

1 Probability

Use the probability table to calculate the following values:

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

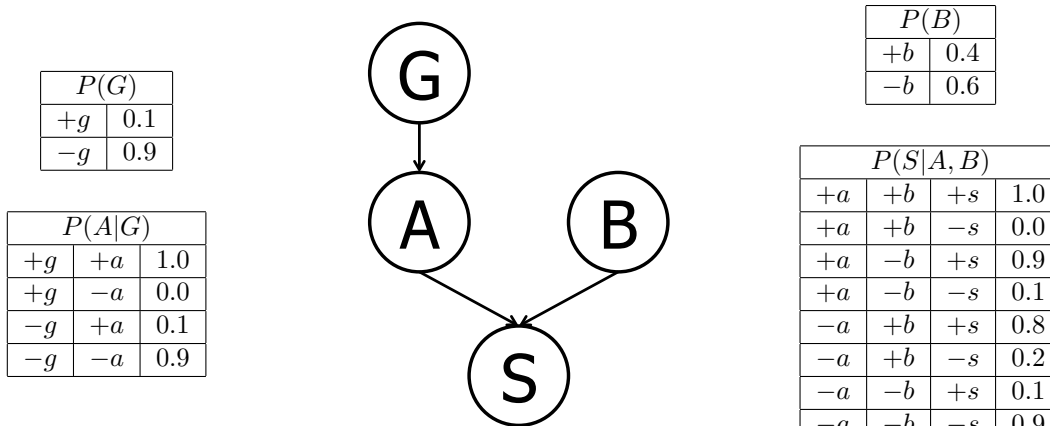
1. $P(X_1 = 1, X_2 = 0) = 0.15$

2. $P(X_3 = 0) = 0.65$

3. $P(X_2 = 1|X_3 = 1) = 0.2/0.35$

4. $P(X_1 = 0|X_2 = 1, X_3 = 1) = 1$

5. $P(X_1 = 0, X_2 = 1|X_3 = 1) = 0.2/0.35$



2 Bayes' Nets Representation and Probability

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A . The Bayes' Net and corresponding conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

(a) Compute the following entry from the joint distribution:

$$P(+g, +a, +b, +s) = P(+g)P(+a|+g)P(+b)P(+s|+b, +a) = (0.1)(1.0)(0.4)(1.0) = 0.04$$

(b) What is the probability that a patient has disease A ?

$$P(+a) = P(+a|+g)P(+g) + P(+a|-g)P(-g) = (1.0)(0.1) + (0.1)(0.9) = 0.19$$

(c) What is the probability that a patient has disease A given that they have disease B ?

$$P(+a|+b) = P(+a) = 0.19 \text{ The first equality holds true since } A \perp\!\!\!\perp B, \text{ which can be inferred from the graph of the Bayes' net.}$$

(d) What is the probability that a patient has disease A given that they have symptom S and disease B ?

$$\begin{aligned} P(+a|+s, +b) &= \frac{P(+a, +b, +s)}{P(+a, +b, +s) + P(-a, +b, +s)} = \frac{P(+a)P(+b)P(+s|+a, +b)}{P(+a)P(+b)P(+s|+a, +b) + P(-a)P(+b)P(+s|-a, +b)} \\ &= \frac{(0.19)(0.4)(1.0)}{(0.19)(0.4)(1.0) + (0.81)(0.4)(0.8)} = \frac{0.076}{0.076 + 0.2592} \approx 0.2267 \end{aligned}$$

(e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A ?

$$P(+g|+a) = \frac{P(+g)P(+a|+g)}{P(+g)P(+a|+g) + P(-g)P(+a|-g)} = \frac{(0.1)(1.0)}{(0.1)(1.0) + (0.9)(0.1)} = \frac{0.1}{0.1 + 0.09} = 0.5263$$