CS 188 Spring 2021 | Introduction to Artificial Intelligence | **Exam Prep 11**

Q1. Naive Bayes: Pacman or Ghost?

You are standing by an exit as either Pacmen or ghosts come out of it. Every time someone comes out, you get two observations: a visual one and an auditory one, denoted by the random variables X_v and X_a, respectively. The visual observation informs you that the individual is either a Pacman ($X_v = 1$) or a ghost ($X_v = 0$). The auditory observation X_a is defined analogously. Your observations are a noisy measurement of the individual's true type, which is denoted by Y. After the indiviual comes out, you find out what they really are: either a Pacman ($Y = 1$) or a ghost ($Y = 0$). You have logged your observations and the true types of the first 20 individuals:

individual i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
first observation $X_v^{(i)}$	0	0	1	0	1	0	0	1	1	1	0	1	1	0	1	1	1	0	0	0
second observation $X_a^{(i)}$	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
individual's type $Y^{(i)}$	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0

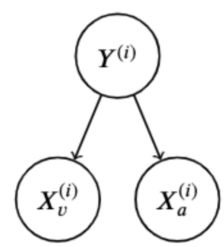
The superscript (i) denotes that the datum is the ith one. Now, the individual with $i = 20$ comes out, and you want to predict the individual's type $Y^{(20)}$ given that you observed $X_v^{(20)} = 1$ and $X_a^{(20)} = 1$.

(a) Assume that the types are independent, and that the observations are independent conditioned on the type. You can model this using naïve Bayes, with $X_v^{(i)}$ and $X_a^{(i)}$ as the features and $Y^{(i)}$ as the labels. Assume the probability distributions take on the following form:

$$P(X_v^{(i)} = x_v | Y^{(i)} = y) = \begin{cases} p_v & \text{if } x_v = y \\ 1 - p_v & \text{if } x_v \neq y \end{cases}$$

$$P(X_a^{(i)} = x_a | Y^{(i)} = y) = \begin{cases} p_a & \text{if } x_a = y \\ 1 - p_a & \text{if } x_a \neq y \end{cases}$$

$$P(Y^{(i)} = 1) = q$$



for $p_v, p_a, q \in [0, 1]$ and $i \in \mathbb{N}$.

(i) What's the maximum likelihood estimate of p_v, p_a and q?

$p_v =$ ________________ $p_a =$ ________________ $q =$ ________________

(ii) What is the probability that the next individual is Pacman given your observations? Express your answer in terms of the parameters p_v, p_a and q (you might not need all of them).

$P(Y^{(20)} = 1 | X_v^{(20)} = 1, X_a^{(20)} = 1) =$ ________________________

Now, assume that you are given additional information: you are told that the individuals are actually coming out of a bus that just arrived, and each bus carries *exactly* 9 individuals. Unlike before, the types of every 9 consecutive individuals are *conditionally* independent given the bus type, which is denoted by Z. Only after all of the 9 individuals have walked out, you find out the bus type: one that carries mostly Pacmans ($Z = 1$) or one that carries mostly ghosts ($Z = 0$). Thus, you only know the bus type in which the first 18 individuals came in:

individual i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
first observation $X_v^{(i)}$	0	0	1	0	1	0	0	1	1	1	0	1	1	0	1	1	1	0	0	0
second observation $X_a^{(i)}$	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
individual's type $Y^{(i)}$	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0

bus j	0	1	
bus type $Z^{(j)}$	0	1	

(b) You can model this using a variant of naïve bayes, where now 9 consecutive labels $Y^{(i)}, \ldots, Y^{(i+8)}$ are *conditionally* independent given the bus type $Z^{(j)}$, for bus j and individual $i = 9j$. Assume the probability distributions take on the following form:

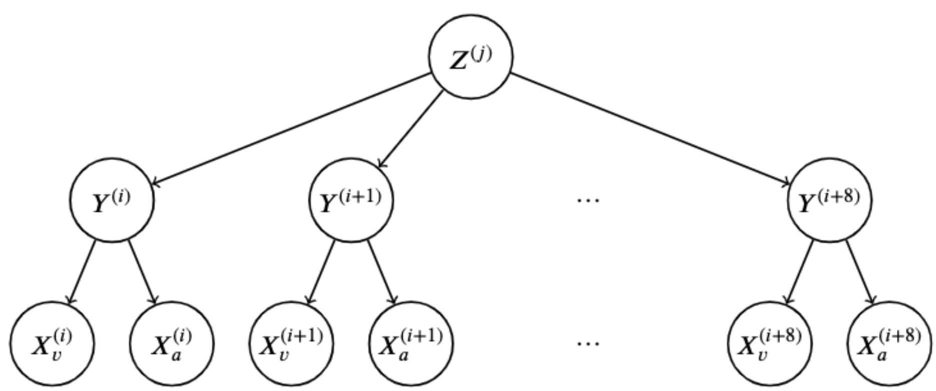
$$P(X_v^{(i)} = x_v | Y^{(i)} = y) = \begin{cases} p_v & \text{if } x_v = y \\ 1 - p_v & \text{if } x_v \neq y \end{cases}$$

$$P(X_a^{(i)} = x_a | Y^{(i)} = y) = \begin{cases} p_a & \text{if } x_a = y \\ 1 - p_a & \text{if } x_a \neq y \end{cases}$$

$$P(Y^{(i)} = 1 | Z^{(j)} = z) = \begin{cases} q_0 & \text{if } z = 0 \\ q_1 & \text{if } z = 1 \end{cases}$$

$$P(Z^{(j)} = 1) = r$$

for $p, q_0, q_1, r \in [0, 1]$ and $i, j \in \mathbb{N}$.



