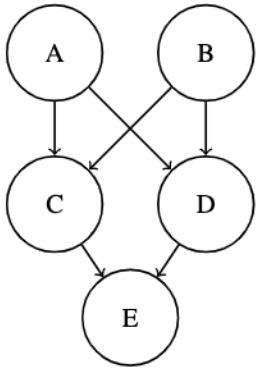
CS 188 Spring 2021 Introduction to Artificial Intelligence

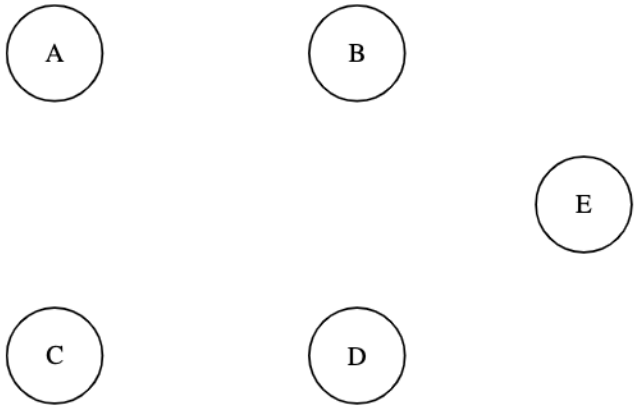
Exam Prep 6

Q1. Bayes Nets and Joint Distributions

(a) Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



(b) Draw the Bayes net associated with the following joint distribution:
$P(A) \cdot P(B) \cdot P(C|A,B) \cdot P(D|C) \cdot P(E|B,C)$



(c) Do the following products of factors correspond to a valid joint distribution over the variables A, B, C, D? (Circle FALSE or TRUE.)

(i) FALSE TRUE $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$

(ii) FALSE TRUE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B,C)$

(iii) FALSE TRUE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$

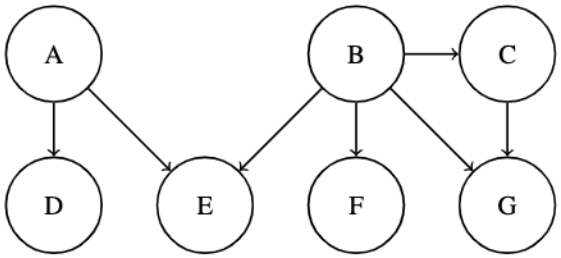
(iv) FALSE TRUE $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$

(d) What factor can be multiplied with the following factors to form a valid joint distribution? (Write "none" if the given set of factors can't be turned into a joint by the inclusion of exactly one more factor.)

(i) $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B, C, D)$

(ii) $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

(e) Answer the next questions based off of the Bayes Net below:
All variables have domains of {-1, 0, 1}



(i) Before eliminating any variables or including any evidence, how many entries does the factor at G have?

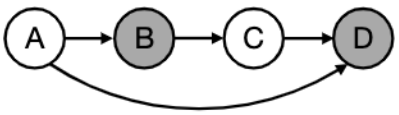
(ii) Now we observe $e = 1$ and want to query $P(D|e = 1)$, and you get to pick the first variable to be eliminated.

- Which choice would create the **largest** factor f_1?

- Which choice would create the **smallest** factor f_1?

Q2. Bayes Nets: Sampling

Consider the following Bayes Net, where we have observed that $B = +b$ and $D = +d$.



$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B\|A)$		
$+a$	$+b$	0.8
$+a$	$-b$	0.2
$-a$	$+b$	0.4
$-a$	$-b$	0.6

$P(C\|B)$		
$+b$	$+c$	0.1
$+b$	$-c$	0.9
$-b$	$+c$	0.7
$-b$	$-c$	0.3

$P(D\|A,C)$			
$+a$	$+c$	$+d$	0.6
$+a$	$+c$	$-d$	0.4
$+a$	$-c$	$+d$	0.1
$+a$	$-c$	$-d$	0.9
$-a$	$+c$	$+d$	0.2
$-a$	$+c$	$-d$	0.8
$-a$	$-c$	$+d$	0.5
$-a$	$-c$	$-d$	0.5

(a) Consider doing Gibbs sampling for this example. Assume that we have initialized all variables to the values $+a, +b, +c, +d$. We then unassign the variable C, such that we have $A = +a$, $B = +b$, $C = ?$, $D = +d$. Calculate the probabilities for new values of C at this stage of the Gibbs sampling procedure.

$P(C = +c$ at the next step of Gibbs sampling$) =$ _____

$P(C = -c$ at the next step of Gibbs sampling$) =$ _____

(b) Consider a sampling scheme that is a hybrid of rejection sampling and likelihood-weighted sampling. Under this scheme, we first perform rejection sampling for the variables A and B. We then take the sampled values for A and B and extend the sample to include values for variables C and D, using likelihood-weighted sampling.

(i) Below is a list of candidate samples. Mark the samples that would be rejected by the rejection sampling portion of the hybrid scheme.

- ☐ $-a$ $-b$
- ☐ $+a$ $+b$
- ☐ $+a$ $-b$
- ☐ $-a$ $+b$

(ii) To decouple from part (i), you now receive a *new* set of samples shown below. Fill in the weights for these samples under our hybrid scheme.

				Weight
$-a$	$+b$	$-c$	$+d$	_____
$+a$	$+b$	$-c$	$+d$	_____
$+a$	$+b$	$-c$	$+d$	_____
$-a$	$+b$	$+c$	$+d$	_____
$+a$	$+b$	$+c$	$+d$	_____

(iii) Use the weighted samples from part (ii) to calculate an estimate for $P(+a \mid +b, +d)$.

The estimate of $P(+a \mid +b, +d)$ is _____

(c) We now attempt to design an alternative hybrid sampling scheme that combines elements of likelihood-weighted and rejection sampling. For each proposed scheme, indicate whether it is valid, i.e. whether the weighted samples it produces correctly approximate the distribution $P(A, C| + b, +d)$.

(i) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are* ***thrown away****. Sampling then restarts from node* ***A****.*

◯ Valid ◯ Invalid

(ii) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are* ***retained****. Sampling then restarts from node* ***C****.*

◯ Valid ◯ Invalid