

Q1. Machine Learning: Potpourri

- (a) What is the **minimum** number of parameters needed to fully model a joint distribution $P(Y, F_1, F_2, \dots, F_n)$ over label Y and n features F_i ? Assume binary class where each feature can possibly take on k distinct values.

- (b) Under the **Naive Bayes assumption**, what is the **minimum** number of parameters needed to model a joint distribution $P(Y, F_1, F_2, \dots, F_n)$ over label Y and n features F_i ? Assume binary class where each feature can take on k distinct values.

- (c) You suspect that you are overfitting with your Naive Bayes with Laplace Smoothing. How would you adjust the strength k in Laplace Smoothing?

Increase k

Decrease k

- (d) While using Naive Bayes with Laplace Smoothing, increasing the strength k in Laplace Smoothing can:

Increase training error

Decrease training error

Increase validation error

Decrease validation error

- (e) It is possible for the perceptron algorithm to never terminate on a dataset that is linearly separable in its feature space.

True

False

- (f) If the perceptron algorithm terminates, then it is guaranteed to find a max-margin separating decision boundary.

True

False

- (g) In multiclass perceptron, every weight w_y can be written as a linear combination of the training data feature vectors.

True

False

- (h) For binary class classification, logistic regression produces a linear decision boundary.

True

False

- (i) In the binary classification case, logistic regression is exactly equivalent to a single-layer neural network with a sigmoid activation and the cross-entropy loss function.

True

False

- (j) (i) You train a linear classifier on 1,000 training points and discover that the training accuracy is only 50%. Which of the following, if done in isolation, has a good chance of improving your training accuracy?

Add novel features

Train on more data

Train on less data

- (ii) You now try training a neural network but you find that the training accuracy is still very low. Which of the following, if done in isolation, has a good chance of improving your training accuracy?

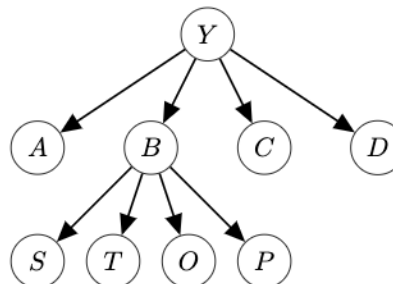
Add more hidden layers

Add more units to the hidden layers

Q2. A Nonconvolutional Nontrivial Network

You have a robotic friend MesutBot who has trouble passing Recaptchas (and Turing tests in general). MesutBot got a 99.99% on the last midterm because he could not determine which squares in the image contained stop signs. To help him ace the final, you decide to design a few classifiers using the below features.

- $A = 1$ if the image contains an octagon, else 0.
- $B = 1$ if the image contains the word STOP, else 0.
 - $S = 1$ if the image contains the letter S, else 0.
 - $T = 1$ if the image contains the letter T, else 0.
 - $O = 1$ if the image contains the letter O, else 0.
 - $P = 1$ if the image contains the letter P, else 0.
- $C = 1$ if the image is more than 50% red in color, else 0.
- $D = 1$ if the image contains a post, else 0.



(a) First, we use a Naive Bayes-inspired approach to determine which images have stop signs based on the features and Bayes Net above. We use the following features to predict $Y = 1$ if the image has a stop sign anywhere, or $Y = 0$ if it doesn't.

(i) Which expressions would a Naive Bayes model use to predict the label for B if given the values for features $S = s, T = t, O = o, P = p$? Choose all valid expressions.

$b = \arg \max_b P(b)P(s|b)P(t|b)P(o|b)P(p|b)$

$b = \arg \max_b P(s|b)P(t|b)P(o|b)P(p|b)$

$b = \arg \max_b P(b|s, t, o, p)$

$b = \arg \max_b P(b, s, t, o, p)$

$b = \arg \max_b P(s, t, o, p|b)$

None

(ii) Which expressions would we use to predict the label for Y with our Bayes Net above? Assume we are given all features except B . So $A = a, S = s, T = t$, etc. For the below choices, the underscore means we are dropping the value of that variable. So $y, _ = (0, 1)$ would mean $y = 0$.

