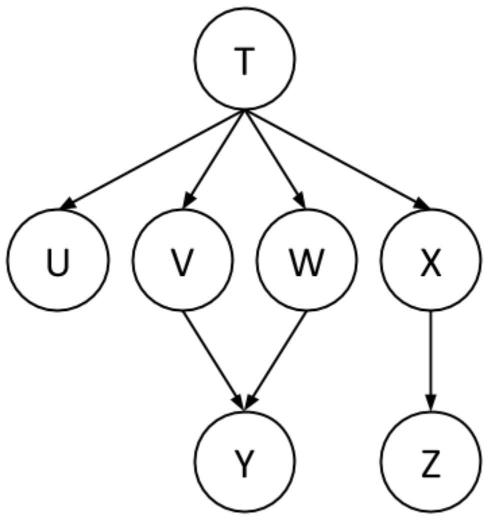
CS 188
Summer 2019

Section Handout 8

1 D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.



1. $U \perp\!\!\!\perp X$

2. $U \perp\!\!\!\perp X | T$

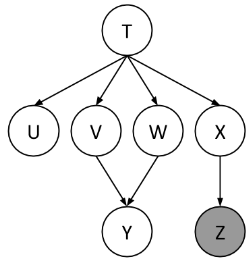
3. $V \perp\!\!\!\perp W | Y$

4. $V \perp\!\!\!\perp W | T$

5. $T \perp\!\!\!\perp Y | V$

6. $Y \perp\!\!\!\perp Z | W$

7. $Y \perp\!\!\!\perp Z | T$



2 Variable Elimination

Using the same Bayes Net (shown below), we want to compute $P(Y \mid +z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: X, T, U, V, W.

Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V, W), P(+z|X)$$

(a) When eliminating X we generate a new factor f_1 as follows, which leaves us with the factors:

$$f_1(+z|T) = \sum_x P(x|T)P(+z|x) \qquad P(T), P(U|T), P(V|T), P(W|T), P(Y|V, W), f_1(+z|T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

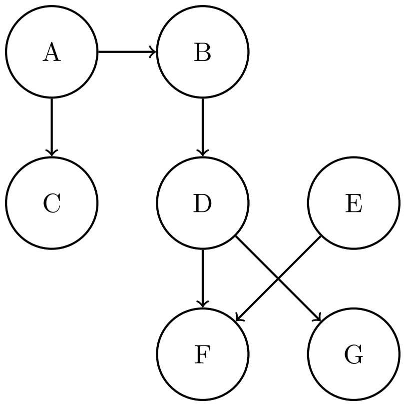
(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

(g) What is the size of the largest factor that gets generated during the above process?

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?

Q3. Bayes' Nets: Representation and Independence

Parts (a), (b), and (c) pertain to the following Bayes' Net.



(a) Express the joint probability distribution as a product of terms representing individual conditional probabilities tables associated with the Bayes Net.

(b) Assume each node can take on 4 values. How many entries do the factors at A, D, and F have?

A: ________________________________

D: ________________________________

F: ________________________________

(c) Mark the statements that are guaranteed to be true. Recall that every variable is conditionally independent of its non-descendants given its parents, and every variable is conditionally independent of all other variables given its Markov blanket.

- ☐ $B \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp F$
- ☐ $D \perp\!\!\!\perp E | F$
- ☐ $E \perp\!\!\!\perp A | D$
- ☐ $F \perp\!\!\!\perp G | D$
- ☐ $B \perp\!\!\!\perp F | D$
- ☐ $C \perp\!\!\!\perp G$
- ☐ $D \perp\!\!\!\perp E$

Parts (d) and (e) pertain to the following probability distribution tables. The joint distribution $P(A, B, C, D)$ is equal to the product of these probability distribution tables.

