What is it?

- New paradigm of fault-tolerant quantum computing
- For efficient quantum computing: error rate $10^{-4}$ (still not achieved) $\rightarrow$ error correction algorithms
- Possible estimated error rate as low as $10^{-30}!!$
- No need for error correction.
What studies Topology?

Properties that are not changed by smooth deformations

Topological properties are robust against small perturbations
The basic idea of TQC

- Create particles from vacuum (initialization)
- Thread their world line (unitary operations)
  This step is not based on interactions between the two particles
- Measure the result (measure)
Physics Particles

- **Fermions** - Particles of Matter:
  - Elementary: Up and Down Quarks, Electrons, Muons
  - Composite: Protons, Neutrons (3 quarks)
    - Obey Fermi-Dirac Statistics
    - Have half-integer spins (½ for electron)
    - No two can be in exactly the same state (Pauli Exclusion Principle)

- **Bosons** (Particles of Force for Elementary):
  - Elementary: Photons, Gluons
  - Composite: Mesons (quark+antiquark), nucleus like He⁴
    - Obey Bose-Einstein Statistics
    - Have integer spins
    - Two can be in the same state
Bosons, fermions, anyons

- If we exchange two fermions: $|\psi\rangle$ becomes $-|\psi\rangle$
  
  Single particle properties: unchanged, but it interferes differently with other particles.

- 3D: only Bosons and fermions

- 2D: we could have also anyons, they can acquire any complex phase $a+ib = \exp(i\pi\theta)$

  **ANY+ons**

- $\theta$ depends on the kind of particle and it’s fixed

- We may have instead of just a phase, an unitary $U$
Braiding in 2+1 dimensions

Clockwise swapping ≠ counterclockwise swapping
Classified by winding number (a topological invariant)
Classical Hall Effect
Fractional Quantum Hall Effect

- Electron gas at the interface in a GaAs heterojunction,
- $T = 10 \text{ mK}$, strong transverse magnetic field $J = \sigma E$

\[ \sigma = \nu \frac{e^2}{h} \]

![Graph showing Integer and Fractional Quantum Hall Effects](image)
Experimental confirmations

- FQH collective excitations are quasiparticles such that the ratio $f$ between electrons and magnetic flux quanta is a fractional number.
- If we circle a $f=1/3$ particle around a $f=2/5$ particle, we can find a relative statistic of $\theta = -1/15$.
- So, they behave like anyons! (Goldman, 2006)
Lattice of Abelian Anyons

- Physical system for complete quantum computation are abelians anyons on a lattice (they could be quantum Hall effect excitations)
- Lattice site: $|+\rangle_j$ is occupied by anyon, $|-\rangle_j$ is not occupied
- $b_j, b_j^\dagger$ are creation and annihilation operators.
  - $A_j = b_j^\dagger b_j$ applied on $|-\rangle_j$ gives 0, while $|+\rangle_j$ is the eigenvector with eigenvalue 1
  - $B_{jk} = b_j^\dagger b_k + b_k^\dagger b_j$ used to swap the states in $j$ and $k$.
- Anyon circled around another anyon gets factor $e^{i\Theta}$
Qubits and 1-qubit Gates

- Qubit is a combination of an anyon-occupied site and a unoccupied one on sites $j$ and $j'$: $|0\rangle_j = |{-+}\rangle_{jj'}$, $|1\rangle_j = |{+-}\rangle_{jj'}$

- This “number operator” generates rotation on z-axis

- Swapping allows to do

- By applying the hamiltonian $B_{jj'}$ for a certain time we can generate the rotation $\exp(i\theta \sigma_x/2)$ around the x axis. Applying $A_j$ will generate the rotations around the z axis.

- The $A_j$ and $B_{jk}$ do not involve interactions with other anyons

- Consequently with rotations about X and Z axis we have a full set of 1-qubit operations
Universal Set of Gates

- Two qubit control gate for $|x\rangle_j |y\rangle_k$ is done by repeated swaps - we circle the content of the first site of j qubit around the first site of the k qubit.

- The phase of -1 for state of two quibits is obtained if and only if the second sites of j and k both contain anyons corresponding to $|1\rangle = |-+\rangle$ giving CPHASE gate:

- The size of the orbits used when circling is unimportant: the phase is a topological effect, not due to interactions.

- We have a universal set of quantum gates!

- 1 qubit operations: local $\rightarrow$ unprotected

- 2 qubit operations: topologically protected
Non Abelian Anyons

- For Abelian anyons 2 qubit operations are topologically protected from errors but single qubit operations are local and unprotected.
- To have fully protected operations we need Non-Abelian anyons. They operate on fusion spaces (interaction of 3 or more anyons) and acquire a unitary instead of a phase.

The “braid group” is generated by moving up/down every thread.

With opportune generators, we can build a dense subset of SU(4) and SU(2): topologically protected single and two qubit operators!
Fibonacci Model

- A simplest of Non-Abelian anyons models is Yang-Lee (Fibonacci) model.
- Two anyons can fuse in either of two ways i.e. $1 \times 1 = 0 + 1$.
- The resulting Hilbert space has dimensions that are Fibonacci numbers.
- Qubits encoded in one anyon: $\log_2 \phi = \log_2 \left(\frac{1 + \sqrt{5}}{2}\right) = \log_2 (1.618) = .694$
- Fibonacci anyons CNOT, accurate to $10^{-3}$:
Future Perspectives

- Experimentalists think that the FQH excitations with \( f=5/2 \) (easier to investigate) and \( 12/5 \) (harder to investigate) are non abelians.
- The \( 5/2 \) particles will not (probably) generate a dense subspace of SU(2). The \( 12/5 \) particle should work.
- At the moment: still trying to measure non-abelian anyons.
- In 2000 they proved that topological quantum computers and ordinary quantum computer are equivalent (can simulate each other).
Possible Error Sources

- Errors occur if thermal fluctuations generate pair of anyons.
- Errors are exponentially small for low temperature.
- Probability of errors decreases exponentially with distance.
- The rate of errors can be minimized to almost 0 with low temperature and keeping anyons sufficiently far apart.
References

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  http://info.phys.unm.edu/~thedude/topo/phystodayTQC.pdf


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