$y, _ = \arg \max_{y, b} P(y)P(a|y)P(b|y)P(c|y)P(d|y)P(s|b)P(t|b)P(o|b)P(p|b)$

$y, _ = \arg \max_{y, b} P(s)P(t)P(o)P(p)P(a)P(b|s, t, o, p)P(c)P(d)P(y|a, b, c, d)$

First compute $b' = \arg \max_b$ of the formula chosen in part (ii).

Then compute $y = \arg \max_y P(y)P(a|y)P(b'|y)P(c|y)P(d|y)$

First compute $b' = \arg \max_b$ of the formula chosen in part (ii).

Then compute $y = \arg \max_y P(y|a, b', c, d)$

$y = \arg \max_y \sum_{b'} P(y)P(a|y)P(b'|y)P(c|y)P(d|y)P(s|b')P(t|b')P(o|b')P(p|b')$

None

(iii) One day MesutBot got allergic from eating too many cashews. The incident broke his letter S

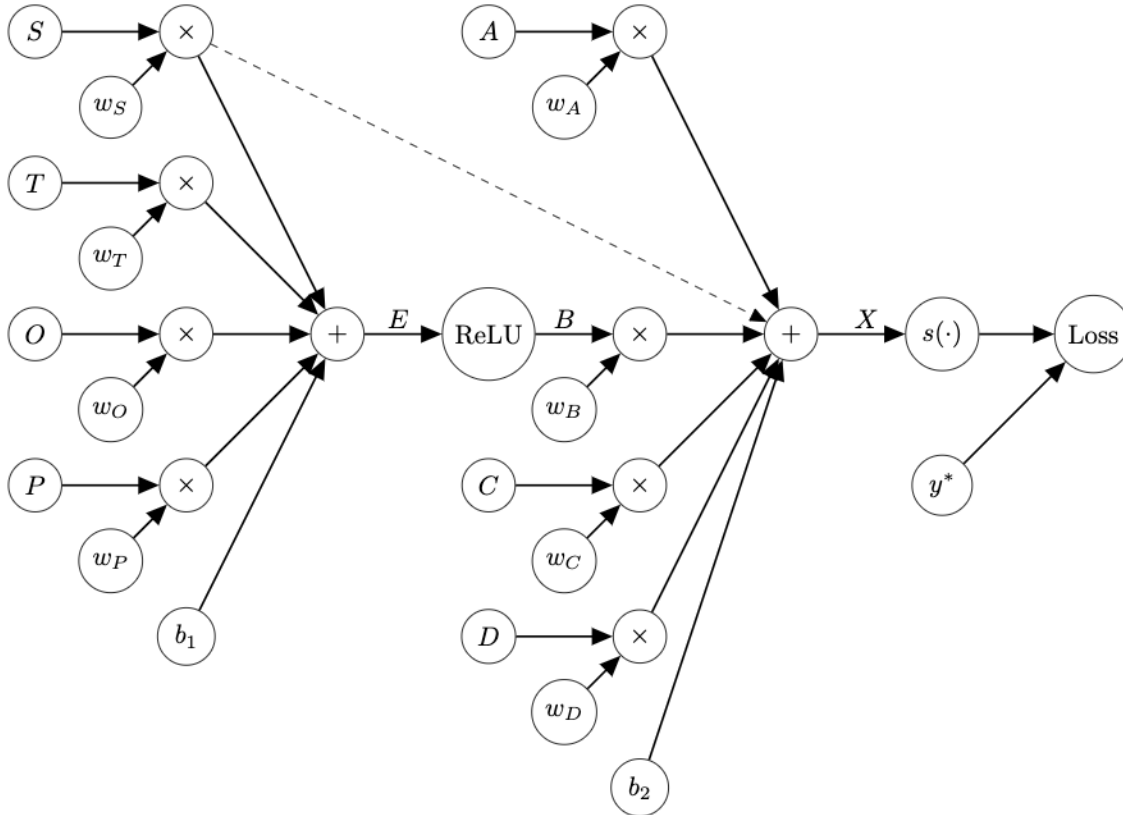
detector, so that he no longer gets reliable S features. Now what expressions would we use to predict the label for Y ? Assume all features except B, S are given. So $A = a, T = t, O = o$, etc.

