CS 188: Artificial Intelligence

Naïve Bayes



Pre-scan attendance QR code now!



Spring 2024

Announcements

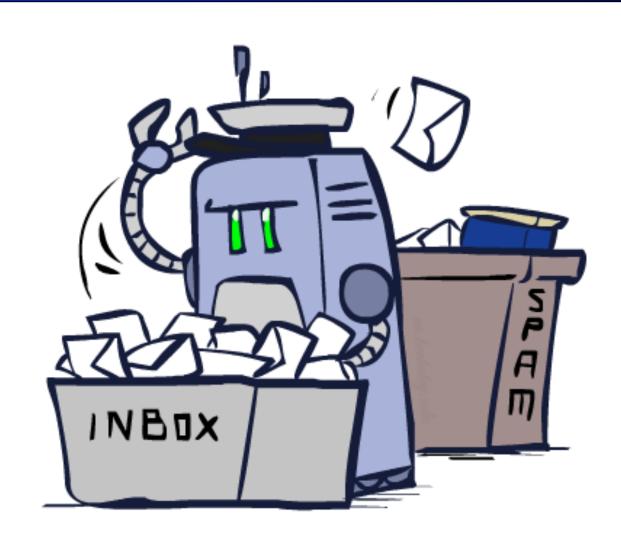
- Project 4: due (tomorrow!) Friday, March 22, 11:59 PM PT
- HW7: due Tuesday, Apr 2, 11:59 PM PT

- Spring break!
 - No additional assignments
 - No office hours / discussions
 - No expectation of staff availability on Ed plan accordingly!
 - No solutions will be released

Machine Learning

- Up until now: how use a model to make optimal decisions
- Machine learning: how to acquire a model from data / experience
 - Learning parameters (e.g. probabilities)
 - Learning structure (e.g. BN graphs)
 - Learning hidden concepts (e.g. clustering, neural nets)
- Today: model-based classification with Naive Bayes

Classification



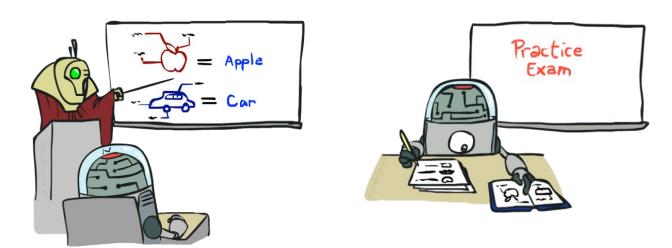
Classification and Machine Learning

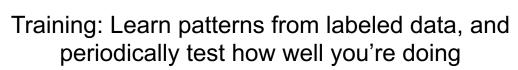
- Dataset: each data point, x, is associated with some label (aka class), y
- Goal of classification: given inputs x, write an algorithm to predict labels y
- Workflow of classification process:
 - Input is provided to you
 - Extract features from the input: attributes of the input that characterize each x and hopefully help with classification
 - Run some machine learning algorithm on the features: today, Naïve Bayes
 - Output a predicted label y



Training and Machine Learning

- Big idea: ML algorithms learn patterns between features and labels from data
 - You don't have to reason about the data yourself
 - You're given training data: lots of example datapoints and their actual labels







Eventually, use your algorithm to predict labels for unlabeled data

Example: Spam Filter

- Input: an email
- Output: spam/ham



- Get a large collection of example emails, each labeled "spam" or "ham"
- Note: someone has to hand label all this data!
- Want to learn to predict labels of new, future emails
- Features: The attributes used to make the ham / spam decision
 - Words: FREE!
 - Text Patterns: \$dd, CAPS
 - Non-text: SenderInContacts, WidelyBroadcast
 - -



Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. ...

TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES FOR ONLY \$99

Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.





Example: Digit Recognition

0

35

- Input: images / pixel grids
- Output: a digit 0-9
- Setup:
 - Get a large collection of example images, each labeled with a digit
 - Note: someone has to hand label all this data!
 - Want to learn to predict labels of new, future digit images

- Features: The attributes used to make the digit decision
 - Pixels: (6,8)=ON
 - Shape Patterns: NumComponents, AspectRatio, NumLoops
 - **-** ...
 - Features are increasingly induced rather than crafted

Other Classification Tasks

Classification: given inputs x, predict labels (classes) y

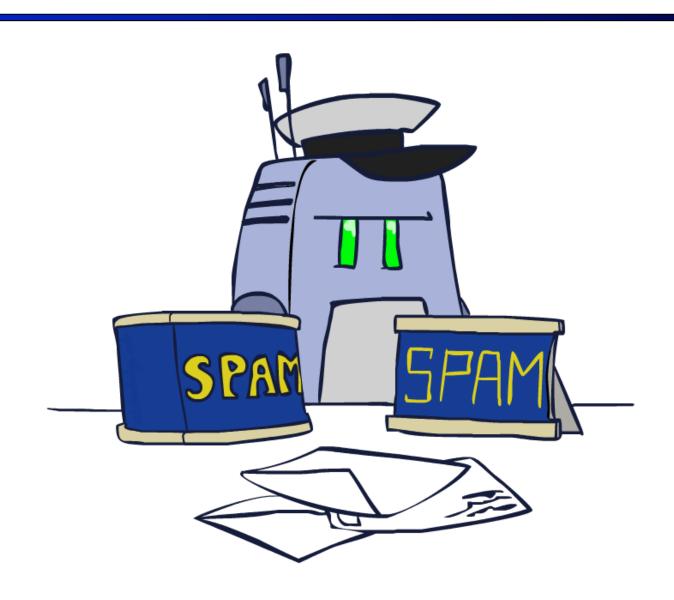
Examples:

- Medical diagnosis (input: symptoms, classes: diseases)
- Fraud detection (input: account activity, classes: fraud / no fraud)
- Automatic essay grading (input: document, classes: grades)
- Customer service email routing
- Review sentiment
- Language ID
- ... many more



Classification is an important commercial technology!

