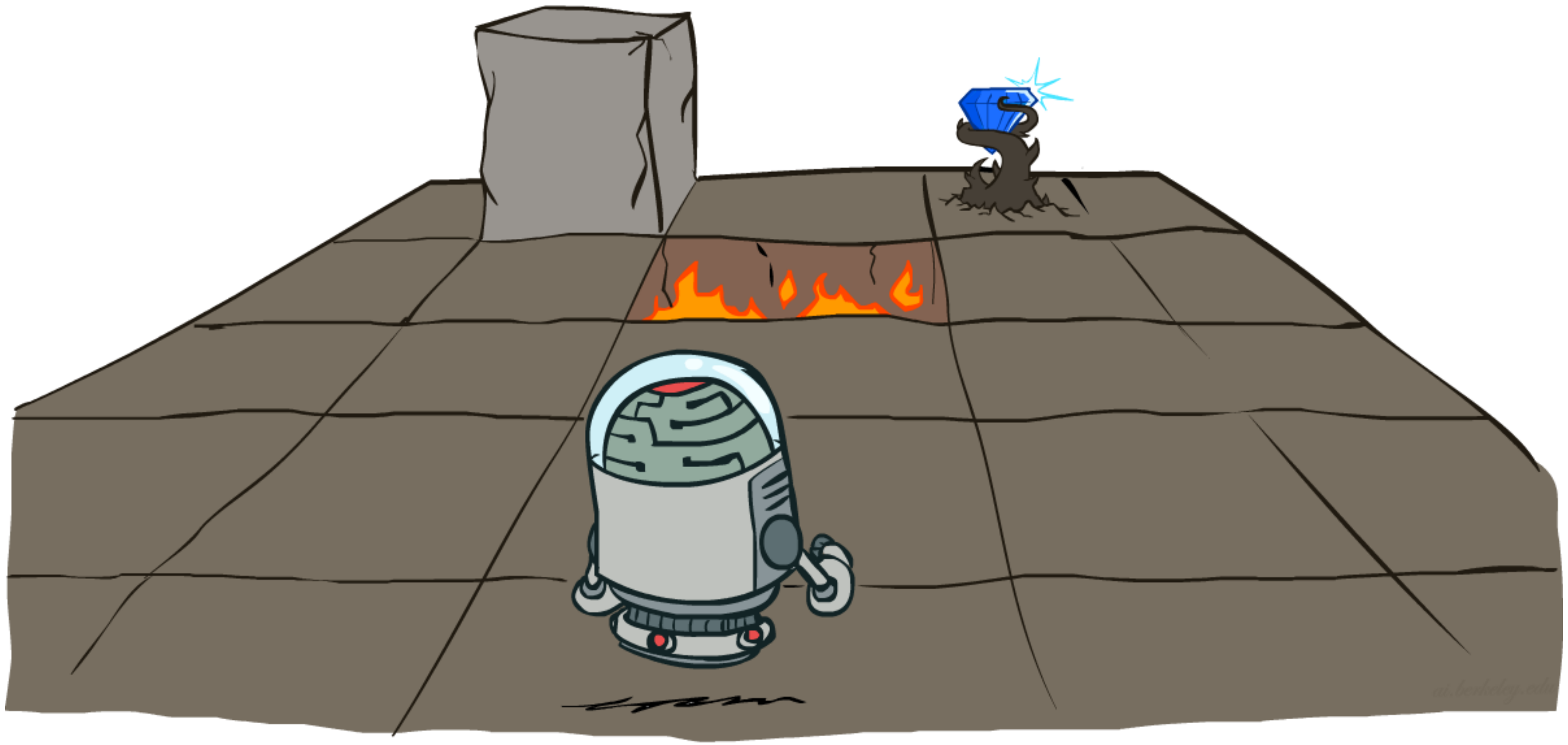
CS 188: Artificial Intelligence

Markov Decision Processes

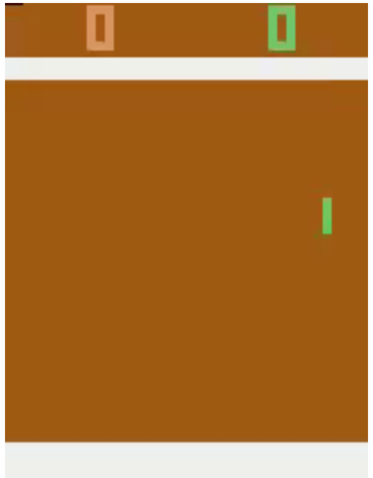


Instructor: Saagar Sanghavi – UC Berkeley

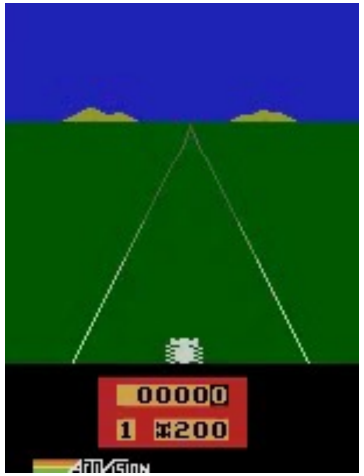
[Slides adapted from Dan Klein, Pieter Abbeel, Ketrina Yim, Stuart Russell, Satish Rao, and many others.]

Deep Reinforcement Learning

2013 **Atari (DQN)**
[Deepmind]

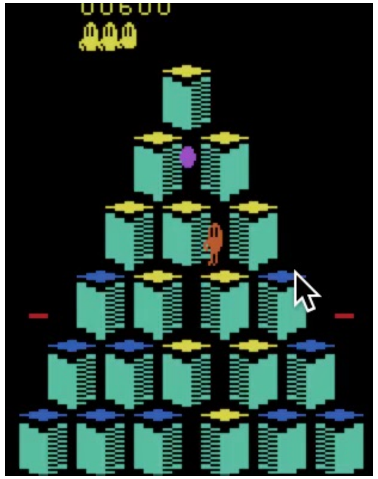


Pong



Enduro

Beamrider



Q*bert

Deep Reinforcement Learning

2013 Atari (DQN)
[Deepmind]

2015 **AlphaGo**
[Deepmind]

AlphaGo Silver et al, Nature 2015
AlphaGoZero Silver et al, Nature 2017
AlphaZero Silver et al, 2017
Tian et al, 2016; Maddison et al, 2014; Clark et al, 2015

Deep Reinforcement Learning

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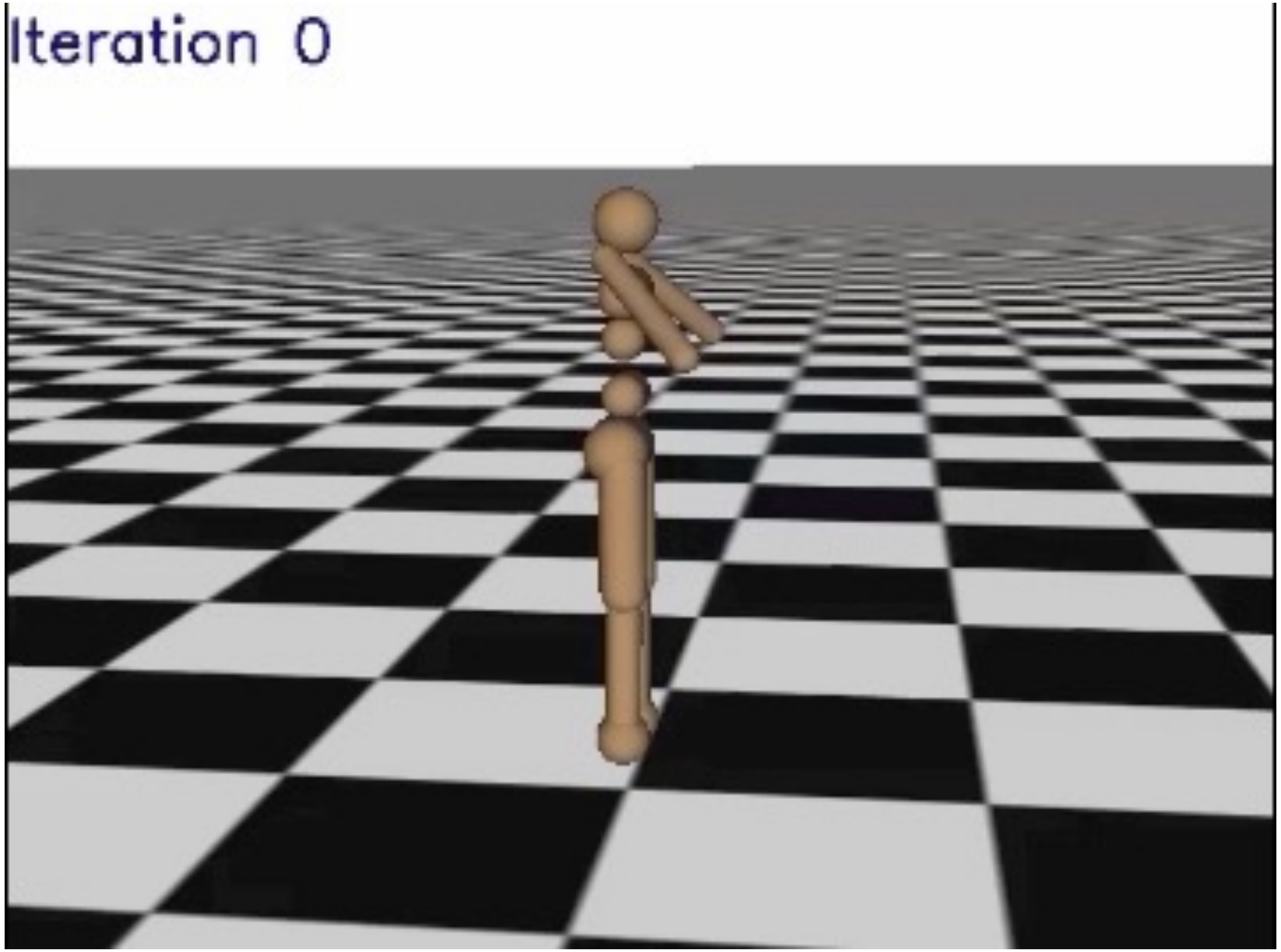
2016 3D locomotion (TRPO+GAE)
[Berkeley]



[Schulman, Moritz, Levine, Jordan, Abbeel, ICLR 2016]

Deep Reinforcement Learning

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2016 3D locomotion (TRPO+GAE)
[Berkeley]

2016 Real Robot Manipulation (GPS)
[Berkeley]

[Levine*, Finn*, Darrell, Abbeel, JMLR 2016]

Deep Reinforcement Learning

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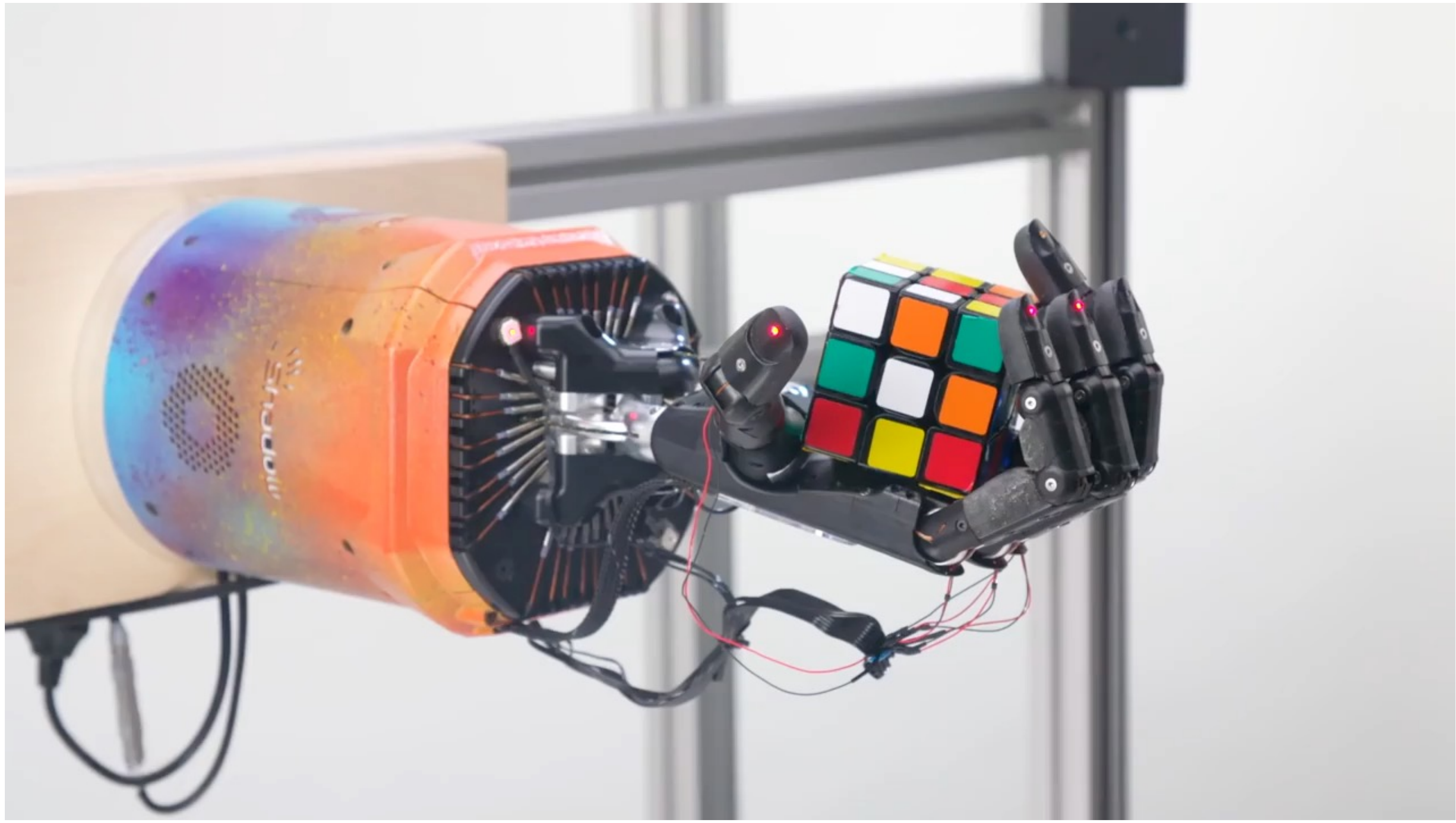
[Berkeley]

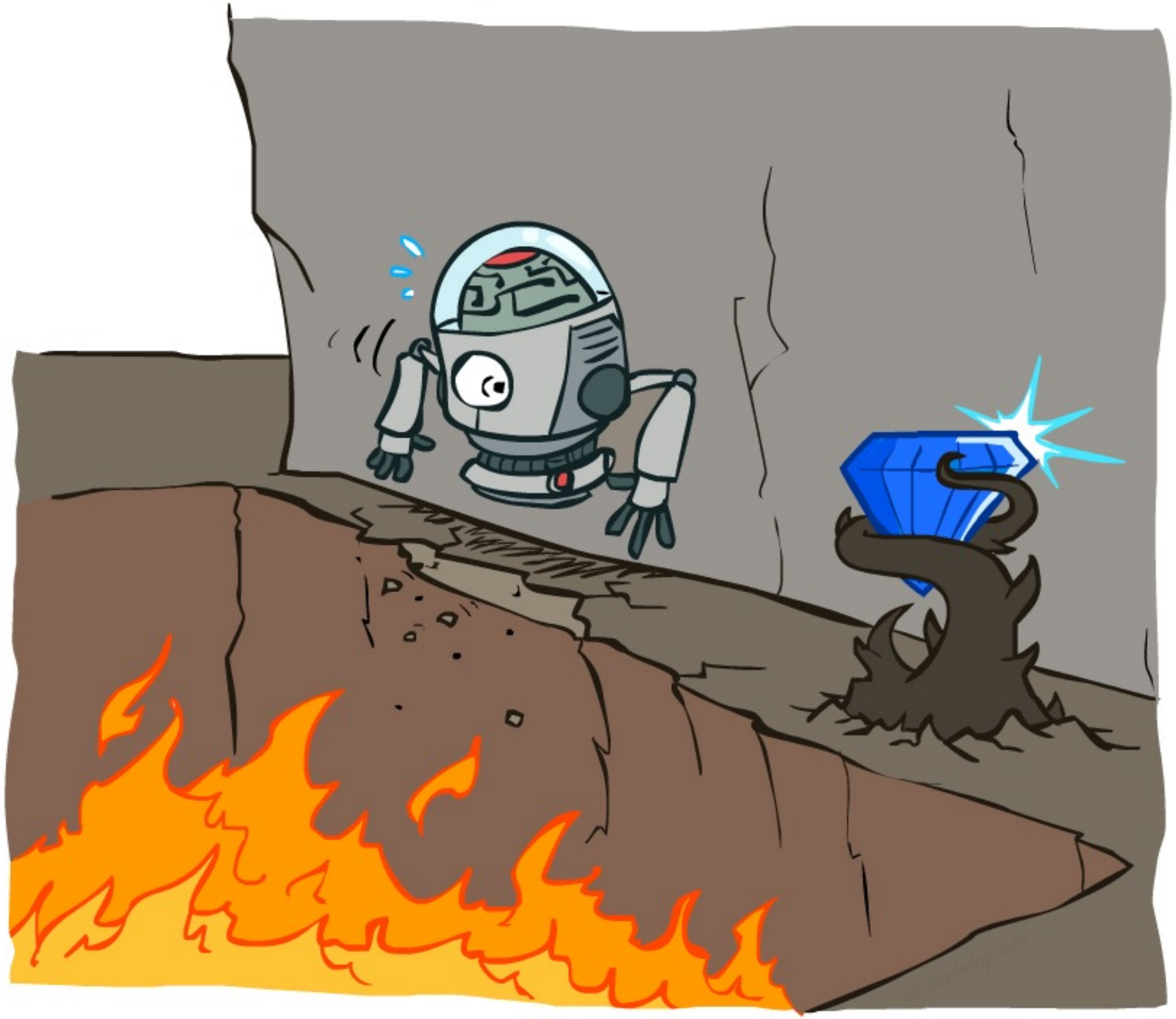
2016 Real Robot Manipulation (GPS)
[Berkeley]

2019 Rubik's Cube (PPO+DR)
[OpenAI]



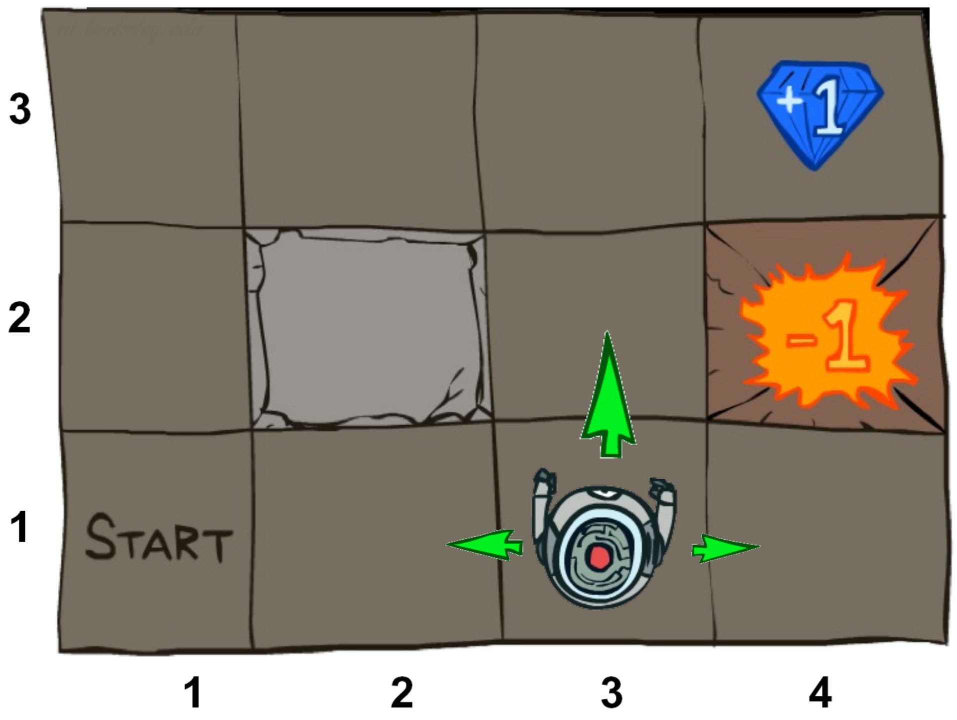
OpenAI

Non-Deterministic Search



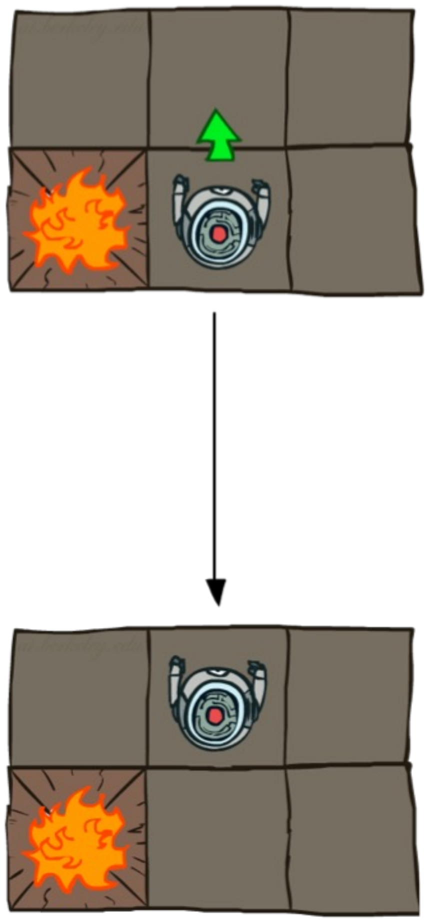
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

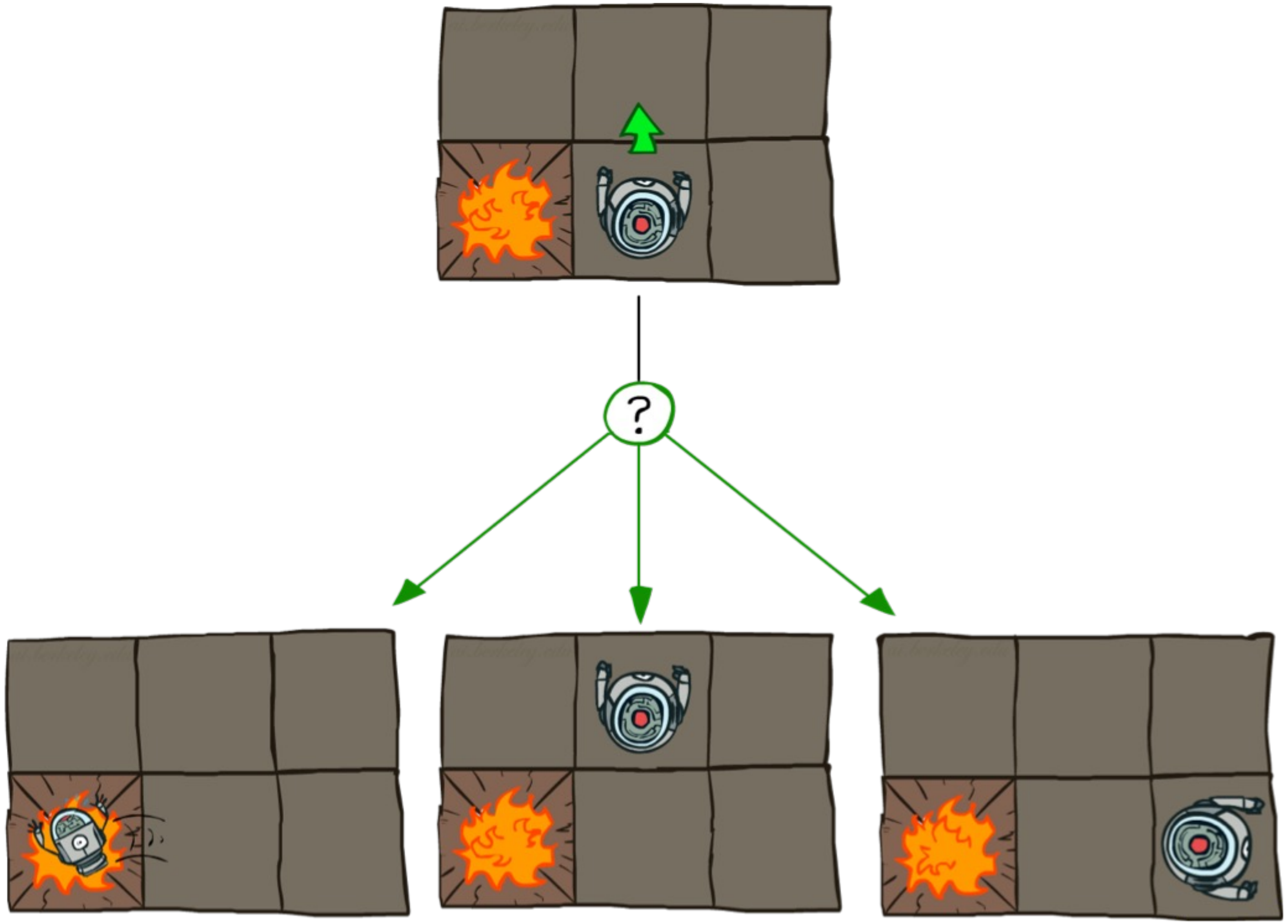


Grid World Actions

Deterministic Grid World

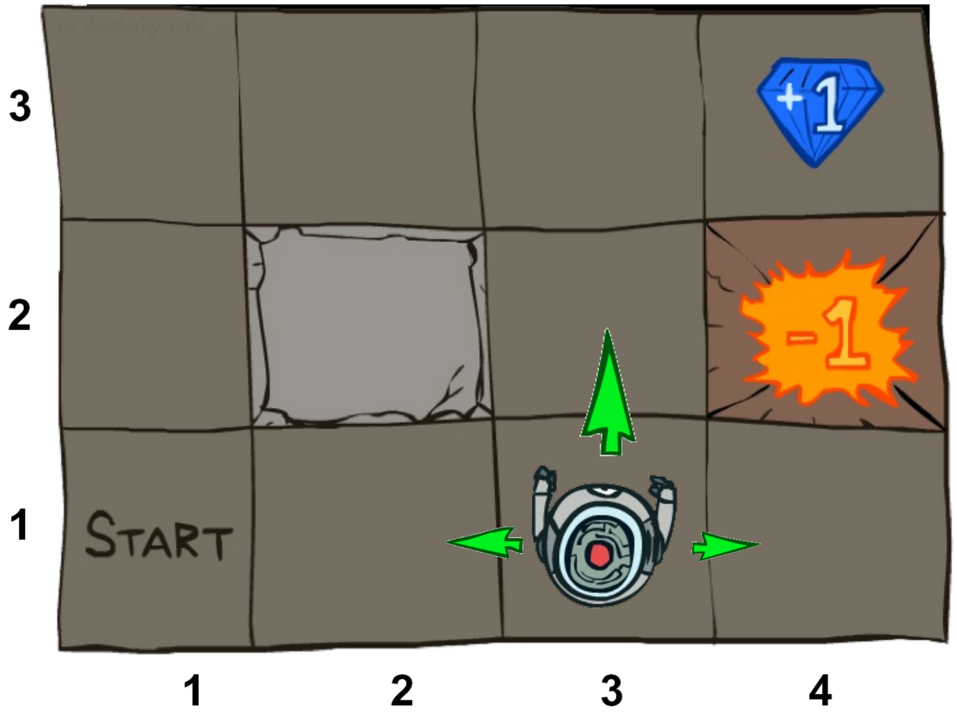


Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
  - A set of states $s \in S$
  - A set of actions $a \in A$
  - A transition function T(s, a, s')
    - Probability that a from s leads to s', i.e., P(s' | s, a)
    - Also called the model or the dynamics
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state
  - Maybe a terminal state



[Demo – gridworld manual intro (L8D1)]

Video of Demo Gridworld Manual Intro

What is Markov about MDPs?

