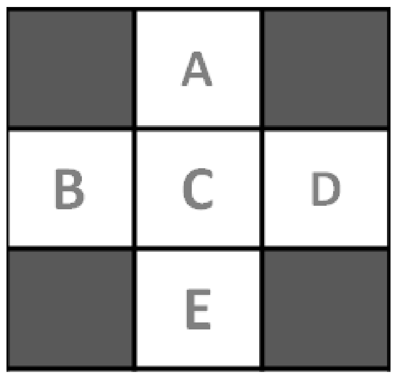
CS 188
Spring 2020

Exam Prep 6 Solutions

1 Learning in Gridworld

Consider the example gridworld that we looked at in lecture. We would like to use TD learning and q-learning to find the values of these states.



Suppose that we have the following observed transitions:
(B, East, C, 2), (C, South, E, 4), (C, East, A, 6), (B, East, C, 2)

The initial value of each state is 0. Assume that $\gamma = 1$ and $\alpha = 0.5$.

(a) What are the learned values from TD learning after all four observations?

$V(B) = 3.5$
$V(C) = 4$
All other states have a value of 0.

(b) What are the learned Q-values from Q-learning after all four observations?

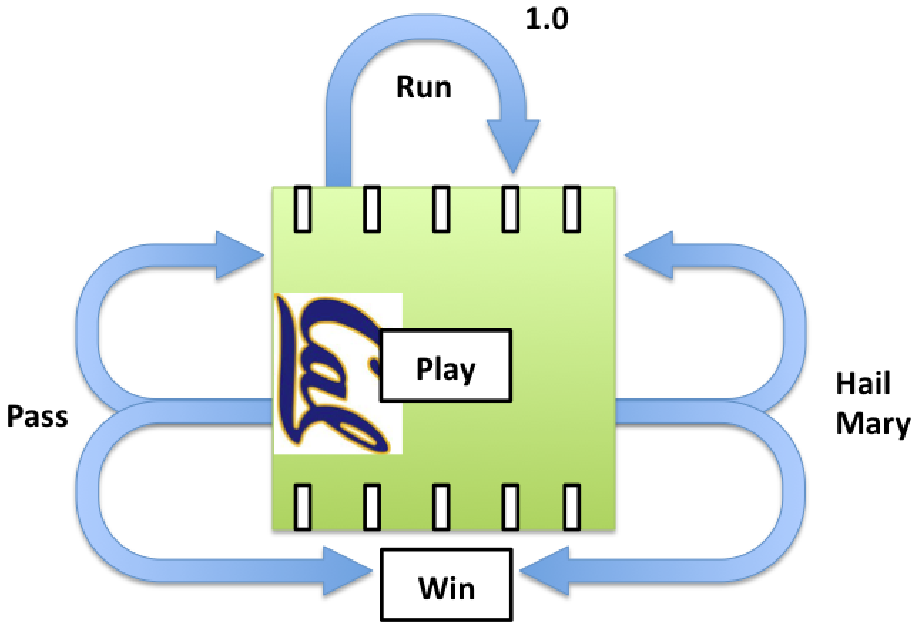
$Q(B, East) = 3$
$Q(C, South) = 2$
$Q(C, East) = 3$
All other q-states have a value of 0.

Q2. MDPs and RL: Go Bears!

Cal's Football team is playing against UCLA for the big homecoming game Saturday night. With a lot of losses in the season so far, Cal needs to switch up their strategy to get any hope of winning this game.

Luckily, the Quarterback (Joe) is a star student in CS188 and has decided to model the game as a Markov Decision Process. There are only two states – the *Play* state (shown as the field in the diagram) and the *Win* State. Although the connectivity of the states is known, the probabilities for each are not.

There are no actions available from the *Win* state – the game simply ends.



From the *Play* state there are three actions: *Run*, *Pass*, and *HailMary*. The connectivity of each action to the two states is shown above.

Reward Values:

State	Action	State'	R(s,a,s')
Play	Run	Play	2
Play	Pass	Play	4
Play	Pass	Win	10
Play	Hail Mary	Play	0
Play	Hail Mary	Win	100

(a) Learning Values Joe wants to learn the value of the play state so he can estimate the outcome of the game. He uses a discount factor of 0.5 for all questions below.

(i) Joe first uses temporal difference value learning to learn the value of the *play* state. After initializing his beliefs to 0, he sees two episodes while in tape review. With a learning rate α of 0.5 what value of the state *play* does he learn?

State	Action	State'
Play	Run	Play
Play	Hail Mary	Play

$$V(play) = 2 + 0.5 * V(play) = 2 + 0.5 * 0 = 2$$
$$V(play) \leftarrow (1 - \alpha) * 0 + \alpha * 2 = 1$$

then

$$V(play) = 0 + 0.5 * 1 = 0.5$$
$$V(play) \leftarrow (1 - \alpha) * 1 + \alpha * 0.5 = 0.75$$

$$V(play) = 0.75$$

(ii) Coach Tedford decides to give Joe a fixed policy instead:

$$\pi(s) \qquad = \qquad Run$$

What value for the state *play* would Joe calculate if he ran value iteration until convergence? Keep in mind that $\sum_{n=0}^{\infty}(\frac{1}{2})^n = 2 - (\frac{1}{2})^n = 1 + 0.5 + 0.25 + 0.125 + ...$

$$V(play) = 2 + 0.5 * (2 + 0.5 * (2 + 0.5 * (2 + 0.5 * ....)))$$
$$V(play) = 2 + 1 + 0.5 + 0.25 + 0.125... = 4$$

$$V^{\pi}(play) \qquad = 4$$

(b) Game Time Joe watches the next lecture video from class and now wants to use Q-learning to compute his optimal strategy.

(i) First Joe uses temporal difference Q-learning to learn the values of the Q nodes. He sees three episodes during the first quarter:

State	Action	State'
Play	Run	Play
Play	Hail Mary	Play
Play	Pass	Win

Update the Q node values after processing each episode (in order). Use a learning rate of 0.5 and a discount rate of 0.5.

$$Q(play, run) = 2 + 0.5 * 0 = 2$$
$$Q(play, hail) = 0 + 0.5 * 1 = 0.5$$
$$Q(play, pass) = 10 + 0.5 * 0 = 10$$

Remember you need to update V(play) as this process continues. and learning rate of alpha of 0.5 means all the above values get averaged against 0 when stored.

(c) Q learning is going well, but it's taking too much time. Thankfully Oski shows up with some special information – he has watched so many games that he know's the true transition probabilities! Here they are:

State	Action	$Q(s,a)$
Play	Run	1
Play	Hail Mary	0.25
Play	Pass	5

State	Action	State'	R(s,a,s')	T(s,a,s')
Play	Run	Play	2	1.0
Play	Pass	Play	4	0.5
Play	Pass	Win	10	0.5
Play	Hail Mary	Play	0	0.9
Play	Hail Mary	Win	100	0.1

(i) Now with these probabilities, what is the optimal policy when there is one time step left? The value?

$$Q(play, hail) = 0.1 * (100) + 0.9 * (0) = 10$$

$$\pi_{k=1}(play) = hail \quad mary$$

$$V_{k=1}(play) = 10$$

(ii) For two time steps left, what is the optimal policy with discount factor 0.5? Hint: you can use your value above to aid in this computation.

$$Q(play, hail) = 0.1 * (100 + 0.5 * 0) + 0.9 * (0 + 0.5 * 10) = 14.5$$

$$\pi_{k=2}(play) = hail \quad mary$$

$$V_{k=2}(play) = 14.5$$