Model-Based Classification



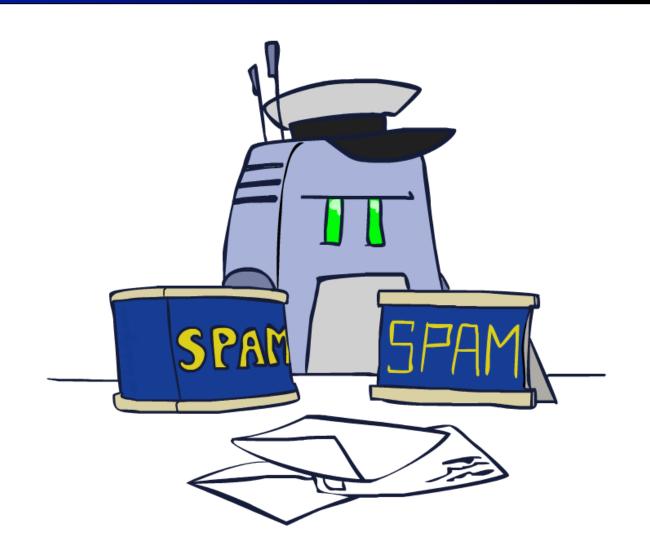
Model-Based Classification

Model-based approach

- Build a model (e.g. Bayes net) where both the output label and input features are random variables
- Instantiate any observed features
- Query for the distribution of the label conditioned on the features

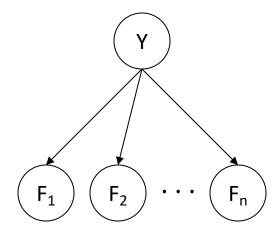
Challenges

- What structure should the BN have?
- How should we learn its parameters?



Naïve Bayes Model

- Random variables in this Bayes net:
 - Y = The label
 - F_1 , F_2 , ..., F_n = The n features
- Probability tables in this Bayes net:
 - P(Y) = Probability of each label occurring, given no information about the features. Sometimes called the *prior*.
 - $P(F_i|Y)$ = One table per feature. Probability distribution over a feature, given the label.



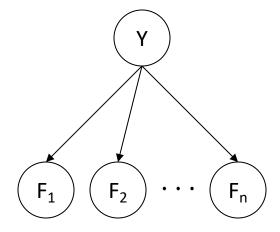
Naïve Bayes Model

To perform training:

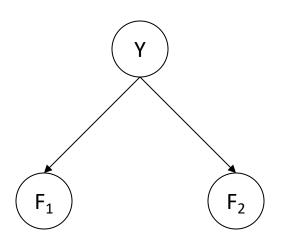
- Use the training dataset to estimate the probability tables.
- Estimate P(Y) = how often does each label occur?
- Estimate $P(F_i|Y)$ = how does the label affect the feature?

To perform classification:

- Instantiate all features. You know the input features, so they're your evidence.
- Query for $P(Y|f_1, f_2, ..., f_n)$. Probability of label, given all the input features. Use an inference algorithm (e.g. variable elimination) to compute this.



- Step 1: Select a ML algorithm. We choose to model the problem with Naïve Bayes.
- Step 2: Choose features to use.



Y: The label (spam or ham)		
Υ	P(Y)	
ham	?	
spam	?	

F_1 : A feature (do I know the sender?)		
F ₁	Υ	$P(F_1 Y)$
yes	ham	?
no	ham	?
yes	spam	?
no	spam	?

F ₂ : Another feature (# of occurrences of FREE)		
F ₂	Υ	P(F ₂ Y)
0	ham	?
1	ham	? ·
2	ham	
0	spam	?
1	spam	?
2	spam	?

Step 3: Training: Use training data to fill in the probability tables.

F ₂ : # of occurrences of FREE				
F ₂	Y P(F ₂ Y)			
0	ham	0.5		
1	ham	0.5		
2	ham	0.0		
0	spam	0.25		
1	spam	0.50		
2	spam	0.25		

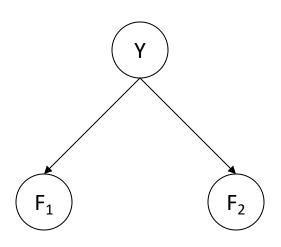
	Training Data		
#	Email Text	Label	
1	Attached is my portfolio.	ham	
2	Are you free for a meeting tomorrow?	ham	
3	Free unlimited credit cards!!!!	spam	
4	Mail \$10,000 check to this address	spam	
5	Sign up now for 1 free Bitcoin	spam	
6	Free money free money	spam	

Row 4: $P(F_2=0 \mid Y=spam) = 0.25$ because 1 out of 4 spam emails contains "free" 0 times.

Row 5: $P(F_2=1 \mid Y=spam) = 0.50$ because 2 out of 4 spam emails contains "free" 1 time.

Row 6: $P(F_2=2 \mid Y=spam) = 0.25$ because 1 out of 4 spam emails contains "free" 2 times.