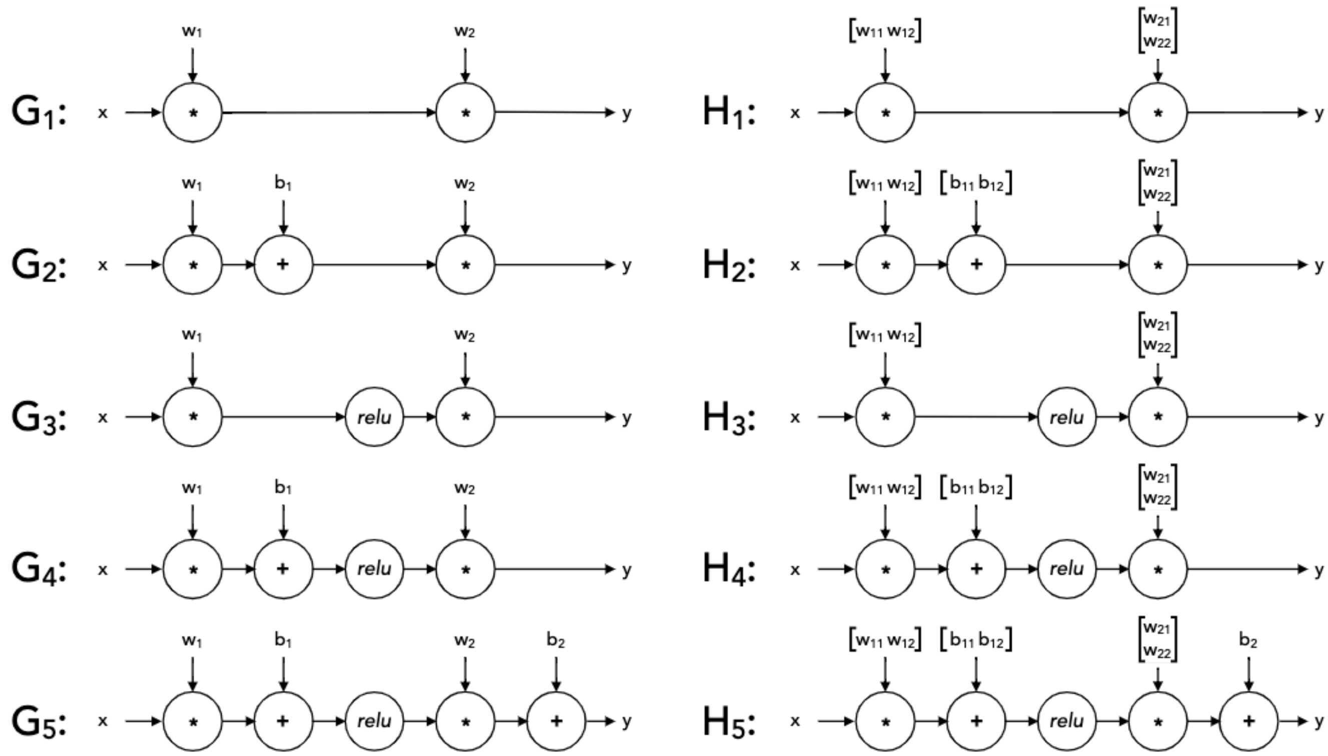
(i) What's the maximum likelihood estimate of q_0, q_1 and r?

$q_0 =$ ________________ $q_1 =$ ________________ $r =$ ________________

(ii) Compute the following joint probability. Simplify your answer as much as possible and express it in terms of the parameters p_v, p_a, q_0, q_1 and r (you might not need all of them).

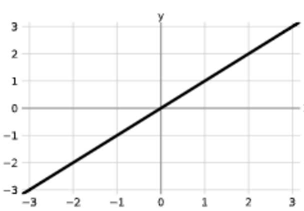
$P(Y^{(20)} = 1, X_v^{(20)} = 1, X_a^{(20)} = 1, Y^{(19)} = 1, Y^{(18)} = 1) =$ ____

Q2. Neural Networks: Representation

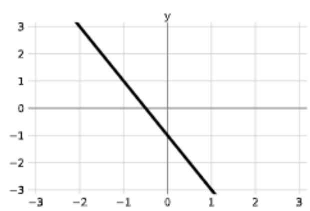


For each of the piecewise-linear functions below, mark all networks from the list above that can represent the function **exactly** on the range $x \in (-\infty, \infty)$. In the networks above, *relu* denotes the element-wise ReLU nonlinearity: $relu(z) = max(0, z)$. The networks G_i use 1-dimensional layers, while the networks H_i have some 2-dimensional intermediate layers.

(a)



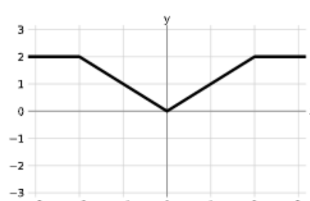
☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above



☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

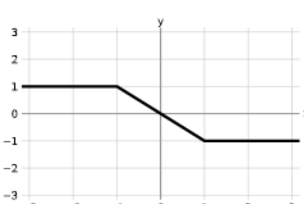
(b)

☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

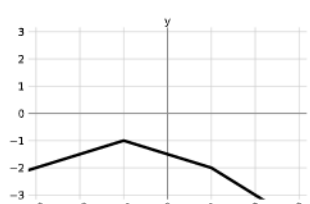


☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

(c)



☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above



☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above