: X_{ij}

Domains: $\{0, 1\}$



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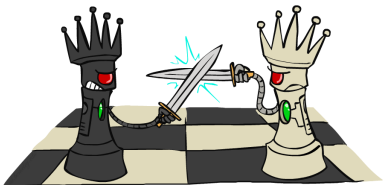

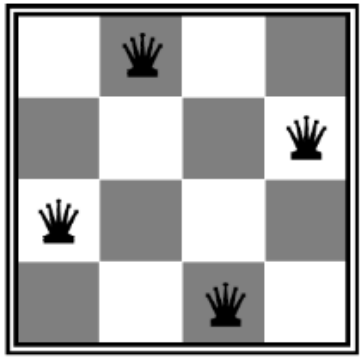

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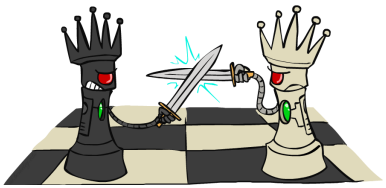

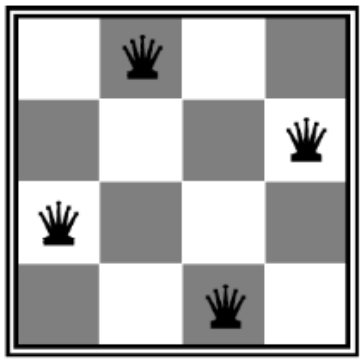

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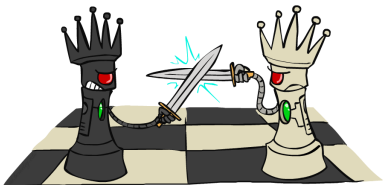

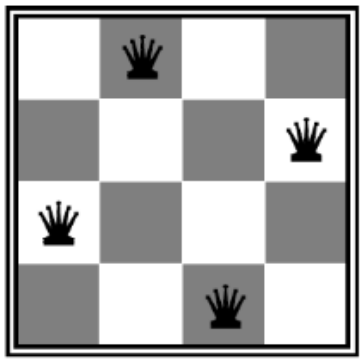

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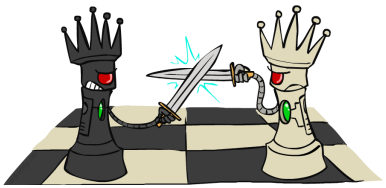

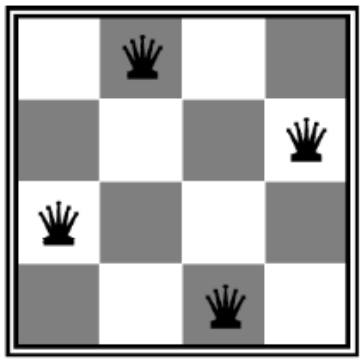

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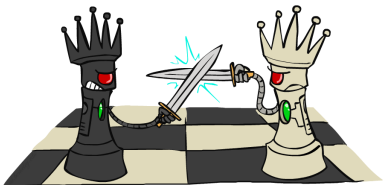

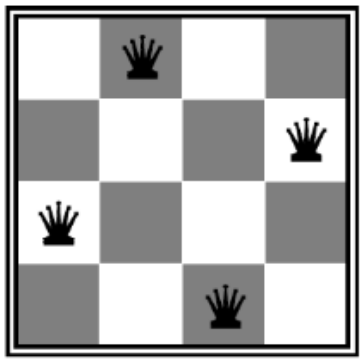

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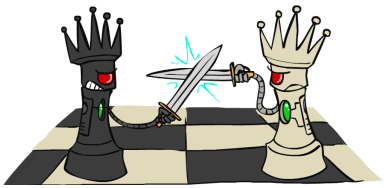

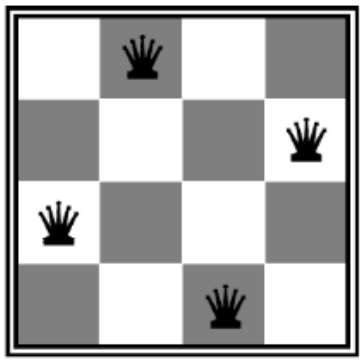

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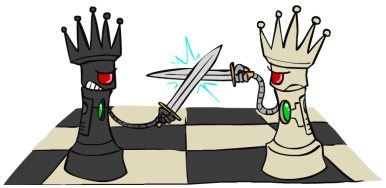

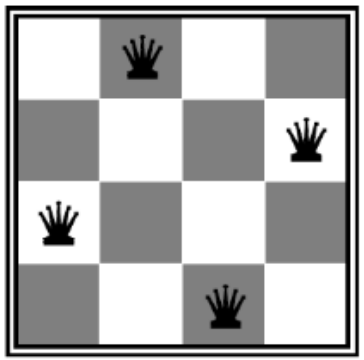

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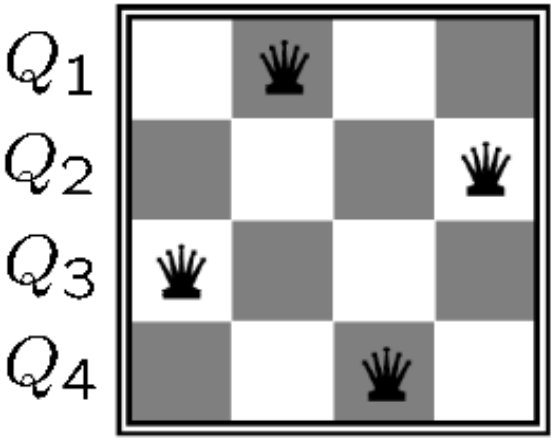
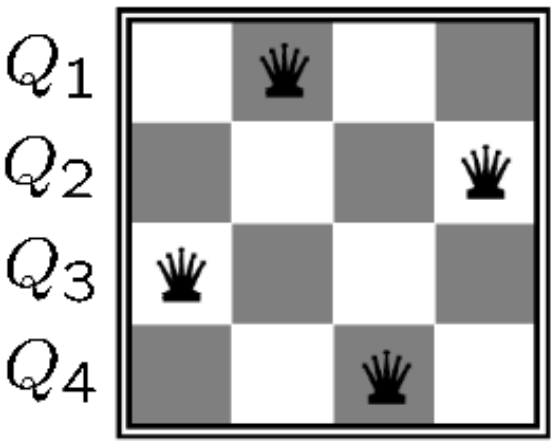
$$\sum_{ij} X_{ij} = N$$

Example: N-Queens



Formulation 2:
Variables: Q_k

Example: N-Queens



Formulation 2:

Variables: Q_k

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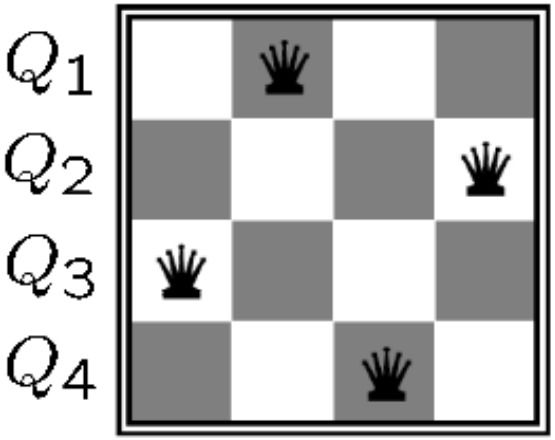

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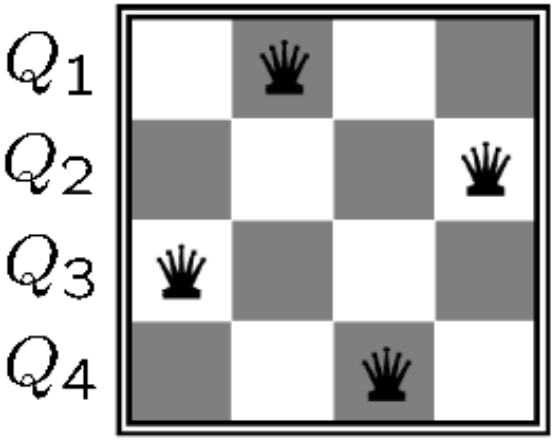

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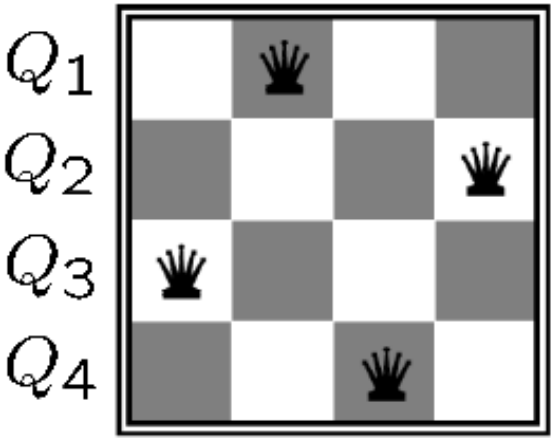

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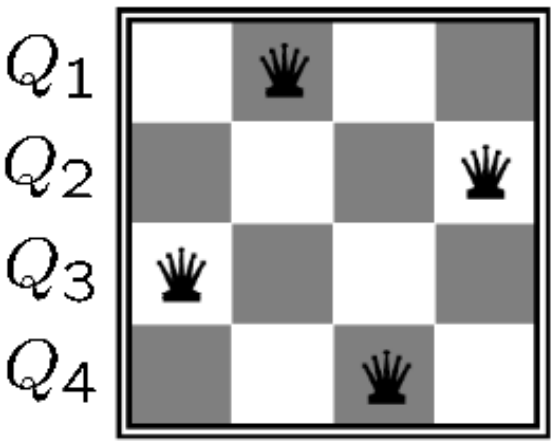