Model trained on a larger dataset:

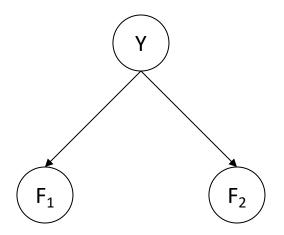


Y: The label (spam or ham)		
Y P(Y)		
ham	0.6	
spam	0.4	

F ₁ : A feature (do I know the sender?)		
F ₁	Υ	$P(F_1 Y)$
yes	ham	0.7
no	ham	0.3
yes	spam	0.1
no	spam	0.9

F ₂ : Another feature (# of occurrences of FREE)		
F ₂	Υ	P(F ₂ Y)
0	ham	0.85
1	ham	0.07
2	ham	0.08
0	spam	0.75
1	spam	0.12
2	spam	0.13

- Step 4: Classification
- Suppose you want to label this email from a known sender:
 "Free food in Soda 430 today"
- Step 4.1: Feature extraction:
 - F_1 = yes, known sender
 - F₂ = 1 occurrence of "free"



- Step 4.2: Inference
- Instantiate features (evidence):
 - $F_1 = yes$
 - $F_2 = 1$



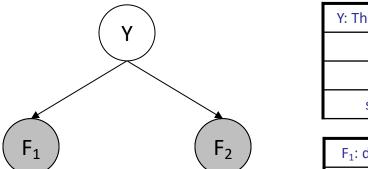
- P(Y = spam, F_1 = yes, F_2 = 1) = P(Y = spam) P(F_1 = yes | spam) P(F_2 = 1 | spam) = 0.4 * 0.1 * 0.12 = 0.0048
- $P(Y = ham, F_1 = yes, F_2 = 1) = P(Y = ham) P(F_1 = yes | ham) P(F_2 = 1 | ham) = 0.6 * 0.7 * 0.07 = 0.0294$

Normalize:

- $P(Y = \text{spam} \mid F_1 = \text{yes}, F_2 = 1) = 0.0048 / (0.0048 + 0.0294) = 0.14$
- $P(Y = ham \mid F_1 = yes, F_2 = 1) = 0.0294 / (0.0048 + 0.0294) = 0.86$

Classification result:

- 14% chance the email is spam. 86% chance it's ham.
- Or, if you don't need probabilities, note that 0.0294 > 0.0048 and guess ham.



Y: The label (spam or ham)		
Y P(Y)		
ham	0.6	
spam	0.4	

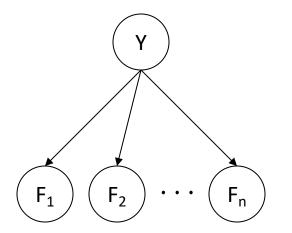
F ₁ : do I know the sender?		
F ₁	Υ	P(F ₁ Y)
yes	ham	0.7
no	ham	0.3
yes	spam	0.1
no	spam	0.9

F ₂ : # of occurrences of FREE		
F ₂	Υ	P(F ₂ Y)
0	ham	0.85
1	ham	0.07
2	ham	0.08
0	spam	0.75
1	spam	0.12
2	spam	0.13

Naïve Bayes for Digits

- Simple digit recognition version:
 - One feature (variable) F_{ij} for each grid position <i,j>
 - Feature values are on / off, based on whether intensity is more or less than 0.5 in underlying image
 - Each input maps to a feature vector, e.g.

$$Arr$$
 Arr
 Arr



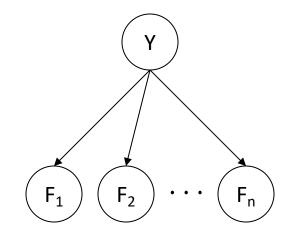
- Here: lots of features, each is binary valued
- Naïve Bayes model: $P(Y|F_{0,0}...F_{15,15}) \propto P(Y) \prod_{i,j} P(F_{i,j}|Y)$
- What do we need to learn?

General Naïve Bayes

- Naïve Bayes assumes that all features are independent effects of the label
- A general Naive Bayes model:

$$P(Y, F_1 ... F_n) = P(Y) \prod_i P(F_i|Y)$$

$$|Y| \times |F|^n \text{ values} \qquad \text{n x } |F| \times |Y| \text{parameters}$$



- We only have to specify how each feature depends on the class
- Total number of parameters is *linear* in n
- Model is very simplistic, but often works anyway

Inference for Naïve Bayes

- Goal: compute posterior distribution over label variable Y
 - Step 1: get joint probability of label and evidence for each label

$$P(Y, f_1 \dots f_n) = \begin{bmatrix} P(y_1, f_1 \dots f_n) \\ P(y_2, f_1 \dots f_n) \\ \vdots \\ P(y_k, f_1 \dots f_n) \end{bmatrix} \longrightarrow \begin{bmatrix} P(y_1) \prod_i P(f_i|y_1) \\ P(y_2) \prod_i P(f_i|y_2) \\ \vdots \\ P(y_k) \prod_i P(f_i|y_k) \end{bmatrix}$$

- Step 2: sum to get probability of evidence
- Step 3: normalize by dividing Step 1 by Step 2

$$V \mid f_{\bullet} = f$$

$$P(Y|f_1 \dots f_n)$$

Naïve Bayes for Text

- Bag-of-words Naïve Bayes:
 - Features: W_i is the word at position i
 - As before: predict label conditioned on feature variables (spam vs. ham)
 - As before: assume features are conditionally independent given label
 - New: each W_i is identically distributed

Word at position *i,* not *i*th word in the dictionary!

- Generative model: $P(Y, W_1 \dots W_n) = P(Y) \prod_i P(W_i|Y)$
- "Tied" distributions and bag-of-words
 - Usually, each variable gets its own conditional probability distribution P(F|Y)
 - In a bag-of-words model
 - Each position is identically distributed
 - All positions share the same conditional probs P(W|Y)
 - Why make this assumption?
 - Called "bag-of-words" because model is insensitive to word order or reordering

Example: Spam Filtering

- Model: $P(Y, W_1 \dots W_n) = P(Y) \prod_i P(W_i | Y)$
- What are the parameters?

