Quantum information processing with atoms and photons

A brief excursion into cavity quantum electrodynamics

Kevin Moore
April 14th, 2005
Atoms and photons as qubits

**Atoms**

Quantum information is encoded in the internal state of an atom (i.e., energy level of electron).

**Photons**

Quantum information can be encoded in photon polarization...

\[ |\psi\rangle = \alpha |\leftrightarrow\rangle + \beta |\downarrow\rangle \]

...or, quantum information can be encoded in photon number!

\[ |\psi\rangle = \alpha |0\text{photons}\rangle + \beta |1\text{photon}\rangle \]
Atoms and photons as qubits

**Atoms**

Atoms are great place to *store* quantum information
- long-lived states
- easy to interact with (lasers, microwaves)
- *hard* to manipulate

**Photons**

Photons are great for *transmitting* quantum information
- travel fast (duh!)
- easy to manipulate (gates = waveplates, measurement = polarizers and photo counters)
- hard to store cause they’re always moving…or are they?
What are our atomic control knobs?

\[
\hat{H} = \sum_i \left( \frac{\hat{p}_i^2}{2m_e} + \frac{Ze^2}{|\hat{r}_i|} \right) + \sum_{i,j \neq i} \frac{e^2}{|\hat{r}_i - \hat{r}_j|} + V_{ext} + \frac{\hat{P}_a^2}{2m_a}
\]
What are our atomic control knobs?

specifies bare energy states for atom (or ion), taken as given

\[ \hat{H} = \sum_i \left( \frac{\hat{p}_i^2}{2m_e} + \frac{Ze^2}{|\hat{r}_i|} \right) + \sum_{i,j \neq i} \frac{e^2}{|\hat{r}_i - \hat{r}_j|} + V_{ext} + \frac{\hat{P}_a^2}{2m_a} \]

center of mass kinetic energy

Examples of external fields one might use

External fields (electric, magnetic, and gravity)

Magnetic fields (Zeeman effect): \( V_{ext} = -\hat{\mu} \cdot \vec{B} \)

Electric fields (Stark effect): \( V_{ext} = -\hat{d} \cdot \vec{E} \)

Oscillating E/M fields (esp. AC Stark shift): \( V_{ext} \) same, but resonance induces dipole moment... use lasers or microwave fields

Gravity: \( V_{ext} = m_a g h \)
What are our atomic control knobs?

specifies bare energy states for atom (or ion), taken as given

\[ \hat{H} = \sum_i \left( \frac{\hat{p}_i^2}{2m_e} + \frac{Ze^2}{|\hat{r}_i|} \right) + \sum_{i,j \neq i} \frac{e^2}{|\hat{r}_i - \hat{r}_j|} + V_{ext} + \frac{\hat{P}_a^2}{2m_a} \]

What can we do about the center of mass motion?

We need the atom to be localized somewhere, as we want to use it for, say, quantum computation.

How can we accomplish this?
Controlling the motion of atoms

Atoms on surfaces don’t move... could we use them?

Answer: Maybe, but the energy levels get more complicated as the atom is strongly interacting with the surface (ask Mike Crommie just how complicated things get!)

What we would really like is an atom in free space: (energy levels are unperturbed)

What determines the motion of a free atom? Temperature!

\[
\frac{\hat{P}_a^2}{2m_a} \approx kT
\]
Just how fast do atoms move?
(in other words, how cold are we talking here?)

Surface temperature (300 K) N₂ gas has an average speed of 421 m/s, which would put it into the stratosphere (nearly outer space).

Liquid helium is pretty cold (4 K), but a 4 K nitrogen molecule would still easily clear the Campanile at 49 m/s!
Cooling atoms

In order for atoms not to zip away so fast that we can’t do any quantum information processing, we must find a way to reduce their temperature (i.e. average speed).

Enter laser cooling!

\[ p_a = mv \]

\[ p_\gamma = \frac{h}{\lambda} \]
Cooling atoms

In order for atoms not to zip away so fast that we can’t do any quantum information processing, we must find a way to reduce their temperature (i.e. average speed).

Enter laser cooling!

\[ p_a = m v \]

\[ p_{tot} = N \gamma \frac{h}{\lambda} \]

With laser cooling you can get atoms as cold as 0.000002 K!!

\[ v_{avg} = \sqrt{\frac{2kT}{m}} \approx 2\text{cm/sec} \]

that’s more like it!
So we have a way to cool atoms down, but how do we keep them in one place?

We need to (cleverly) apply external fields to accomplish this task.

Example: How might you trap a *chargeless* spin-$\frac{1}{2}$ particle with a magnetic field?

$$\hat{H} = -\hat{\mu} \cdot \vec{B}$$
Trapping atoms

So we have a way to cool atoms down, but how do we keep them in one place?

We need to (cleverly) apply external fields to accomplish this task.

Example: How might you trap a chargeless spin-$\frac{1}{2}$ particle with a magnetic field?

\[ \hat{H} = -\frac{\hbar e B}{2m} | \uparrow \rangle \langle \uparrow | + \frac{\hbar e B}{2m} | \downarrow \rangle \langle \downarrow | \]

Particles aligned with field have lower energies in high magnetic fields.

Particles anti-aligned with field have lower energies in high magnetic fields.
Suppose I have a ball of spin-$\frac{1}{2}$ particles with a mixture of both up and down spin states.
Trapping atoms

| ↓⟩, | ↑⟩
Trapping atoms

down particles are repelled from high magnetic fields

up particles are attracted to high magnetic fields
Trapping atoms

\[ |↓\rangle, |↑\rangle \]
Trapping atoms

- Down particles are repelled from high magnetic fields
- Up particles are attracted to high magnetic fields

$\vec{B}$
Trapping atoms

\[ | \downarrow \rangle, | \uparrow \rangle \]
Trapping atoms
Trapping atoms
Trapping atoms
Trapping atoms
Trapping atoms

field minimum in the center traps weak field seekers!

high field seekers are expelled
Cooling and trapping together

Laser cooling and trapping
Nobel Prize 1997

Bose-Einstein condensation
Nobel Prize 2001

Coldest stuff in the universe... $N_2$ at these temperatures would only travel 3 millionths of a meter!
Photons

Thanks to this guy, we found out 100 years ago that light comes in little packets of energy known as photons:

The energy of a photon is, from Einstein’s relation, determined by its frequency. For a given frequency, the energy of the system is given by the total number of photons of that frequency times the energy per photon ($\hbar \nu$).

Quantum states of the photon field are given by eigenstates of a number operator which counts the number of photons in a mode.

$$ U_{\text{classical}} = \frac{1}{8\pi} \int d^3\vec{r} \left( \vec{E}^2 + \frac{1}{c^2} \vec{B}^2 \right) \rightarrow \hat{H} = \hat{N} \hbar \omega_o $$
Atoms and photons together

- Ingredients:
  - two-level atoms
  - discrete light quanta

What does the Hamiltonian look like?

\[ \hat{H} = \hbar \omega_a |b\rangle \langle b| + \hbar \omega_o |1\rangle \langle 1| + 2\hbar \omega_o |2\rangle \langle 2| + \ldots \]
A single atom and single photon

\[ |b\rangle \]
\[ |a\rangle \]
\[ \omega_a \]

\[ |0\rangle \]

\[ |1\rangle \]

\[ \omega_o \]

4-level state space: \{ |a, 0\rangle, |a, 1\rangle, |b, 0\rangle, |b, 1\rangle \} 

\[ \hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1| \]

\[ \begin{pmatrix}
\hbar \omega_o & 0 & 0 \\
0 & \hbar \omega_a & 0 \\
0 & 0 & 0
\end{pmatrix} \]

No off diagonal elements... boring!
Atoms and photons \textit{interacting}

- **Ingredients:**
  - two-level atoms
  - discrete light quanta

\[
\hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1|
\]
Atoms and photons *interacting*

**Ingredients:**
- two-level atoms
- discrete light quanta
- interactions

\[
\hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1| - \hat{d} \cdot \vec{E}
\]

(it’s like \(\hat{H} = -\hat{\mu} \cdot \vec{B}\) !)

Claim: interaction term should yield *off diagonal elements*!
Atoms and photons *interacting*

- **Ingredients:**
  - two-level atoms
  - discrete light quanta
  - interactions

\[
\hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1| + \hbar g_o (|b, 0\rangle \langle a, 1| + |a, 1\rangle \langle b, 0|)
\]

- destroys a photon, raises atom to excited state
- brings an atom to the ground state, creates a photon
Atoms and photons *interacting*

\[ \hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1| + \hbar g_o (|b, 0\rangle \langle a, 1| + |a, 1\rangle \langle b, 0|) \]

Who cares about the ground state?

\[
\begin{pmatrix}
\hbar \omega_o & \hbar g_o & 0 \\
\hbar g_o & \hbar \omega_a & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

Now that's a nice (and familiar) Hamiltonian!
Atoms and photons *not* interacting

\[ \hat{H} = \hbar \omega_a |b, 0\rangle \langle b, 0| + \hbar \omega_o |a, 1\rangle \langle a, 1| \]

- This is the photon frequency, which is usually under our control (a tunable laser or whatever)
- This is the a property of the atom, not under our control (although we can choose which atom!)
Atoms and photons not interacting

\[ \hat{H} = \hbar \omega_a \langle b, 0 | b, 0 \rangle + \hbar \omega_a \langle a, 1 | a, 1 \rangle \]

Tune the light to resonance!

set \( \omega_o = \omega_a \)!

\[
\begin{pmatrix}
\hbar \omega_a & 0 \\
0 & \hbar \omega_a
\end{pmatrix}
\]

these two states have the same energy
Atoms and photons *interacting*

\[
\hat{H} = \hbar \omega_a \langle b, 0 | b, 0 \rangle + \hbar \omega_a \langle a, 1 | a, 1 \rangle + \hbar g_o \left( \langle b, 0 | a, 1 \rangle + \langle a, 1 | b, 0 \rangle \right)
\]

What are the energy eigenstates of \( \hat{H} \)?

\[
| E_+ = \hbar \left( \omega_a + g_o \right) \rangle = \frac{| b, 0 \rangle + | a, 1 \rangle}{\sqrt{2}}
\]

\[
| E_+ = \hbar \left( \omega_a - g_o \right) \rangle = \frac{| b, 0 \rangle - | a, 1 \rangle}{\sqrt{2}}
\]
Atoms and photons interacting

\[ \hat{H} = \hbar \omega_a |b, 0 \rangle \langle b, 0| + \hbar \omega_a |a, 1 \rangle \langle a, 1| + \hbar g_o (|b, 0 \rangle \langle a, 1| + |a, 1 \rangle \langle b, 0|) \]

Energy eigenstates are superpositions of having a photon and not having a photon!

There's no state where the atom's energy level used to be!
How I learned to stop worrying and make $d \cdot E$ large

Single atom/photon Hamiltonian $\rightarrow$ (low excitation regime)

We want $g_o$ as big as possible... but how big is $g_o$?

Take my word for it...

$$g_o = -\hat{d} \cdot \vec{E} = -d \sqrt{\frac{\hbar \omega_o}{2 \epsilon_o V}} \sim d \sqrt{\frac{\omega_o}{V}}$$

$d$ is the electric dipole moment of the atomic transition... your choice of which atom, which levels

$V$ is the mode volume of the photon (i.e. how much space does the photon occupy)
Okay, seriously, what is this guy talking about?

- Can get interesting quantum states if we can get a single atom to feel the effect of a single photon

\[ \begin{pmatrix} \hbar \omega_o & \hbar g_o \\ \hbar g_o & \hbar \omega_a \end{pmatrix} \]

- Those eigenstates have both an atom character (storage) and photon character (potential for transmission)

\[ \left| b,0 \right\rangle \pm \left| a,1 \right\rangle \frac{\sqrt{2}}{} \]

- At least two things must occur for this to make any sense
  1. Photons must have large electric field
  2. Photon must hang around the atom for longer than \(1/g_o\)

\[ \frac{|\vec{E}|}{\text{photon}} \propto d \sqrt{\frac{\omega_o}{V}} \]
Enter cavity quantum electrodynamics!

Get really shiny mirrors and make a cavity... as small as possible!

Place atom inside cavity

Like a guitar string, the length of the cavity “tunes” the resonant frequencies... set the length to have a resonance at $\omega_o$.
Some notable cavity QED experiments

Serge Haroche at ENS (France)

Jeff Kimble at Caltech

Robert Schoelkopf and Steve Girvin at Yale
Let’s bring it all together!

- Advances in laser cooling and *ultracold* atomic physics allow us to cool, trap, and control the position and velocity of atoms.

- Shiny mirrors and small volumes (cavity QED) allows us to get a single photon (quantum information transmitter) to interact strongly with a single atom (quantum information storage).

- The ability to transfer quantum information between quantum systems is unique to cavity QED... this is the big payoff!
Let’s bring it all together!

- Advances in laser cooling and *ultracold* atomic physics allow us to cool, trap, and control the position and velocity of atoms.

- Shiny mirrors and small volumes (cavity QED) allows us to get a single photon (quantum information transmitter) to interact strongly with a single atom (quantum information storage).

- The ability to transfer quantum information between quantum systems is unique to cavity QED... this is the big payoff!

- Great! So how to do we do it?
magnetic trap spectrum

enter the “millitrap”

B'' = 5 x 10^4 G/cm^2

B'' = 10^4 G/cm^2

B'' = 10^2 G/cm^2

cavity spectrum

e.g. Vahala
e.g. Mabuchi
e.g. Vahala
e.g. Kimble

e.g. Haroche

magnetic trapping tough with superconducting \( \mu \)-wave cavities
Millimeter cavities and millimeter traps

- Anti-bias coils: (6 mm ID, 8 mm OD), 1 mm separation
- Curvature coils: (3 mm ID, 4 mm OD), 4.5 mm separation
- Lofte bars: 2 mm x 2 mm cross-section

2.5 mm separation

Strongly coupled cavity has ~100 µm separation
Millimeter cavities and millimeter traps

anti-bias coils (6 mm ID, 8 mm OD)
1 mm separation

curvature coils (3 mm ID, 4 mm OD)
4.5 mm separation

Ioffe bars
2 mm x 2 mm cross-section

1 μK cloud in inch-scale IP trap (10^2 G/cm²)

1 μK cloud in millitrap (10^4 G/cm²)

~100 μm

cavity mirror
Millitrap assembly

1. Winding anodized aluminum coils
   - 5 turns
   - Current in
   - Current out

2. Mounting coils

3. Integrate Ioffe bars

4. Mounting trap in UHV chamber
   - LN2 in
   - LN2 out
Millitrap assembly (continued)

hand-winding aluminum foil leads to lots of dead coils

after 100 bad coils, six good coils make a trap!

trap is mounted in LN$_2$ cooled assembly

LN$_2$ circulation
Ultracold atom production

(1) load MOT

(2) capture atoms in a quadrupole trap

(3) transfer 3 inches to millitrap region with quad coil pair

(4) atoms are transferred from external quadrupole trap to millitrap
Ultracold atom production

MOT coils

transfer coils
Ultracold atom production

- MOT
- millitrap
- Zeeman-slowed atom beam
Ultracold atom production (continued)

October 28th, 2004 – first magnetically-trapped atoms

Dec. 2nd, 2004 - Pure BECs of 3 million atoms produced
The cavity
http://physics.berkeley.edu/research/ultracold