- "Markov" generally means that given the present state, the future and the past are independent

- For Markov decision processes, "Markov" means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

$$=$$

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

- This is just like search, where the successor function could only depend on the current state (not the history)

Andrey Markov
(1856-1922)

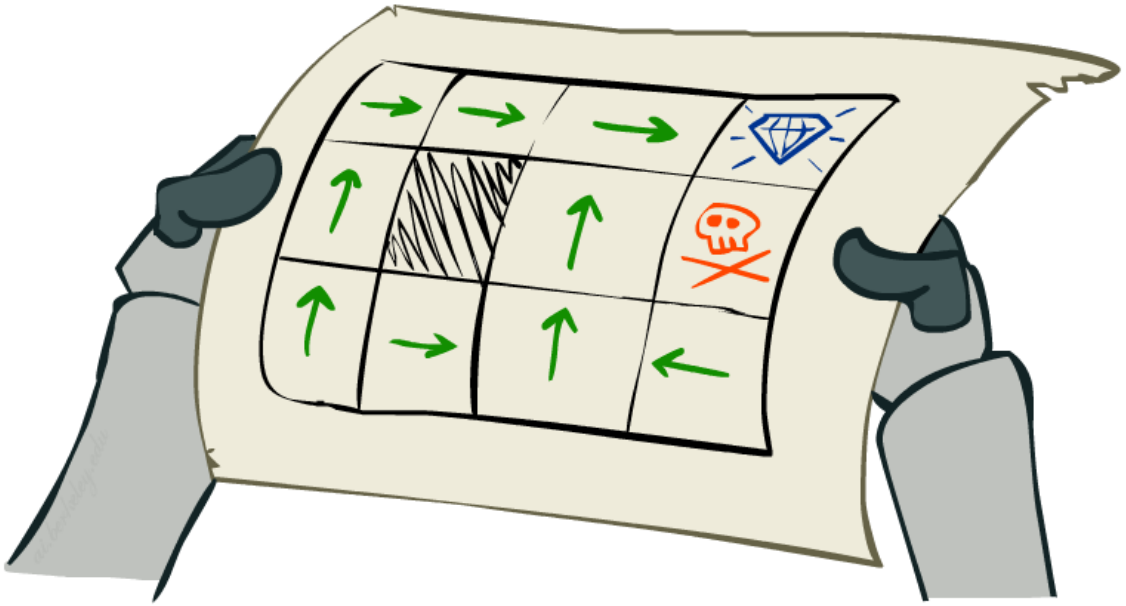
Policies

- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

- For MDPs, we want an optimal

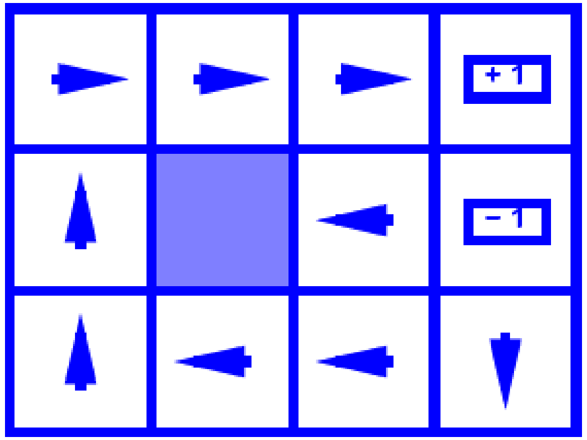
  policy $\pi^*: S \rightarrow A$

  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed
  - An explicit policy defines a reflex agent

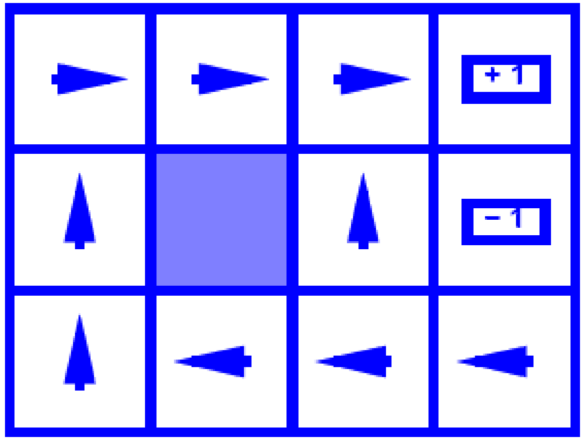


Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

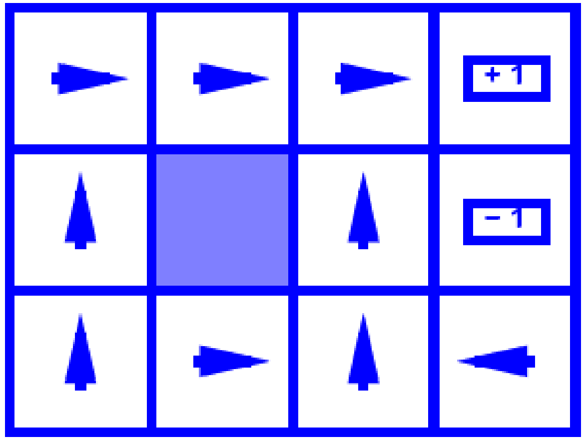
Optimal Policies



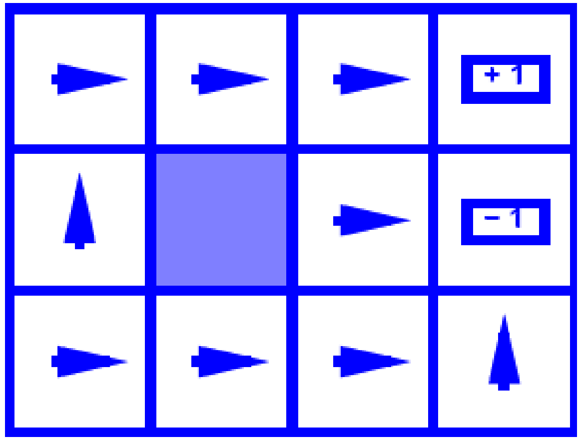
R(s) = -0.01



R(s) = -0.03

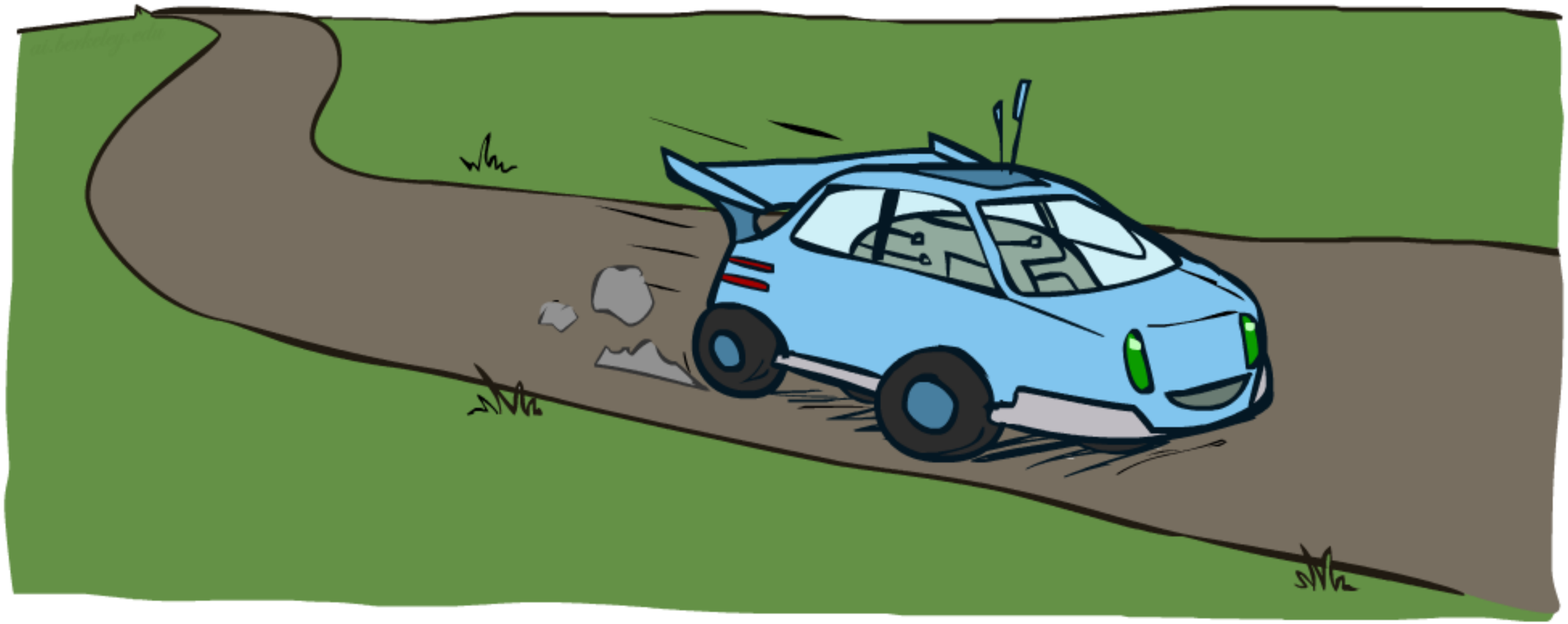


R(s) = -0.4



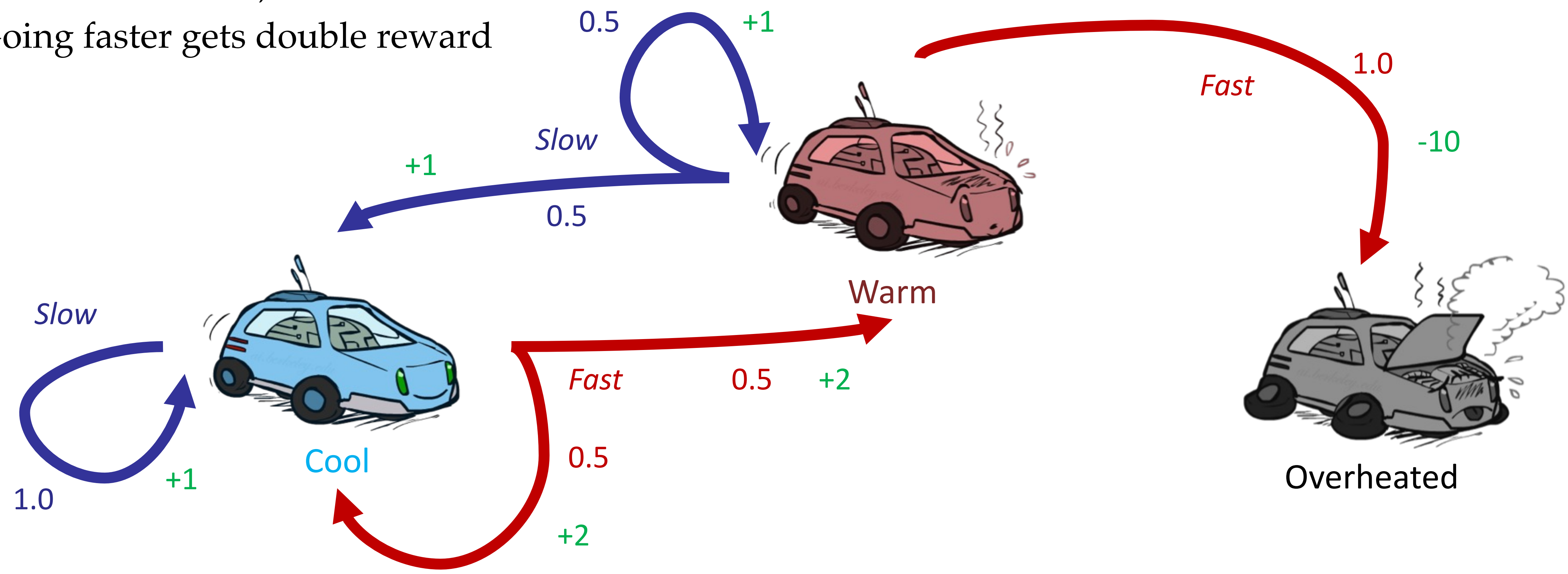
R(s) = -2.0

Example: Racing

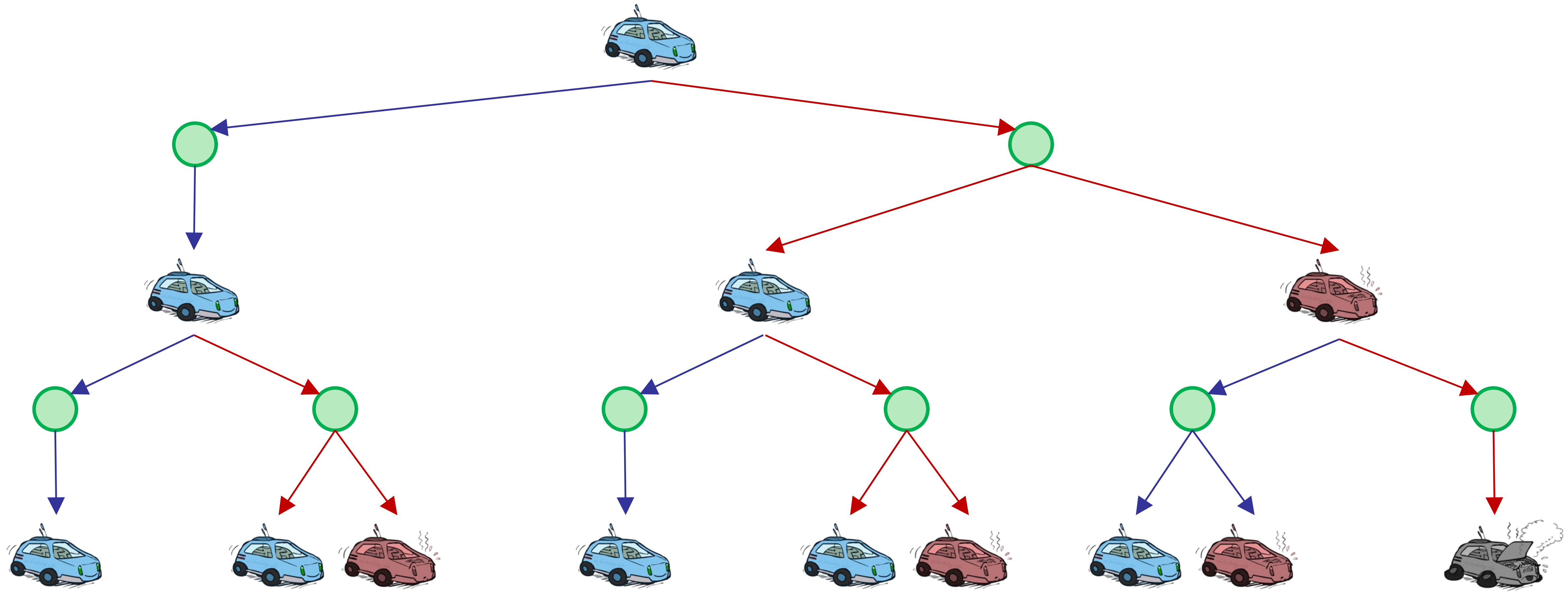


Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
- Going faster gets double reward

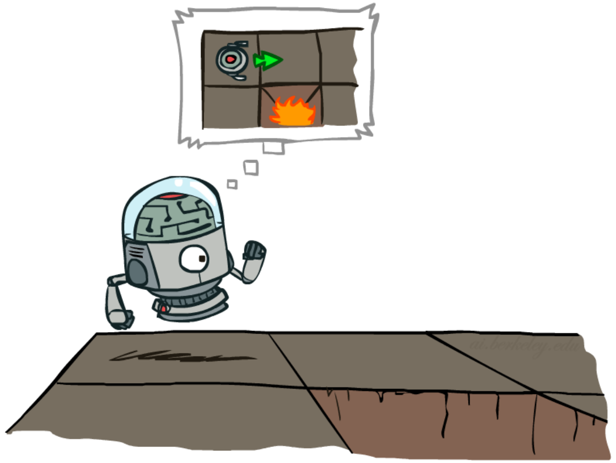


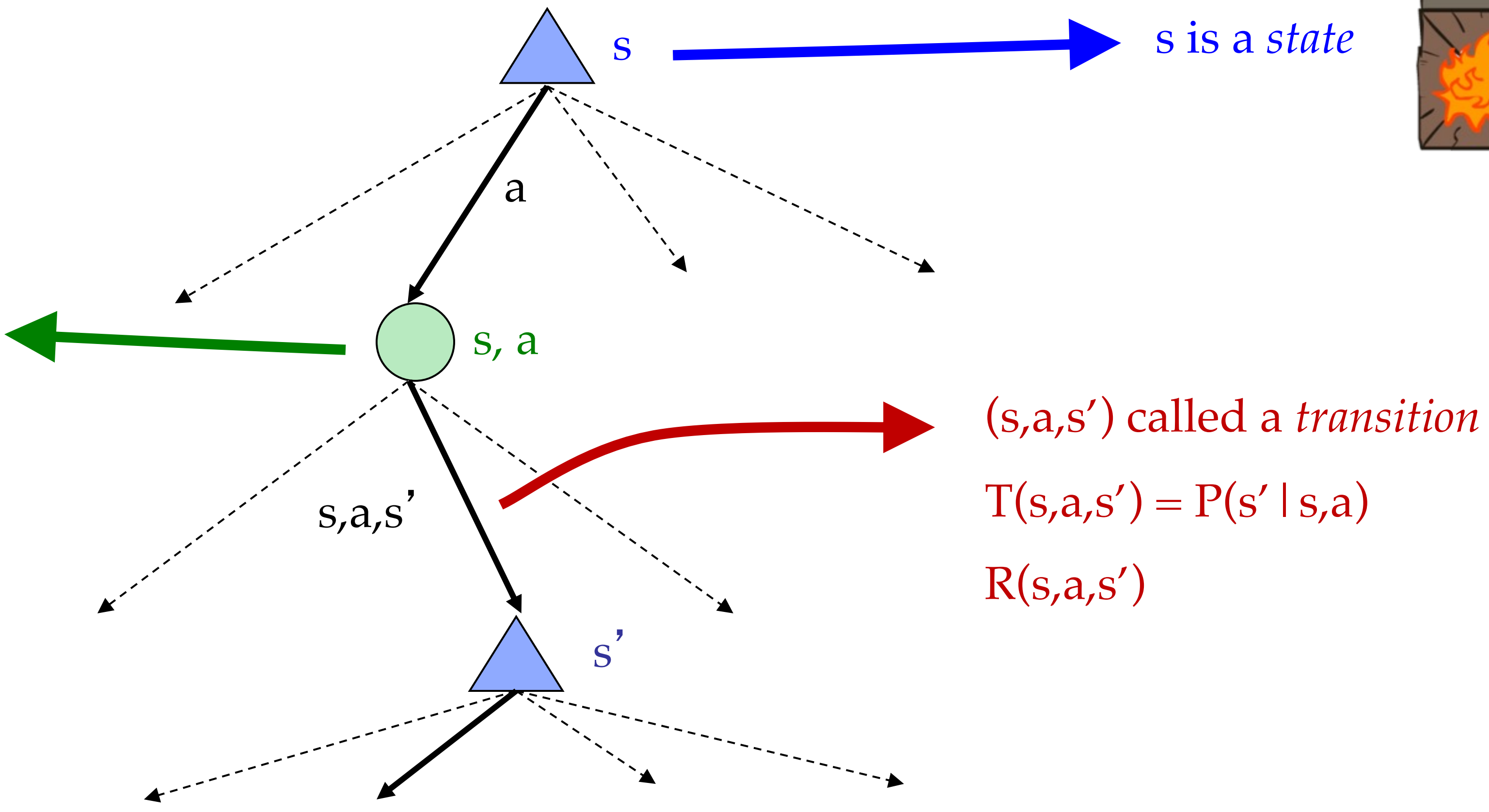
Racing Search Tree



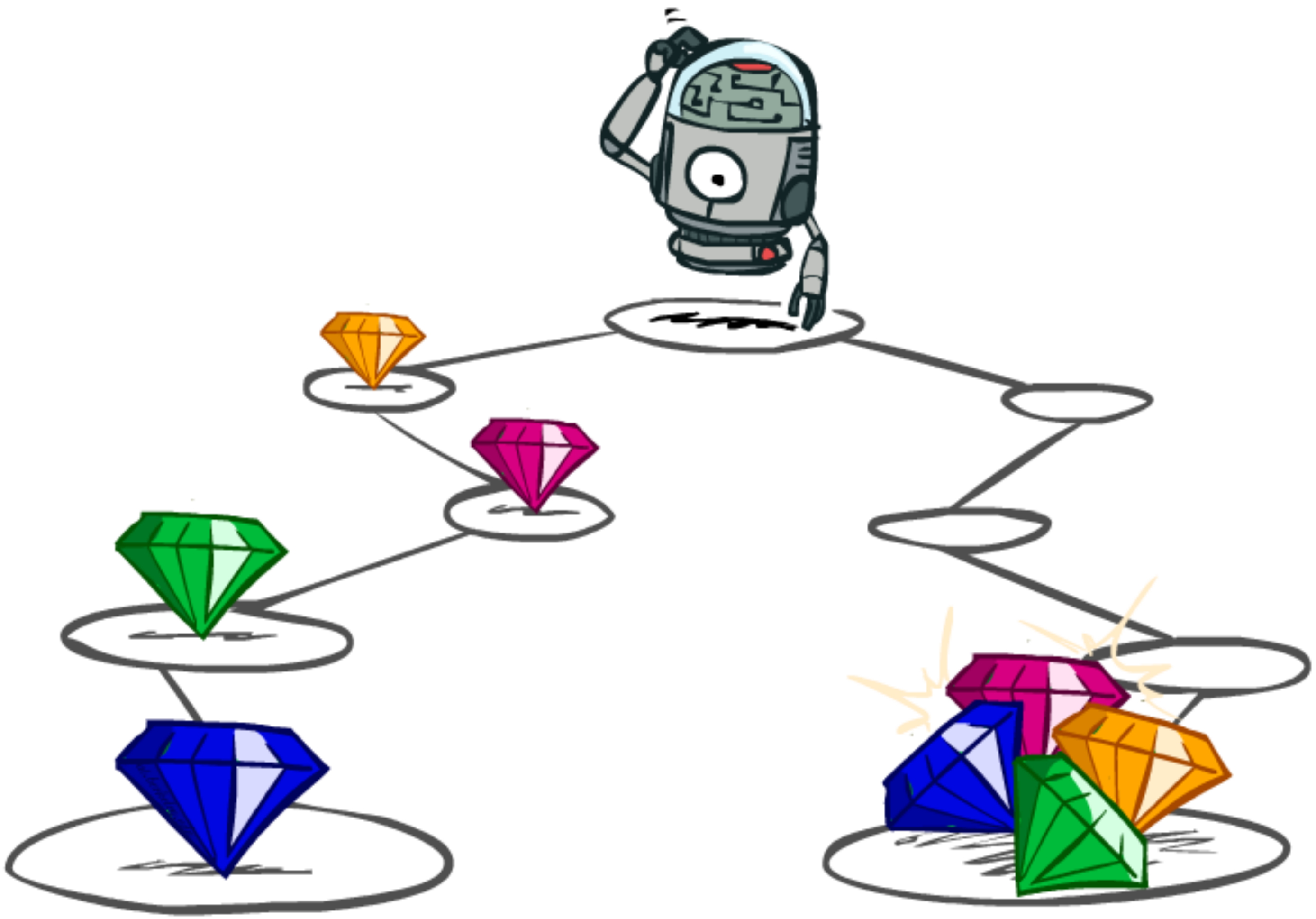
MDP Search Trees

- Each MDP state projects an expectimax-like search tree



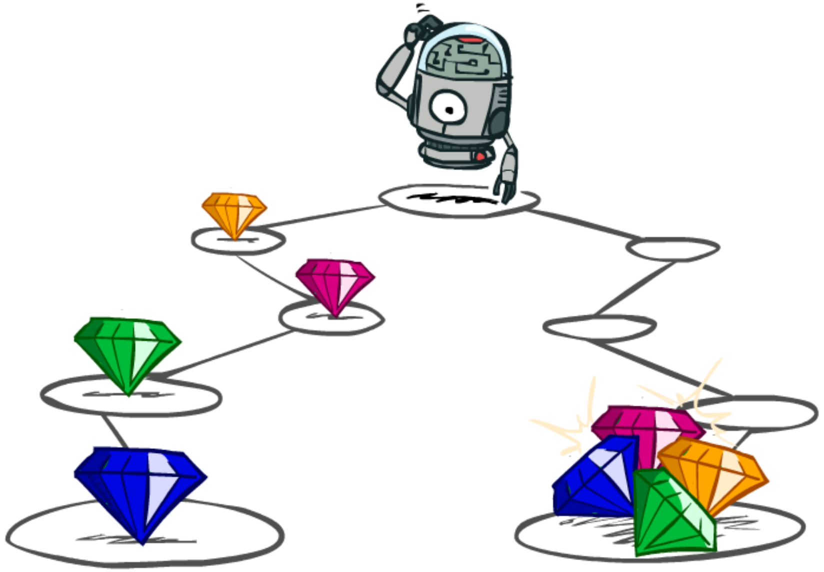


Utilities of Sequences



Utilities of Sequences

- What preferences should an agent have over reward sequences?
- More or less? [1, 2, 2] or [2, 3, 4]
- Now or later? [0, 0, 1] or [1, 0, 0]



Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

1

Worth Now

γ

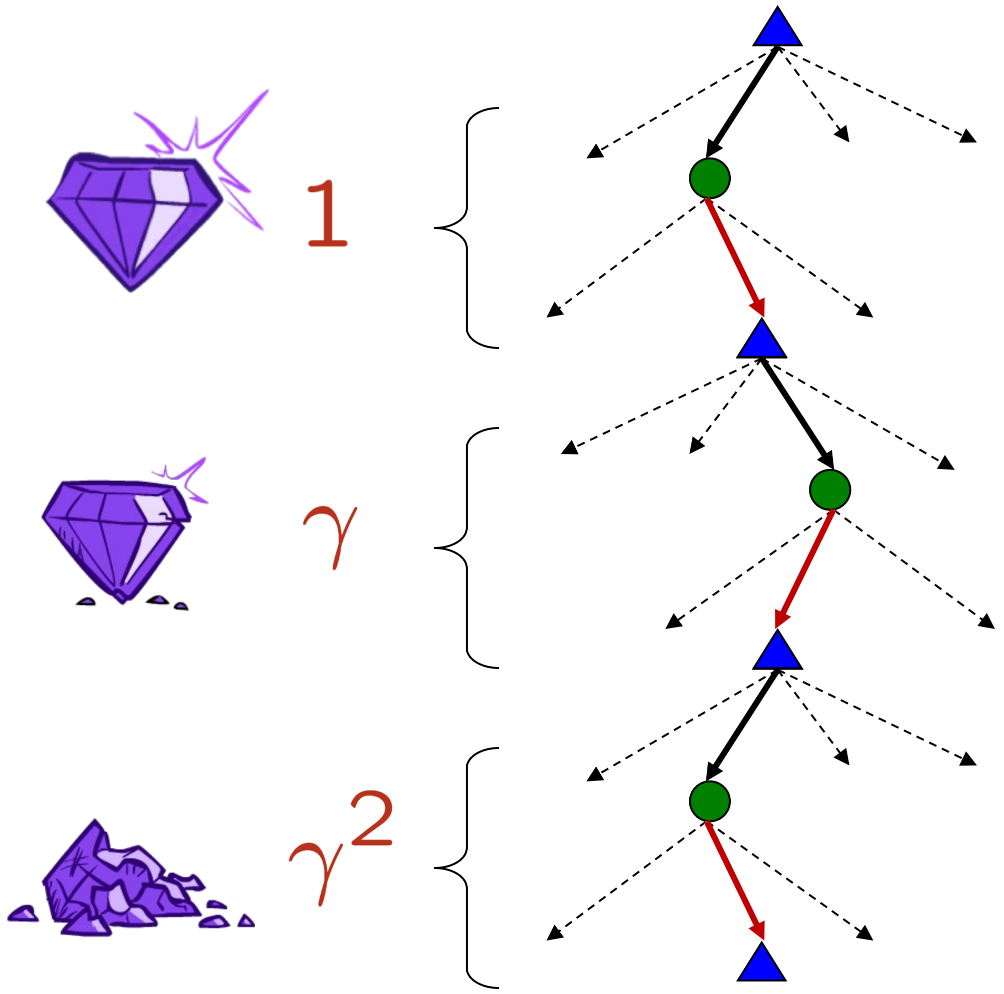
Worth Next Step

γ^2

Worth In Two Steps

Discounting

- How to discount?
  - Each time we descend a level, we multiply in the discount once
- Why discount?
  - Reward now is better than later
  - Can also think of it as a 1-gamma chance of ending the process at every step
  - Also helps our algorithms converge
- Example: discount of 0.5
  - U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3
  - U([1,2,3]) < U([3,2,1])



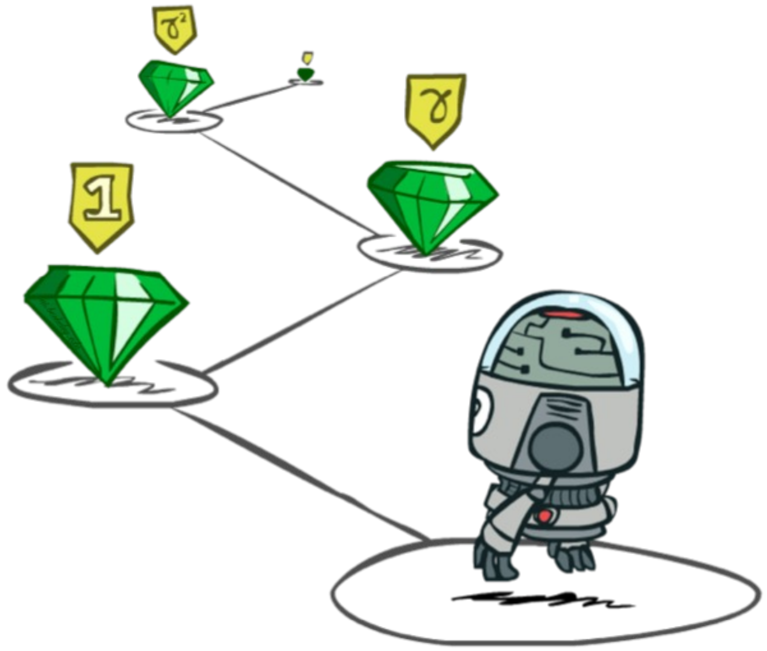
Stationary Preferences

- Theorem: if we assume stationary preferences:

$$[a_1, a_2, \ldots] \succ [b_1, b_2, \ldots]$$

$$\Updownarrow$$

$$[r, a_1, a_2, \ldots] \succ [r, b_1, b_2, \ldots]$$



- Then: there is only ways to define utilities
  - Additive discounted utility:

$$U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \cdots$$

Quiz: Discounting

- Given:

10				1
a	b	c	d	e

  - Actions: East, West, and Exit (only available in exit states a, e)
  - Transitions: deterministic

- Quiz 1: For $\gamma = 1$, what is the optimal policy?