P(Y)

ham: 0.66 spam: 0.33

P(W|spam)

the: 0.0156
to: 0.0153
and: 0.0115
of: 0.0095
you: 0.0093
a: 0.0086
with: 0.0080
from: 0.0075

$P(W|\mathsf{ham})$

the: 0.0210
to: 0.0133
of: 0.0119
2002: 0.0110
with: 0.0108
from: 0.0107
and: 0.0105
a: 0.0100

Where do these tables come from?

Spam Example

Word	P(w spam)	P(w ham)	Tot Spam	Tot Ham
(prior)	0.33333	0.66666	-1.1	-0.4

General Naïve Bayes

- What do we need in order to use Naïve Bayes?
 - Inference method
 - Start with a bunch of probabilities: P(Y) and the P(F_i|Y) tables
 - Use standard inference to compute $P(Y|F_1...F_n)$
 - Nothing new here
 - Estimates of local conditional probability tables
 - P(Y), the prior over labels
 - P(F_i|Y) for each feature (evidence variable)
 - lacktriangle These probabilities are collectively called the *parameters* of the model and denoted by $oldsymbol{ heta}$
 - Up until now, we assumed these appeared by magic, but they typically come from training data counts

Parameter Estimation



Parameter Estimation

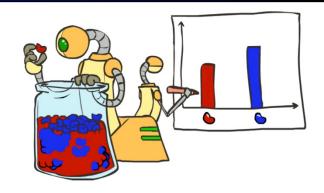
- Estimating the distribution of a random variable
- Elicitation: ask a human (why is this hard?)
- Empirically: use training data (learning!)
 - Example: The parameter θ is the true fraction of red beans in the jar.
 You don't know θ but would like to estimate it.
 - Collecting training data: You randomly pull out 3 beans:







- Estimating θ using counts, you guess 2/3 of beans in the jar are red.
- Can we mathematically show that using counts is the "right" way to estimate θ ?



- Can we mathematically show that using counts is the "right" way to estimate θ ?
- Maximum likelihood estimation: Choose the θ value that maximizes the probability of the observation
 - In other words, choose the θ value that maximizes P(observation $\mid \theta$)
 - For our problem:

```
P(observation | \theta)

= P(randomly selected 2 red and 1 blue | \theta of beans are red)

= P(red | \theta) P(red | \theta) P(blue | \theta)

= \theta^2 (1-\theta)
```

We want to compute:

```
\underset{\theta}{\text{argmax } \theta^2 \text{ (1- }\theta)}
```

We want to compute:

```
\underset{\theta}{\operatorname{argmax}} \theta^2 (1-\theta)
```

- Set derivative to 0, and solve!
 - Common issue: The likelihood (expression we're maxing) is the product of a lot of probabilities.
 This can lead to complicated derivatives.
 - Solution: Maximize the log-likelihood instead. Useful fact:

```
\underset{\theta}{\operatorname{argmax}} f(\theta) = \underset{\theta}{\operatorname{argmax}} \ln f(\theta)
```

$$\operatorname{argmax} \theta^{2}(1 - \theta)$$

$$= \operatorname{argmax} \ln \left(\theta^{2}(1 - \theta)\right)$$

$$\frac{d}{d\theta} \ln \left(\theta^{2}(1 - \theta)\right) = 0$$

$$\frac{d}{d\theta} \left[\ln(\theta^{2}) + \ln(1 - \theta)\right] = 0$$

$$\frac{d}{d\theta} \left[2\ln(\theta) + \ln(1 - \theta)\right] = 0$$

$$\frac{d}{d\theta} 2\ln(\theta) + \frac{d}{d\theta} \ln(1 - \theta) = 0$$

$$\frac{2}{\theta} - \frac{1}{1 - \theta} = 0$$

Find θ that maximizes likelihood

Find θ that maximizes log-likelihood (will be the same θ)