A	$P(A)$
+a	0.8
–a	0.2

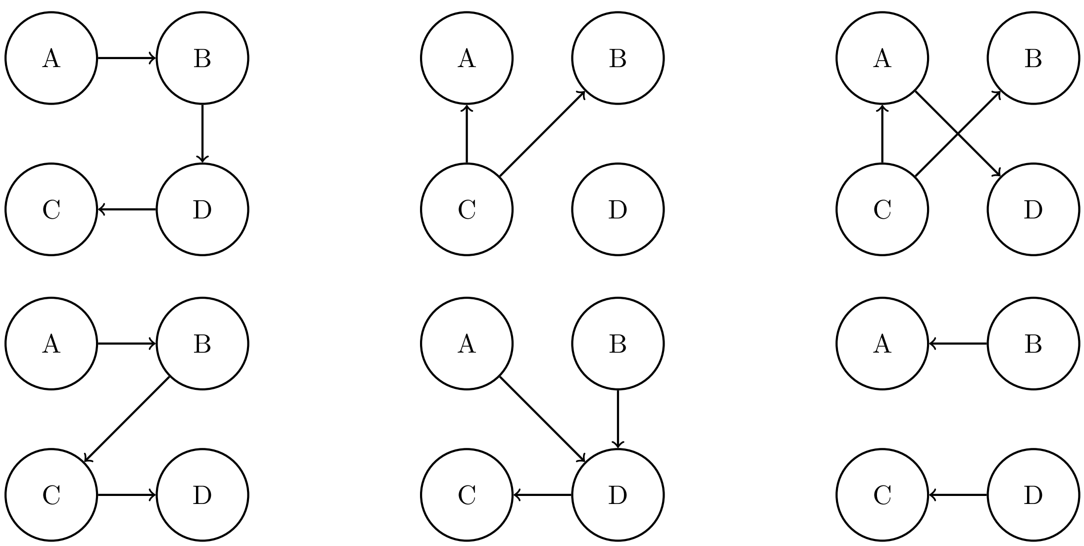
A	B	$P(B\|A)$
+a	+b	0.9
+a	–b	0.1
–a	+b	0.6
–a	–b	0.4

B	C	$P(C\|B)$
+b	+c	0.8
+b	–c	0.2
–b	+c	0.8
–b	–c	0.2

C	D	$P(D\|C)$
+c	+d	0.25
+c	–d	0.75
–c	+d	0.5
–c	–d	0.5

(d) State all non-conditional independence assumptions that are implied by the probability distribution tables.

(e) Circle all the Bayes net(s) that can represent a distribution that is consistent with the tables given.

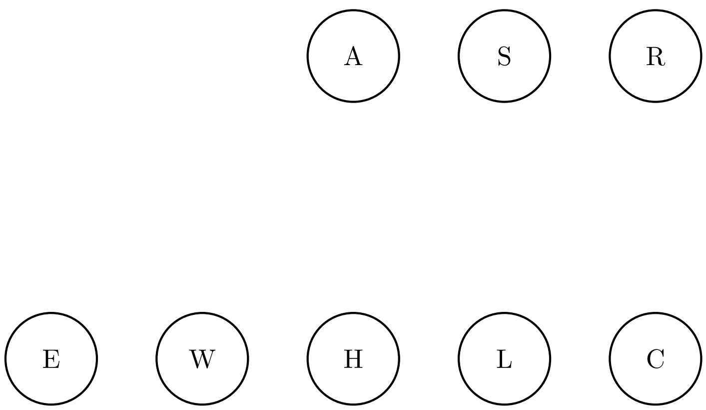


You are building advanced safety features for cars that can warn a driver if they are falling asleep (A) and also calculate the probability of a crash (C) in real time. You have at your disposal 6 sensors (random variables):

- E: whether the driver's eyes are open or closed
- W: whether the steering wheel is being touched or not
- L: whether the car is in the lane or not
- S: whether the car is speeding or not
- H: whether the driver's heart rate is somewhat elevated or resting
- R: whether the car radar detects a close object or not

A influences $\{E, W, H, L, C\}$. C is influenced by $\{A, S, L, R\}$.

(f) Draw the Bayes Net associated with the description above by adding edges between the provided nodes where appropriate.



(g) Mark all the independence assumptions that must be true.

☐ $E \perp\!\!\!\perp S$	☐ $L \perp\!\!\!\perp R \mid C$
☐ $W \perp\!\!\!\perp H \mid A$	☐ $W \perp\!\!\!\perp R$
☐ $S \perp\!\!\!\perp R$	☐ $A \perp\!\!\!\perp C$
☐ $E \perp\!\!\!\perp L$	☐ $E \perp\!\!\!\perp C \mid L$

(h) The car's sensors tell you that the car is in the lane ($L = +l$) and that the car is not speeding ($S = -s$). Now you would like to calculate the probability of crashing, $P(C \mid +l, -s)$. We will use the variable elimination ordering R, A, E, W, H. Write down the largest factor generated during variable elimination. Box your answer.

(i) Write down a more efficient variable elimination ordering, i.e. one whose largest factor is smaller than the one generated in the previous question.