10	<-	<-	<-	1

- Quiz 2: For $\gamma = 0.1$, what is the optimal policy?

10	<-	<-	->	1

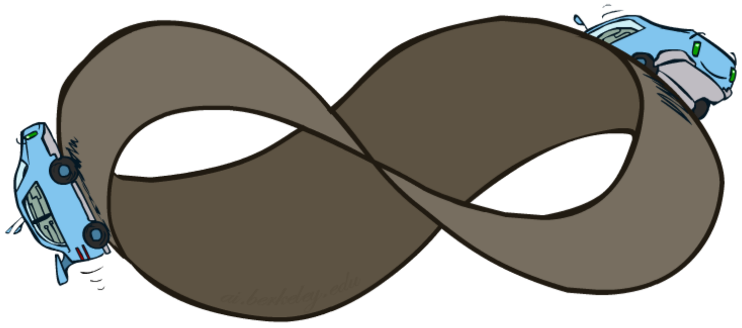
- Quiz 3: For which γ are West and East equally good when in state d?

$1\gamma = 10\,\gamma^3$

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?

- Solutions:
  - Finite horizon: (similar to depth-limited search)
    - Terminate episodes after a fixed T steps (e.g. life)
    - Gives nonstationary policies (π depends on time left)



  - Discounting: use $0 < \gamma < 1$

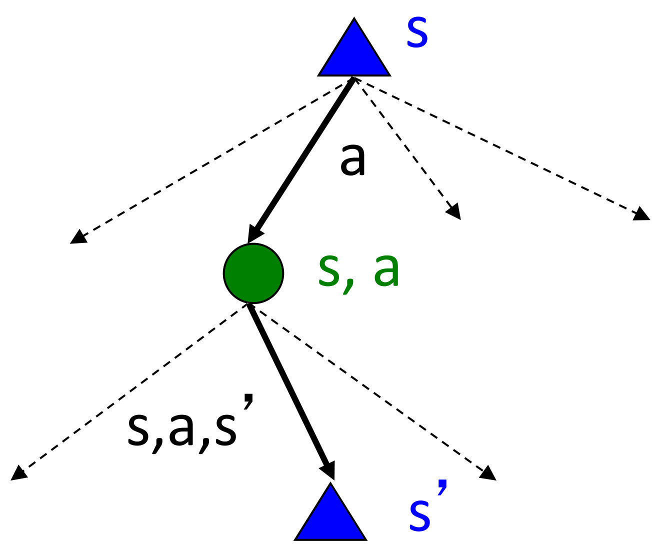
$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max}/(1-\gamma)$$

    - Smaller γ means smaller "horizon" – shorter term focus

  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

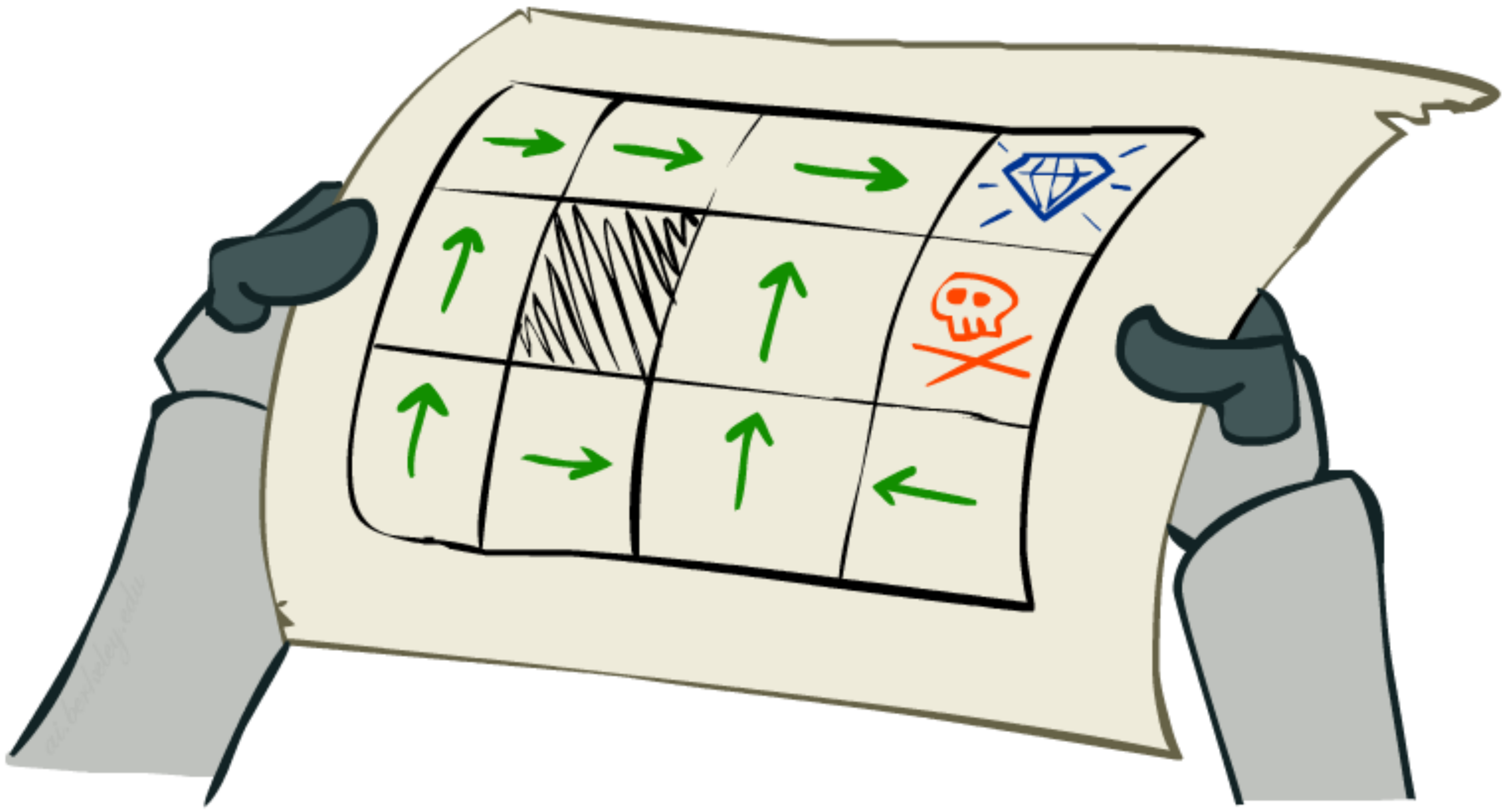
Recap: Defining MDPs

- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s' | s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards

Solving MDPs



Recall: Racing MDP

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
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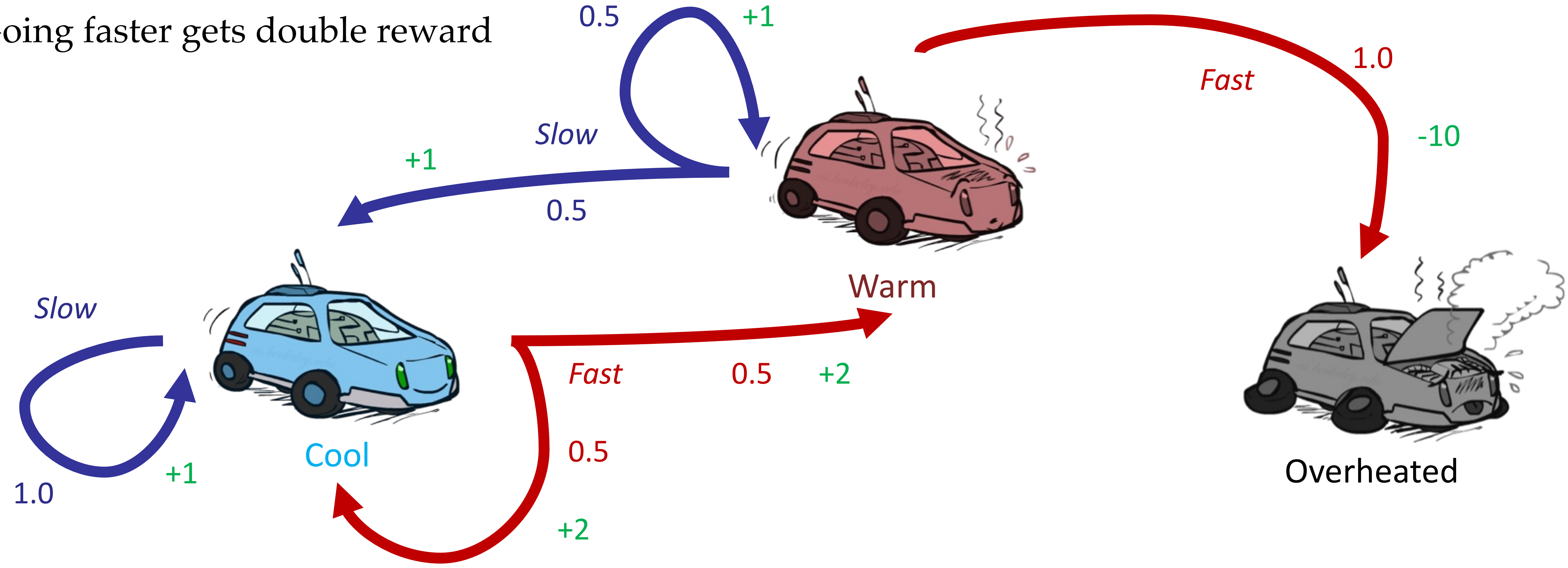
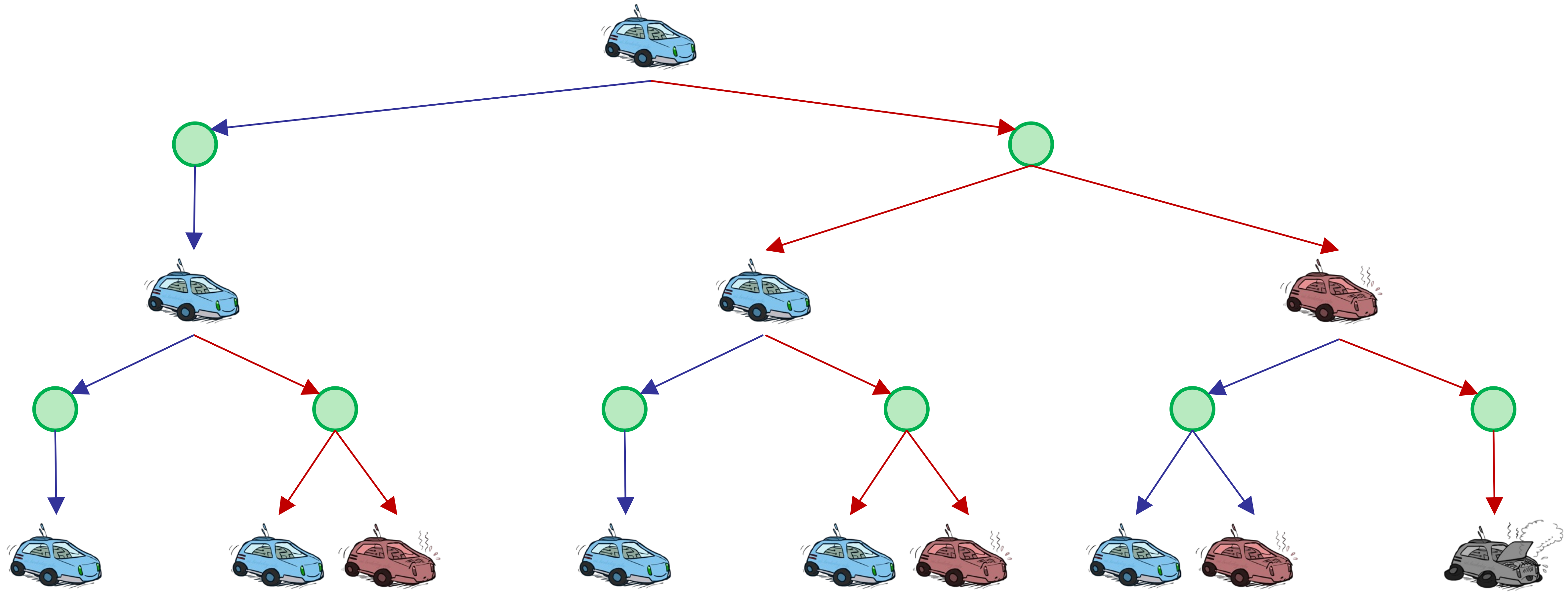


Racing Search Tree



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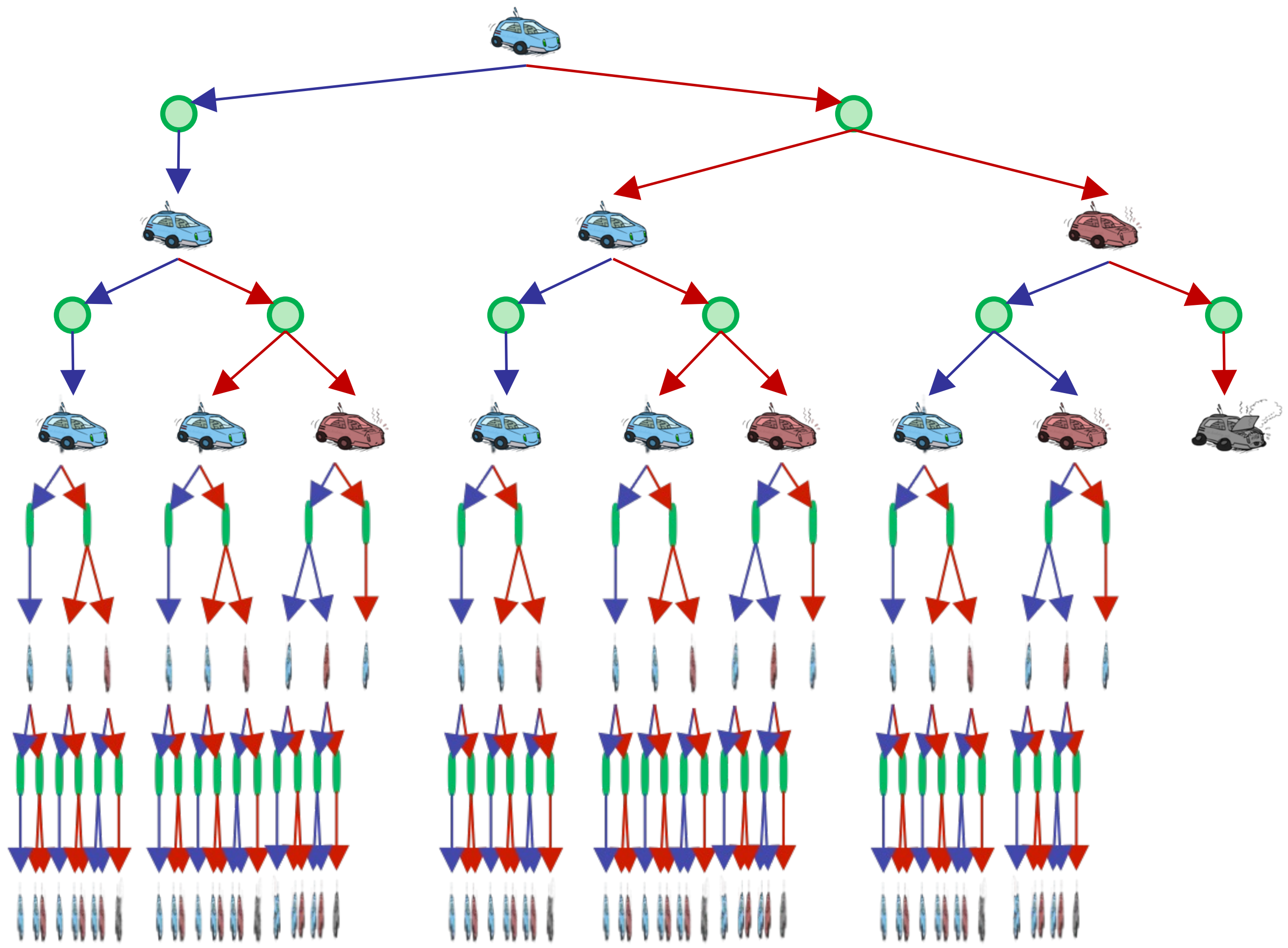
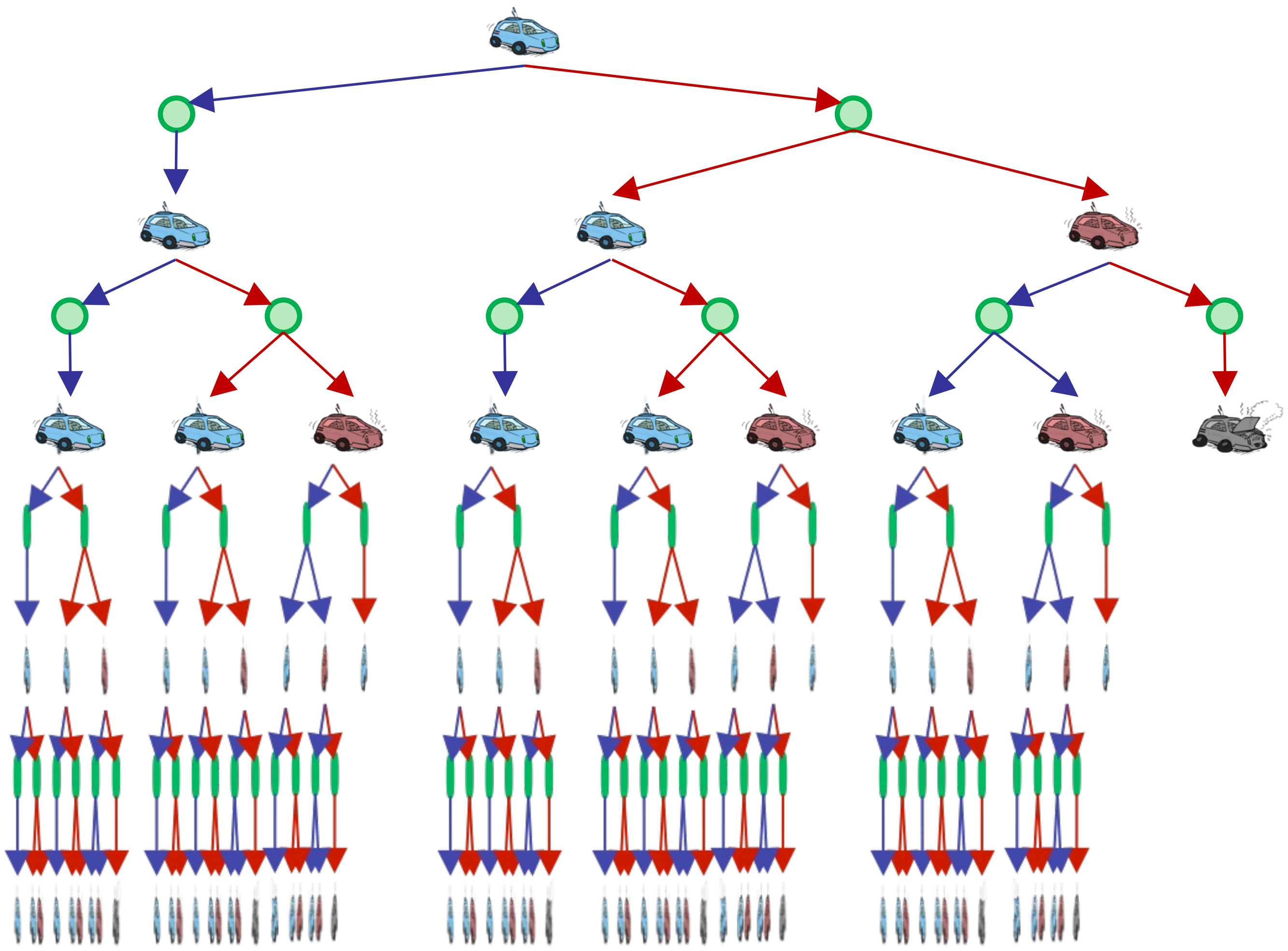


Racing Search Tree

- We're doing way too much work with expectimax!
- Problem: States are repeated
  - Idea: Only compute needed quantities once
- Problem: Tree goes on forever
  - Idea: Do a depth-limited computation, but with increasing depths until change is small
  - Note: deep parts of the tree eventually don't matter if $\gamma < 1$



Optimal Quantities

- The value (utility) of a state s:

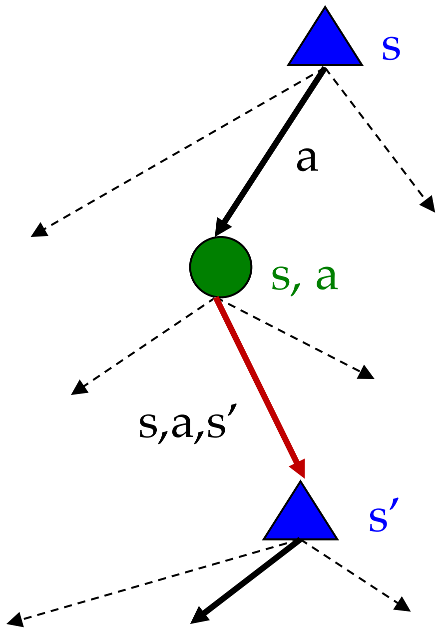
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

  $\pi^*(s)$ = optimal action from state s

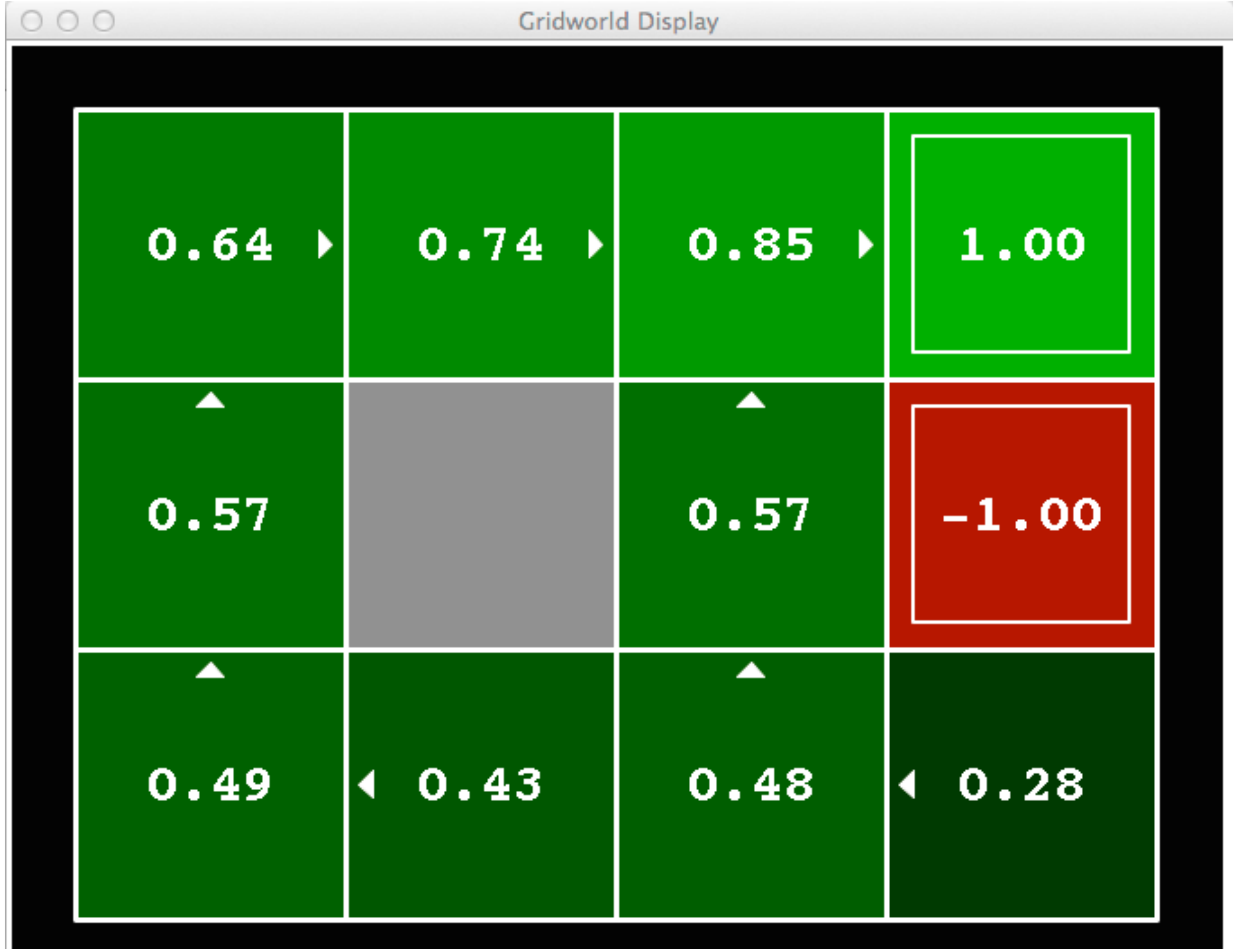


s is a *state*

(s, a) is a *q-state*

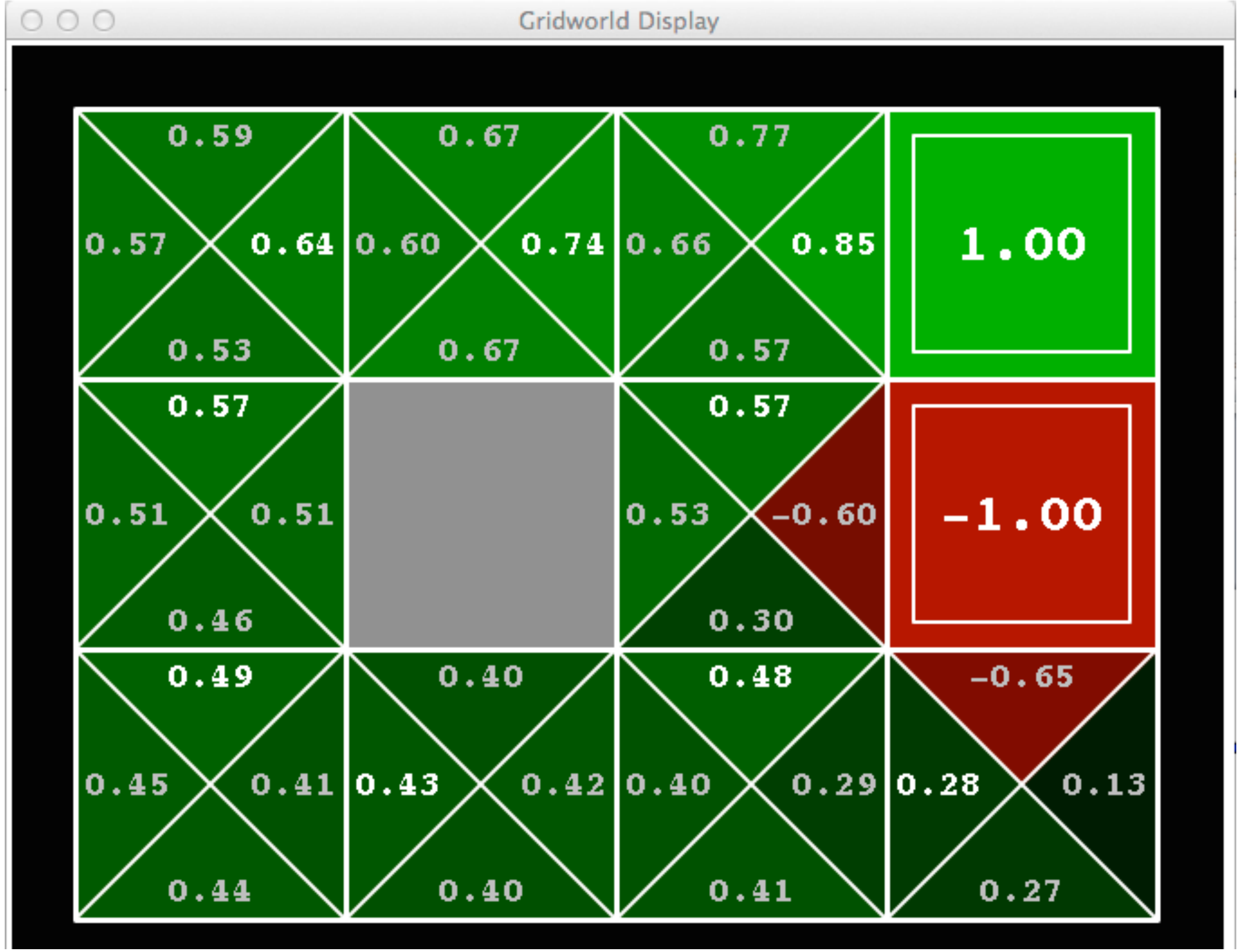
(s,a,s') is a *transition*

Gridworld V* Values



Noise = 0.2
Discount = 0.9
Living reward = 0

Gridworld Q* Values



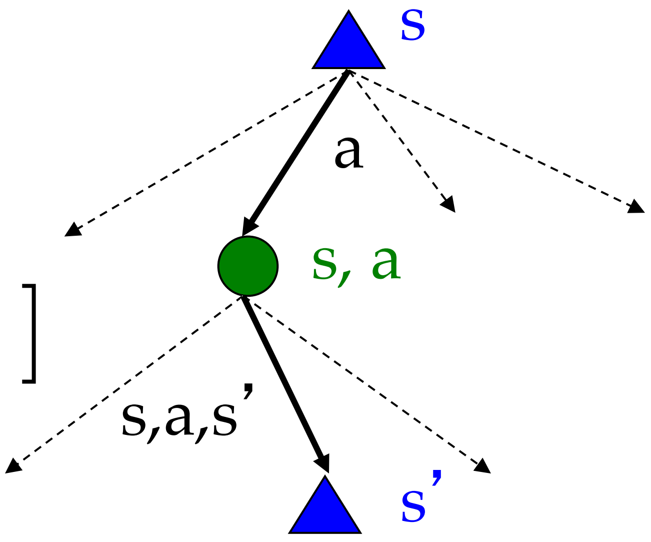
Noise = 0.2
Discount = 0.9
Living reward = 0