Set derivative to 0

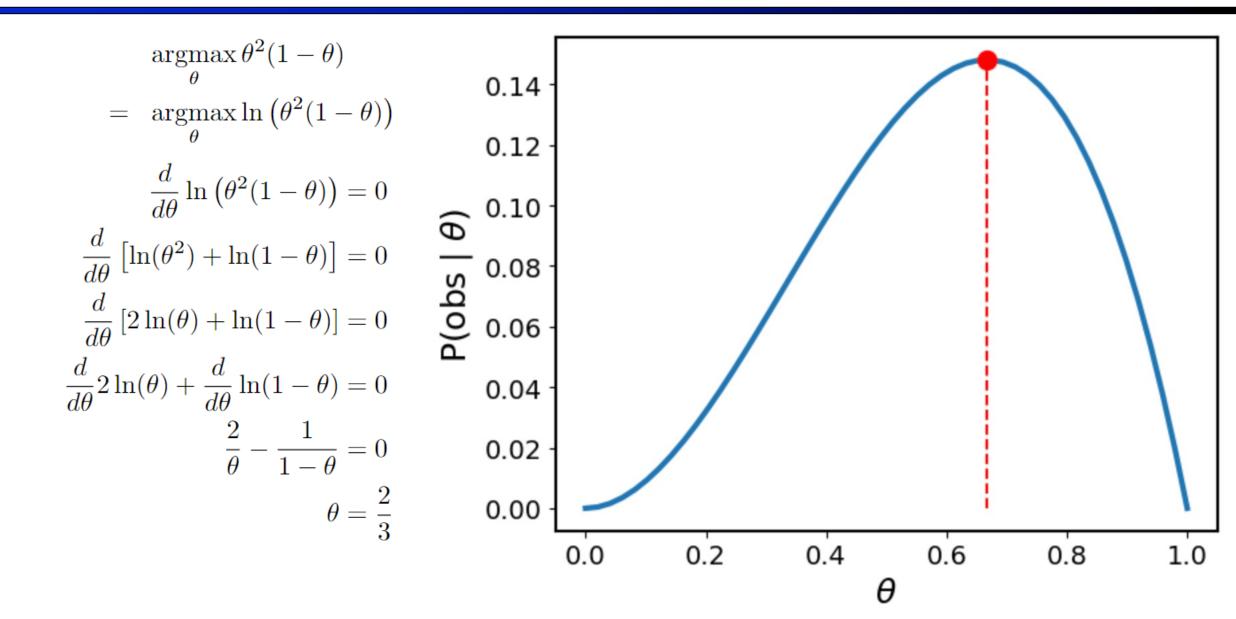
Logarithm rule: products become sums

 $\frac{d}{d\theta} \left[2 \ln(\theta) + \ln(1 - \theta) \right] = 0$ Logarithm rule: exponentiation becomes multiplication

Now we can derive each term of the original product separately

Reminder: Derivative of $ln(\theta)$ is $1/\theta$

Use algebra to solve for θ . If we used arbitrary red and blue counts r and b instead of r=2 and b=1, we'd get θ = r / (r+b), the count estimate.



Maximum Likelihood?

Relative frequencies are the maximum likelihood estimates

$$\theta_{ML} = \arg\max_{\theta} P(\mathbf{X}|\theta)$$

$$= \arg\max_{\theta} \prod_{i} P_{\theta}(X_{i})$$

$$P_{\text{ML}}(x) = \frac{\text{count}(x)}{\text{total samples}}$$

Another option is to consider the most likely parameter value given the data

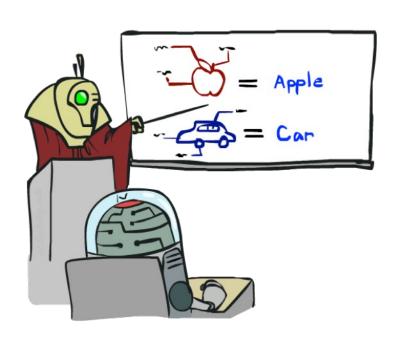
$$\theta_{MAP} = \arg\max_{\theta} P(\theta|\mathbf{X})$$

$$= \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta)/P(\mathbf{X})$$

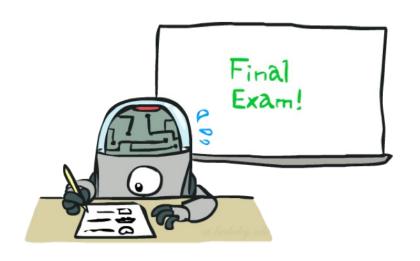
$$= \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta)$$
?????

- How do we estimate the conditional probability tables?
 - Maximum Likelihood, which corresponds to counting
- Need to be careful though ... let's see what can go wrong..

Training and Testing







Empirical Risk Minimization

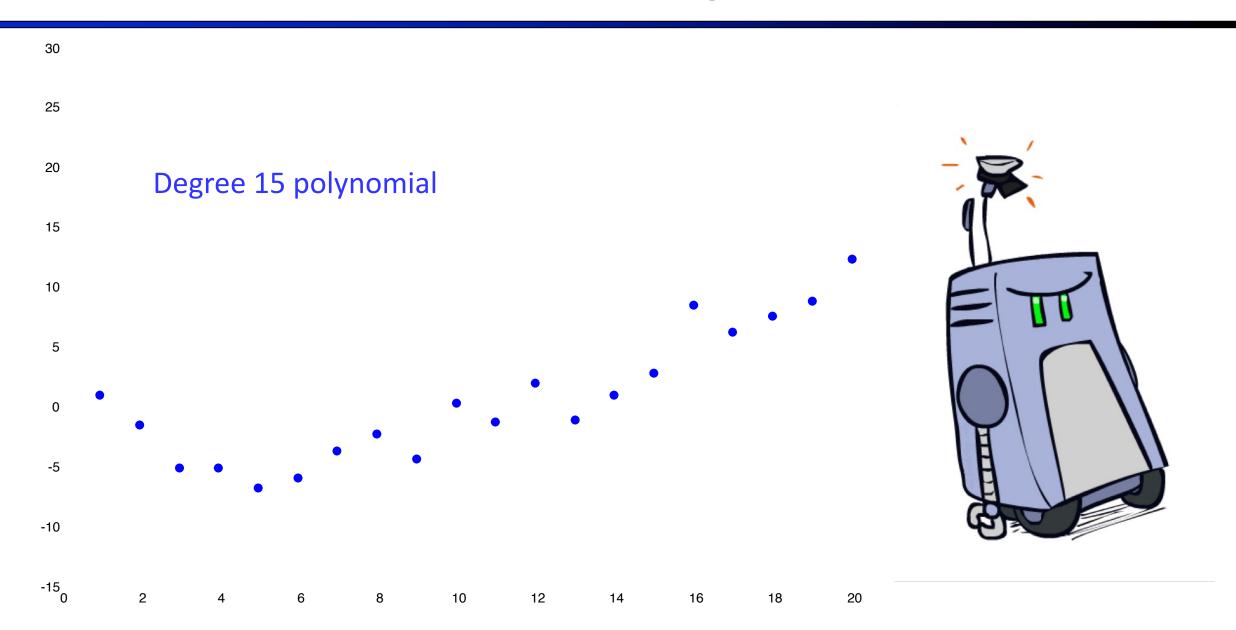
Empirical risk minimization

- Basic principle of machine learning
- We want the model (classifier, etc) that does best on the true test distribution
- Don't know the true distribution so pick the best model on our actual training set
- Finding "the best" model on the training set is phrased as an optimization problem

Main worry: overfitting to the training set

- Better with more training data (less sampling variance, training more like test)
- Better if we limit the complexity of our hypotheses (regularization and/or small hypothesis spaces)

Overfitting



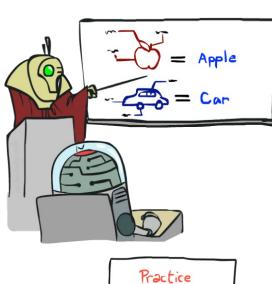
Important Concepts

- How do we check that we're not overfitting during training?
- Split training data into 3 different sets:
 - Training set
 - Held out set (more on this later)
 - Test set
- Experimentation cycle
 - Learn parameters (e.g. model probabilities) on training set
 - Compute accuracy of test set
 - Very important: never "peek" at the test set!
- Evaluation (many metrics possible, e.g. accuracy)
 - Accuracy: fraction of instances predicted correctly
- Overfitting and generalization
 - Want a classifier which does well on test data
 - Overfitting: fitting the training data very closely, but not generalizing well
 - We'll investigate overfitting and generalization formally in a few lectures

Training Data

Held-Out Data

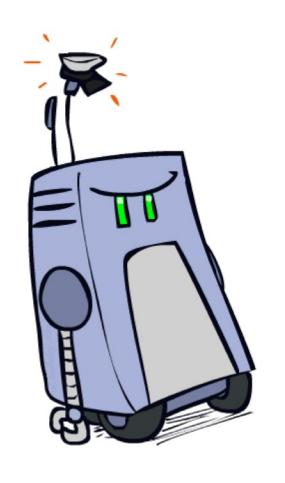
> Test Data

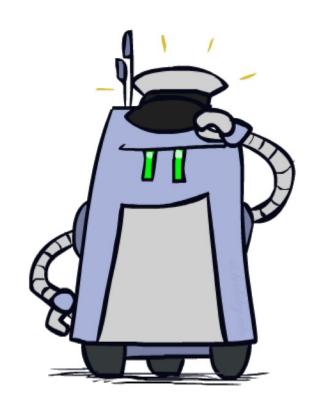


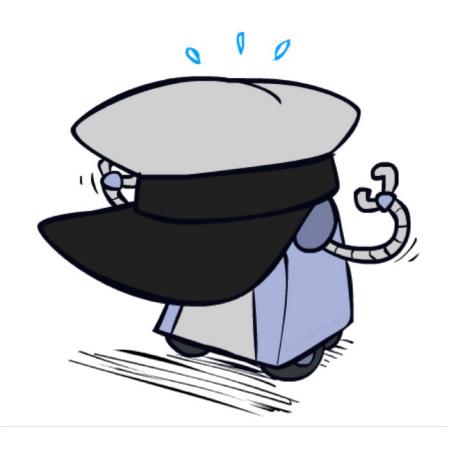




Generalization and Overfitting







Example: Overfitting

P(features, C = 2)

$$P(C = 2) = 0.1$$

$$P(\text{on}|C=2) = 0.8$$

$$P(\text{on}|C=2)=0.1$$

$$P(\text{off}|C=2) = 0.1$$

$$P(\mathsf{on}|C=2) = 0.01$$

P(features, C = 3)

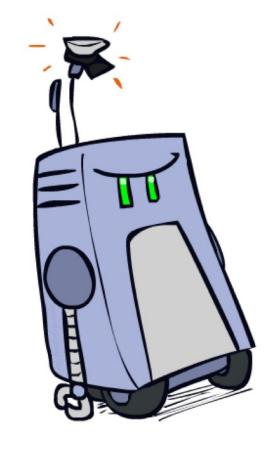
$$P(C = 3) = 0.1$$

$$P(\text{on}|C=3)=0.8$$

$$-P(\text{on}|C=3)=0.9$$

$$P(\text{off}|C=3)=0.7$$

$$-P(\text{on}|C=3)=0.0$$



2 wins!!

Example: Overfitting

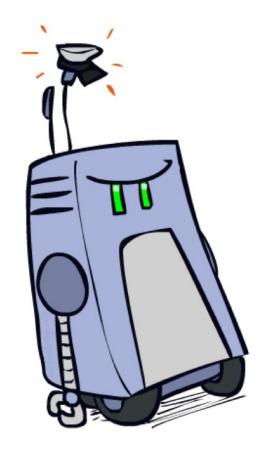
Posteriors determined by relative probabilities (odds ratios):

$$\frac{P(W|\mathsf{ham})}{P(W|\mathsf{spam})}$$

```
south-west : inf
nation : inf
morally : inf
nicely : inf
extent : inf
seriously : inf
```

```
\frac{P(W|\text{spam})}{P(W|\text{ham})}
```

```
screens : inf
minute : inf
guaranteed : inf
$205.00 : inf
delivery : inf
signature : inf
```