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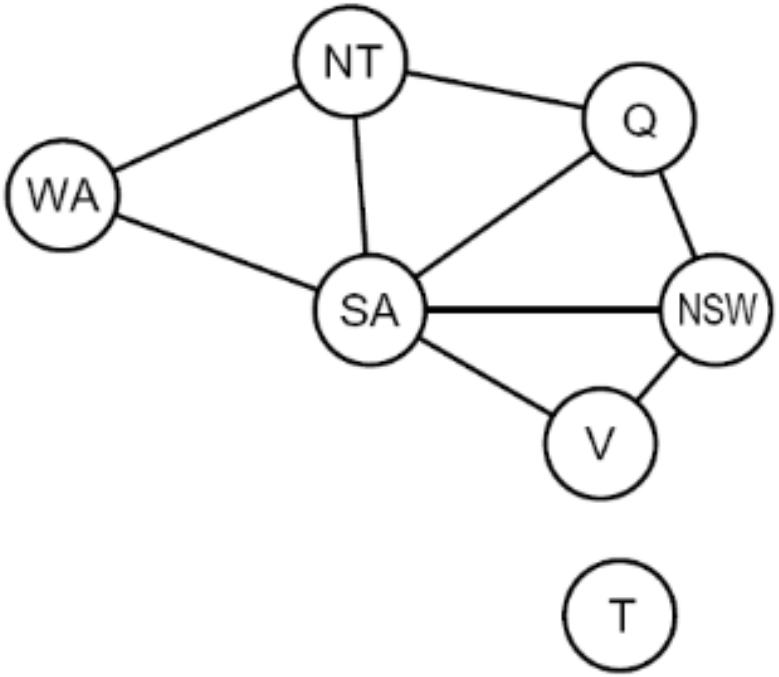
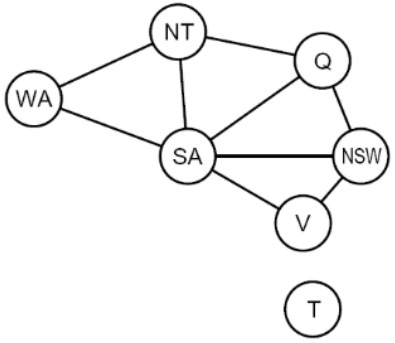
Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

Constraint Graphs



Constraint Graphs

Binary CSP: each constraint relates (at most) two variables



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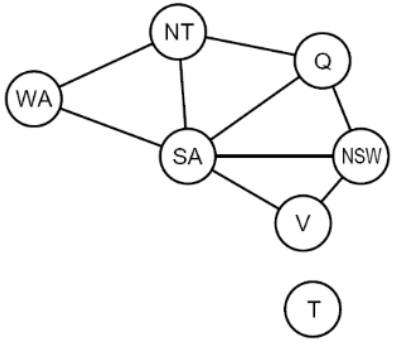

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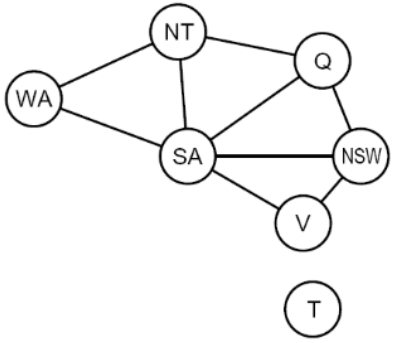

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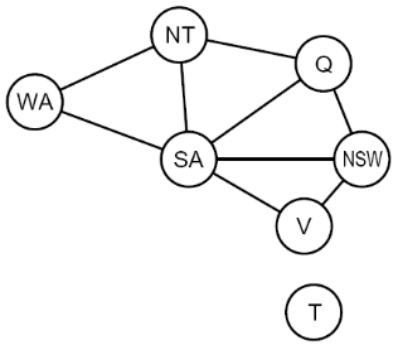
Constraint Graphs



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E.g., Tasmania is an independent subproblem!

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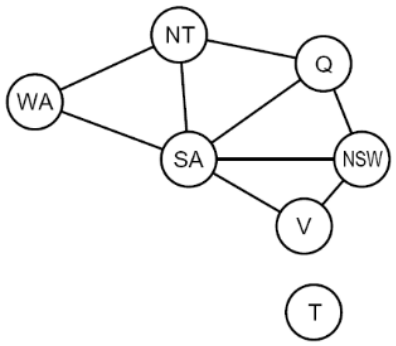

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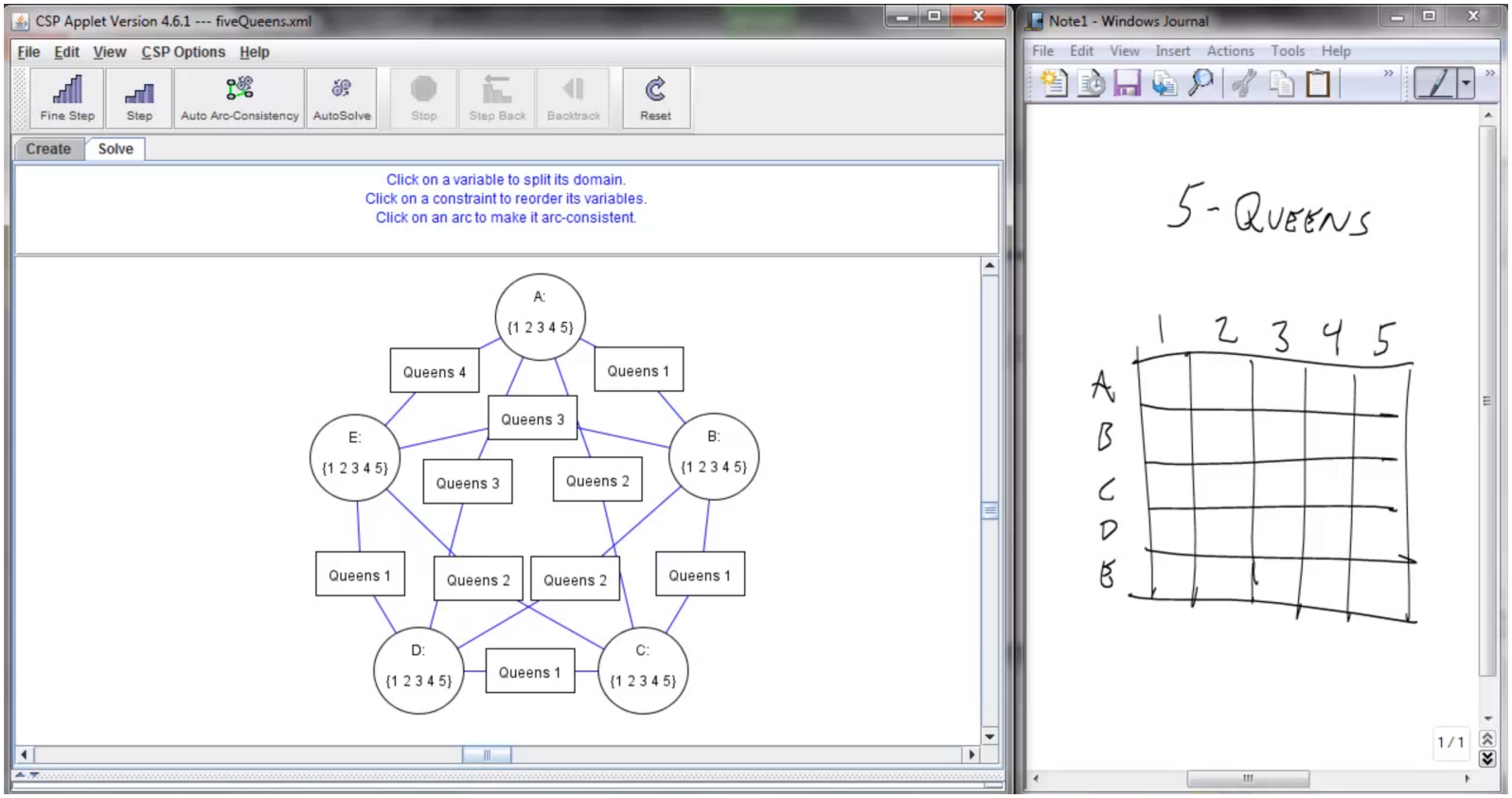
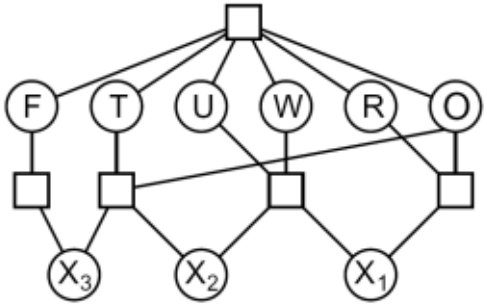
[Demo: CSP applet (made available by aispace.org) – n-queens]

5-Queens


CSP Applet Version 4.6.1 --- fiveQueens.xml
File Edit View CSP Options Help
Fine Step
Step
Auto Arc-Consistency
AutoSolve
Stop
Step Back
Backtrack
Reset
Create
Solve
Click on a variable to split its domain.
Click on a constraint to reorder its variables.
Click on an arc to make it arc-consistent.
A: {1 2 3 4 5}
B: {1 2 3 4 5}
C: {1 2 3 4 5}
D: {1 2 3 4 5}
E: {1 2 3 4 5}
Queens 1
Queens 2
Queens 3
Queens 4
Note1 - Windows Journal
File Edit View Insert Actions Tools Help
5-QUEENS
1 2 3 4 5
A
B
C
D
E
1 / 1

Example: Cryptarithmetic

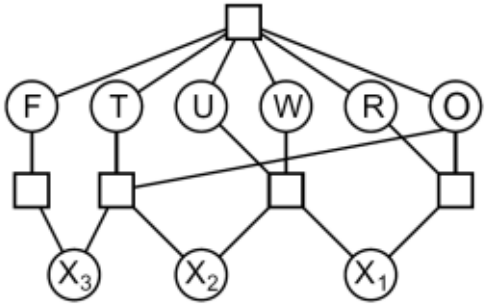
```
   T W O
+  T W O
-------
F O U R
```



Variables:

Example: Cryptarithmetic

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$



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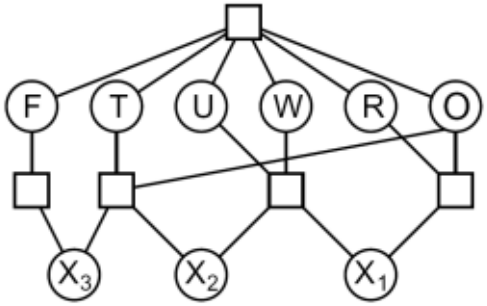

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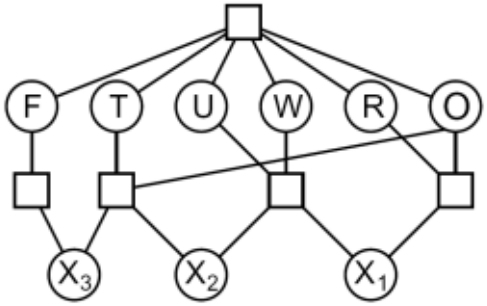

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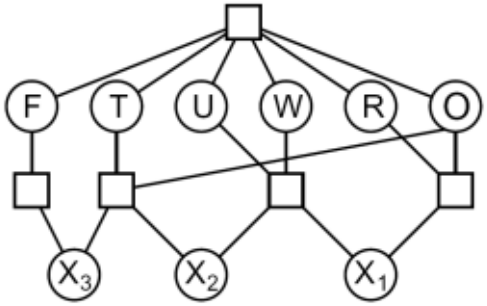

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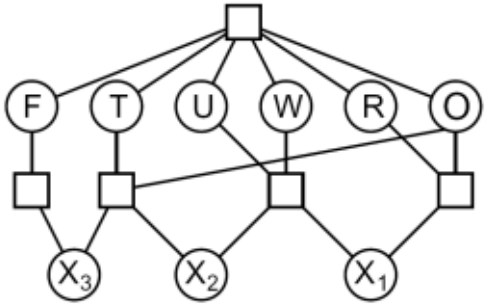

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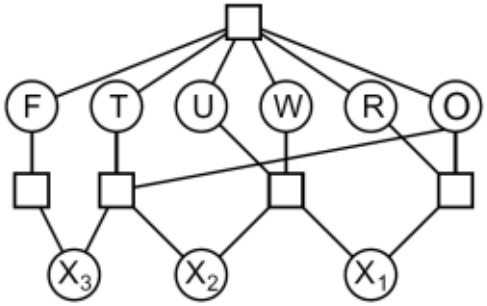

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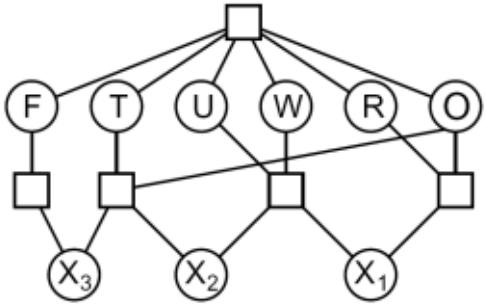

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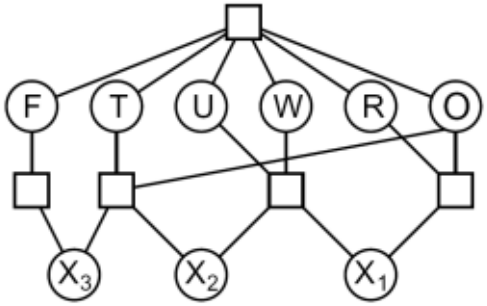

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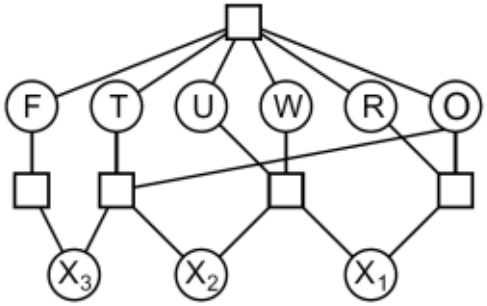

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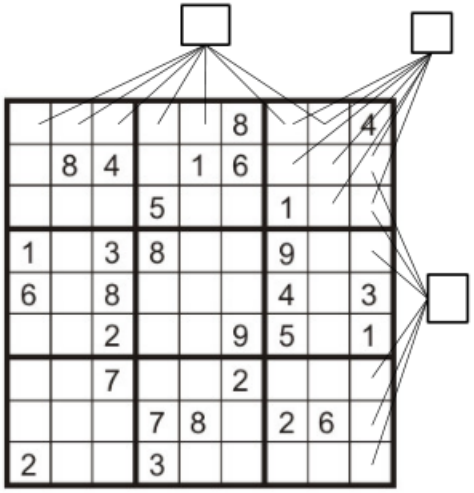
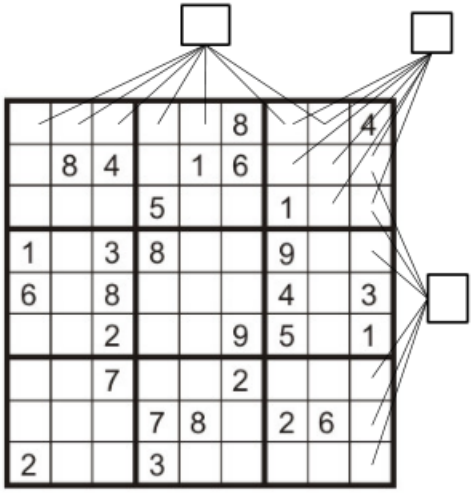
$\ldots$

Example: Sudoku



Variables:

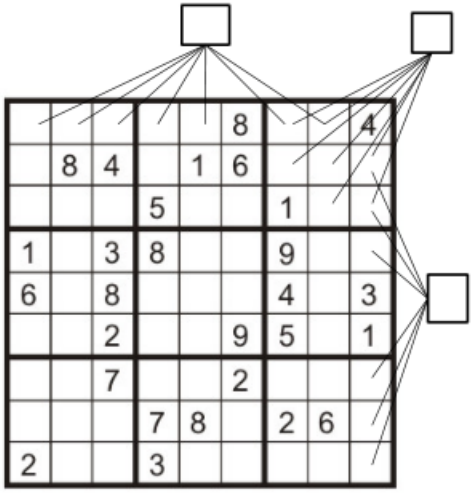
Example: Sudoku



Variables:

Each (open) square.

Example: Sudoku



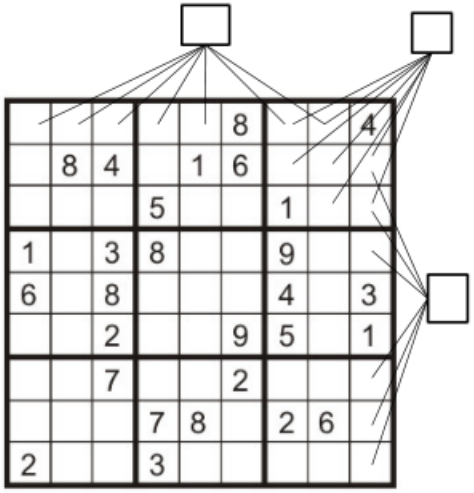
Variables:

Each (open) square.

Domains:

$\{1, 2, \ldots, 9\}$

Example: Sudoku



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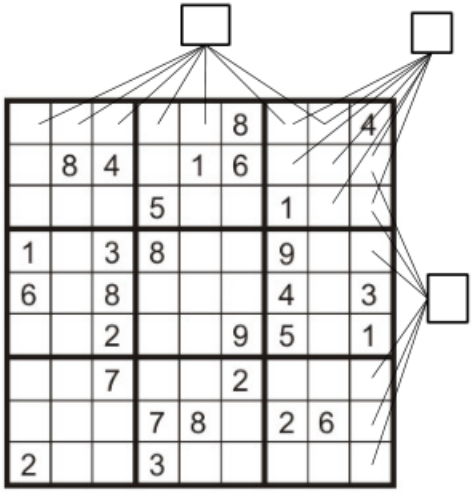

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Constraints:

9-way alldiff for each row

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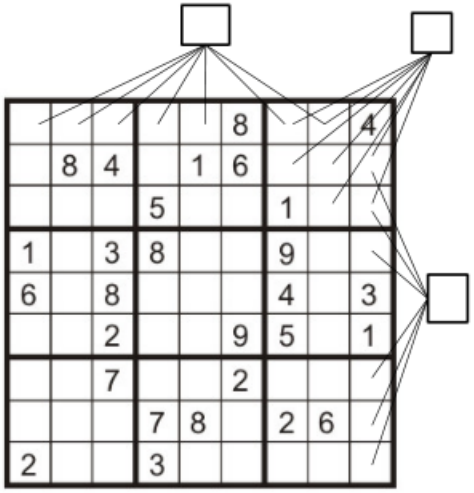

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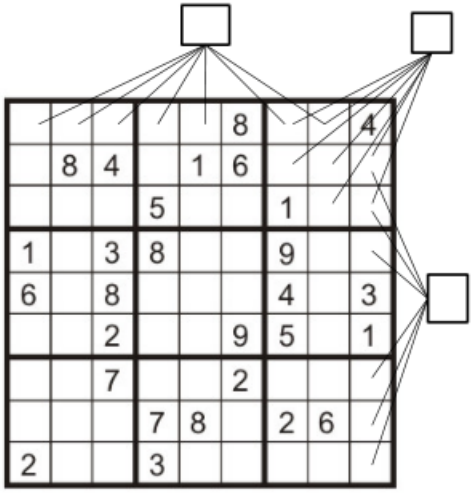

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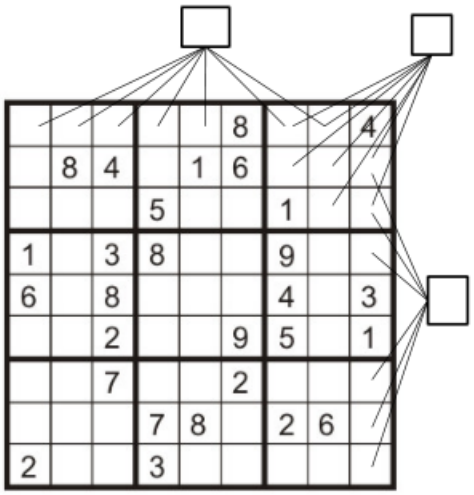

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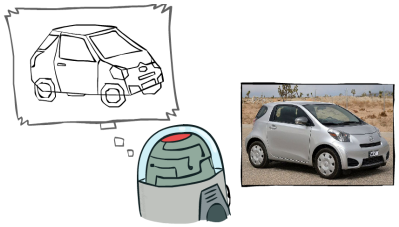
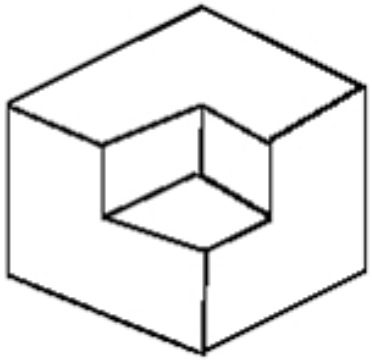
(or can have a bunch of pairwise inequality constraints)

Example: The Waltz Algorithm



The Waltz algorithm is for interpreting line drawings of solid polyhedra as 3D objects



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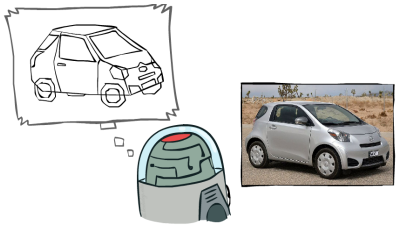

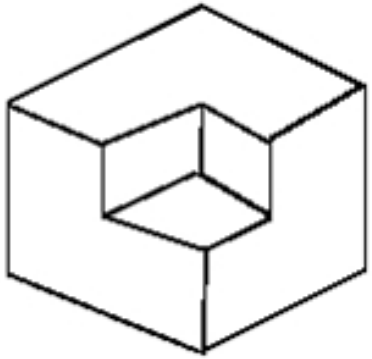

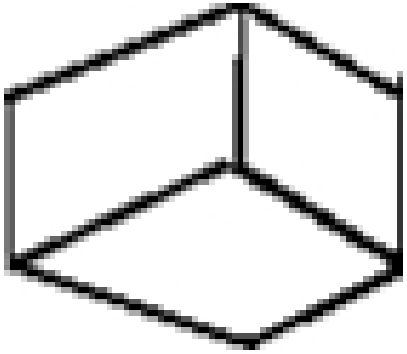

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An early example of an AI computation posed as a CSP

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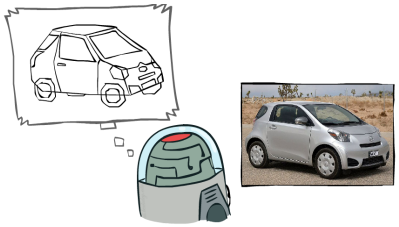

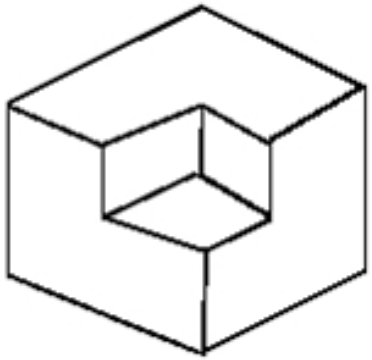

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Approach:

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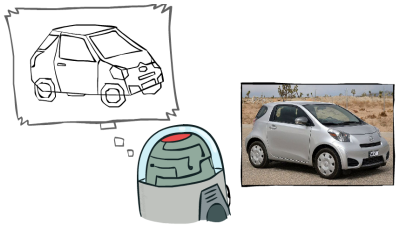

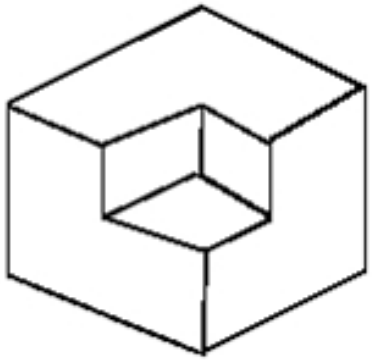

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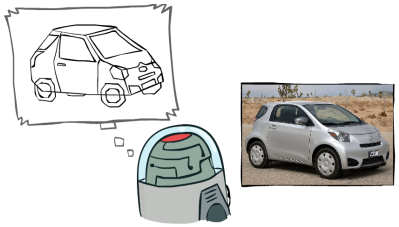

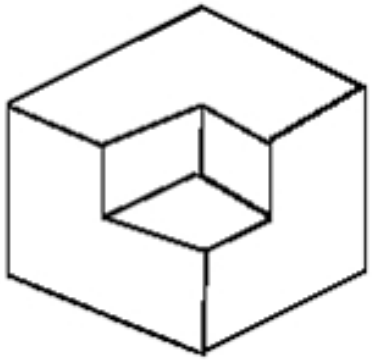

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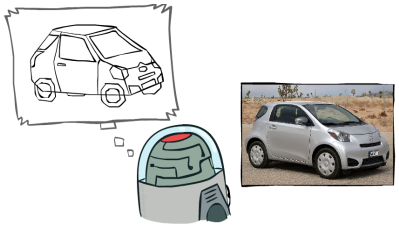

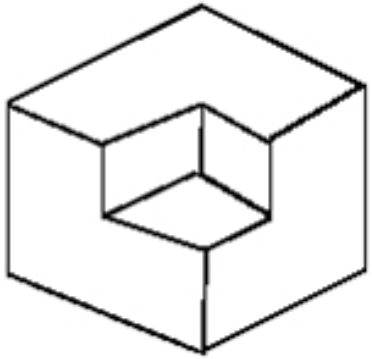

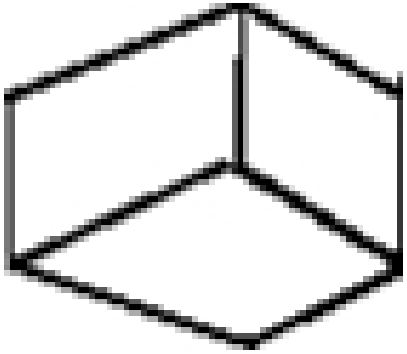

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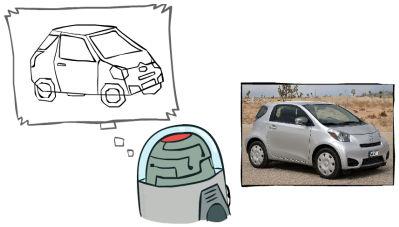

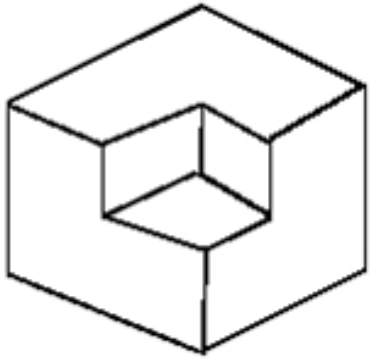

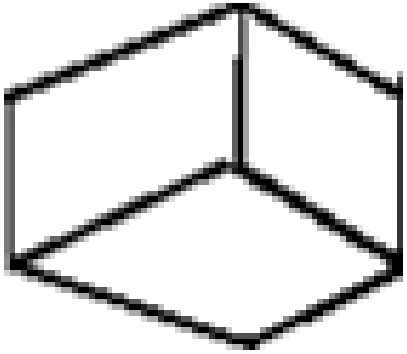

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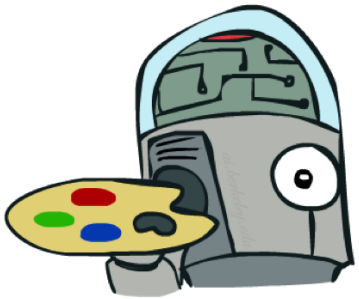
Varieties of CSPs and Constraints

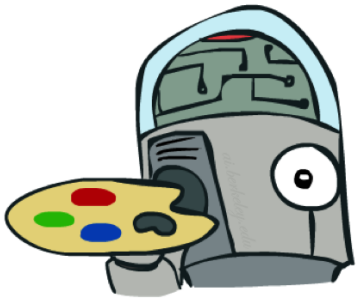
Varieties of CSPs

Discrete Variables



Varieties of CSPs

Discrete Variables
Finite domains



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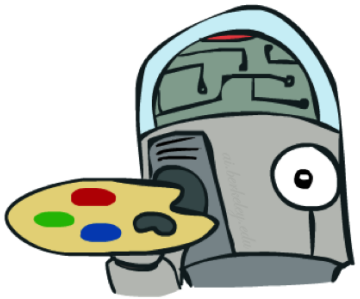

Discrete Variables

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  - Size d means $O(d^n)$ complete assignments.

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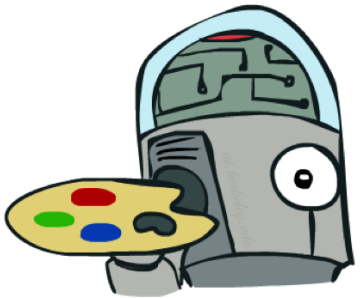

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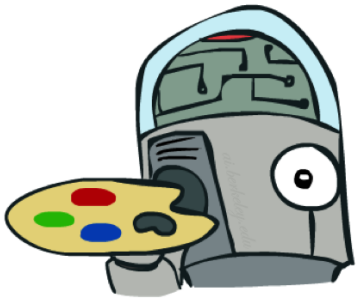

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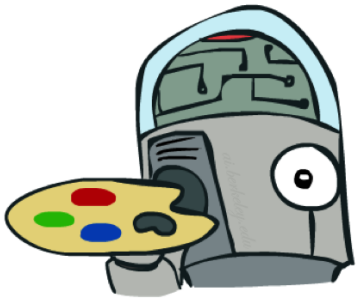

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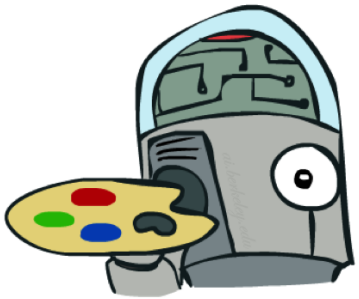

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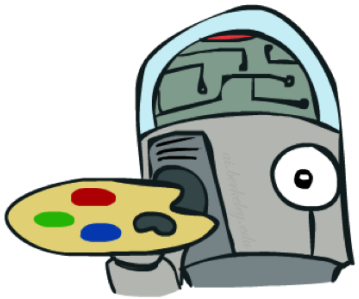

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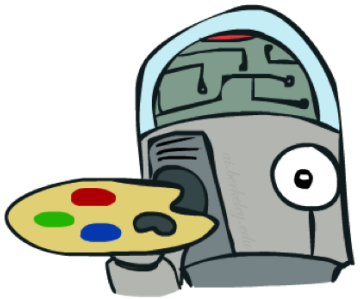

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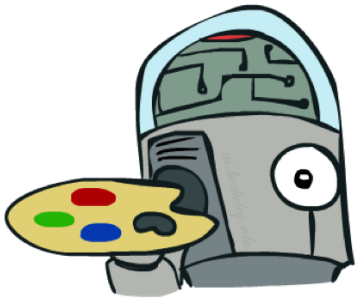

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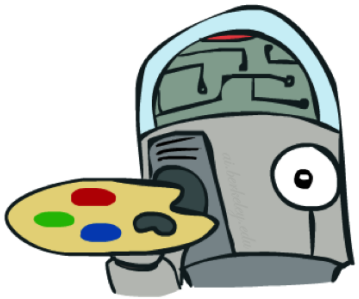

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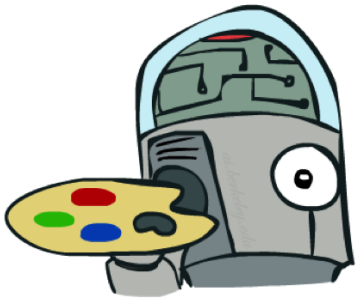

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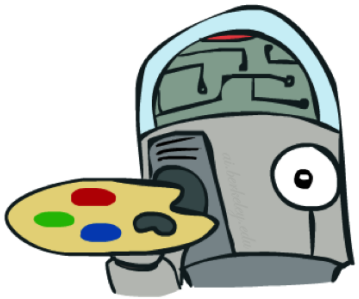
Varieties of CSPs



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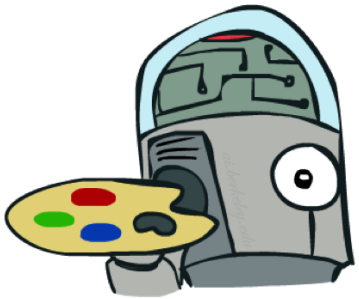

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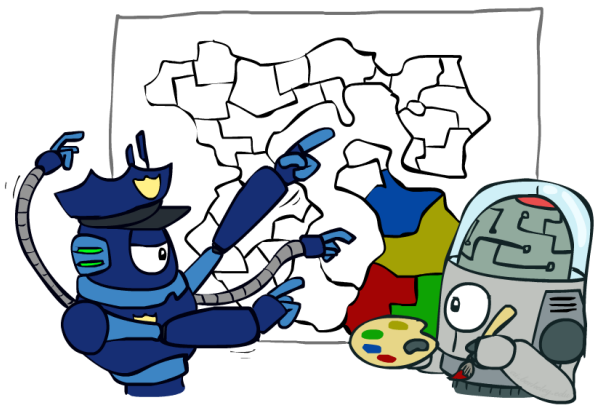