Values of States: Bellman Equation

- Recursive definition of value:

$$V^*(s) = \max_a Q^*(s,a)$$

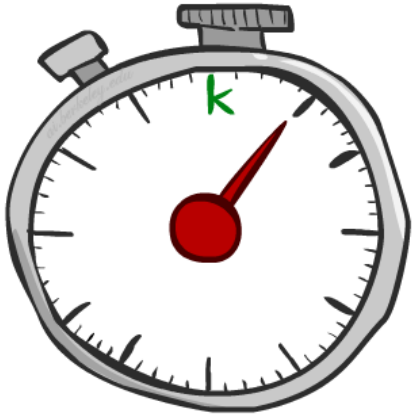
$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

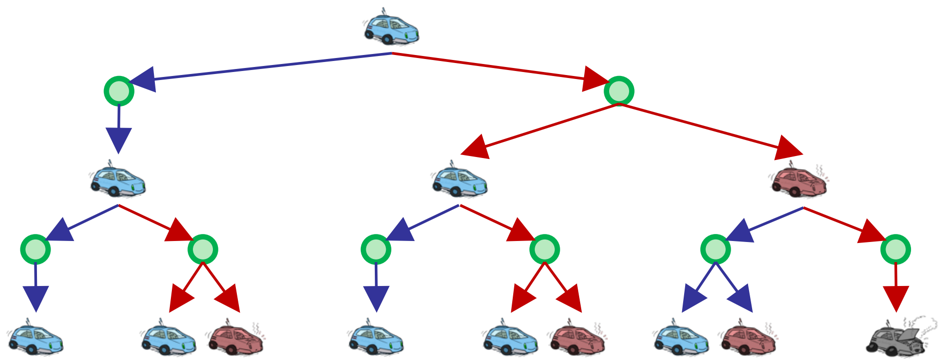


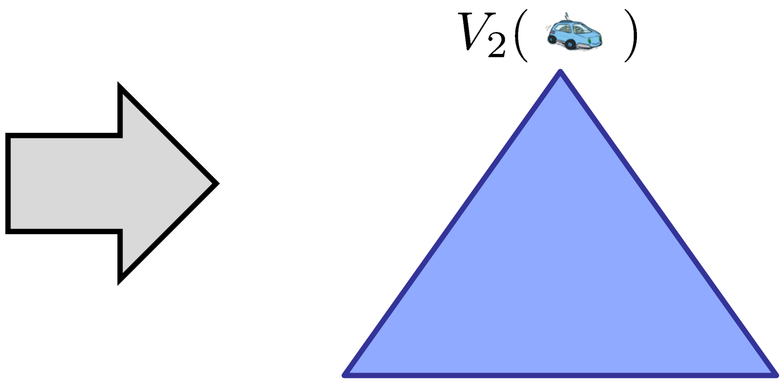
$$V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

Time-Limited Values

- Key idea: time-limited values
- Define $V_k(s)$ to be the optimal value of s if the game ends in k more time steps
  - Equivalently, it's what a depth-k expectimax would give from s

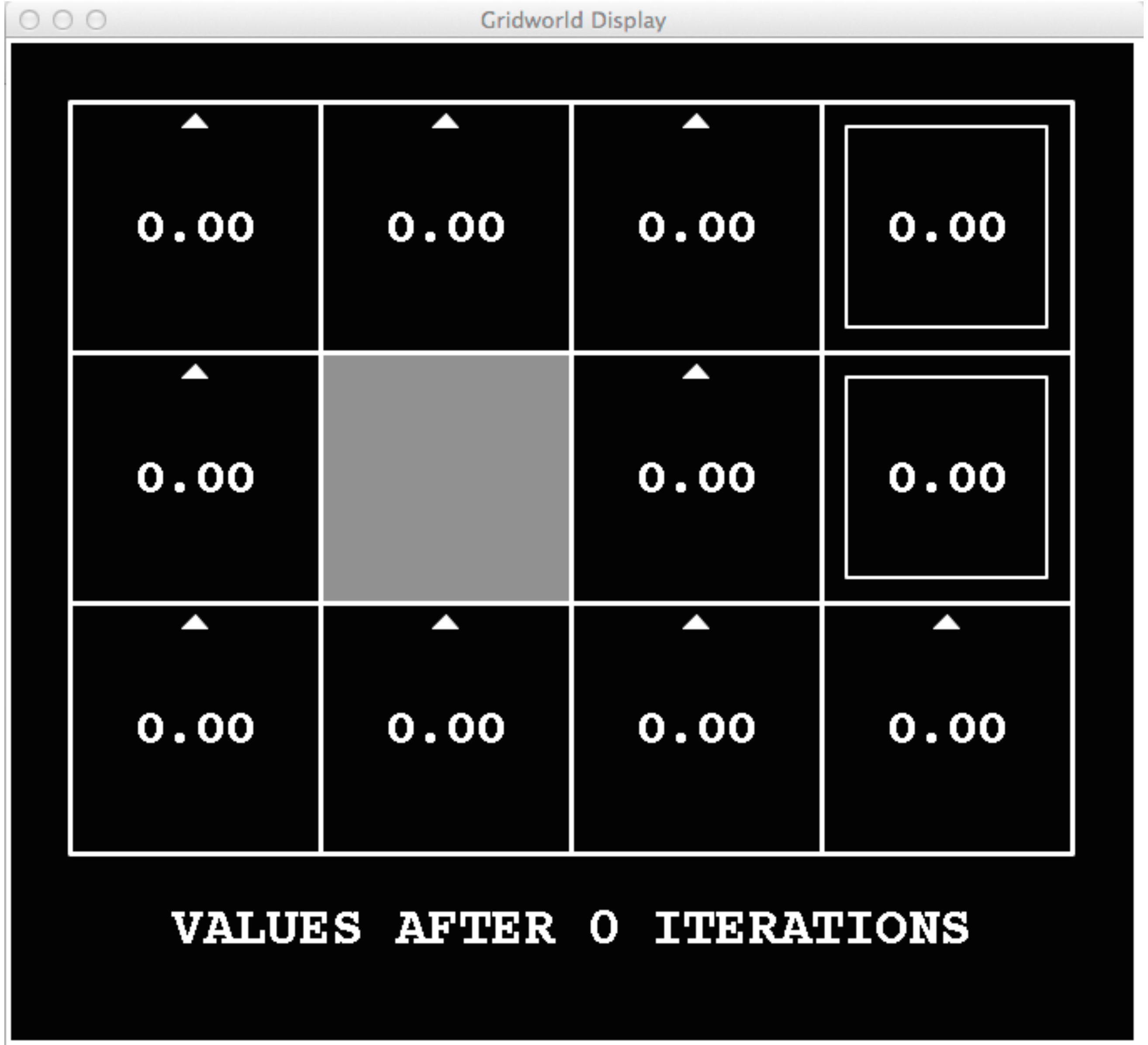






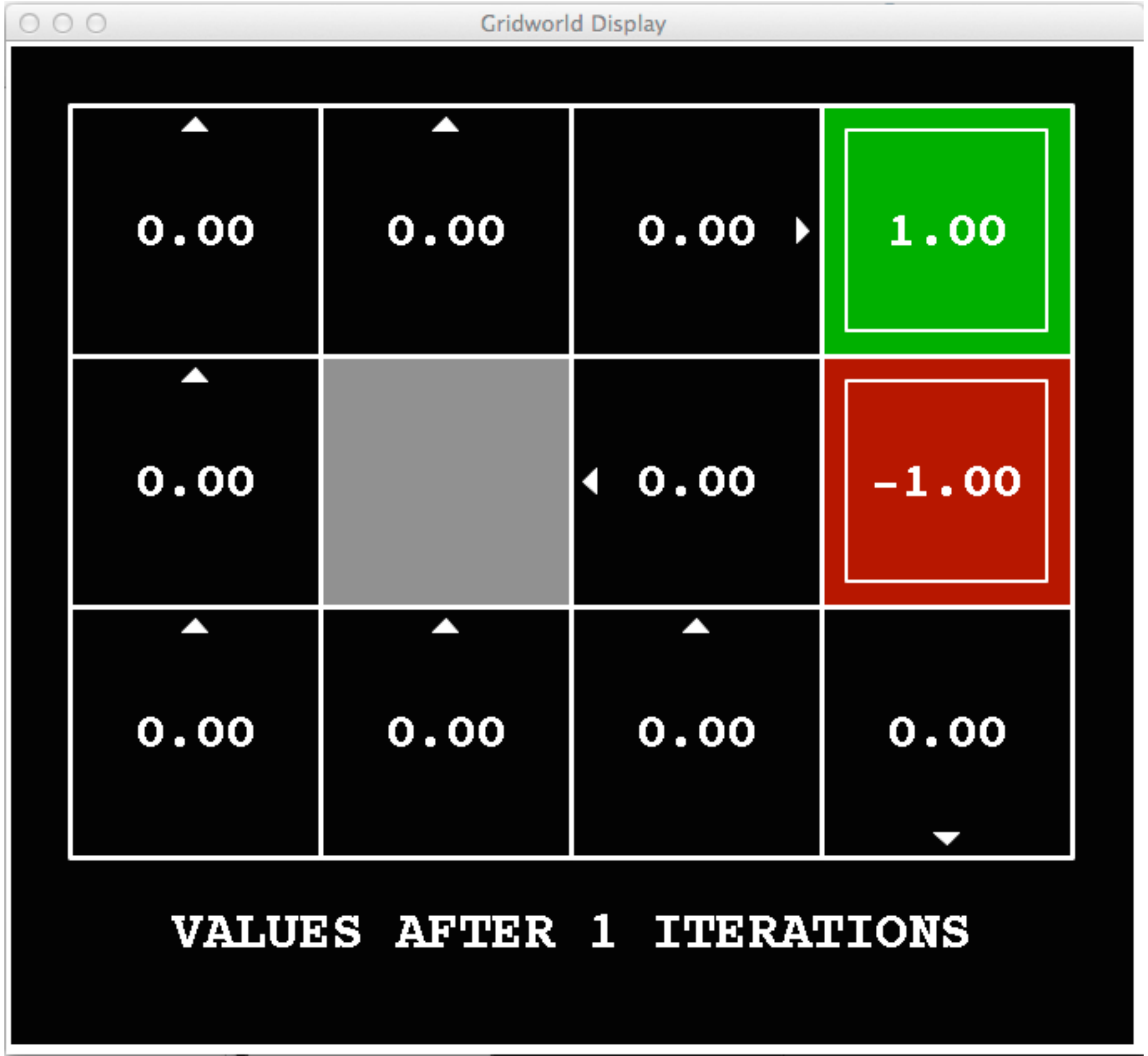
[Demo – time-limited values (L8D4)]

k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

k=1



Noise = 0.2
Discount = 0.9
Living reward = 0

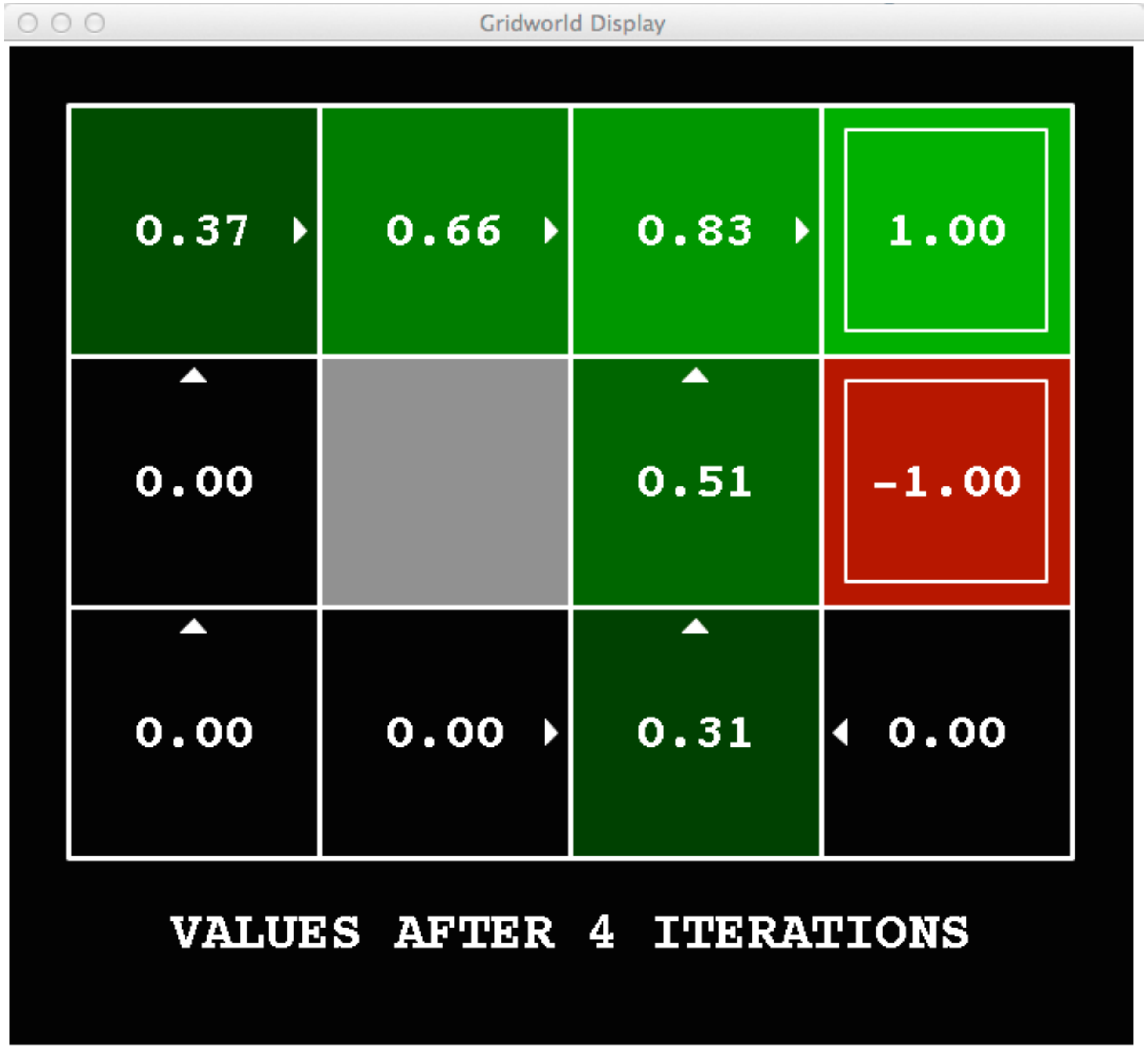
k=2

Noise = 0.2
Discount = 0.9
Living reward = 0

k=3

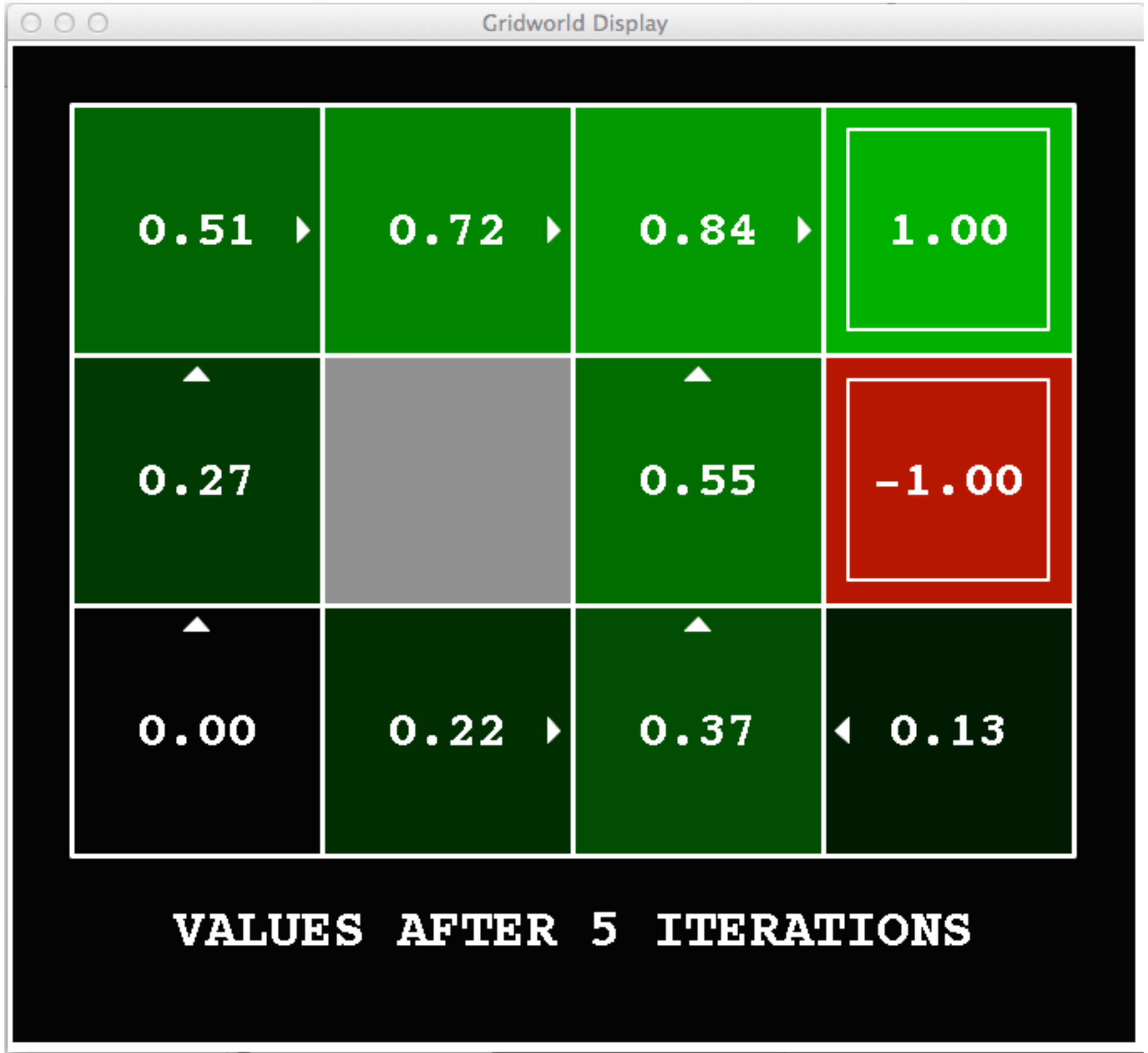
Noise = 0.2
Discount = 0.9
Living reward = 0

k=4



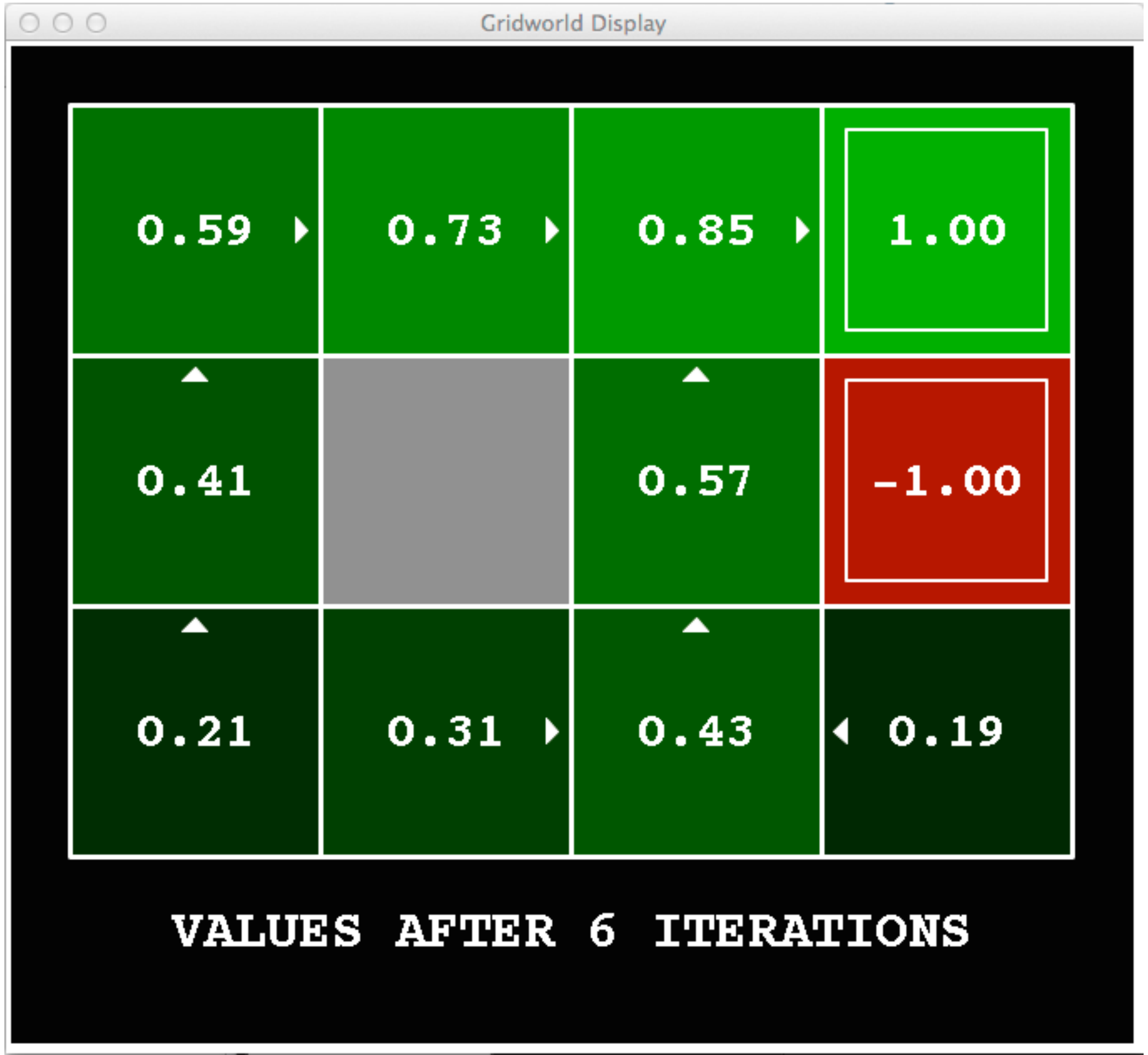
Noise = 0.2
Discount = 0.9
Living reward = 0

k=5



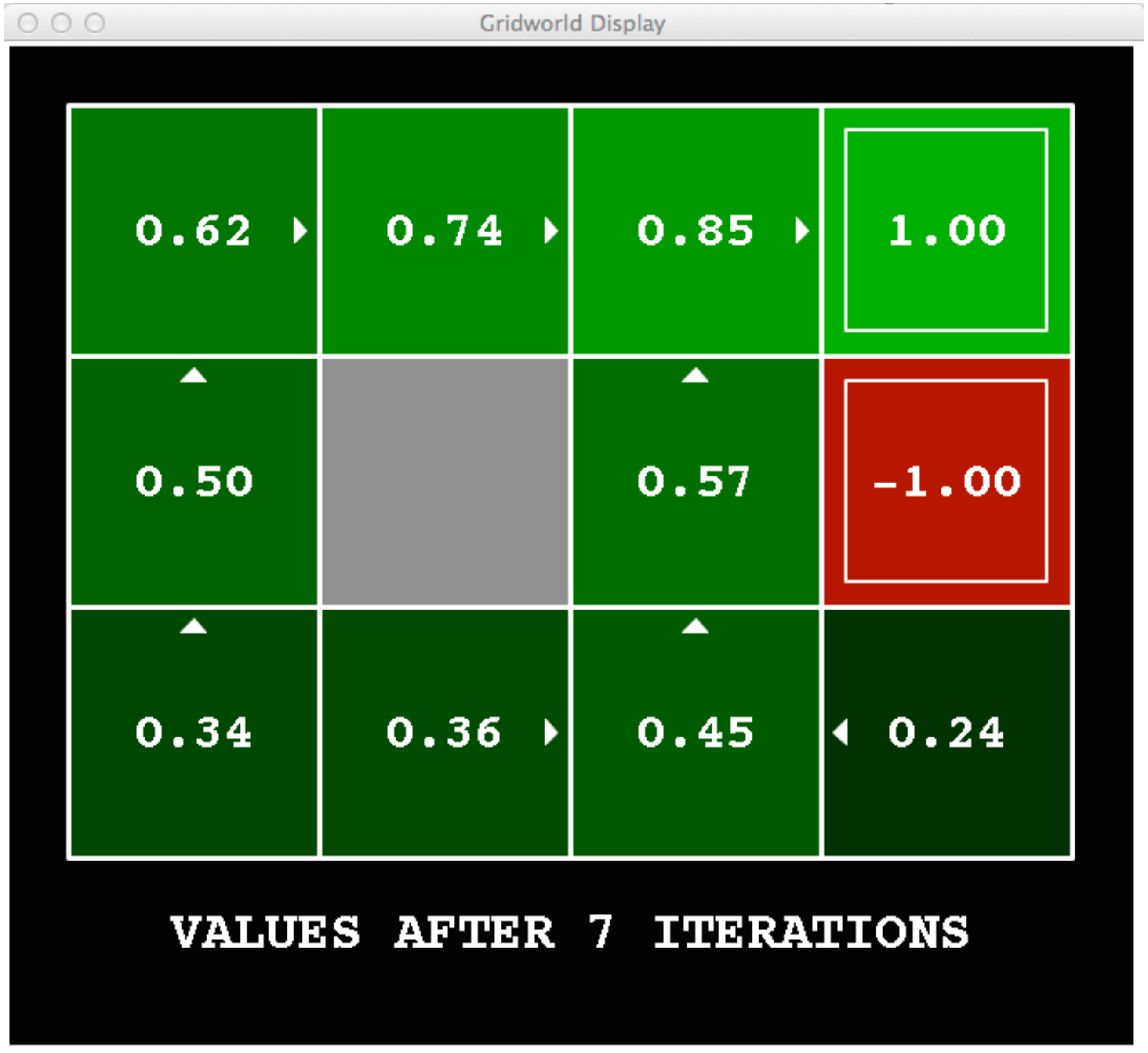
Noise = 0.2
Discount = 0.9
Living reward = 0

