1. (a) Consider a CNOT gate whose control qubit (first input) is $1/\sqrt{2}|0\rangle + 1/\sqrt{2}|1\rangle$ and target qubit (second input) is $1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. What are the states of the two output qubits? Repeat when the control qubit is $|-\rangle = 1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. Can you describe the action of the CNOT gate on the first qubit when the target qubit is $|-\rangle = 1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. What about when the target qubit is $|+\rangle$.

   (b) This brings us to an important concept. We can consider what the action of a quantum gate looks like when we write the qubits in a different basis. Show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

2. Suppose $|\psi\rangle = \alpha_{00}|\phi^+\rangle + \alpha_{10}|\phi^-\rangle + \alpha_{01}|\psi^+\rangle + \alpha_{11}|\psi^-\rangle$, where $\{|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle\}$ is the Bell basis. Give a quantum circuit that maps $|\psi\rangle$ to $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$.

3. (a) Create a "copying circuit" that correctly copies the states $|+\rangle$ and $|-\rangle$. i.e. it takes as input two qubits $|\psi\rangle$ and $|0\rangle$, and in the case that $|\psi\rangle$ is either $|+\rangle$ or $|-\rangle$ it outputs $|\psi\rangle$ and $|\psi\rangle$.

   (b) Now suppose that the two states in question are $|+\rangle$ and $|0\rangle$. Can you create a "copying circuit" that correctly copies these two states?

   Hint: What is the inner product between $|+\rangle|0\rangle$ and $|0\rangle|0\rangle$? And what is the inner product between $|+\rangle|+\rangle$ and $|0\rangle|0\rangle$?

4. Suppose you have two qubits in some arbitrary entangled state $|\psi\rangle$. You apply the teleportation protocol to each of the two qubits separately. What is the resulting state obtained by the other party? Why?