- $y = \arg \max_y P(y)P(a|y)P(c|y)P(d|y)$
- $y, _ , _ = \arg \max_{y,b,s} P(y)P(a|y)P(b|y)P(c|y)P(d|y)P(s|b)P(t|b)P(o|b)P(p|b)$
- $y, _ = \arg \max_{y,s} P(y)P(a|y)P(b|y)P(c|y)P(d|y)P(s|b)P(t|b)P(o|b)P(p|b)$
- $y, _ = \arg \max_{y,b} P(y)P(a|y)P(b|y)P(c|y)P(d|y)P(t|b)P(o|b)P(p|b)$
- $y, _ = \arg \max_{y,b} P(y)P(a|y)P(b|y)P(c|y)P(d|y)P(s|b)P(t|b)P(o|b)P(p|b)$
- $y, _ = \arg \max_{y,b} P(y|a, b, c, d)$
- $y = \arg \max_y P(y)P(a|y)P(c|y)P(d|y) \sum_{b',s'} P(b'|y)P(s'|b')P(t|b')P(o|b')P(p|b')$
- None

(b) You decide to try to output a probability $P(Y|features)$ of a stop sign being in the picture instead of a discrete ± 1 prediction. We denote this probability as $P(Y|\vec{f}(x))$. Which of the following functions return a **valid** probability distribution for $P(Y = y|\vec{f}(x))$? Recall that $y \in \{-1, 1\}$.

- $\frac{e^{y \cdot \vec{w}^T \vec{f}(x)}}{e^{-y \cdot \vec{w}^T \vec{f}(x)} + e^{y \cdot \vec{w}^T \vec{f}(x)}}$
- $\frac{1}{2}$
- $\frac{0.5}{1 + e^{-\vec{w}^T \vec{f}(x)}}$
- $\frac{-1}{1 + e^{\vec{w}^T \vec{f}(x)}} + 1$
- None

Unimpressed by the perceptron, you note that features are inputs into a neural network and the output is a label, so you modify the Bayes Net from above into a Neural Network computation graph. Recall the logistic function $s(x) = \frac{1}{1+e^{-x}}$ has derivative $\frac{\partial s(x)}{\partial x} = s(x)[1 - s(x)]$



(c) For this part, ignore the dashed edge when calculating the below.

(i) What is $\frac{\partial \text{Loss}}{\partial w_A}$?

- $\frac{\partial \text{Loss}}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot A$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot A$
- $\frac{\partial \text{Loss}}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot 2A + 1$
- $\frac{\partial \text{Loss}}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot 2A$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot A + 1$
- $\frac{\partial \text{Loss}}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot A + 1$
- None

(ii) What is $\frac{\partial Loss}{\partial w_s}$? Keep in mind we are still ignoring the dotted edge in this subpart.

- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot 2S + S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot 2S$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S + S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S + S$
- None

(d) MesutBot is having trouble paying attention to the S feature because sometimes it gets zeroed out by the ReLU, so we connect it directly to the input of $s(\cdot)$ via the dotted edge. For the below, treat the dotted edge as a regular edge in the neural net.

(i) Which of the following is equivalent to $\frac{\partial Loss}{\partial w_A}$?

- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot A$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot A$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot 2A + A$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot 2A$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot A + A$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot A + A$
- None

(ii) Which of the following is equivalent to $\frac{\partial Loss}{\partial w_s}$? Keep in mind we are still treating the dotted edge as a regular edge.

- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot 2S + S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot 2S$
- $2(s(X) - y^*) \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S + S$
- $\frac{\partial Loss}{\partial s(X)} \cdot [s(X) \cdot (1 - s(X))] \cdot w_B \cdot \begin{pmatrix} 1 & E \geq 0 \\ 0 & E < 0 \end{pmatrix} \cdot S + S$
- None