k=6



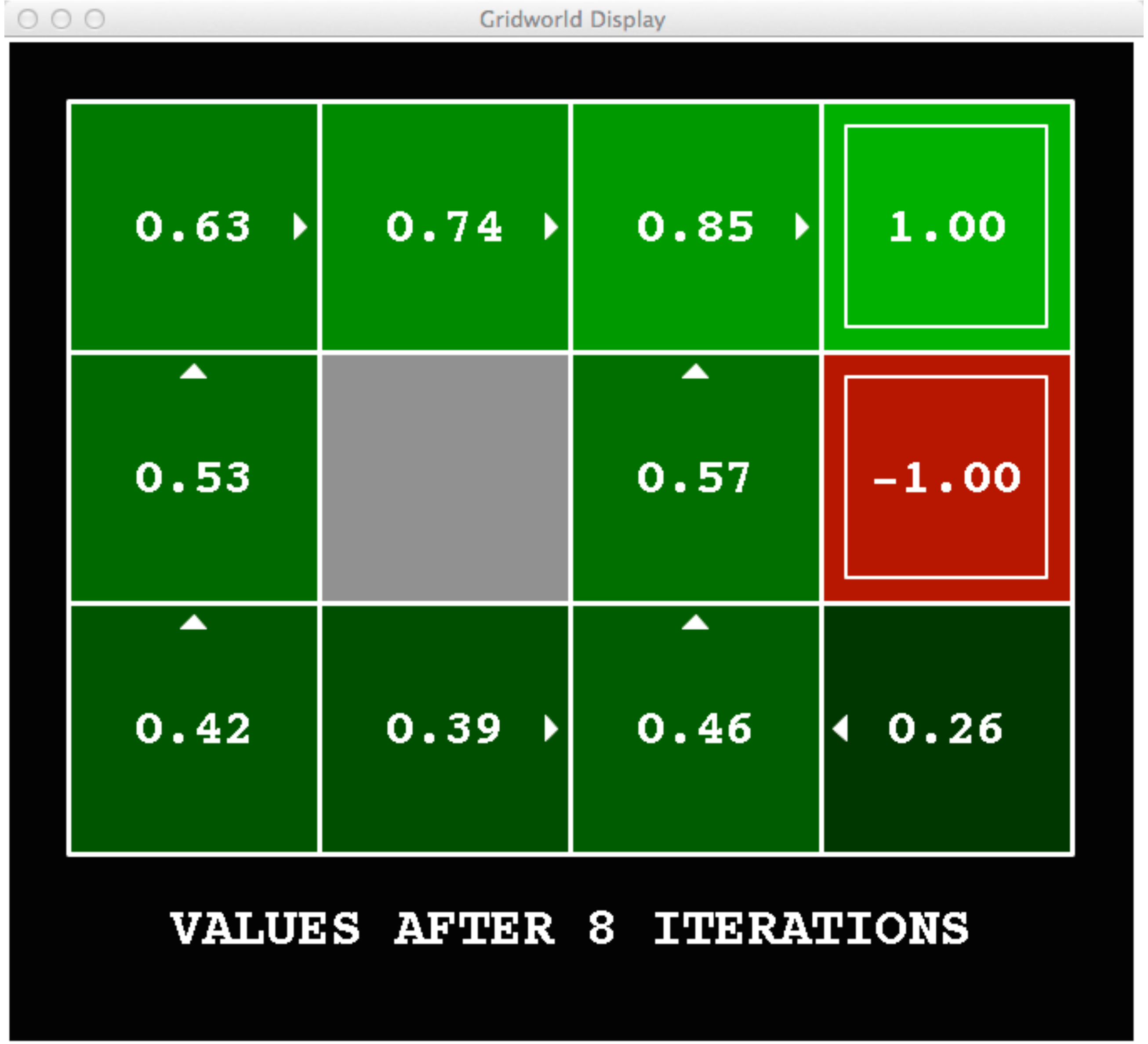
Noise = 0.2
Discount = 0.9
Living reward = 0

k=7



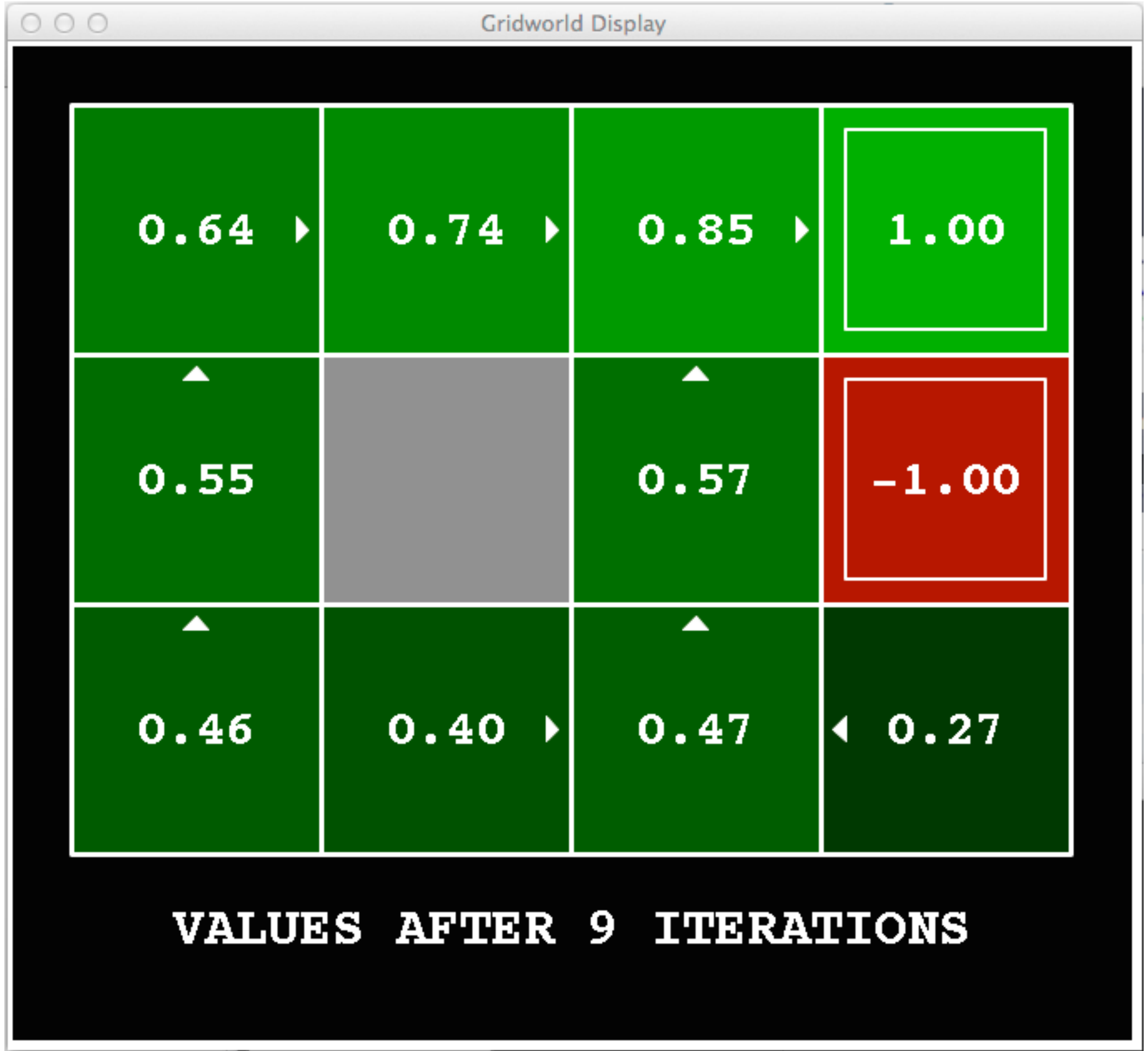
Noise = 0.2
Discount = 0.9
Living reward = 0

k=8



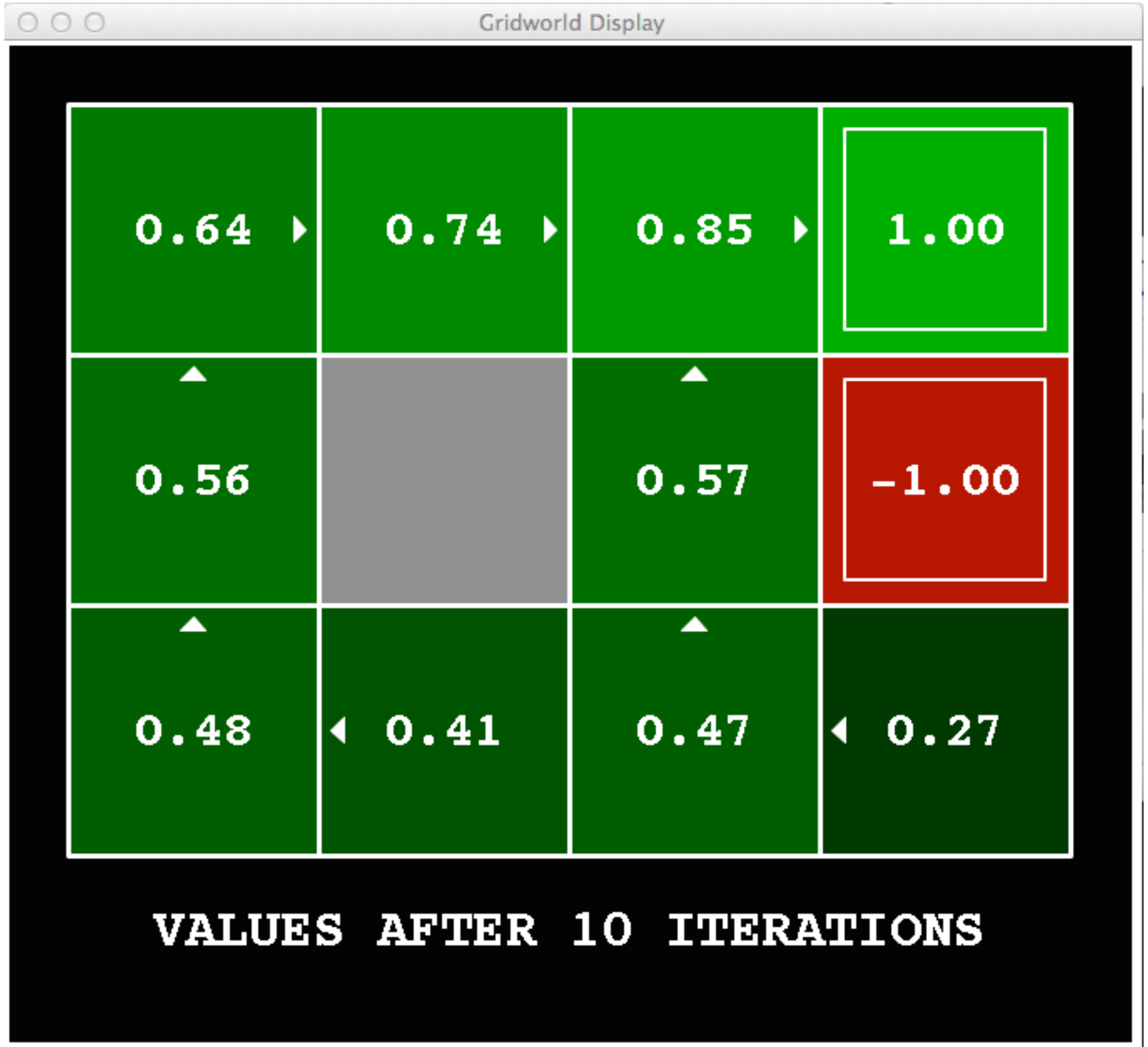
Noise = 0.2
Discount = 0.9
Living reward = 0

k=9



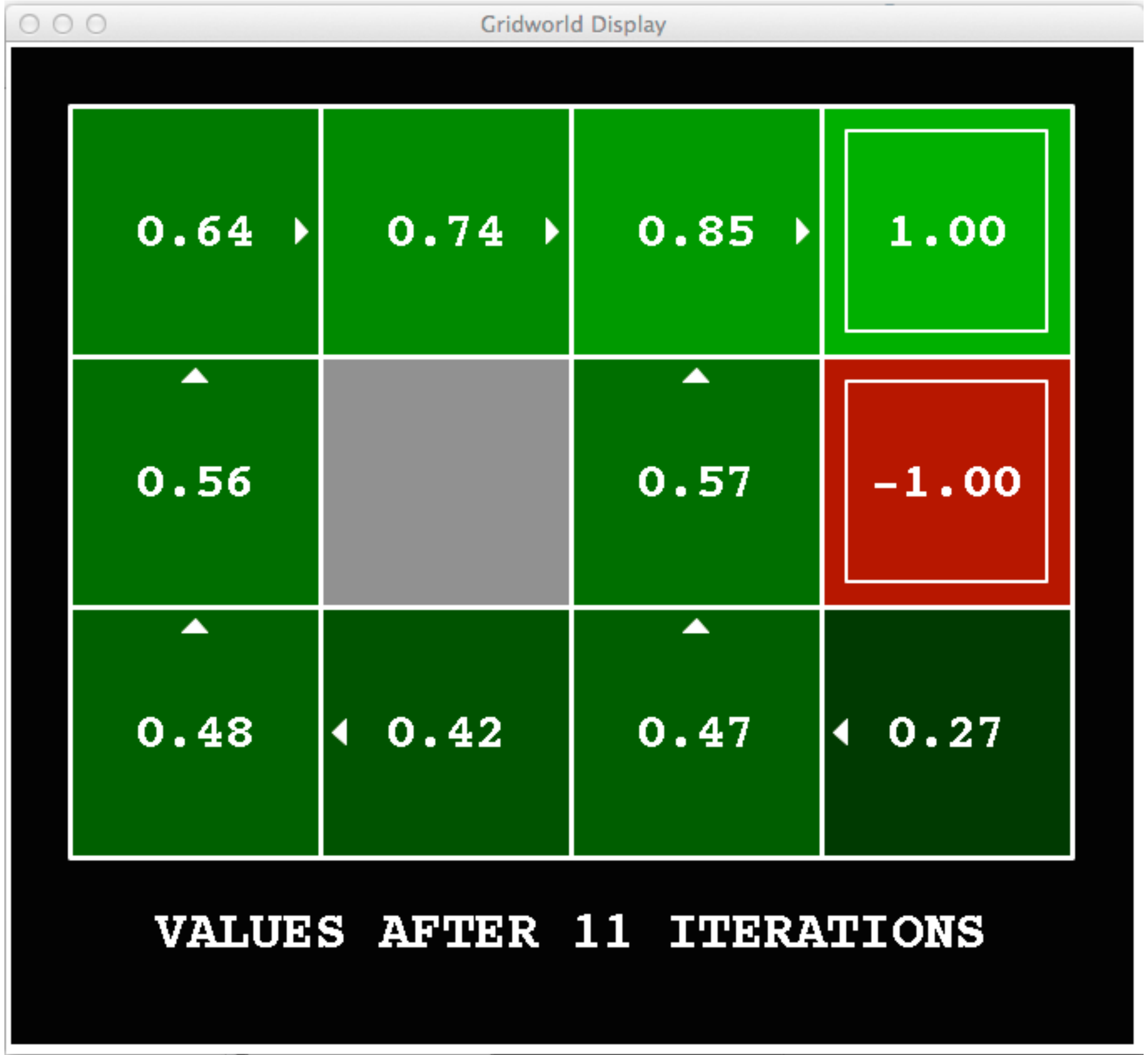
Noise = 0.2
Discount = 0.9
Living reward = 0

k=10



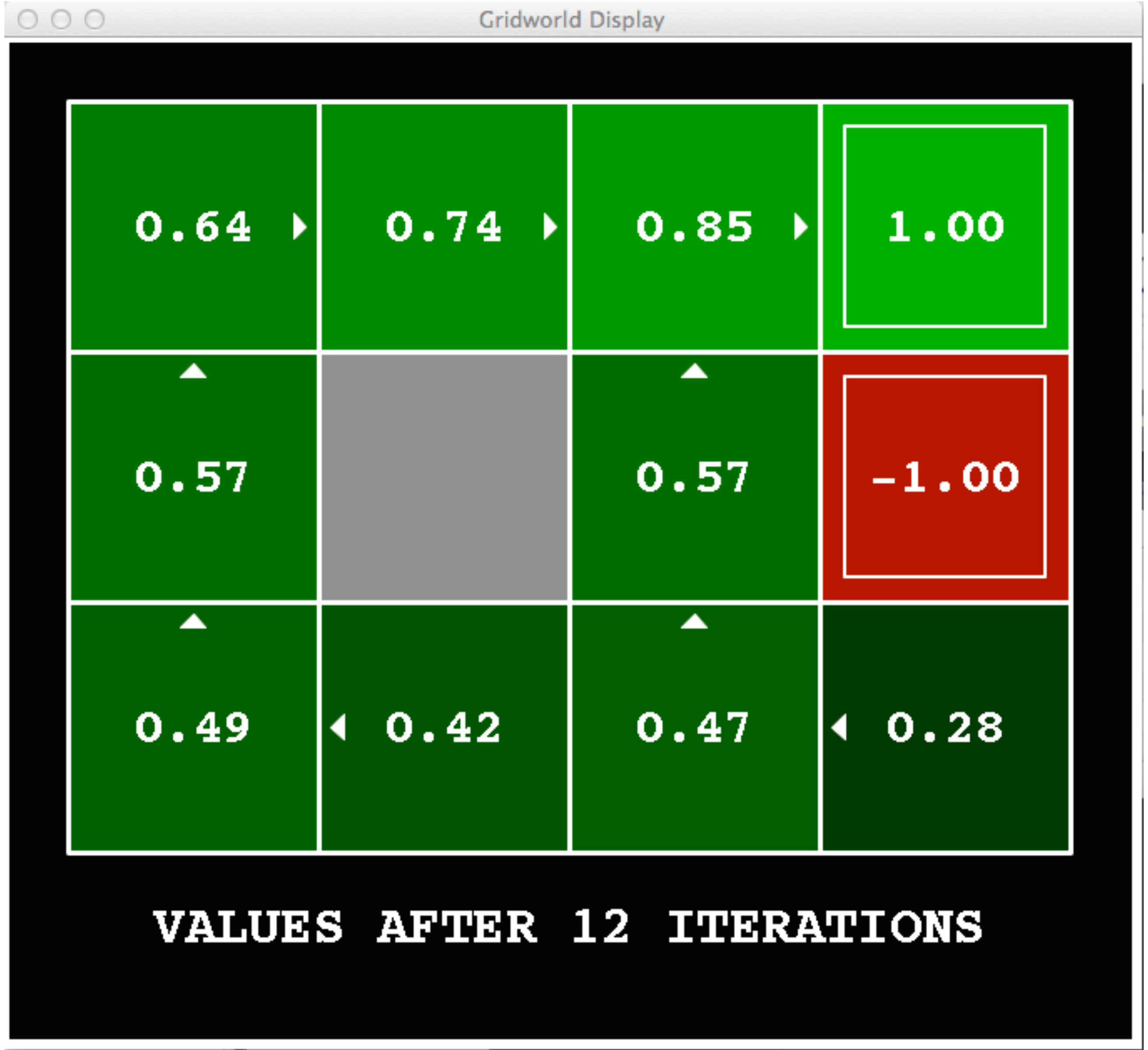
Noise = 0.2
Discount = 0.9
Living reward = 0

k=11



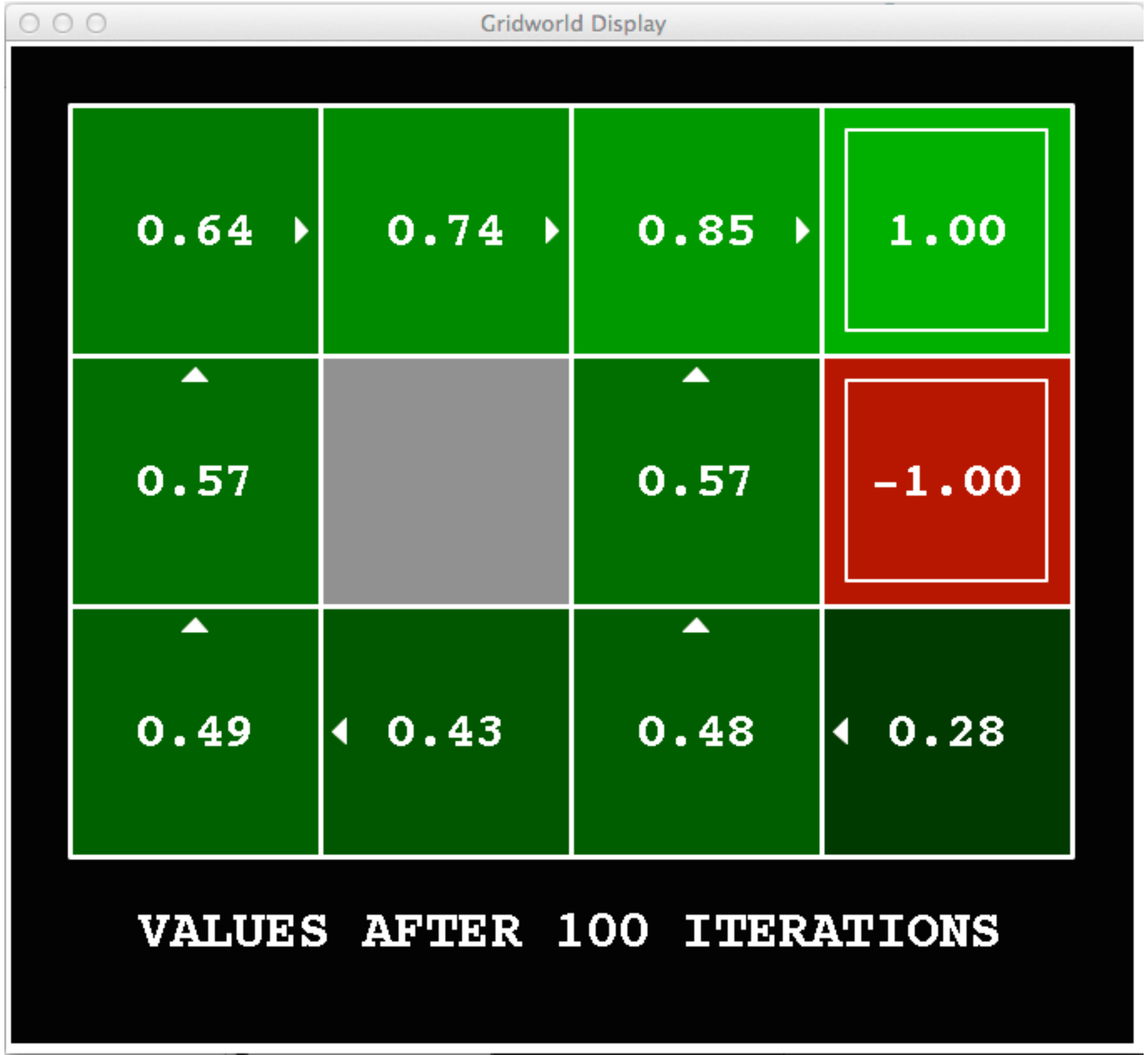
Noise = 0.2
Discount = 0.9
Living reward = 0

k=12



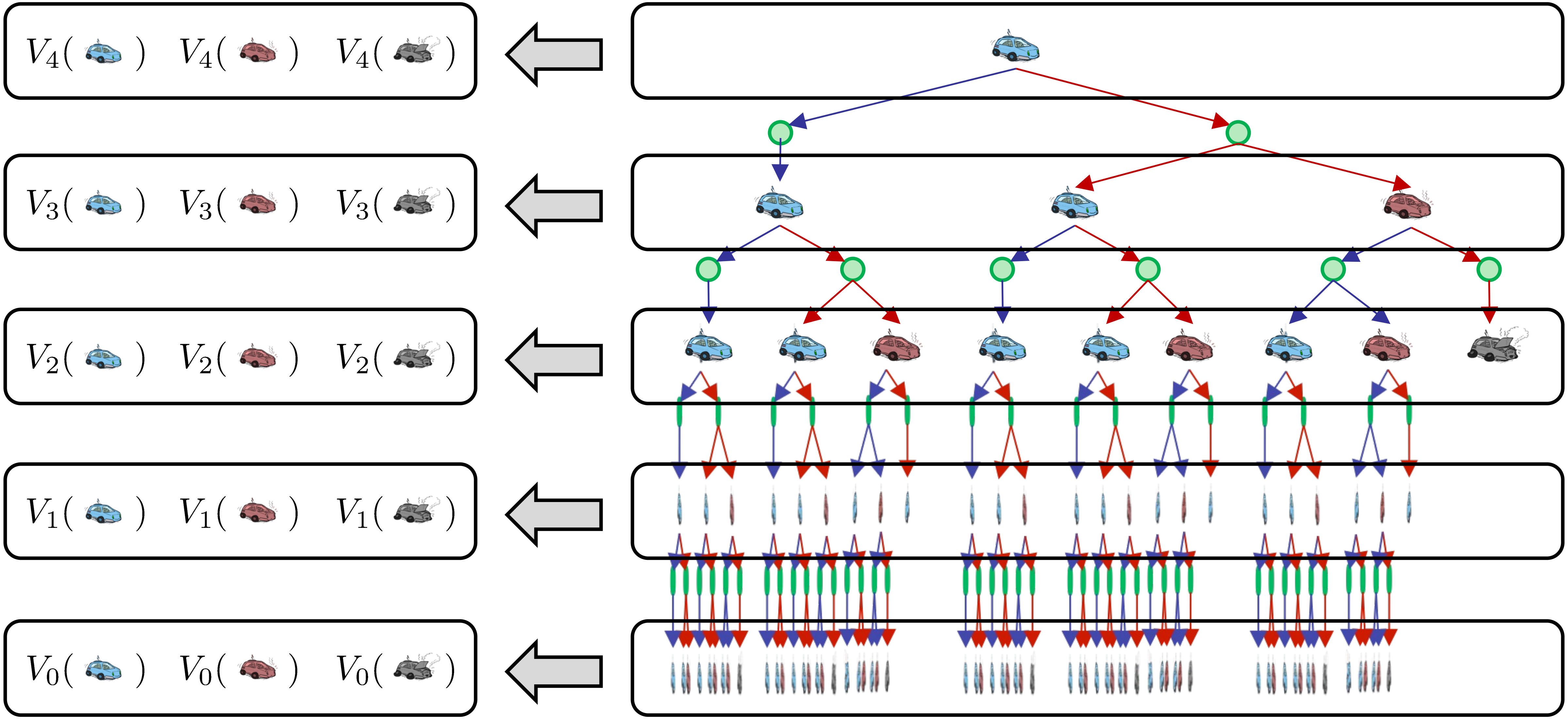
Noise = 0.2
Discount = 0.9
Living reward = 0

