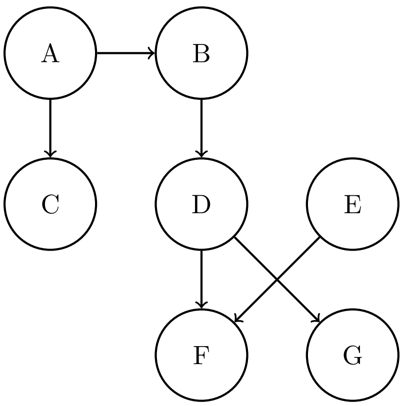
Discussion 3A

1 Bayes Nets: Representation

Parts (a), (b), and (c) pertain to the following Bayes' Net.



(a) Express the joint probability distribution as a product of terms from the Bayes Nets CPTs.

(b) Assume each node can take on 4 values. How many entries do the factors at A, D, and F have?

A: D: F:

(c) Mark all that are guaranteed to be true:

- ☐ $B \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp F$
- ☐ $D \perp\!\!\!\perp E|F$
- ☐ $E \perp\!\!\!\perp A|D$
- ☐ $F \perp\!\!\!\perp G|D$
- ☐ $B \perp\!\!\!\perp F|D$
- ☐ $C \perp\!\!\!\perp G$
- ☐ $D \perp\!\!\!\perp E$

Parts (d) and (e) pertain to the following CPTs.

A	$P(A)$
+a	0.8
−a	0.2

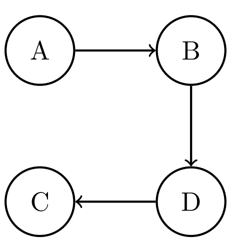
A	B	$P(B\|A)$
+a	+b	0.9
+a	−b	0.1
−a	+b	0.6
−a	−b	0.4

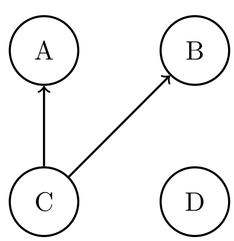
B	C	$P(C\|B)$
+b	+c	0.8
+b	−c	0.2
−b	+c	0.8
−b	−c	0.2

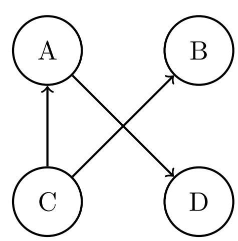
C	D	$P(D\|C)$
+c	+d	0.25
+c	−d	0.75
−c	+d	0.5
−c	−d	0.5

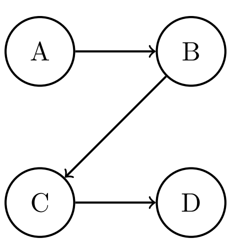
(d) State all non-conditional independence assumptions that are implied by the probability distribution tables.

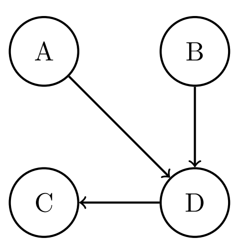
(e) Circle all the Bayes net(s) that can represent a distribution that is consistent with the tables given.

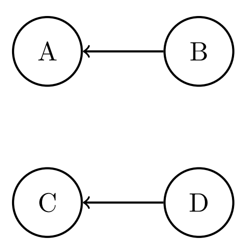








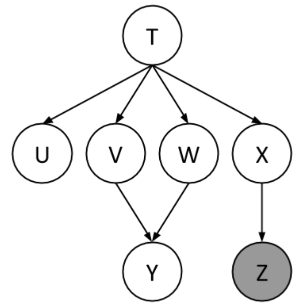




2 Variable Elimination

Using the Bayes Net shown below, we want to compute $P(Y \mid +z)$. All variables have **binary domains**. We run variable elimination, with the following variable elimination ordering: X, T, U, V, W.

After inserting evidence, we have the following factors to start out with:



$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V, W), P(+z|X)$$

(a) When eliminating X we generate a new factor f_1 as follows,

$$f_1(+z, T) = \sum_x P(x|T)P(+z|x)$$

which leaves us with the factors:

$$P(T), P(U|T), P(V|T), P(W|T), P(Y|V, W), f_1(+z, T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

(g) What is the size of the largest factor that gets generated during the above process?

(h) Does there exist a better elimination ordering (one which generates smaller largest factors)?