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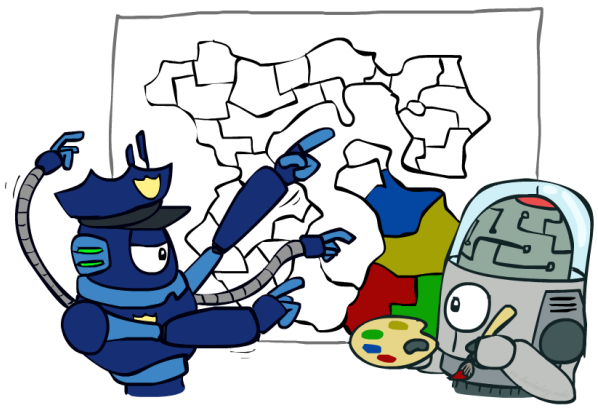

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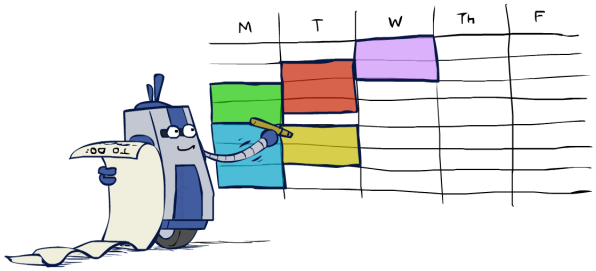
(We'll ignore these until we get to Bayes' nets)

Real-World CSPs

Assignment problems: e.g., who teaches what class



Real-World CSPs

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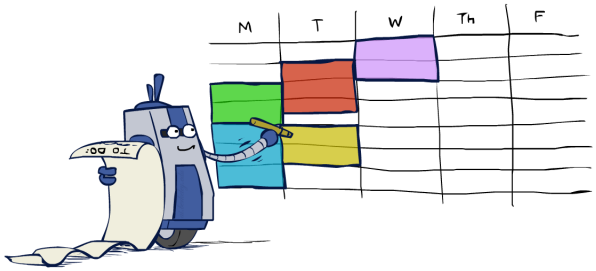

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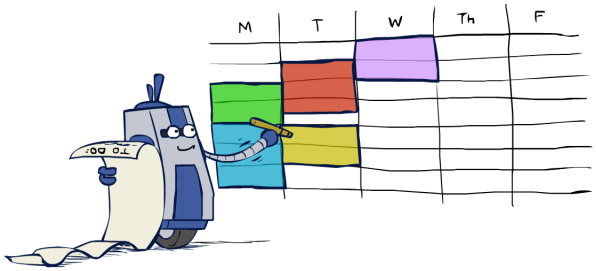

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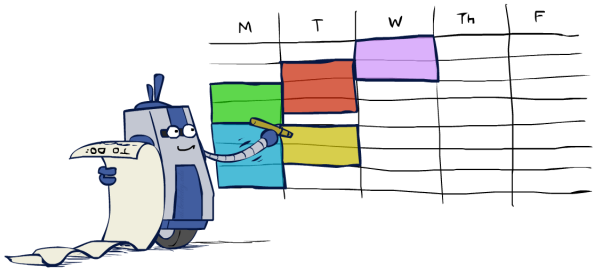

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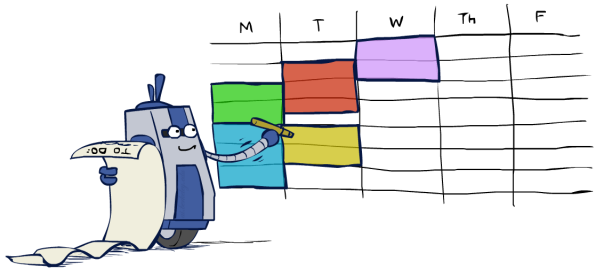

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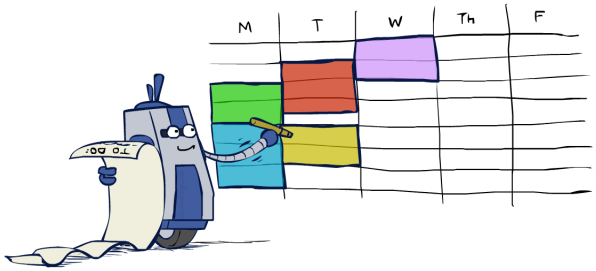

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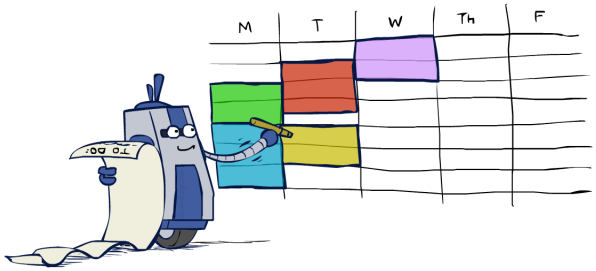

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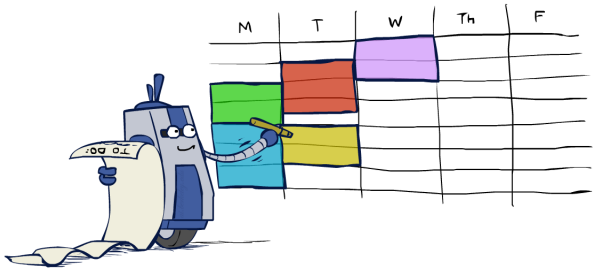

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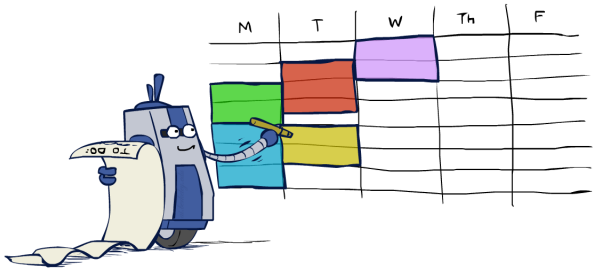

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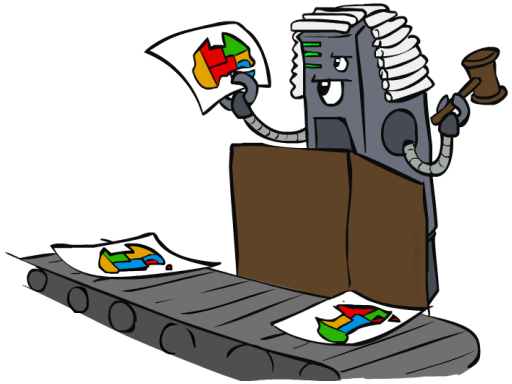
Many real-world problems involve real-valued variables. . .

Solving CSPs

Standard Search Formulation

Standard search formulation of CSPs



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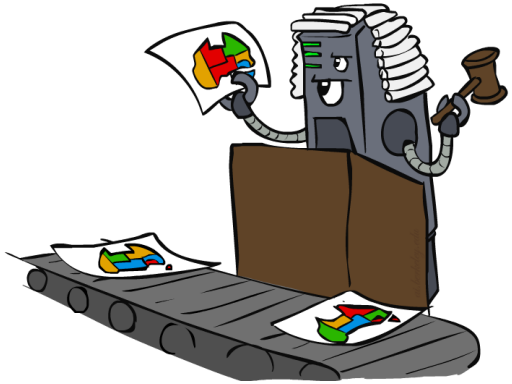

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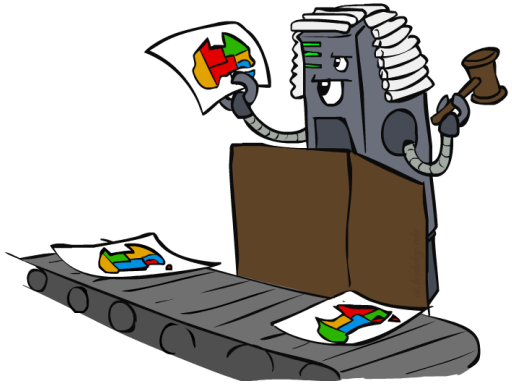

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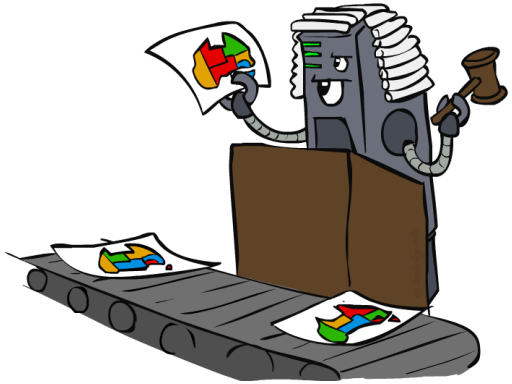

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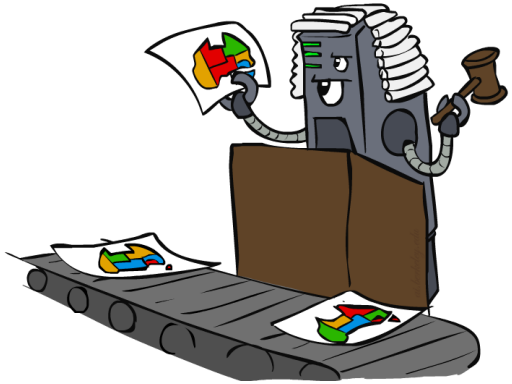

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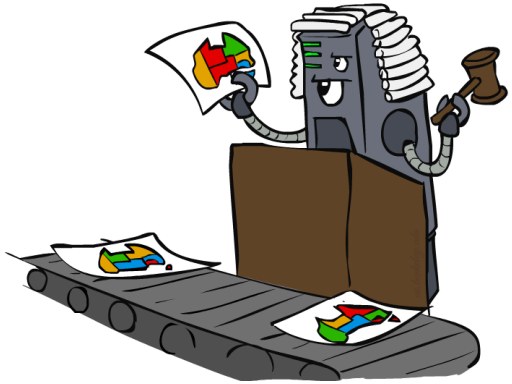

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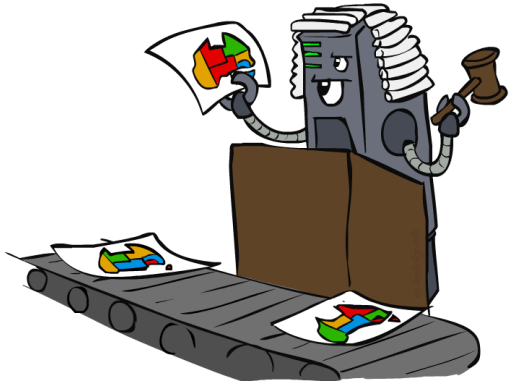

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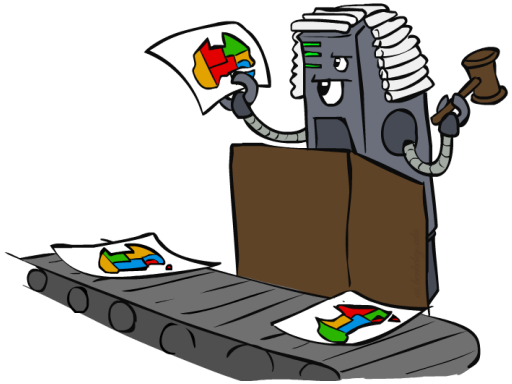

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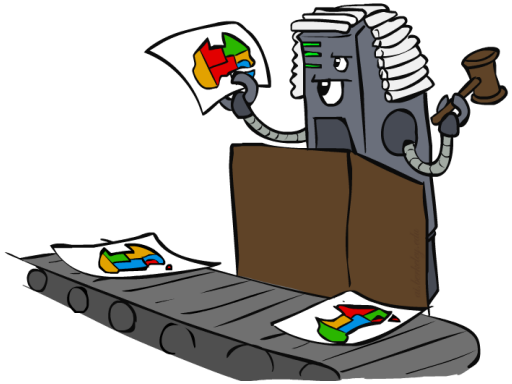

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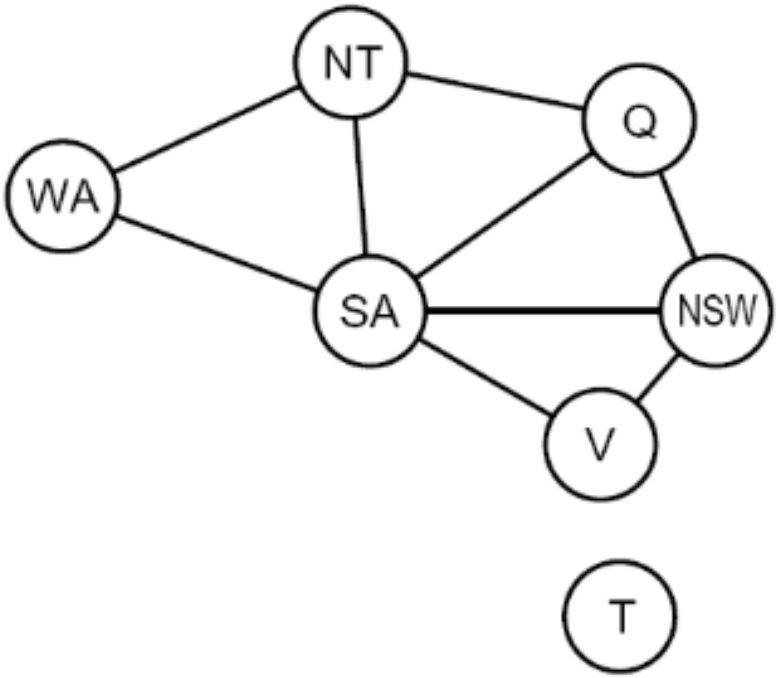
We'll start with the straightforward, naive approach, then improve it

Search Methods



What would BFS do?

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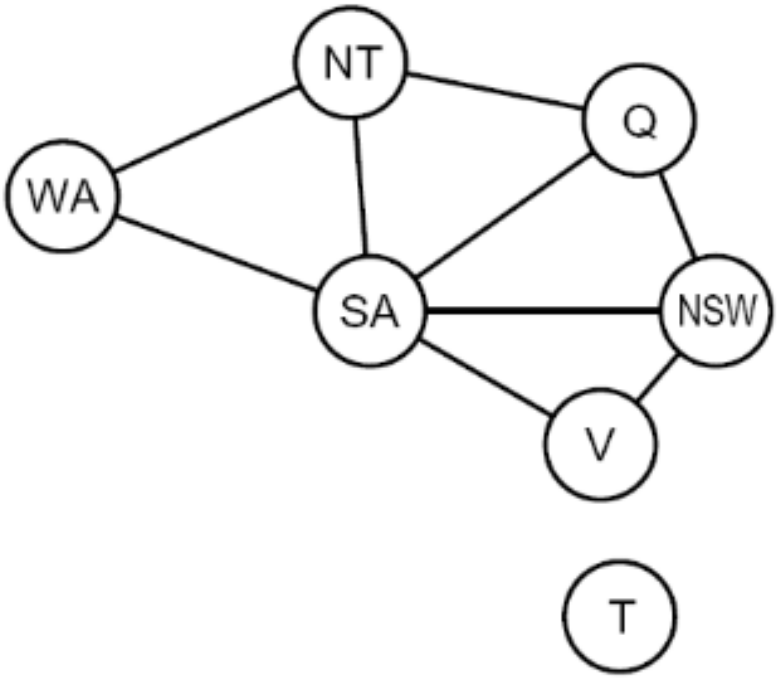

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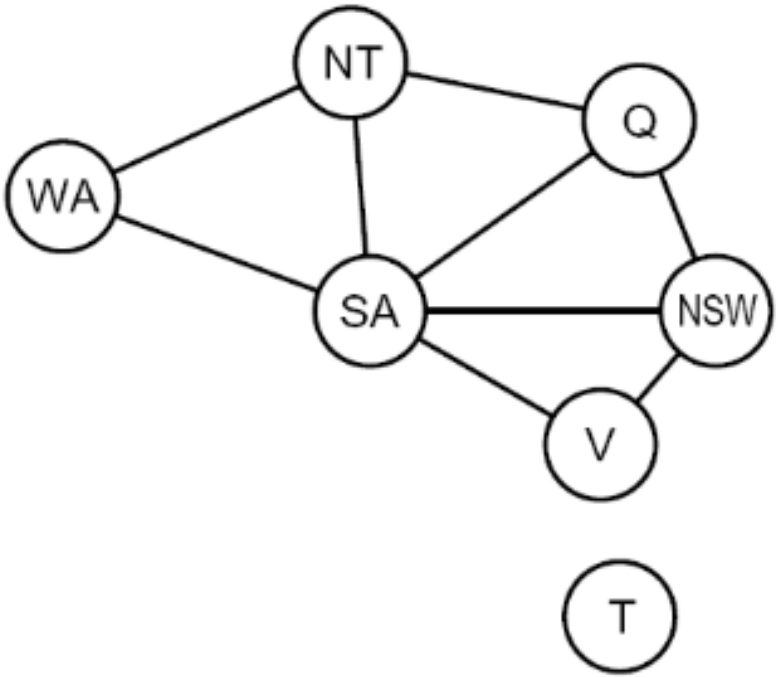
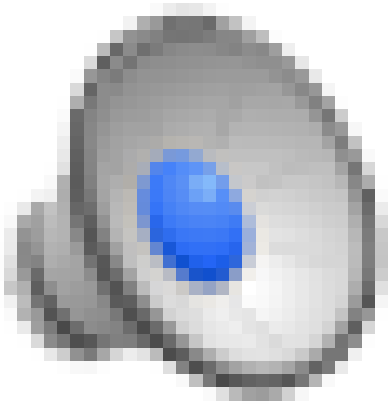


What would BFS do?

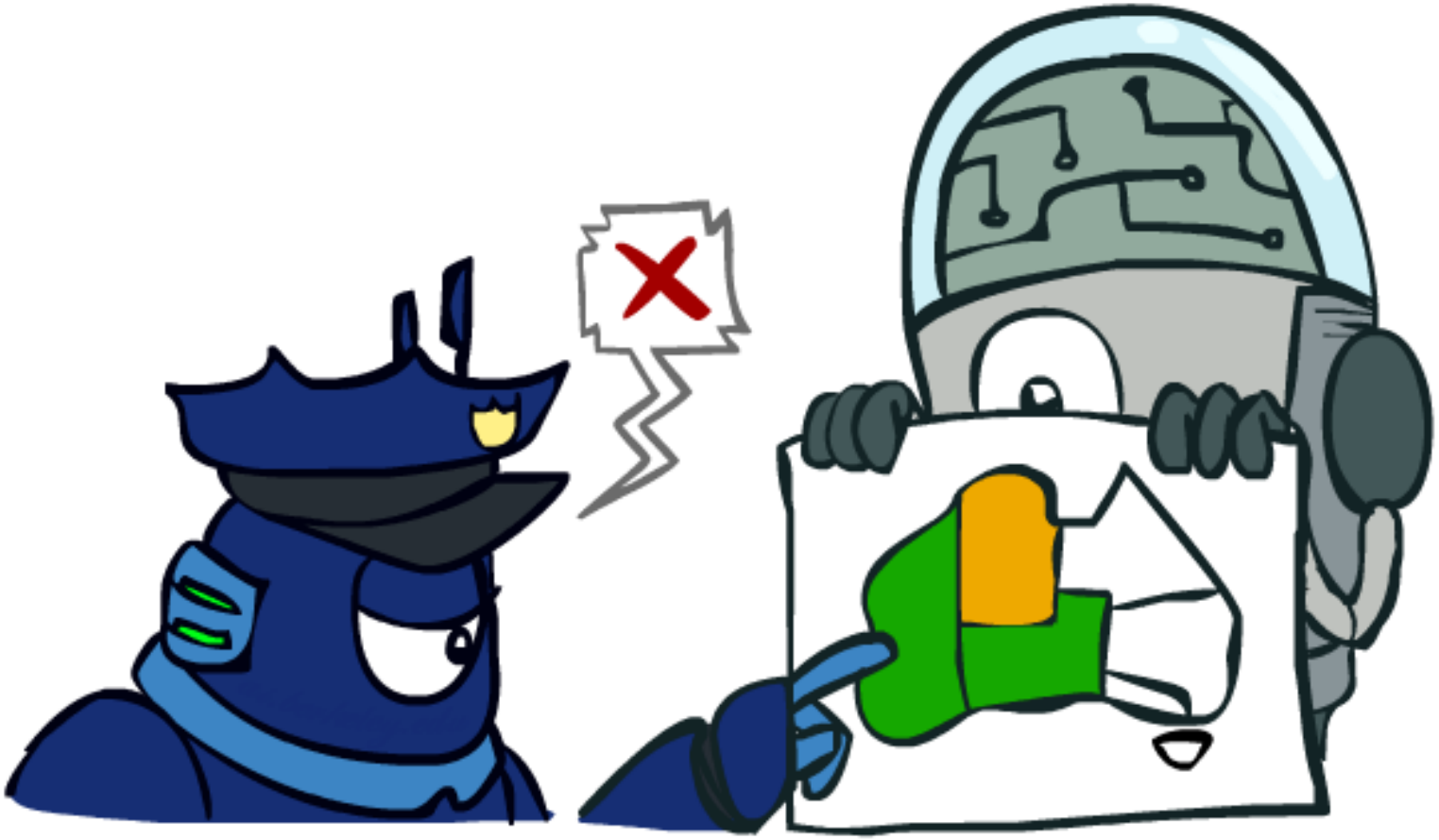
What would DFS do?