k=100

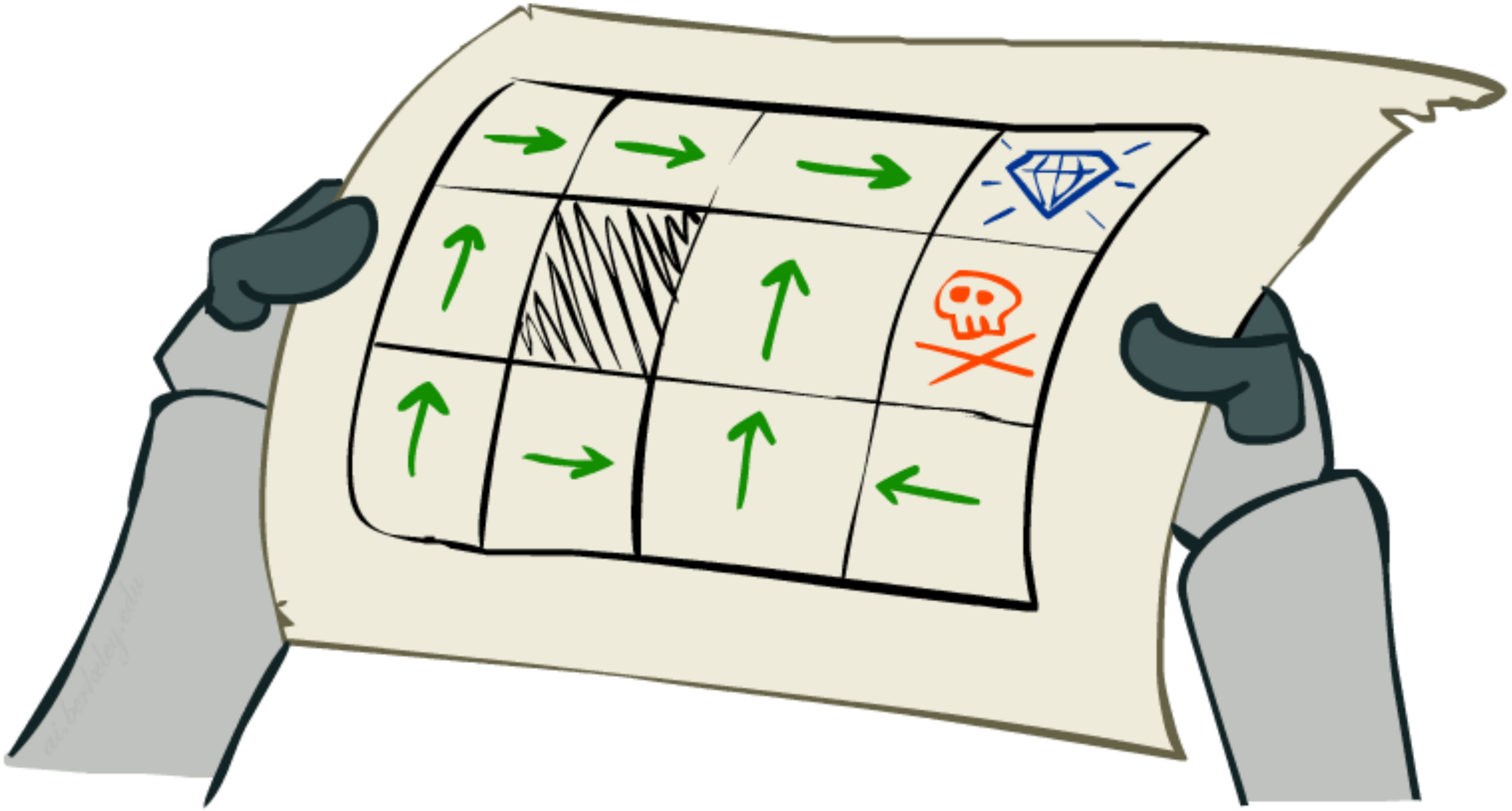


Noise = 0.2
Discount = 0.9
Living reward = 0

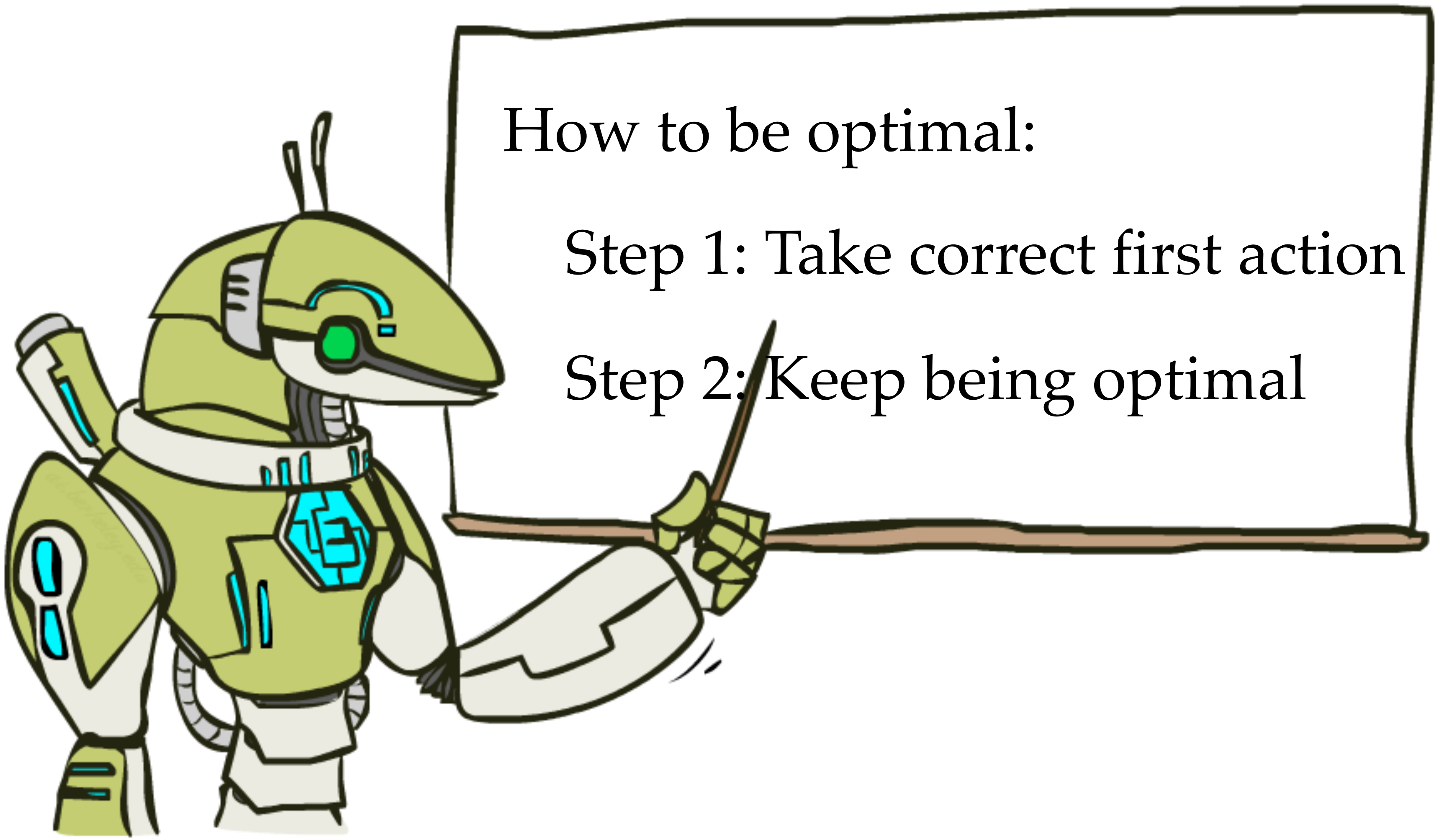
Computing Time-Limited Values



Solving MDPs



The Bellman Equations

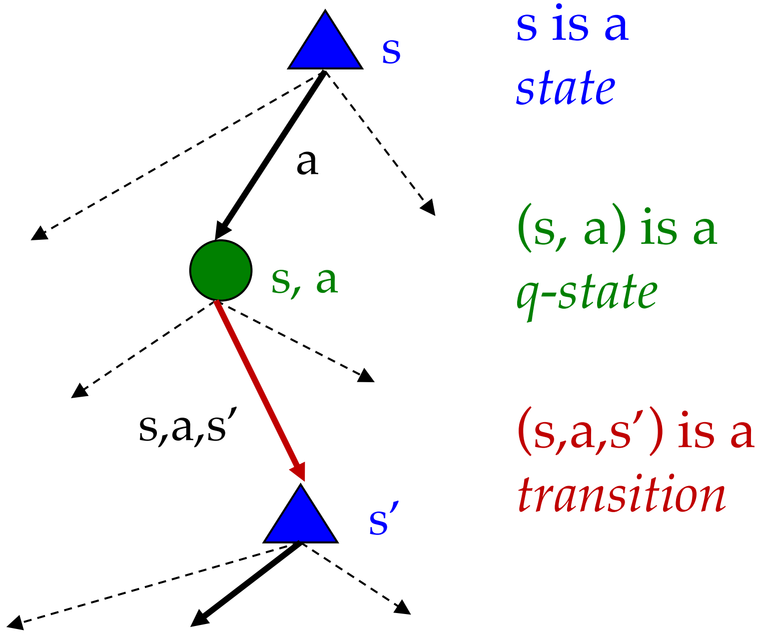


Optimal Quantities

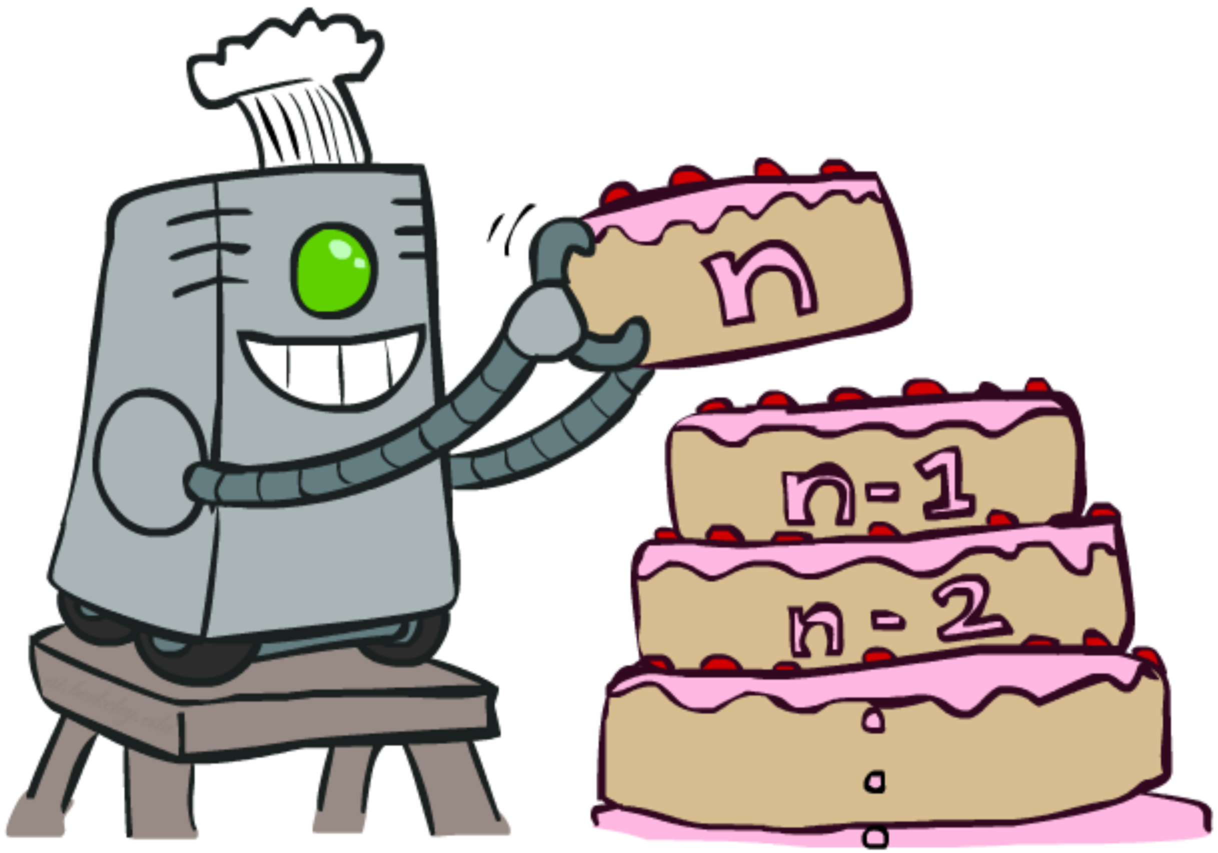
- The value (utility) of a state s:
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):
  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:
  $\pi^*(s)$ = optimal action from state s



Value Iteration



Value Iteration

- Bellman equations characterize the optimal values:

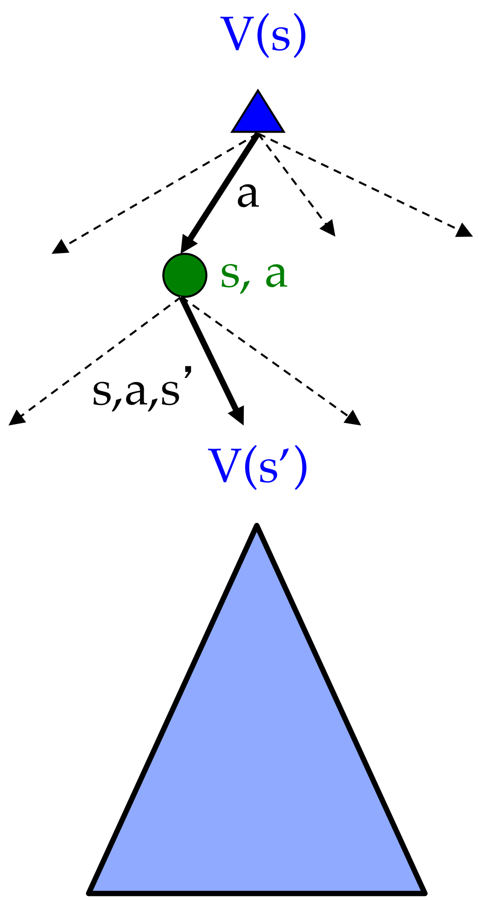
$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

- Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

"Bellman Update"

- Value iteration is just a fixed point solution method
  - … though the V_k vectors are also interpretable as time-limited values

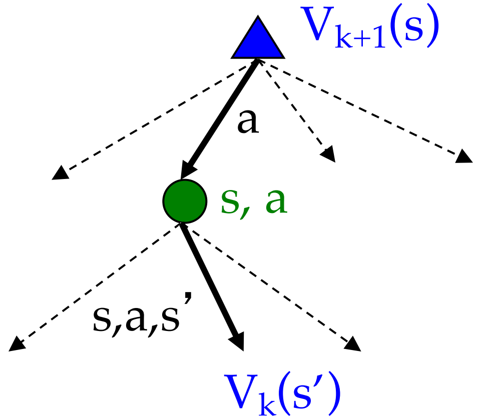


Similarly, can have Q-Value Iteration

- Bellman Equation: recursive definition of Q-values
  - $Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')]$
- Q-Value Iteration: Dynamic Programming
  - $Q_{k+1}(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]$

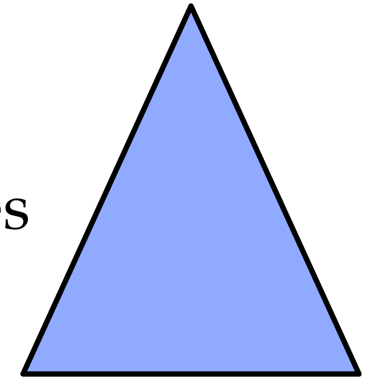
Value Iteration: Dynamic Programming

- Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero
- Given vector of $V_k(s)$ values, do one ply of expectimax from each state:



$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

  - V = B(V) Where B is the Bellman update operator
- Repeat until convergence, which yields V*
- Complexity of each iteration: $O(S^2A)$
- Theorem: Value Iteration will converge to unique optimal values
  - Basic idea: approximations get refined towards optimal values
  - Policy may converge long before values do



Example: Value Iteration

V_2

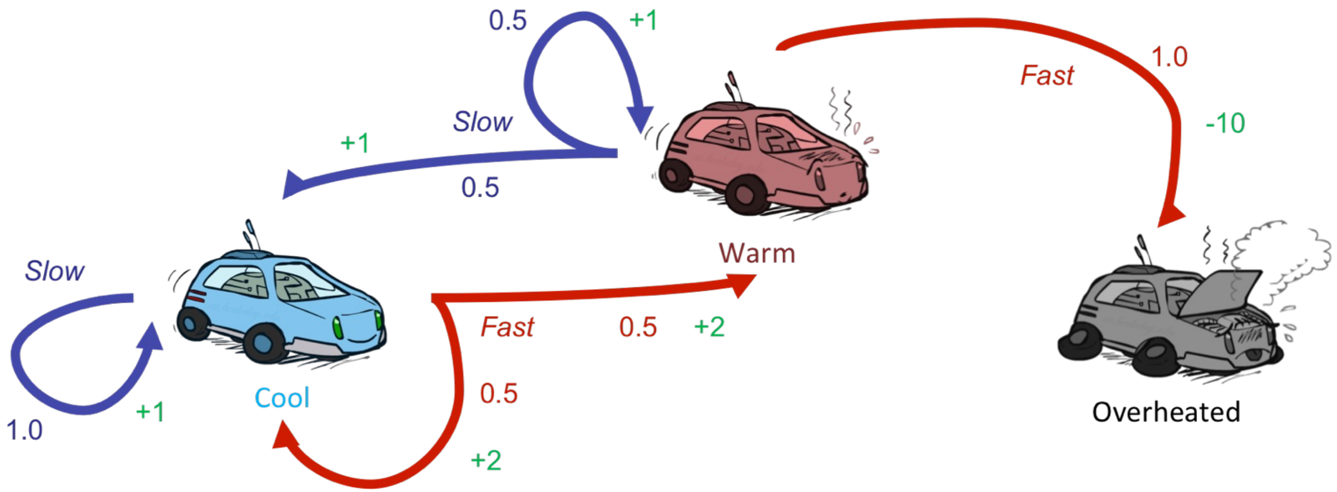
V_1

S: 1

F: .5*2+.5*2=2

V_0

0	0	0

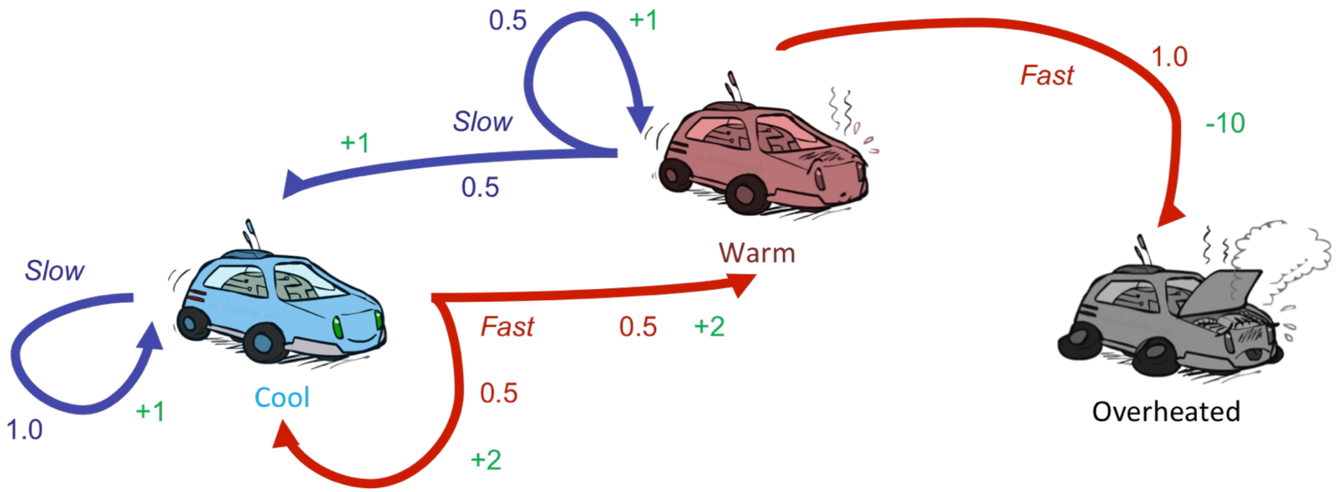


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

Example: Value Iteration

V_2			
V_1	2	S: .5*1+.5*1=1 F: -10	
V_0	0	0	0

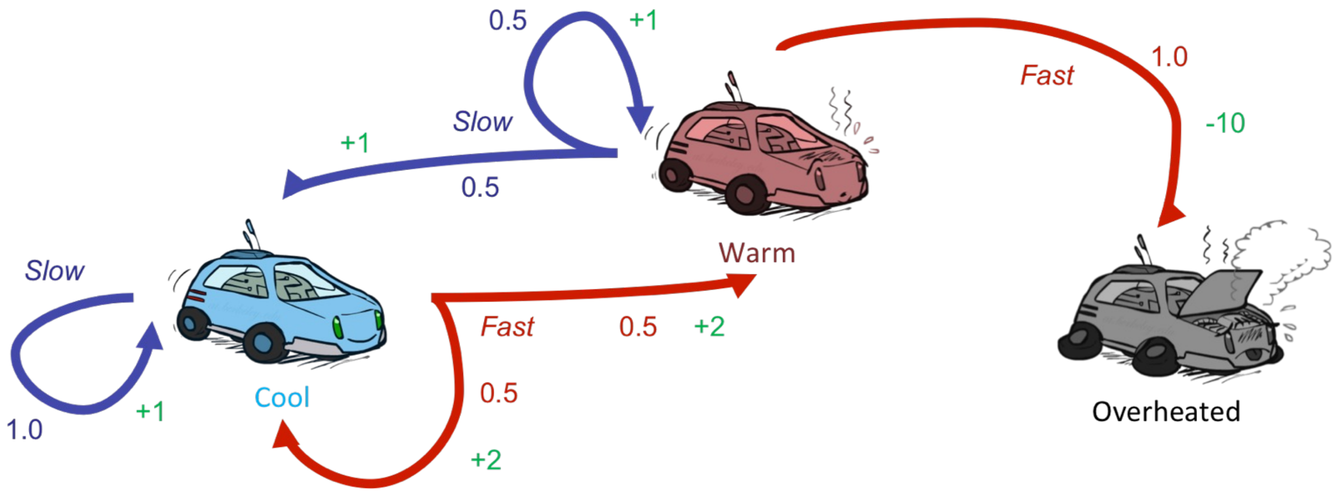


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

Example: Value Iteration

V_2			
V_1	2	1	0
V_0	0	0	0

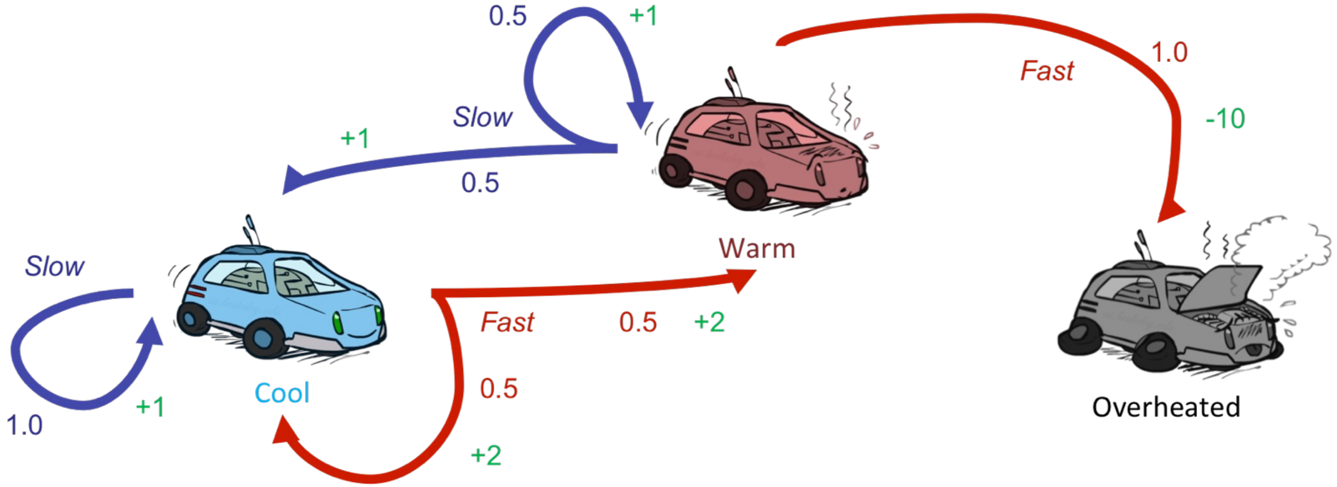


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Example: Value Iteration

V_2	S: 1+2=3 F: .5*(2+2)+.5*(2+1)=3.5		
V_1	2	1	0
V_0	0	0	0

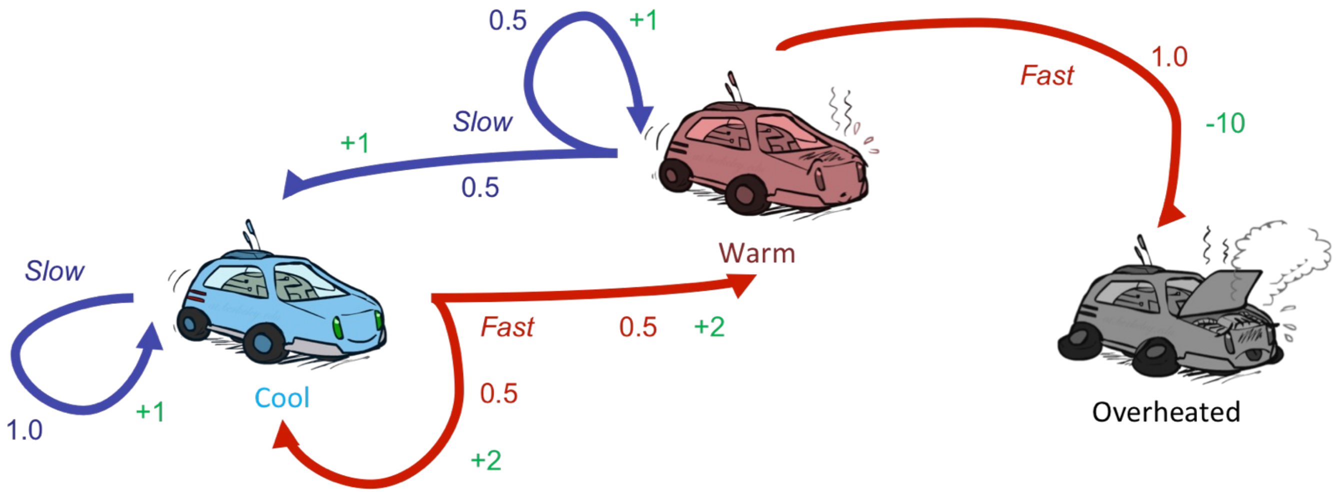


Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



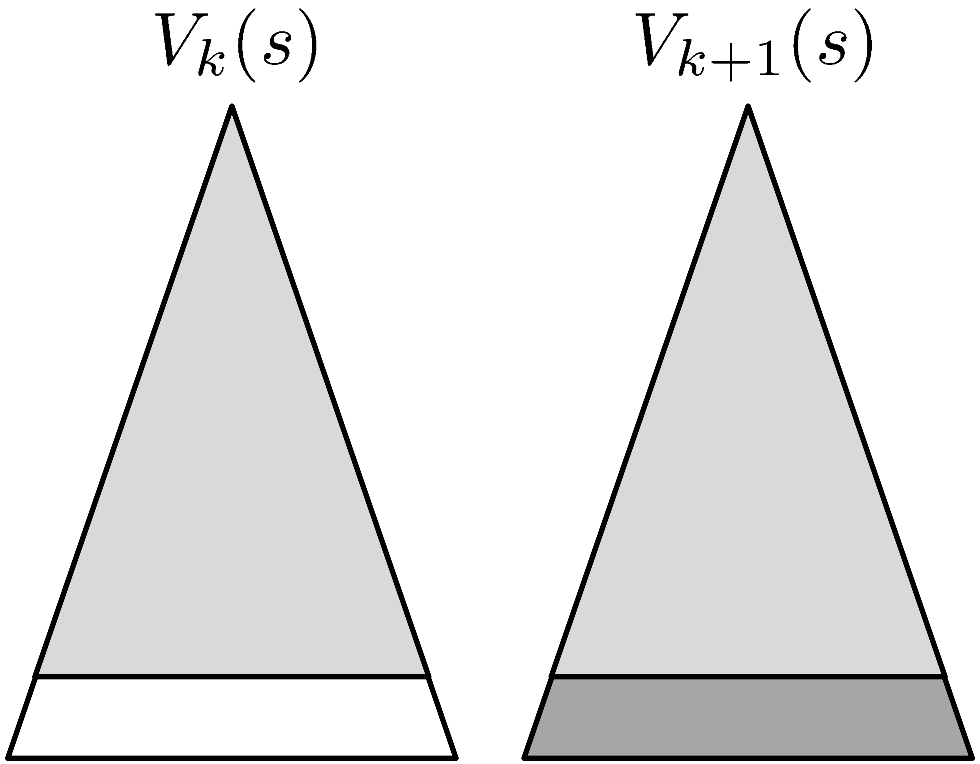
Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

Convergence

- How do we know the V_k vectors are going to converge? (assuming $0 < \gamma < 1$)

- Proof Sketch:
  - For any state V_k and V_{k+1} can be viewed as depth k+1 expectimax results in nearly identical search trees
  - The difference is that on the bottom layer, V_{k+1} has actual rewards while V_k has zeros
  - That last layer is at best all R_{MAX}
  - It is at worst R_{MIN}
  - But everything is discounted by γ^k that far out
  - So V_k and V_{k+1} are at most $\gamma^k \max|R|$ different
  - So as k increases, the values converge



Convergence of Value Iteration: Contraction

- New concept: contraction
  - If some operator F is a contraction by a factor, it brings any pair of objects closer to each other (according to some metric d(,))
    - For any x, y, we have $d(F(x), F(y)) < cd(x, y)$ where c < 1
  - If F is a contraction it has a unique fixed point z (i.e., F(z)=z)
- Since Value iteration is just $V_{k+1} = B(V_k)$, the Bellman update B is a contraction by γ
- Metric is the max norm: $||V - W|| = \max_s |V(s) - W(s)|$
- What's the fixed point for B?
- $BV^* = V^*$

Speed of Convergence

- Look at what happens to the distance between V_k and V^*
- $||V_{k+1} - V^*||$
- $= ||BV_k - V^*||$ (definition of V_{k+1} from VI update)
- $= ||BV_k - BV^*||$ (V^* is the fixed point of B)
- $\leq \gamma ||V_k - V^*||$ (B is a contraction by γ)
- I.e., the error is reduced by at least a factor γ on every iteration
- Exponentially fast convergence!