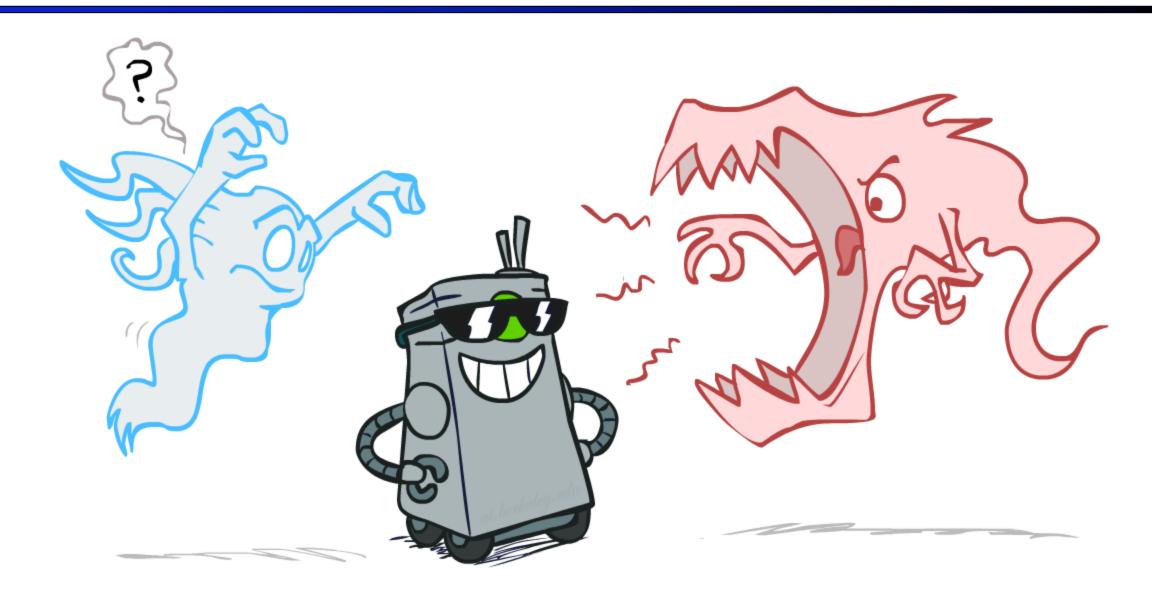


What went wrong here?

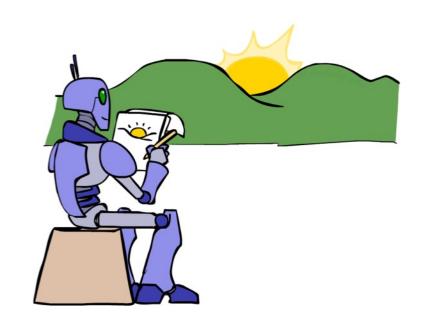
Generalization and Overfitting

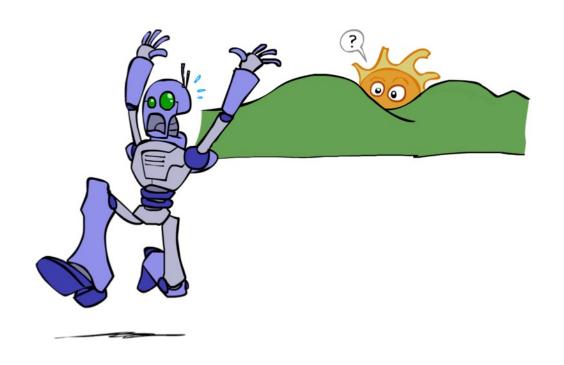
- Relative frequency parameters will overfit the training data!
 - Just because we never saw a 3 with pixel (15,15) on during training doesn't mean we won't see it at test time
 - Unlikely that every occurrence of "minute" is 100% spam
 - Unlikely that every occurrence of "seriously" is 100% ham
 - What about all the words that don't occur in the training set at all?
 - In general, we can't go around giving unseen events zero probability
- As an extreme case, imagine using the entire email as the only feature (e.g. document ID)
 - Would get the training data perfect (if deterministic labeling)
 - Wouldn't generalize at all
 - Just making the bag-of-words assumption gives us some generalization, but isn't enough
- To generalize better: we need to smooth or regularize the estimates

Smoothing



Unseen Events





Laplace Smoothing

Laplace's estimate:

 Pretend you saw every outcome once more than you actually did

$$P_{LAP}(x) = \frac{c(x) + 1}{\sum_{x} [c(x) + 1]}$$
$$= \frac{c(x) + 1}{N + |X|}$$

 Can derive this estimate with Dirichlet priors (see cs281a)

$$P_{ML}(X) =$$

$$P_{LAP}(X) =$$

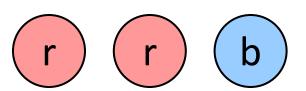
Laplace Smoothing

- Laplace's estimate (extended):
 - Pretend you saw every outcome k extra times

$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

- What's Laplace with k = 0?
- k is the strength of the prior
- Laplace for conditionals:
 - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y) + k}{c(y) + k|X|}$$



$$P_{LAP,0}(X) =$$

$$P_{LAP,1}(X) =$$

$$P_{LAP,100}(X) =$$

Real Naïve Bayes: Smoothing

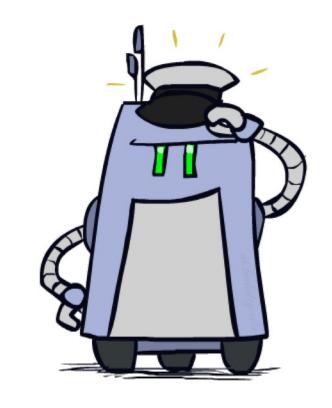
- For real classification problems, smoothing is critical
- New odds ratios:

$$\frac{P(W|\mathsf{ham})}{P(W|\mathsf{spam})}$$

helvetica: 11.4
seems: 10.8
group: 10.2
ago: 8.4
areas: 8.3

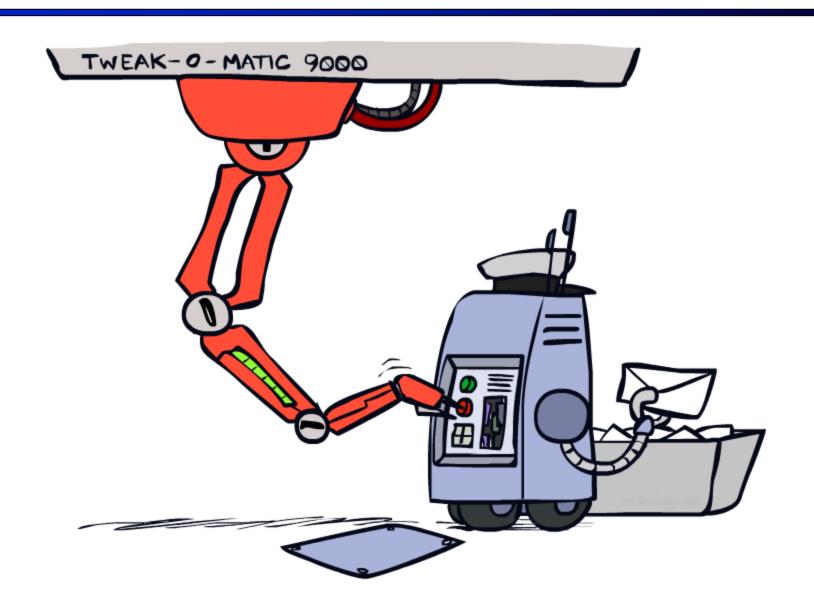
$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

verdana : 28.8
Credit : 28.4
ORDER : 27.2
 : 26.9
money : 26.5
...



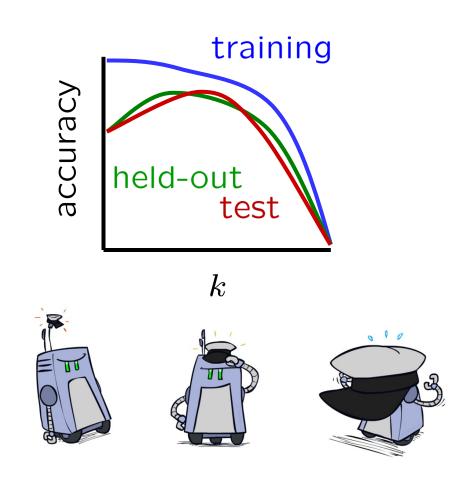
Do these make more sense?

Tuning

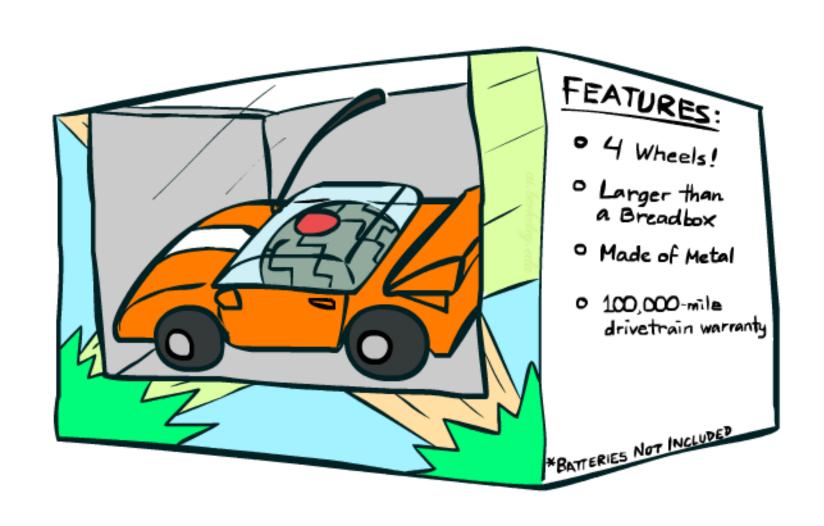


Tuning on Held-Out Data

- Now we've got two kinds of unknowns
 - Parameters: the probabilities P(X|Y), P(Y)
 - Hyperparameters: e.g. the amount / type of smoothing to do, k, α
- What should we learn where?
 - Learn parameters from training data
 - Tune hyperparameters on different data
 - Why?
 - For each value of the hyperparameters, train and test on the held-out data
 - Choose the best value and do a final test on the test data



Features



Errors, and What to Do

Examples of errors

Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the latest version of OmniPage Pro, for just \$99.99* - the regular list price is \$499! The most common question we've received about this offer is - Is this genuine? We would like to assure you that this offer is authorized by ScanSoft, is genuine and valid. You can get the . . .

. . . To receive your \$30 Amazon.com promotional certificate, click through to

http://www.amazon.com/apparel

and see the prominent link for the \$30 offer. All details are there. We hope you enjoyed receiving this message. However, if you'd rather not receive future e-mails announcing new store launches, please click . . .

What to Do About Errors?

- Need more features— words aren't enough!
 - Have you emailed the sender before?
 - Have 1K other people just gotten the same email?
 - Is the sending information consistent?
 - Is the email in ALL CAPS?
 - Do inline URLs point where they say they point?
 - Does the email address you by (your) name?
- Can add these information sources as new variables in the NB model
- Next class we'll talk about classifiers which let you easily add arbitrary features more easily, and, later, how to induce new features



Baselines

- First step: get a baseline
 - Baselines are very simple "straw man" procedures
 - Help determine how hard the task is
 - Help know what a "good" accuracy is
- Weak baseline: most frequent label classifier
 - Gives all test instances whatever label was most common in the training set
 - E.g. for spam filtering, might label everything as ham
 - Accuracy might be very high if the problem is skewed
 - E.g. calling everything "ham" gets 66%, so a classifier that gets 70% isn't very good...
- For real research, usually use previous work as a (strong) baseline

Summary

- Bayes rule lets us do diagnostic queries with causal probabilities
- The naïve Bayes assumption takes all features to be independent given the class label
- We can build classifiers out of a naïve Bayes model using training data
- Smoothing estimates is important in real systems
- Classifier confidences are useful, when you can get them

Next Time: Discriminative Learning