What problems does naive search have?

Video of Demo Coloring – DFS



Backtracking Search



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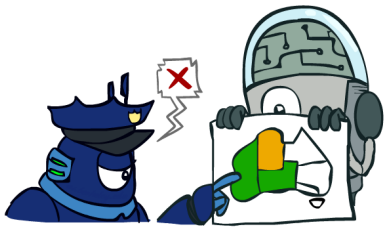
Backtracking search is the basic uninformed algorithm for solving CSPs



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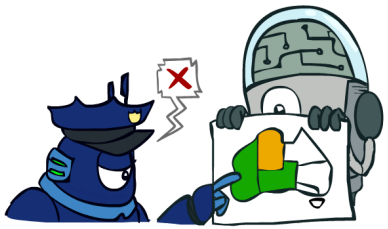

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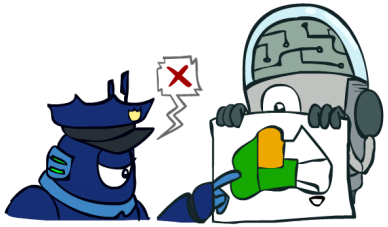

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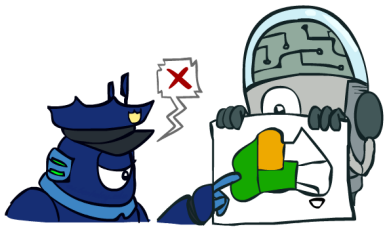

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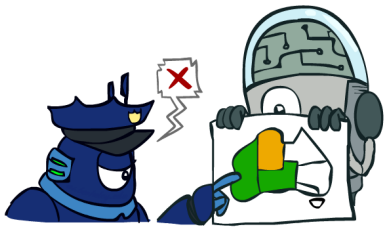

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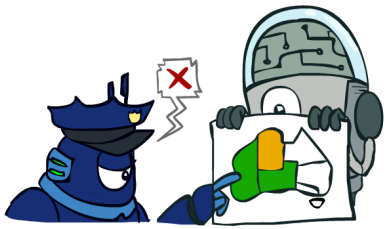

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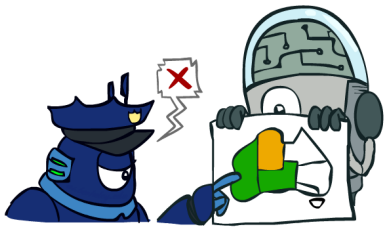

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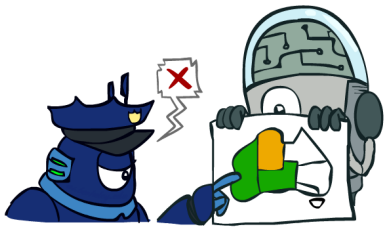

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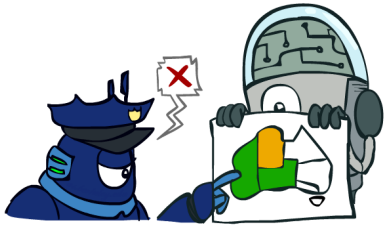

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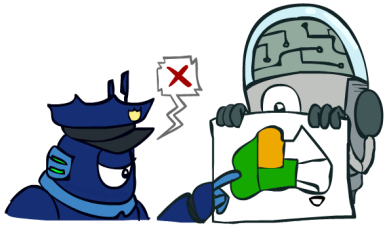

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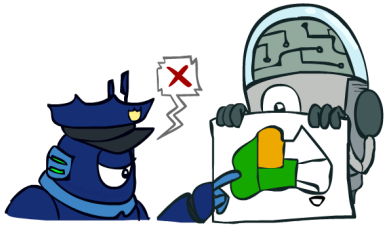

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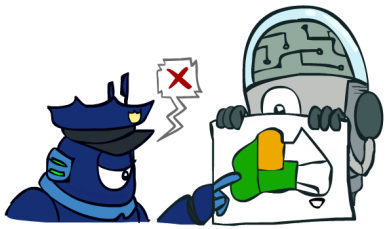

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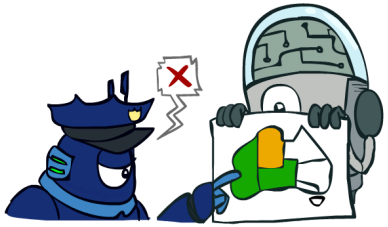

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"Incremental goal test"

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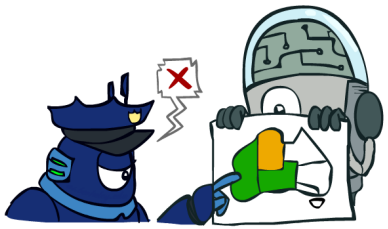

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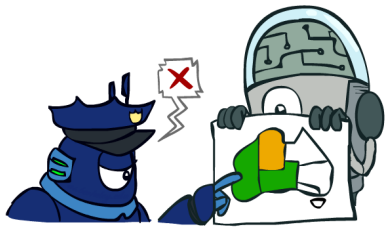

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Can solve n-queens for $n \approx 25$

Backtracking Example

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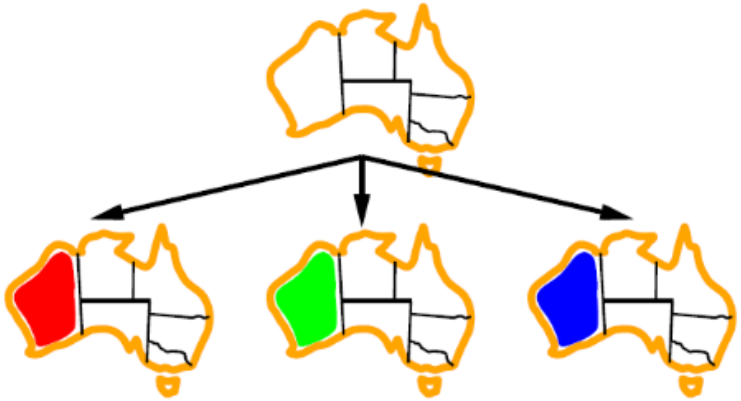

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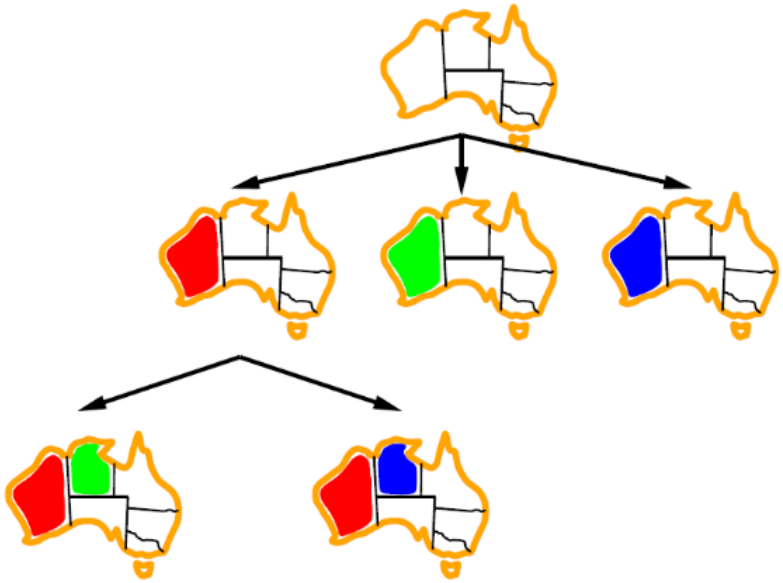

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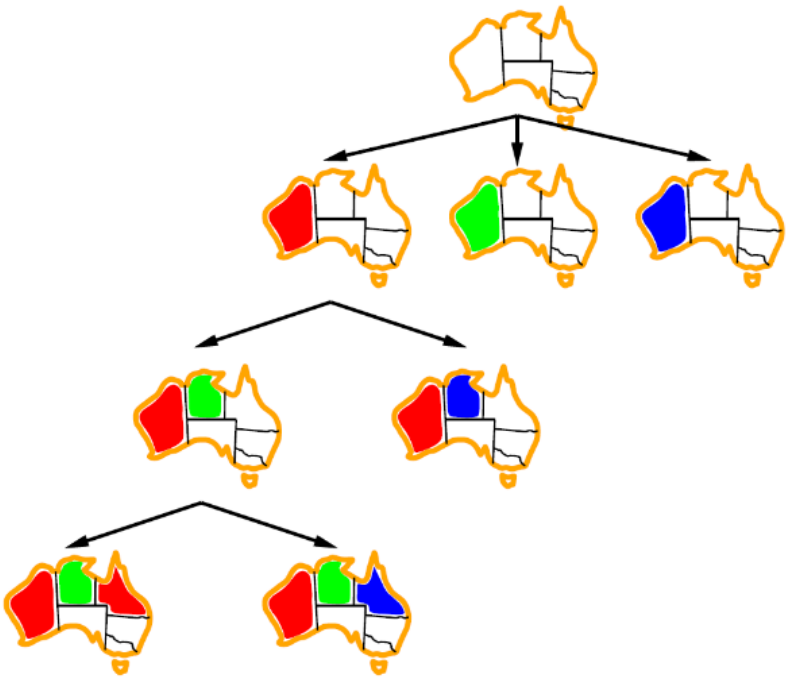

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   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
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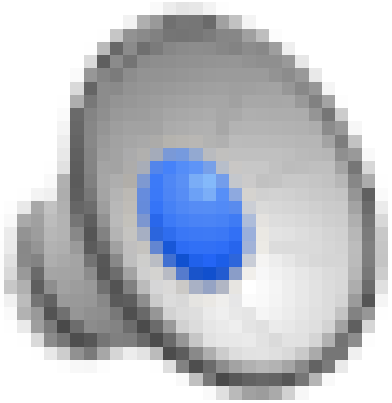
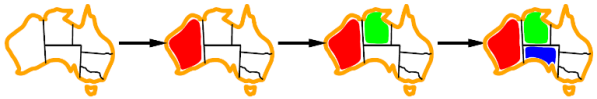
Backtracking = DFS + variable-ordering + fail-on-violation
What are the choice points?

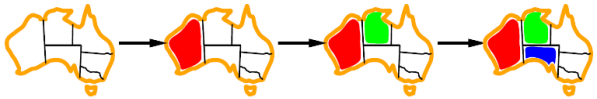
Video of Demo Coloring – Backtracking



CSP-Backtracking Search



CSP-Backtracking Search



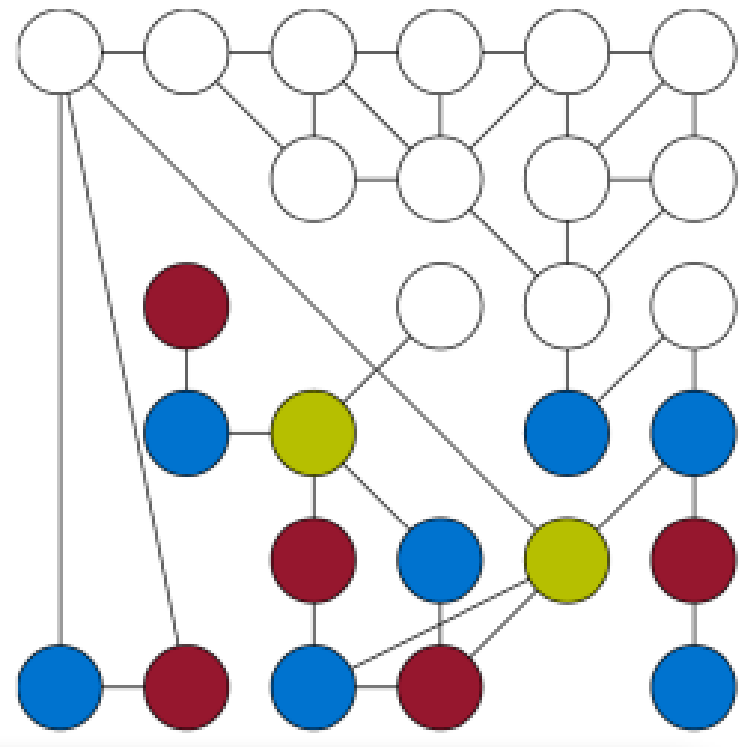
CSP-Backtracking = DFS + variable-ordering + fail-on-violation

CSP-Backtracking Search

CSP-Backtracking = DFS + variable-ordering + fail-on-violation

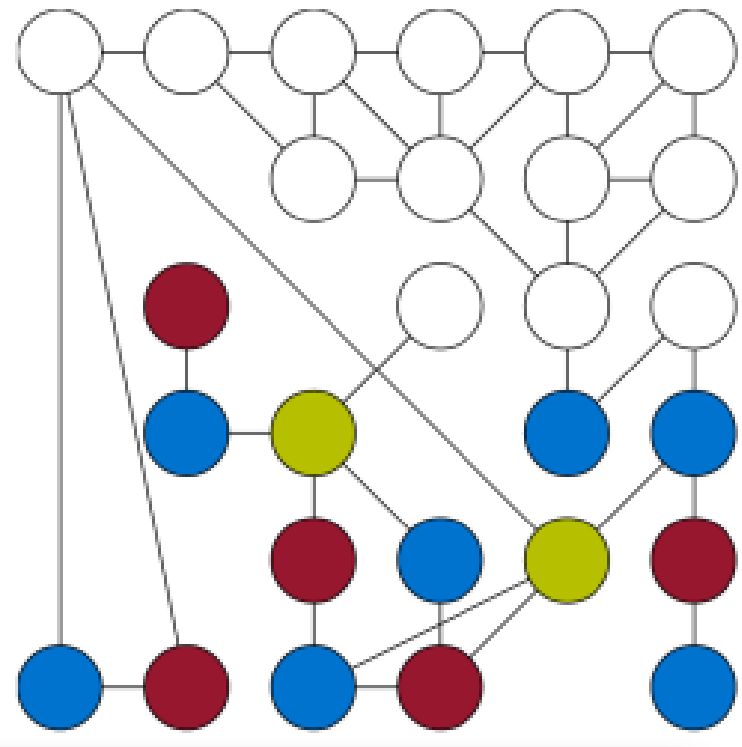
One optimization possibility: Pick “better” variable orderings and value orderings.

Some issues.



Consider the partially completed CSP assignment.

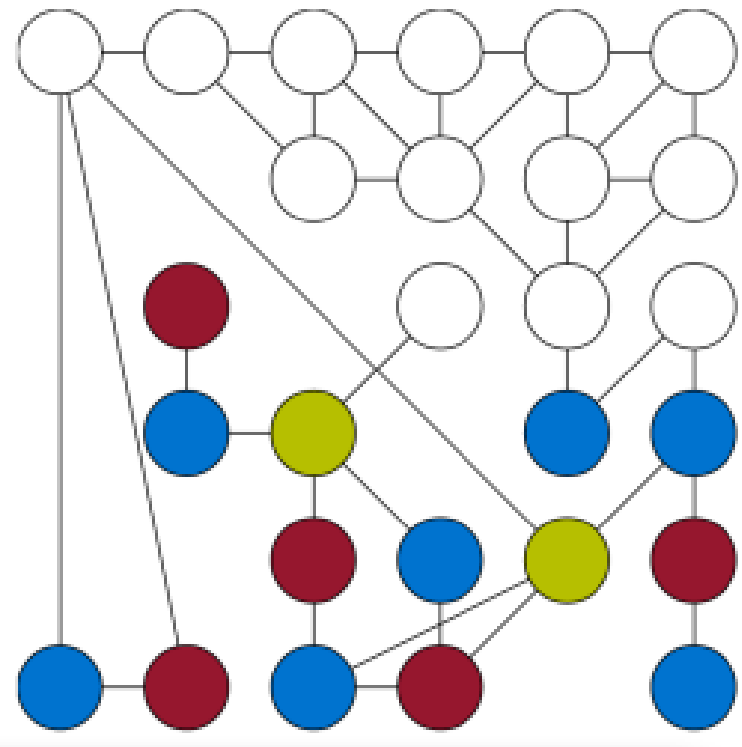
Some issues.



Consider the partially completed CSP assignment.

Decisions made bottom-up, left-to-right.

Some issues.



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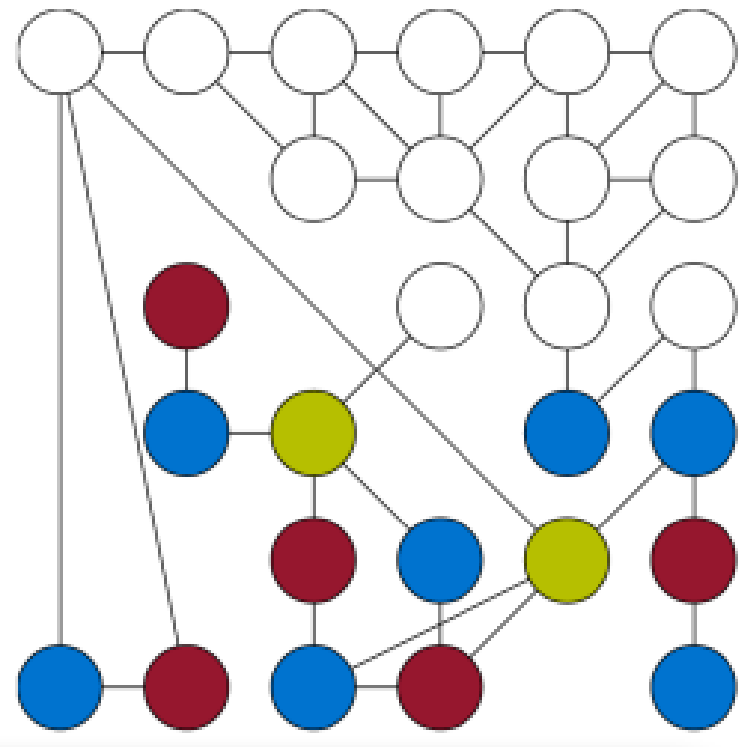
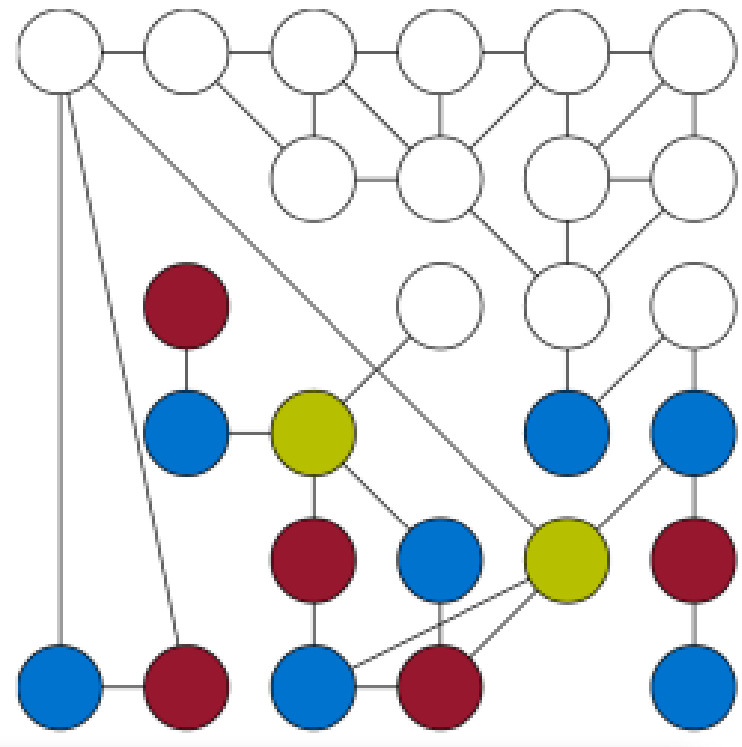


Consider the partially completed CSP assignment.

Decisions made bottom-up, left-to-right. Let X be the decision is obviously doomed in the current assignment.

What is X?

Some issues.



Consider the partially completed CSP assignment.

Decisions made bottom-up, left-to-right. Let X be the decision is obviously doomed in the current assignment.

What is X?

Bonus: How many decisions before CSP-Backtracking search realizes its error?

Improving Backtracking

General-purpose ideas give huge gains in speed

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Ordering:

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Ordering:
  Which variable should be assigned next?

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Ordering:

Which variable should be assigned next?

In what order should its values be tried?

Improving Backtracking

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Ordering:

Which variable should be assigned next?
In what order should its values be tried?

Filtering:

Improving Backtracking

General-purpose ideas give huge gains in speed

Ordering:
Which variable should be assigned next?
In what order should its values be tried?

Filtering:
Can we detect inevitable failure early?

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Ordering:
Which variable should be assigned next?
In what order should its values be tried?

Filtering:
Can we detect inevitable failure early?

Structure:

Improving Backtracking

General-purpose ideas give huge gains in speed

Ordering:
Which variable should be assigned next?
In what order should its values be tried?

Filtering:
Can we detect inevitable failure early?

Structure:
Can we exploit the problem structure?

Next Time.

Heuristic improvements to CSP search.