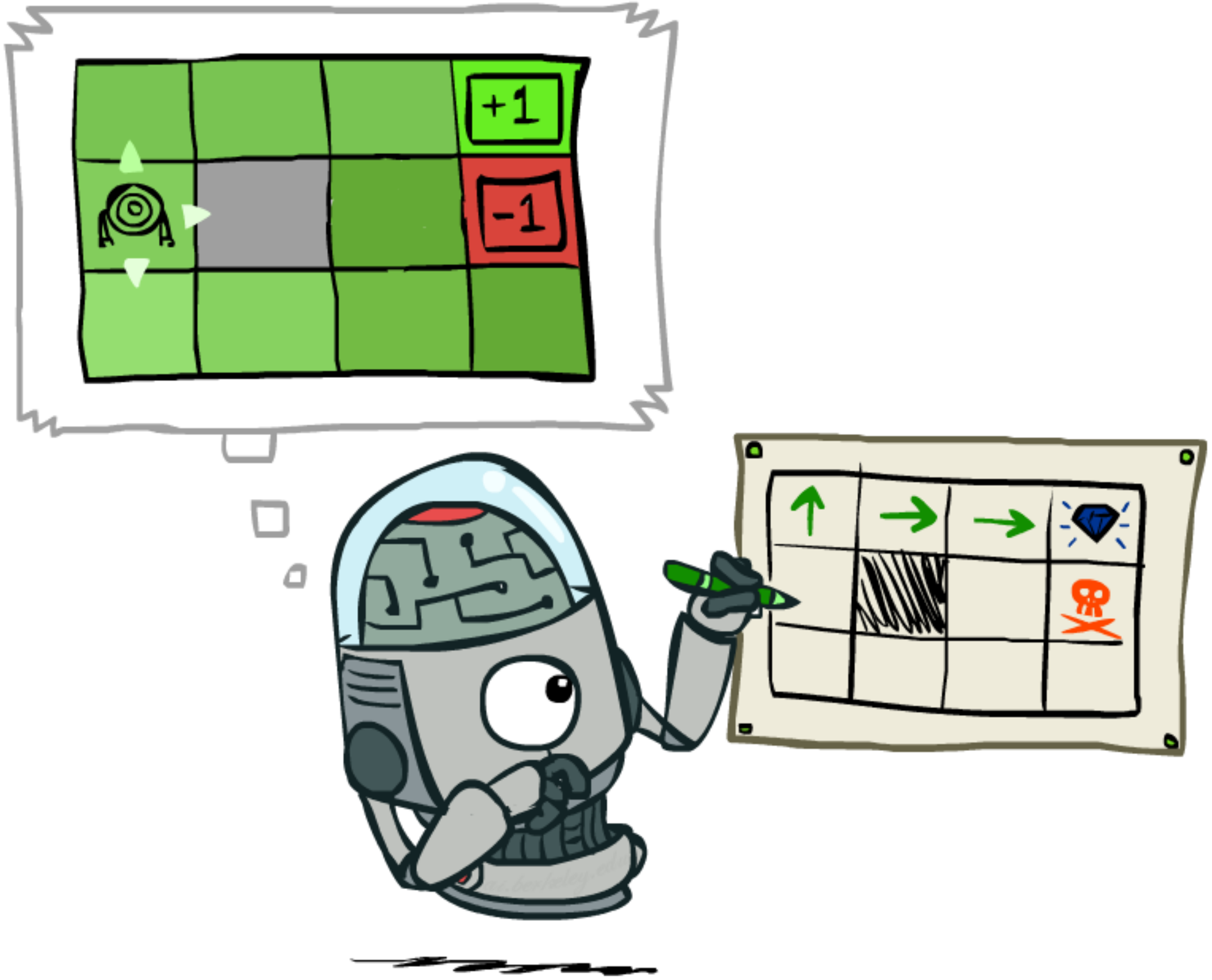
Correctness of Convergence

- Don't usually converge exactly; stops when change $< \frac{\epsilon(1-\gamma)}{\gamma}$
- I.e. $||V_{k+1} - V_k|| < \frac{\epsilon(1-\gamma)}{\gamma}$
- What about $||V_{k+1} - V^*||$ when $||V_{k+1} - V_k|| < \frac{\epsilon(1-\gamma)}{\gamma}$
- Useful properties:
  - Contraction: $\left||V_{k+1} - V^*|\right| \leq \gamma \left||V_k - V^*|\right|$
  - Triangle inequality: $\left||V_k - V^*|\right| \leq \left||V_{k+1} - V_k|\right| + \left||V_{k+1} - V^*|\right|$

Correctness of Convergence

- Value Iteration: stop when $||V_{k+1} - V_k|| < \frac{\epsilon(1-\gamma)}{\gamma}$
- What about $||V_{k+1} - V^*||$ when $||V_{k+1} - V_k|| < \frac{\epsilon(1-\gamma)}{\gamma}$?
- Triangle inequality: $\left||V_k - V^*|\right| \leq \left||V_{k+1} - V_k|\right| + \left||V_{k+1} - V^*|\right|$
- $\frac{1}{\gamma}\left||V_{k+1} - V^*|\right| \leq \left||V_{k+1} - V_k|\right| + \left||V_{k+1} - V^*|\right|$ B(V) is contraction by γ
- $\left(\frac{1}{\gamma} - 1\right)\left||V_{k+1} - V^*|\right| \leq \left||V_{k+1} - V_k|\right|$
- $\left(\frac{1}{\gamma} - 1\right)\left||V_{k+1} - V^*|\right| \leq \frac{\epsilon(1-\gamma)}{\gamma}$ when we have converged
- $\left||V_{k+1} - V^*|\right| \leq \epsilon$

Policy Extraction

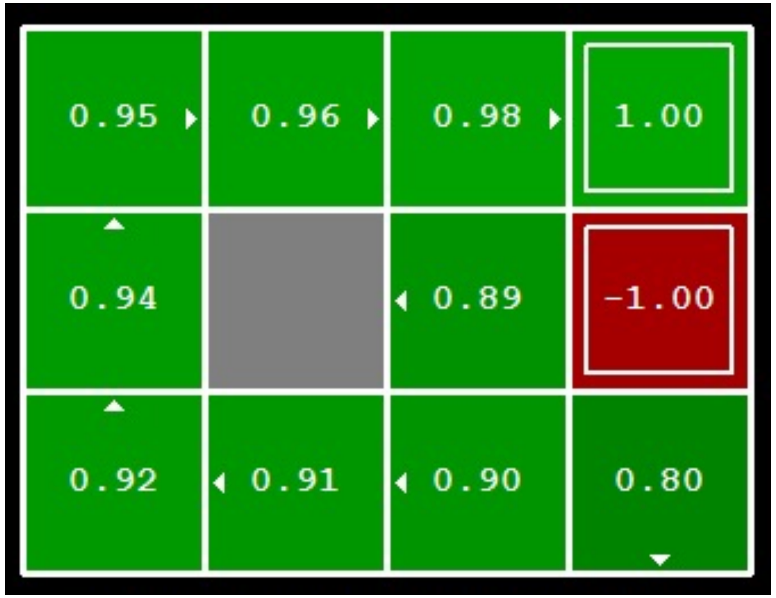


Computing Actions from Values

- Let's imagine we have the optimal values V*(s)

- How should we act?
  - It's not obvious!

- We need to do a mini-expectimax (one step)

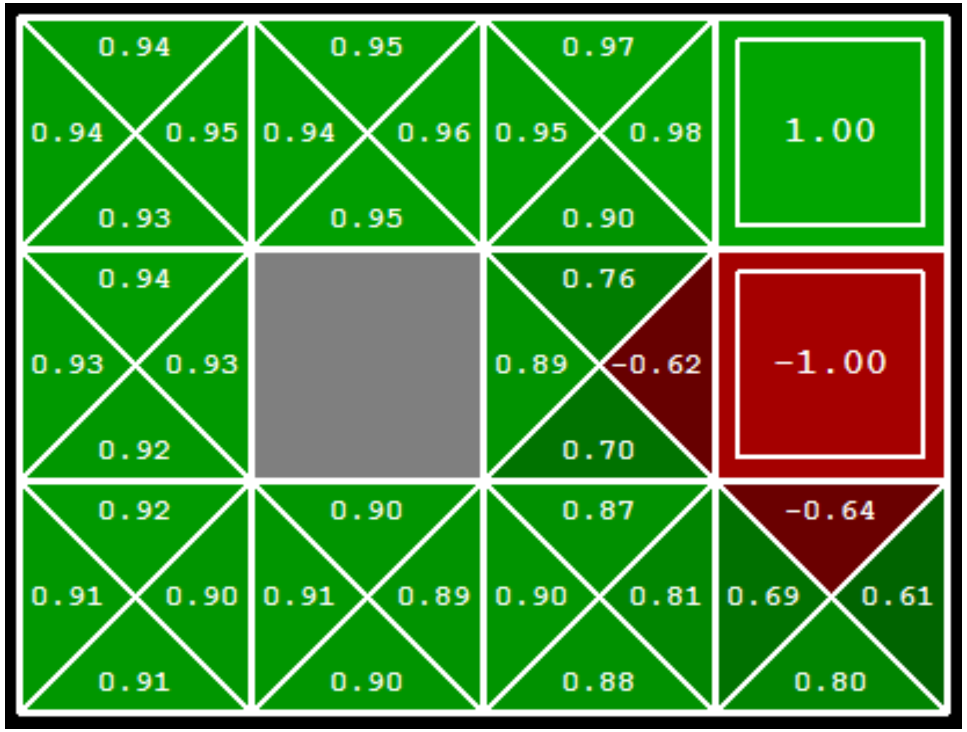


$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

- This is called policy extraction, since it gets the policy implied by the values

Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:

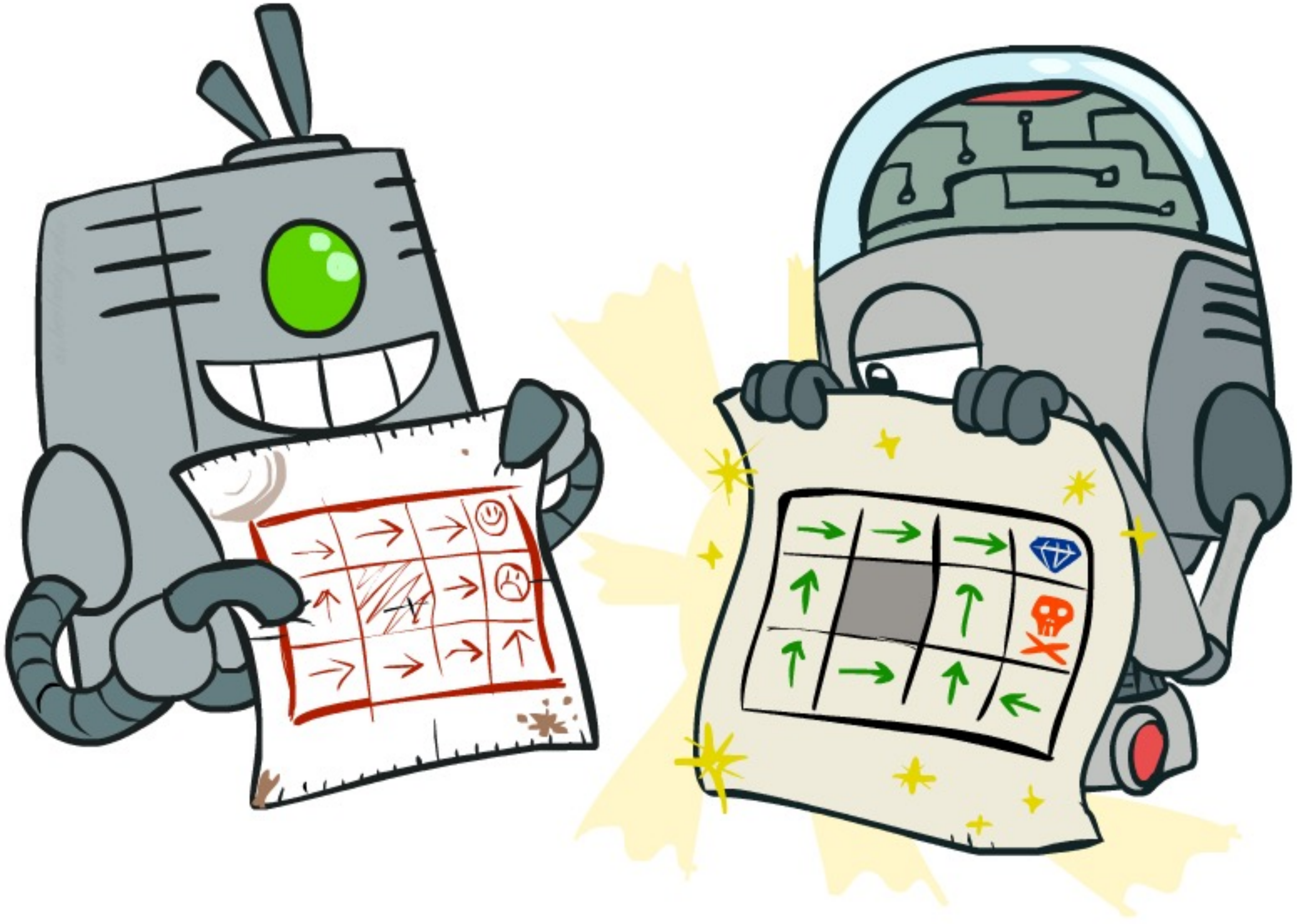


- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

- Important lesson: actions are easier to select from q-values than values!

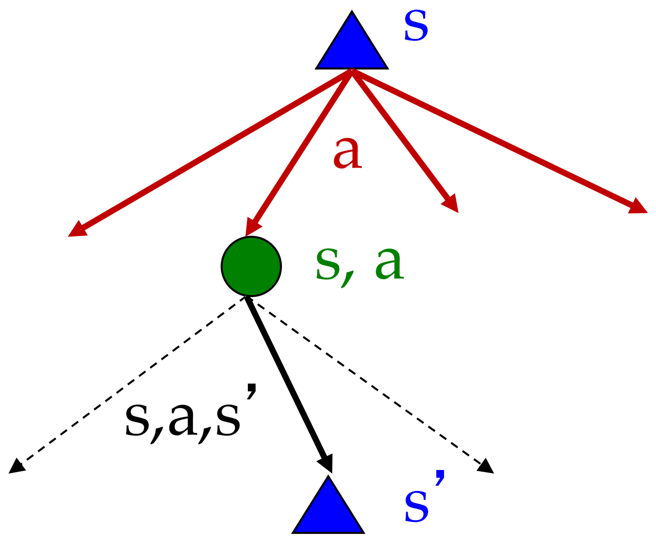
Policy Methods



Problems with Value Iteration

- Value iteration repeats the Bellman updates:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

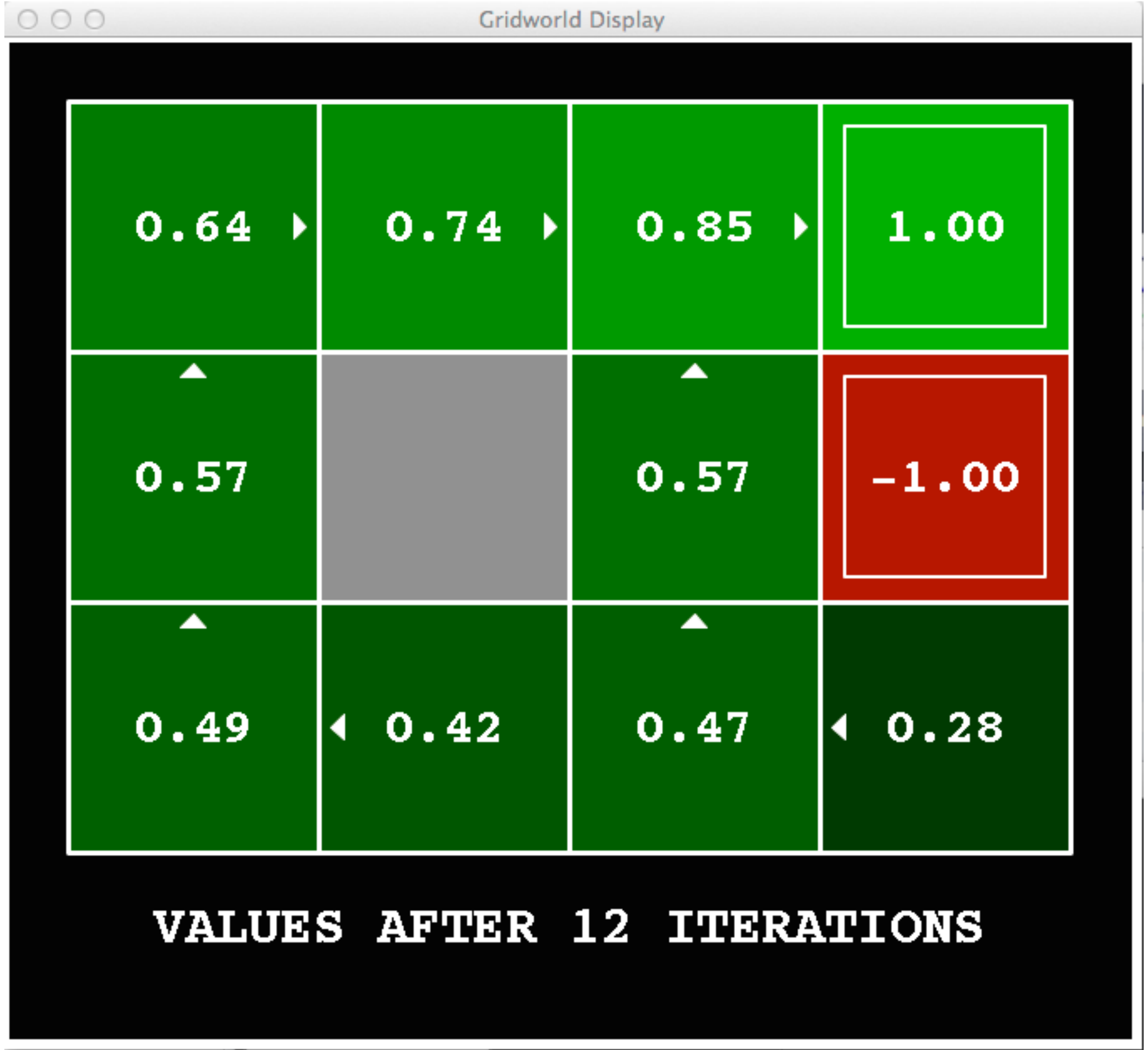


- Problem 1: It's slow – $O(S^2A)$ per iteration

- Problem 2: The "max" at each state rarely changes

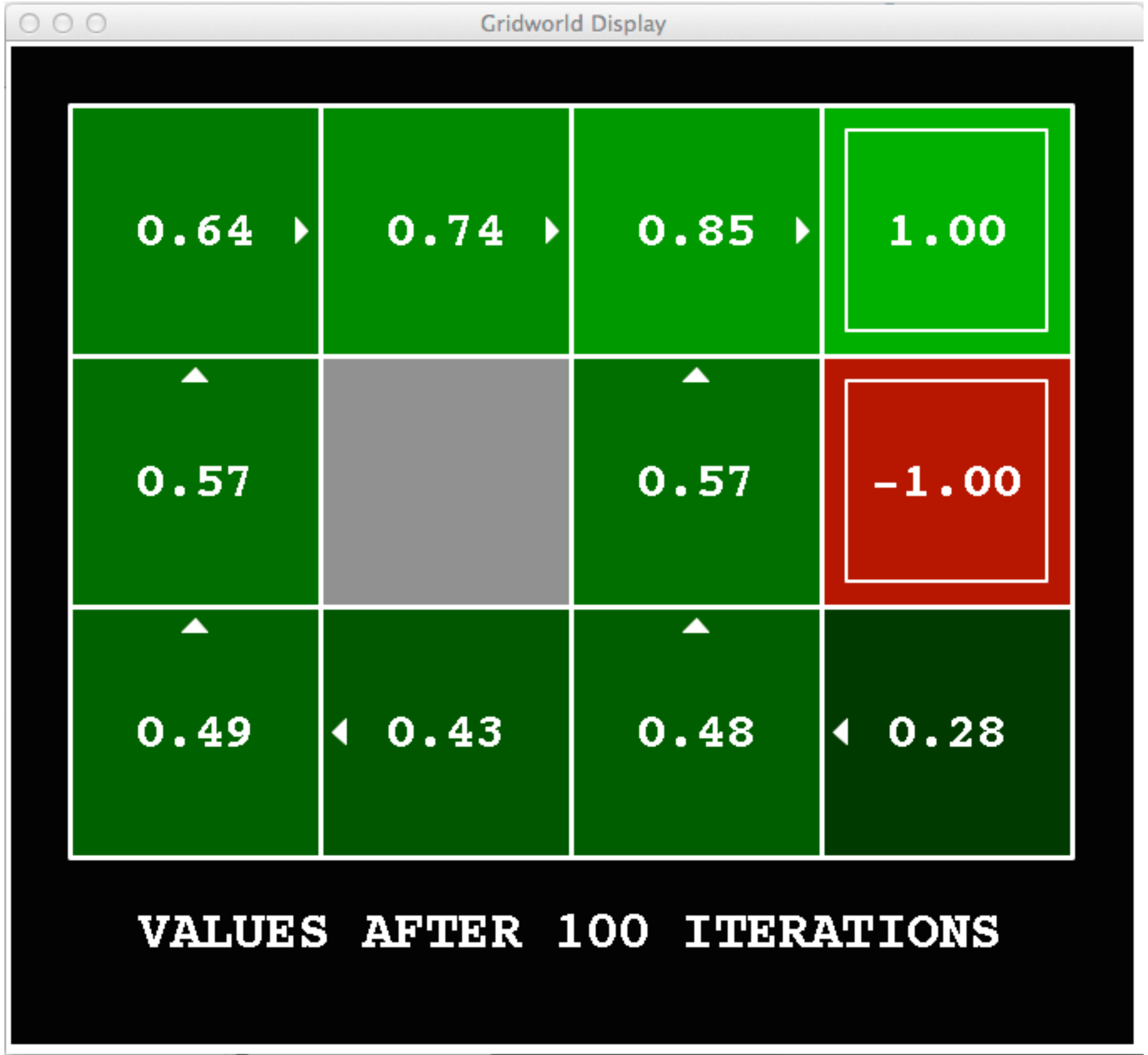
- Problem 3: The policy often converges long before the values

k=12



Noise = 0.2
Discount = 0.9
Living reward = 0

k=100



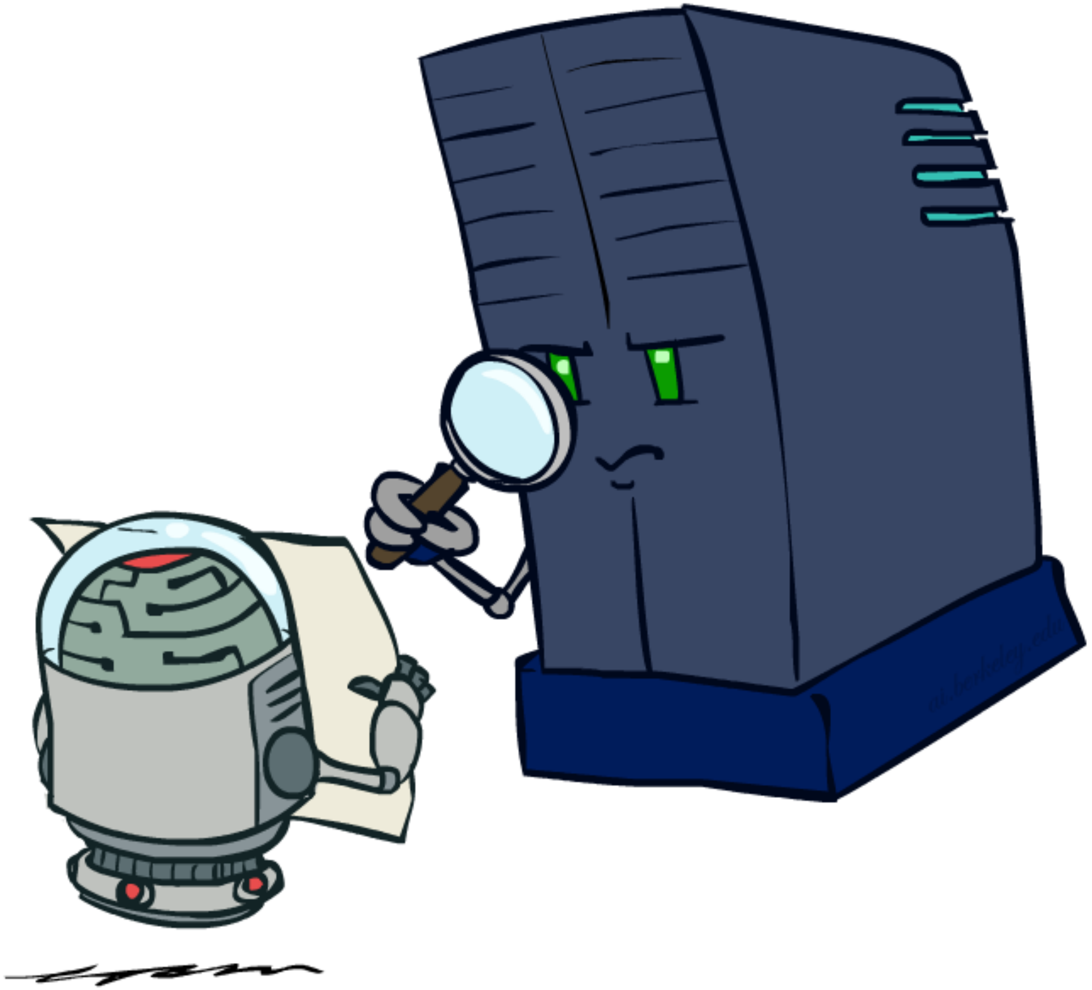
Noise = 0.2
Discount = 0.9
Living reward = 0

Policy Iteration

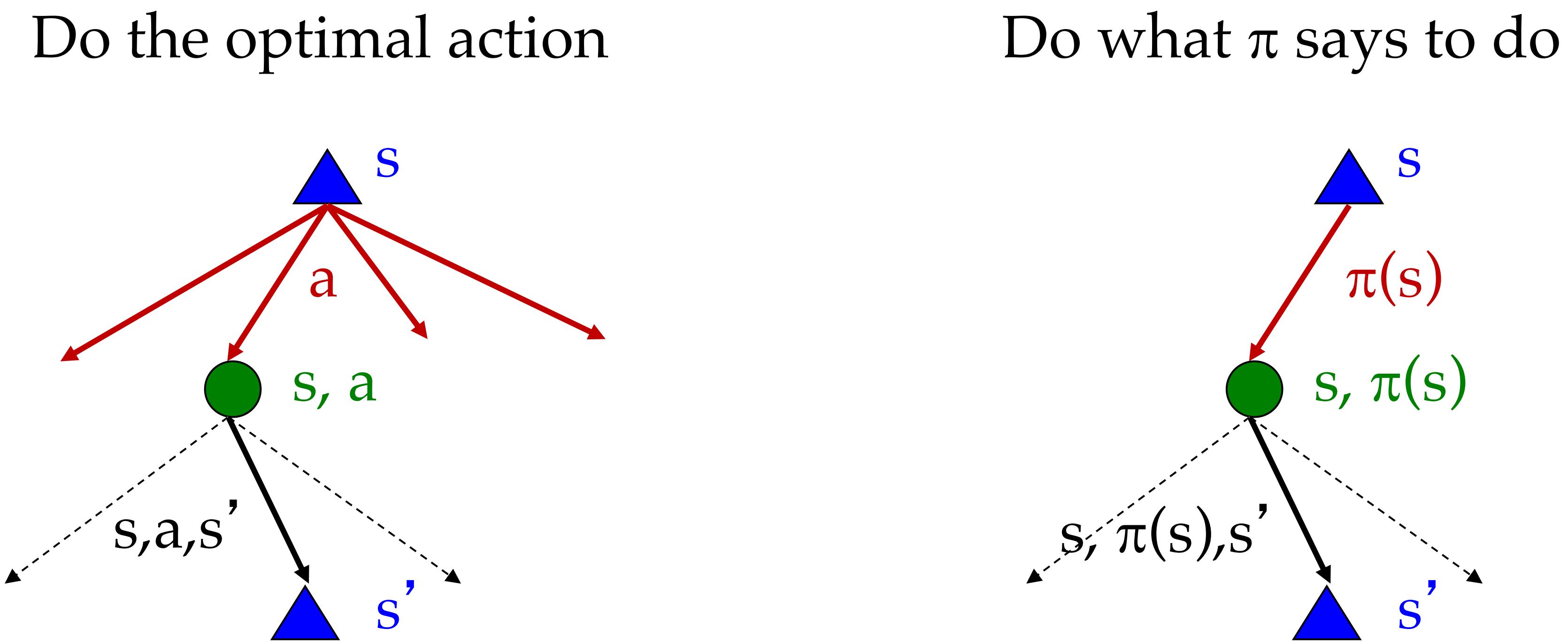
- Alternative approach for optimal values:
  - Step 1: Policy Evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - Step 2: Policy Improvement: update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges

- This is Policy Iteration
  - It's still optimal!
  - Can converge (much) faster under some conditions

Policy Evaluation



Fixed Policies



- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy π(s), then the tree would be simpler – only one action per state
  - … though the tree's value would depend on which policy we fixed

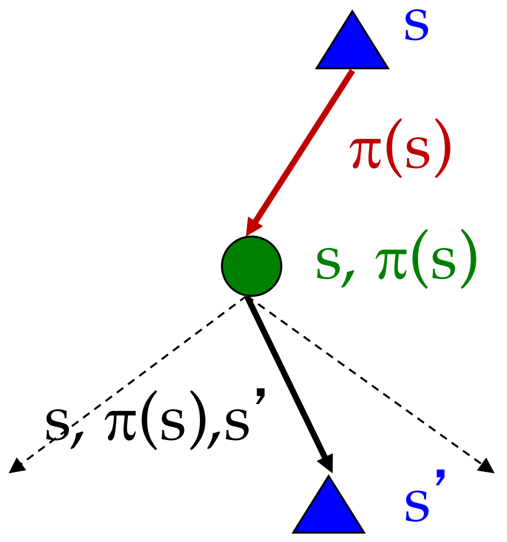
Utilities for a Fixed Policy

- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy

- Define the utility of a state s, under a fixed policy π:

  $V^\pi(s)$ = expected total discounted rewards starting in s and following π

- Recursive relation (one-step look-ahead / Bellman equation):



$$V^\pi(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

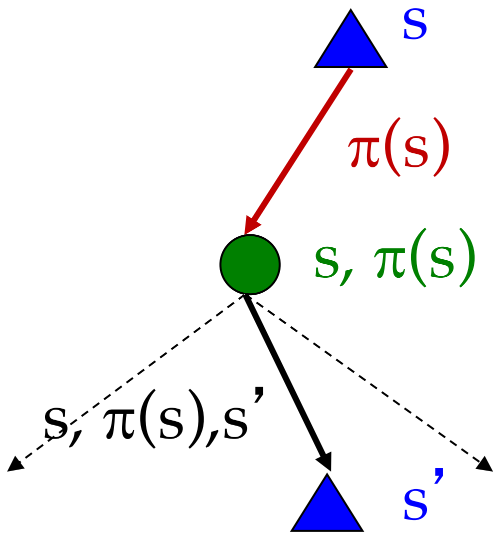
Policy Evaluation

- How do we calculate the V's for a fixed policy π?

- Idea 1: Turn recursive Bellman equations into updates (like value iteration)

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^\pi(s')]$$

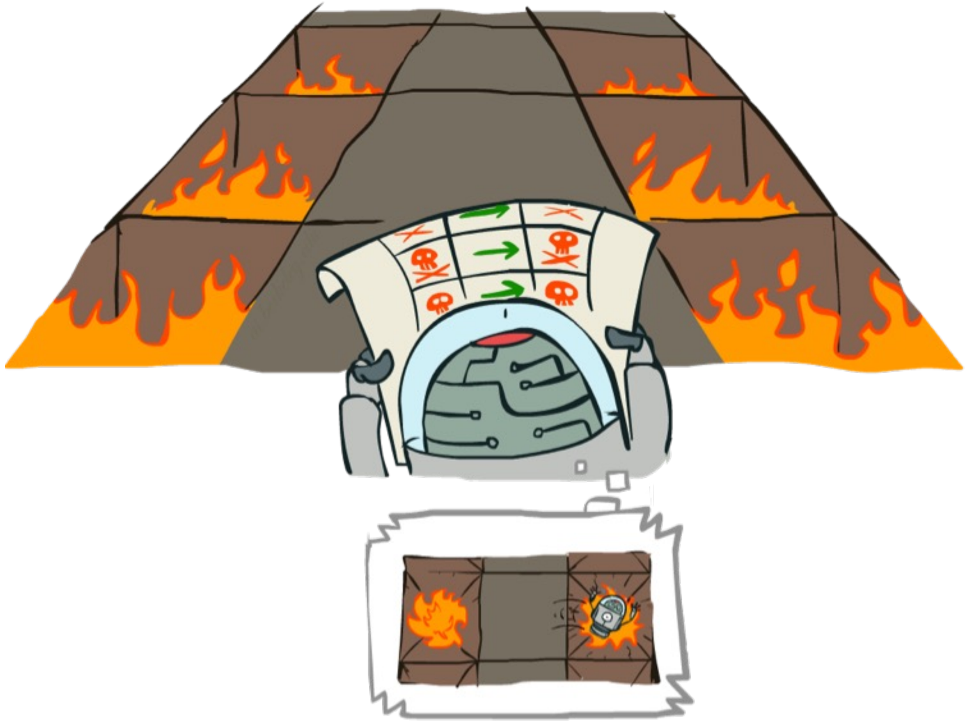


- Efficiency: $O(S^2)$ per iteration

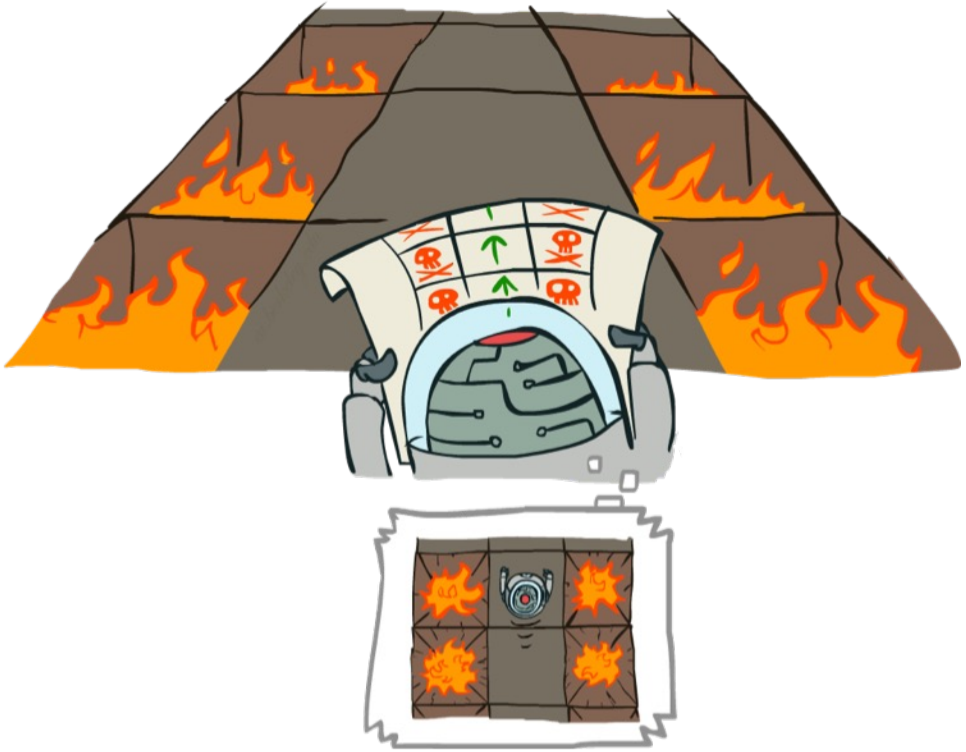
- Idea 2: Without the maxes, the Bellman equations are just a linear system
  - Solve the system of equations

Example: Policy Evaluation

Always Go Right



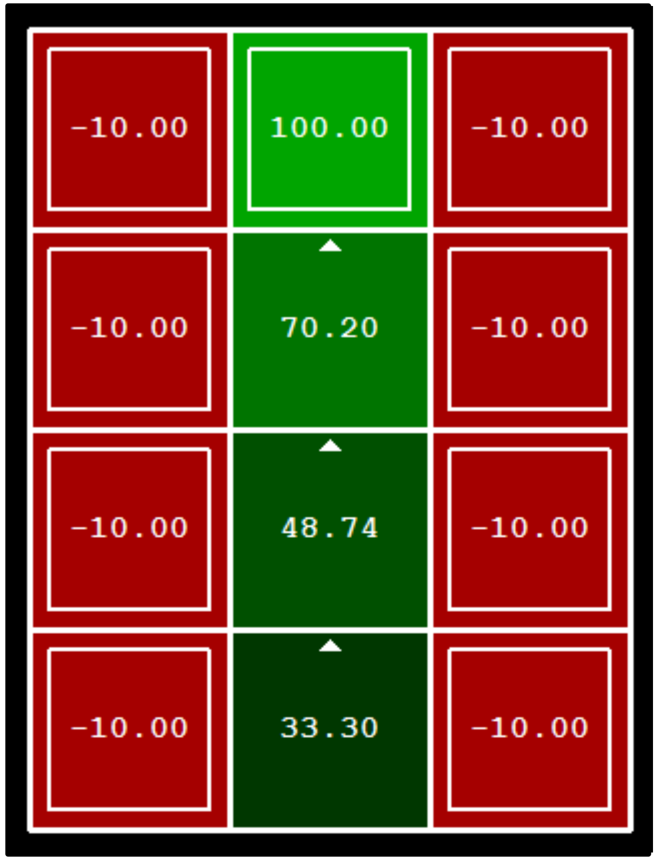
Always Go Forward



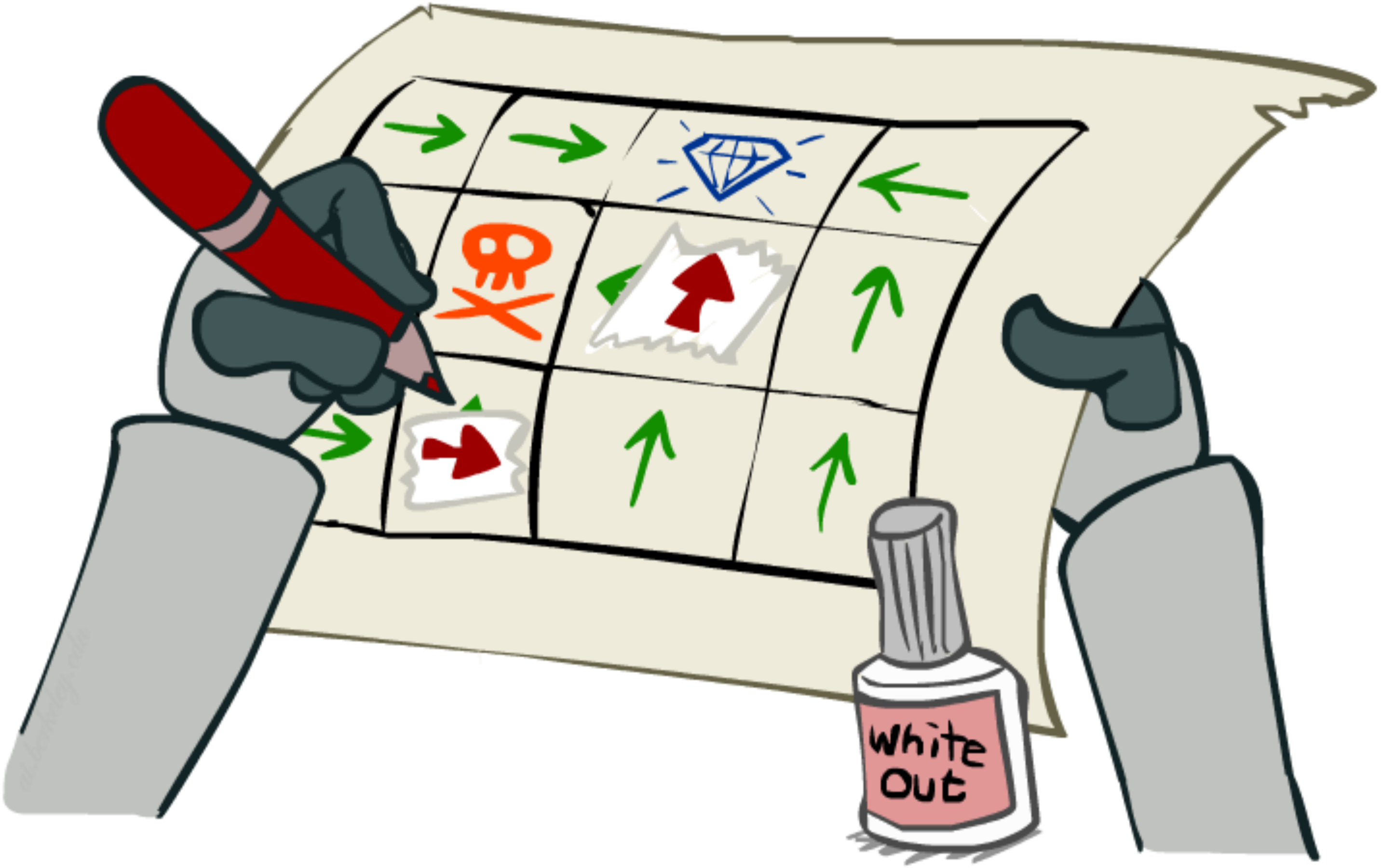
Example: Policy Evaluation

Always Go Right

Always Go Forward



Policy Iteration



Policy Iteration

- Evaluation: For fixed current policy π, find values with policy evaluation:
  - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')\right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$$

Comparison

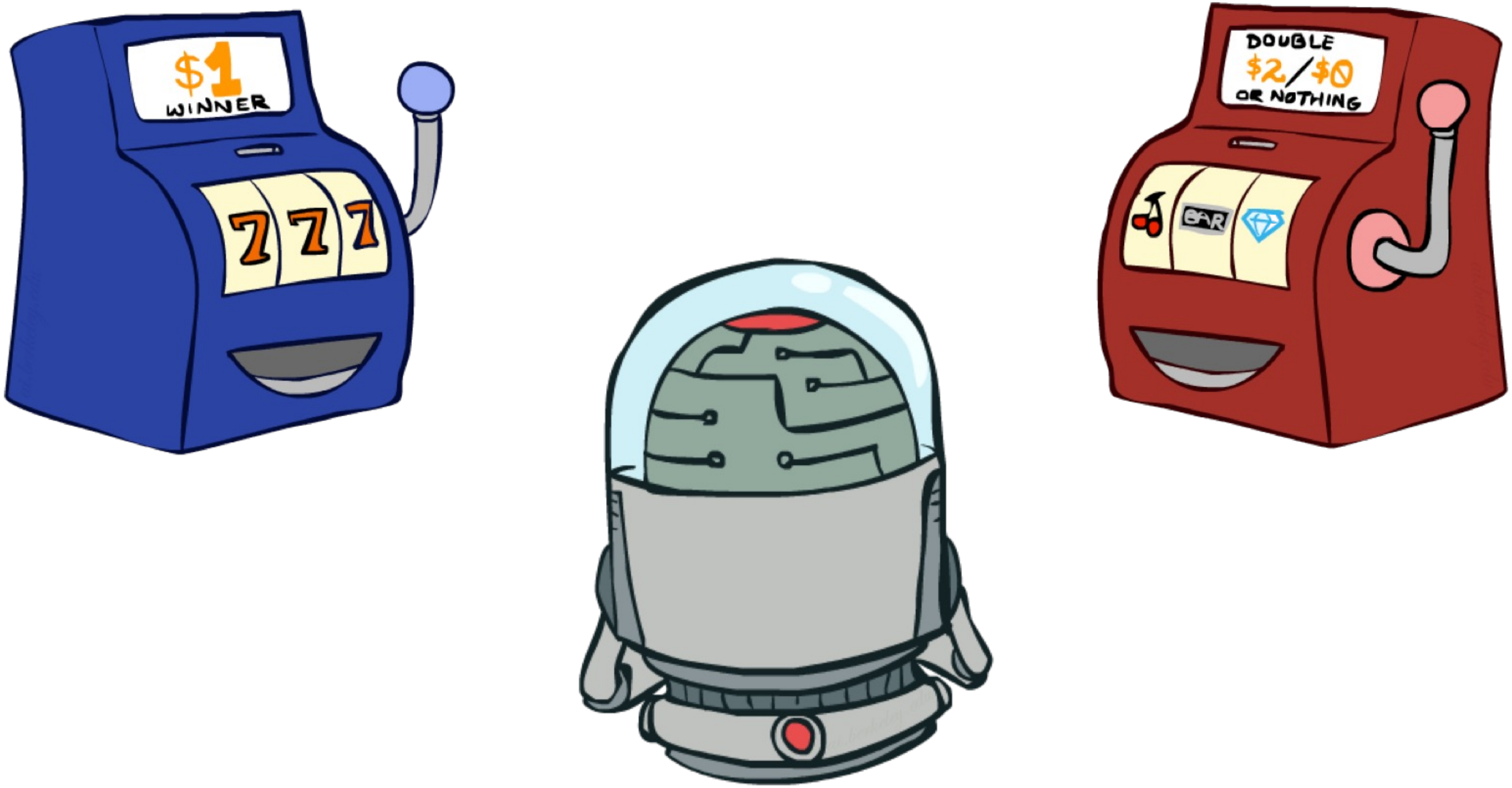
- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
  - Every iteration updates both the values and (implicitly) the policy
  - We don’t track the policy, but taking the max over actions implicitly recomputes it
- In policy iteration:
  - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
  - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
  - The new policy will be better (or we’re done)
- Both are dynamic programming approaches for solving MDPs

Summary: MDP Algorithms

- So you want to….
  - Compute optimal values: use value iteration or policy iteration
  - Compute values for a particular policy: use policy evaluation
  - Turn your values into a policy: use policy extraction (one-step lookahead)

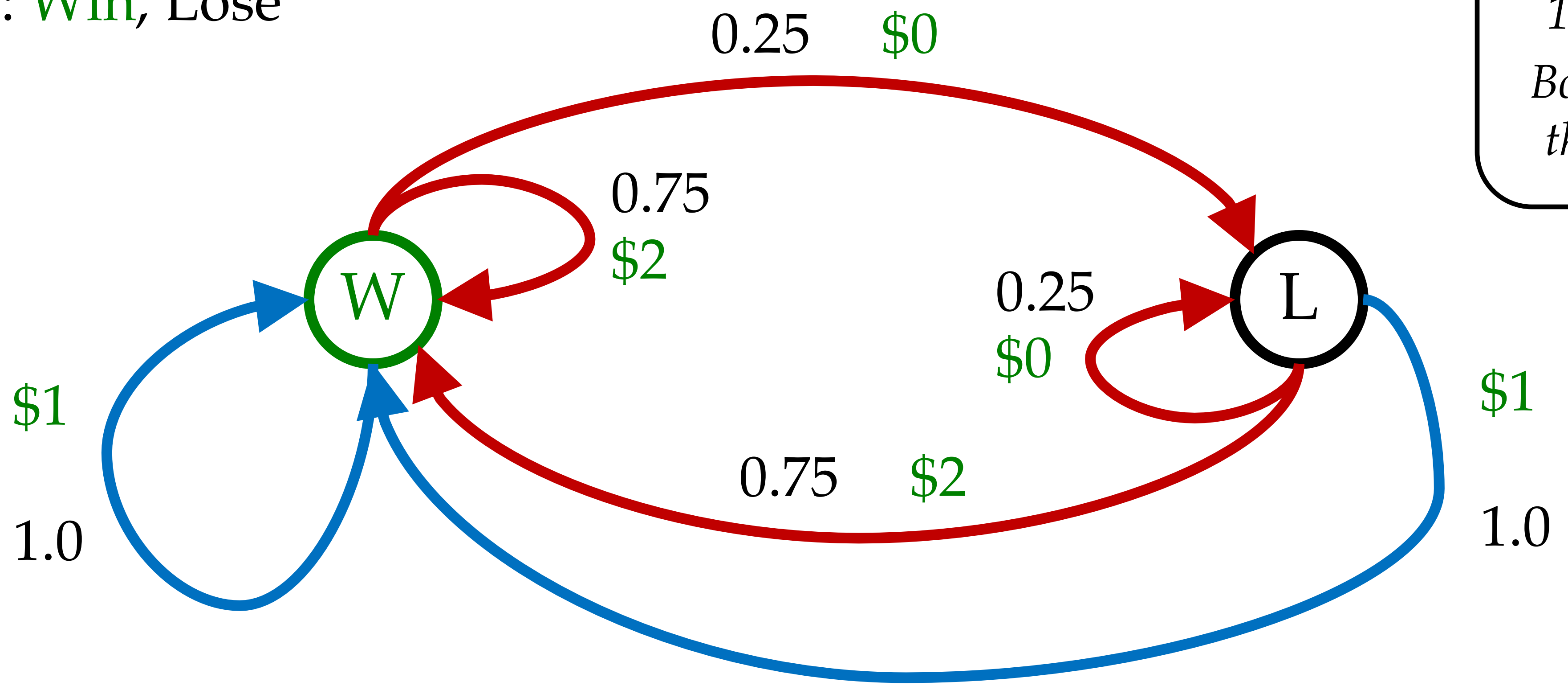
- These all look the same!
  - They basically are – they are all variations of Bellman updates
  - They all use one-step lookahead expectimax fragments
  - They differ only in whether we plug in a fixed policy or max over actions

Double Bandits



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose



No discount

100 time steps

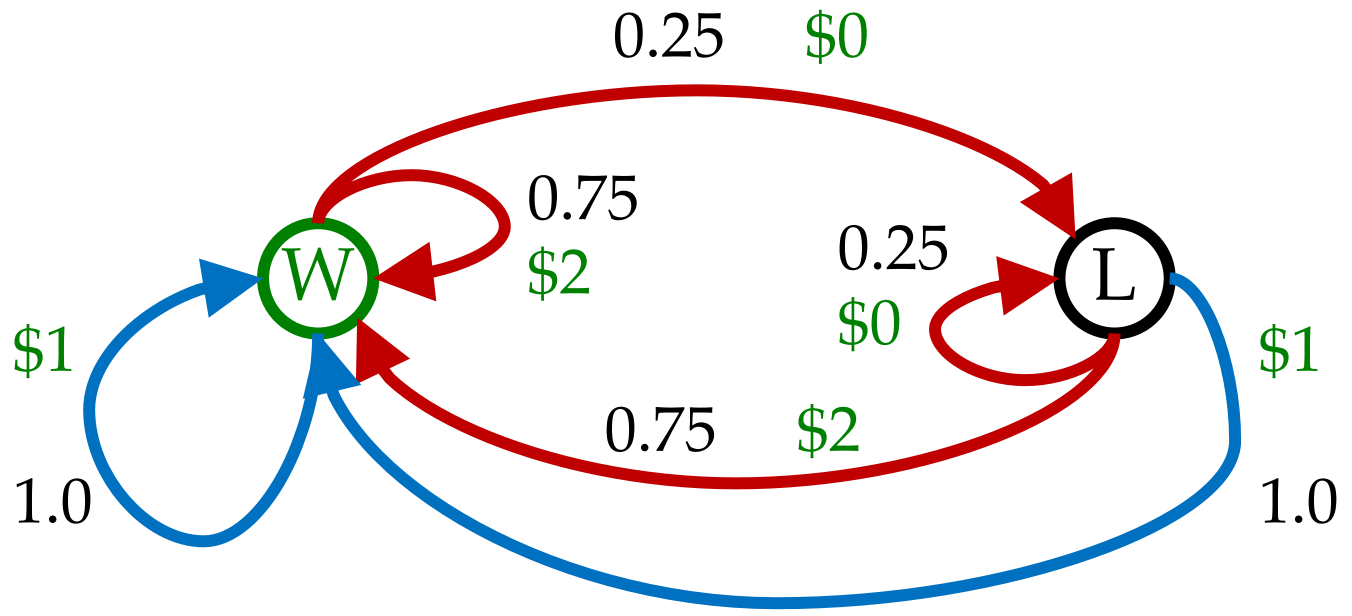
Both states have the same value

Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount
100 time steps
Both states have the same value

	Value
Play Red	150
Play Blue	100



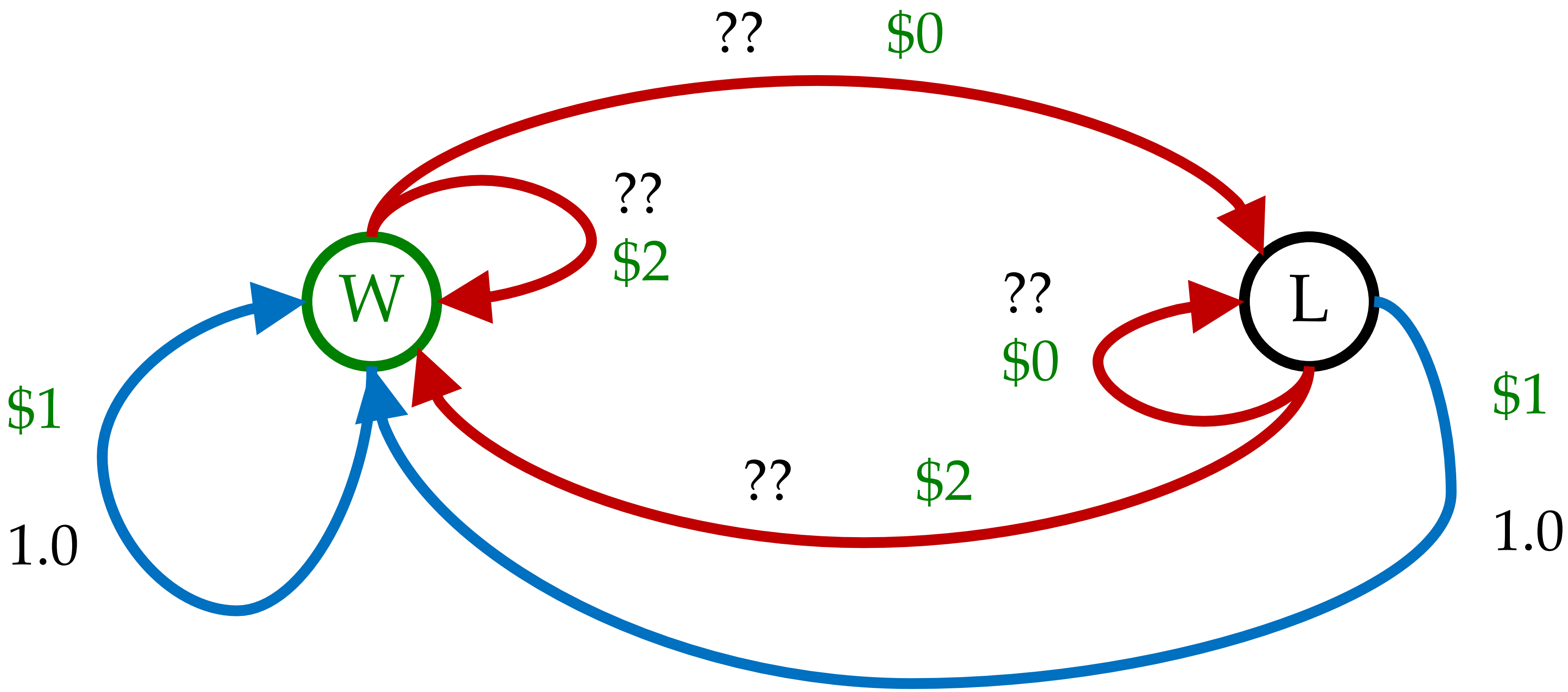
Let's Play!

$2 $2 $0 $2 $2

$2 $2 $0 $0 $0

Online Planning

- Rules changed! Red's win chance is different.



Let's Play!

$0 $0 $0 $2 $0

$2 $0 $0 $0 $0

What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP