Automatic Image Alignment

CS194: Intro to Comp. Vision and Comp. Photo
Alexei Efros, UC Berkeley, Fall 2022

with a lot of slides stolen from Steve Seitz and Rick Szeliski

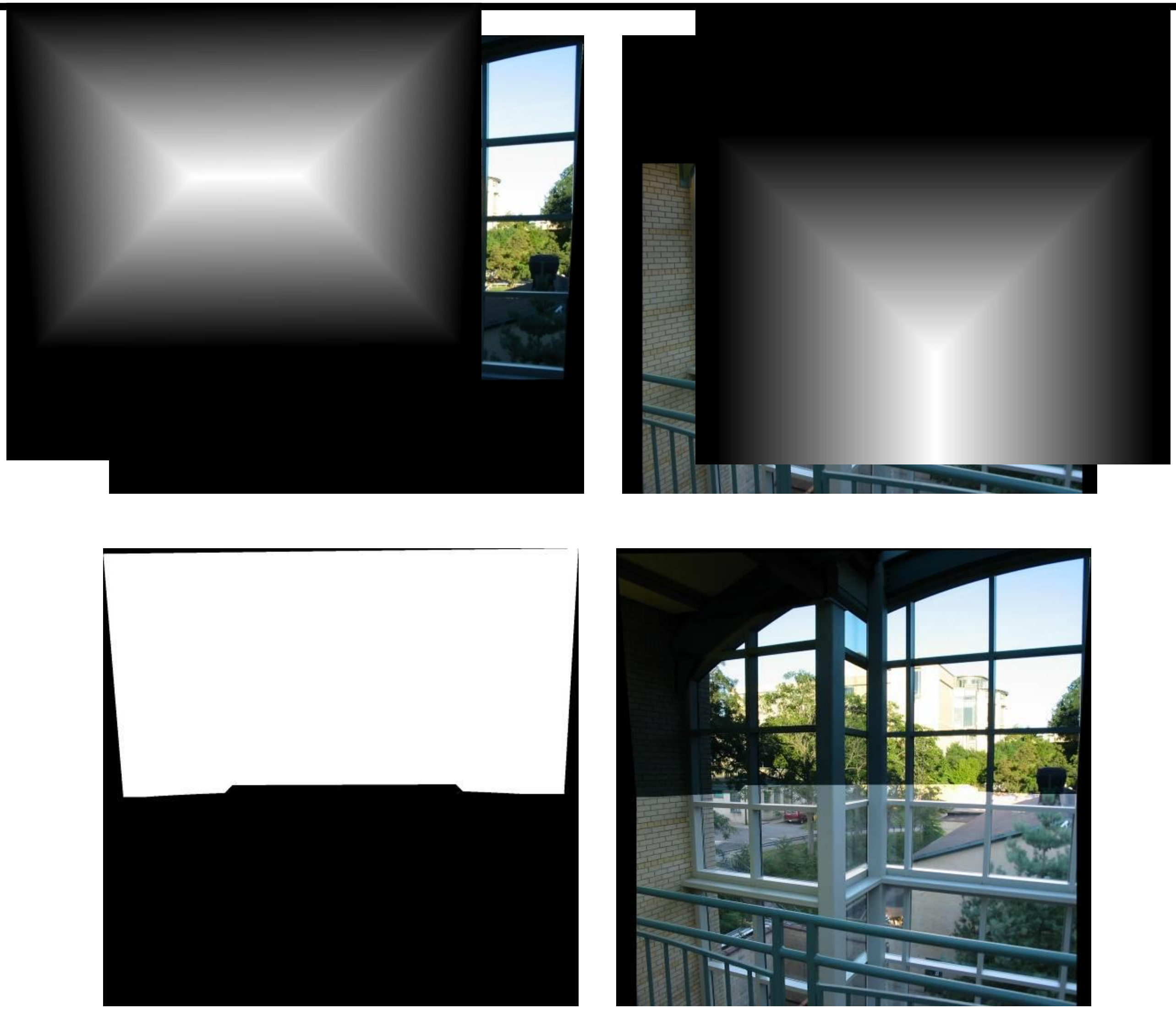
Project Proposals due in a month (11/11)

for cs294-26 and others interested to do more

Alpha blending for Panorama Stitching

Alpha = .5 in overlap region

Setting alpha: center seam



Distance Transform
`bwdist`

Alpha = logical(dtrans1>dtrans2)

Simplification: Two-band Blending

Brown & Lowe, 2003

- Only use two bands: high freq. and low freq.
- Blends low freq. smoothly
- Blend high freq. with no smoothing: use binary alpha

2-band “Laplacian Stack” Blending

Low frequency (λ > 2 pixels)

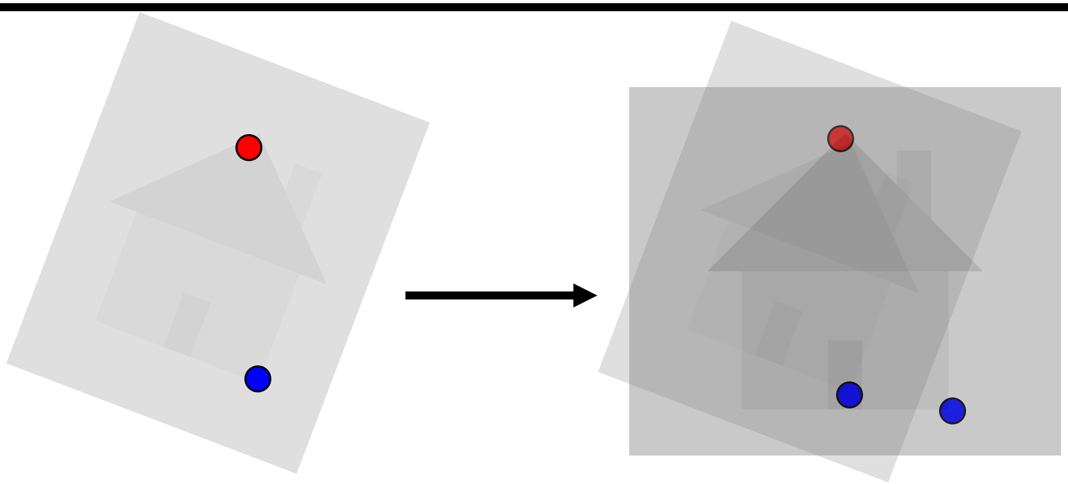
High frequency (λ < 2 pixels)

Linear Blending

2-band Blending

Live Homography…

Image Alignment



How do we align two images automatically?

Two broad approaches:

- Feature-based alignment
  - Find a few matching features in both images
  - compute alignment
- Direct (pixel-based) alignment
  - Search for alignment where most pixels agree

Direct Alignment

The simplest approach is a brute force search (hw1)

- Need to define image matching function
  - SSD, Normalized Correlation, edge matching, etc.
- Search over all parameters within a reasonable range:

e.g. for translation:

```
for tx=x0:step:x1,
  for ty=y0:step:y1,
     compare image1(x,y) to image2(x+tx,y+ty)
  end;
end;
```

Need to pick correct `x0,x1` and `step`

- What happens if `step` is too large?

Direct Alignment (brute force)

What if we want to search for more complicated transformation, e.g. homography?

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

```
for a=a0:astep:a1,
  for b=b0:bstep:b1,
    for c=c0:cstep:c1,
      for d=d0:dstep:d1,
        for e=e0:estep:e1,
          for f=f0:fstep:f1,
             for g=g0:gstep:g1,
                for h=h0:hstep:h1,
     compare image1 to H(image2)
end; end; end; end; end; end; end; end;
```

Problems with brute force

Not realistic

- Search in $O(N^8)$ is problematic
- Not clear how to set starting/stopping value and step

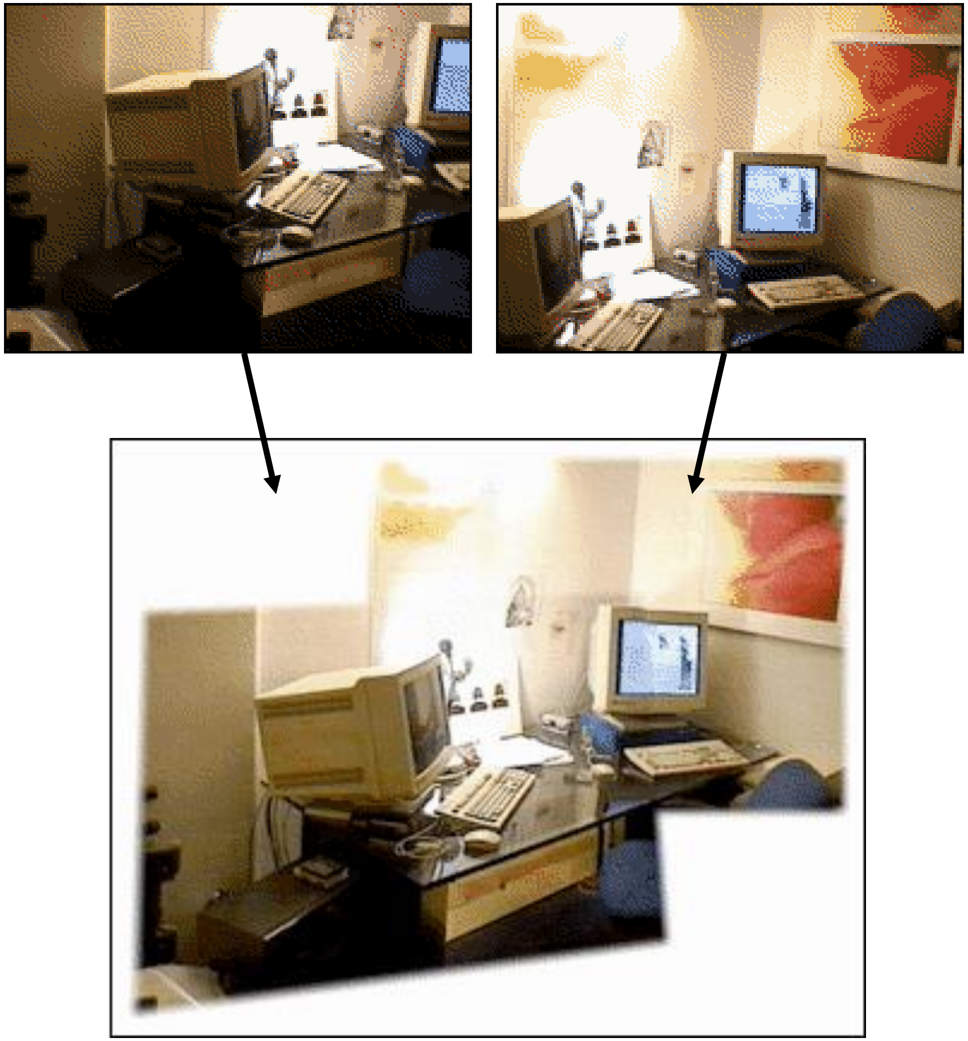
What can we do?

- Use pyramid search to limit starting/stopping/step values

Alternative: gradient decent on the error function

- i.e. how do I tweak my current estimate to make the SSD error go down?
- Can do sub-pixel accuracy
- BIG assumption?
  - Images are already almost aligned (<2 pixels difference!)
  - Can improve with pyramid
- Same tool as in **motion estimation**

Image alignment



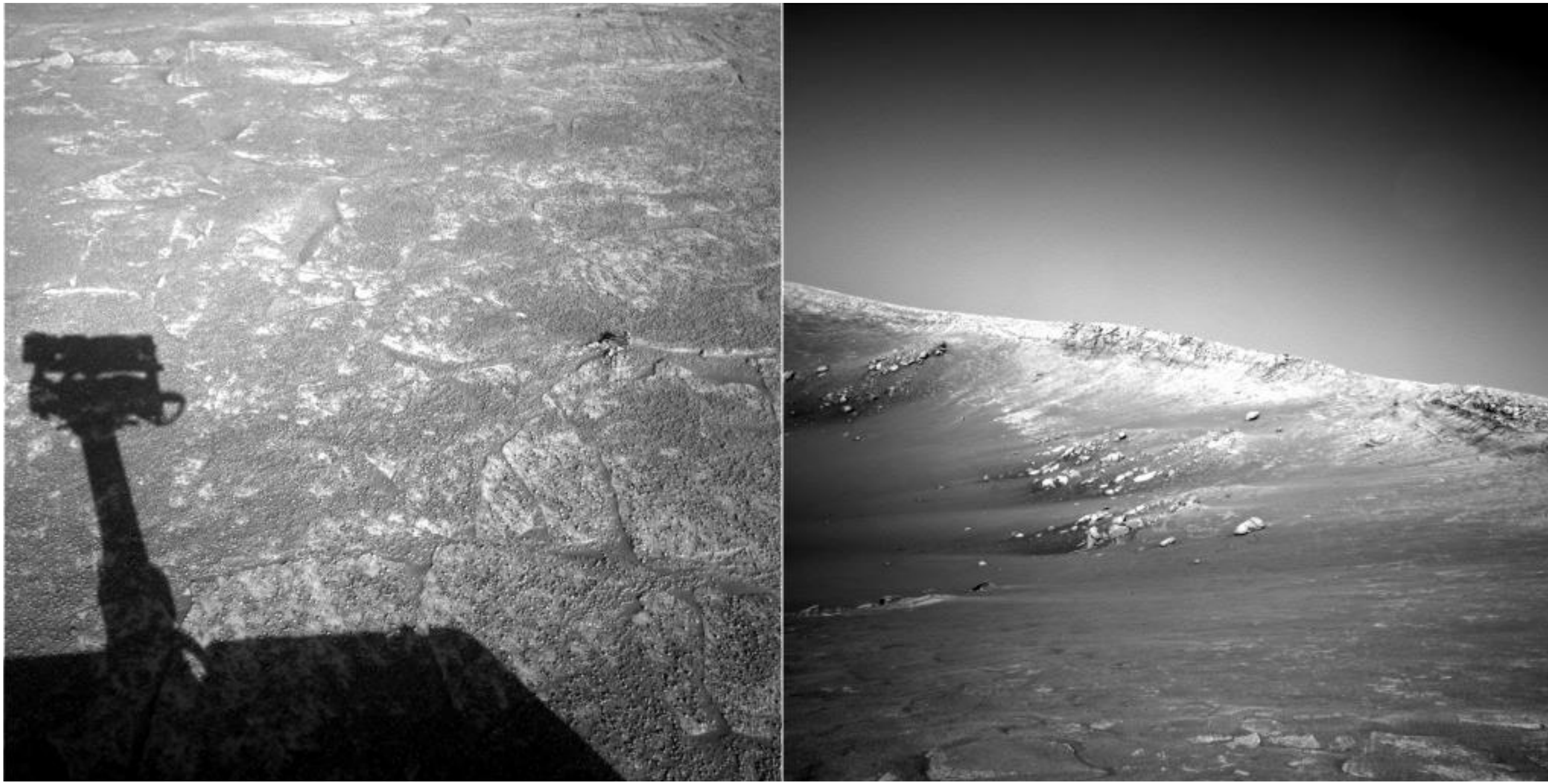
Feature-based alignment

1. **Feature Detection**: find a few important features (aka Interest Points) in each image separately
2. **Feature Matching**: match them across two images
3. **Compute image transformation**: as per Project 5, Part I

How do we choose good features automatically?

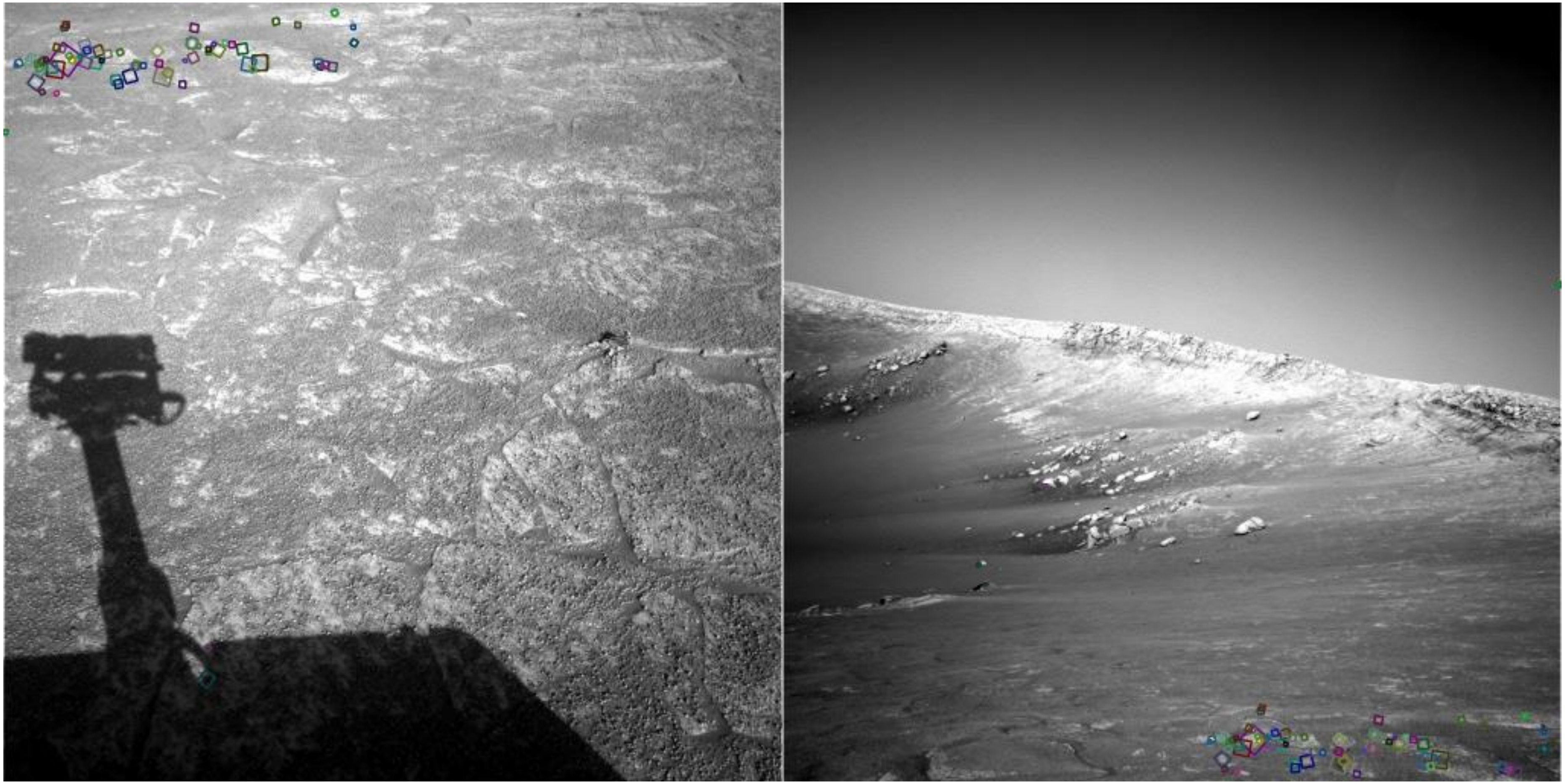
- They must be prominent in both images
- Easy to localize
- Think how you did that by hand in Project #6 Part I
- Corners!

A hard feature matching problem



NASA Mars Rover images

Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Feature Detection

Feature Matching

How do we match the features between the images?

- Need a way to <u>describe</u> a region around each feature
  - e.g. image patch around each feature
- Use successful matches to estimate homography
  - Need to do something to get rid of outliers

Issues:

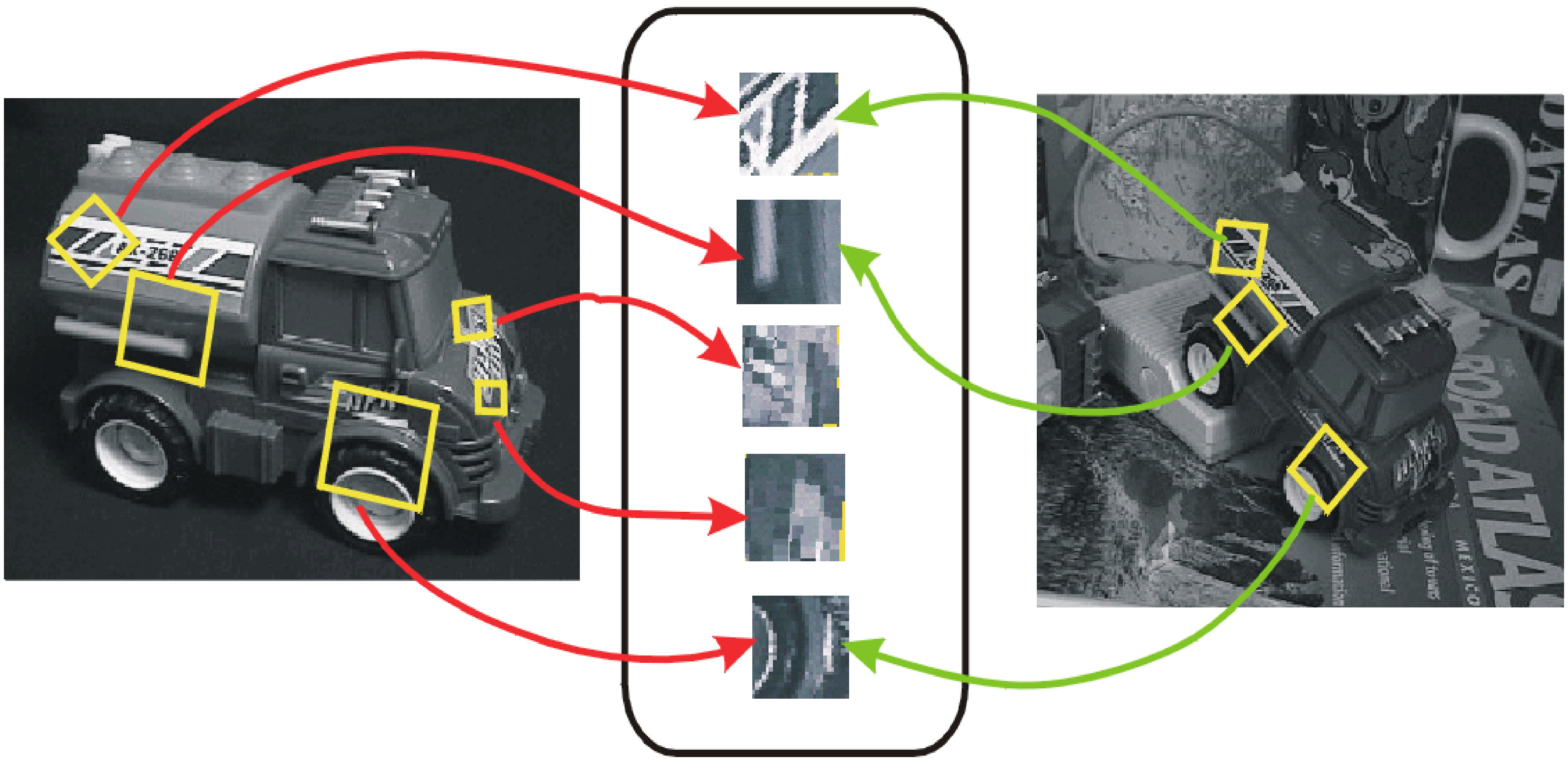
- What if the image patches for several interest points look similar?
  - Make patch size bigger
- What if the image patches for the same feature look different due to scale, rotation, etc.
  - Need an invariant descriptor

Invariant Feature Descriptors

Schmid & Mohr 1997, Lowe 1999, Baumberg 2000, Tuytelaars & Van Gool 2000, Mikolajczyk & Schmid 2001, Brown & Lowe 2002, Matas et. al. 2002, Schaffalitzky & Zisserman 2002

Invariant Local Features

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



Features Descriptors

Applications

Feature points are used for:

- Image alignment (homography, fundamental matrix)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval
- Robot navigation
- … other

Today's lecture

- 1 Feature <u>detector</u>
  - scale invariant Harris corners
- 1 Feature <u>descriptor</u>
  - patches, oriented patches

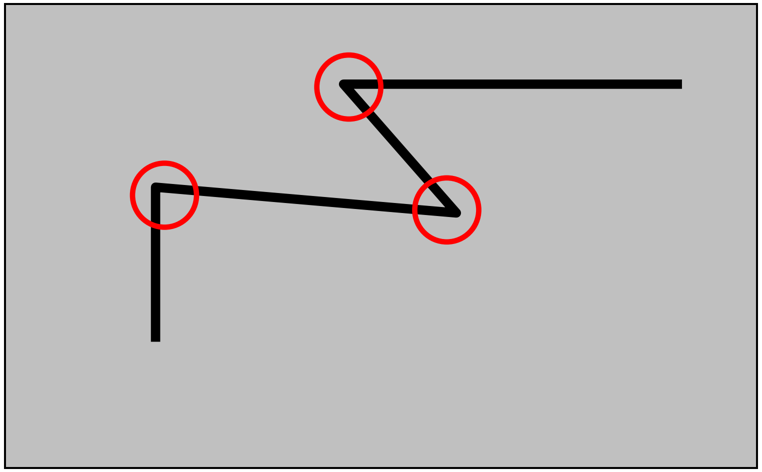
Reading:

Multi-image Matching using Multi-scale image patches, CVPR 2005

Feature Detector – Harris Corner

Harris corner detector

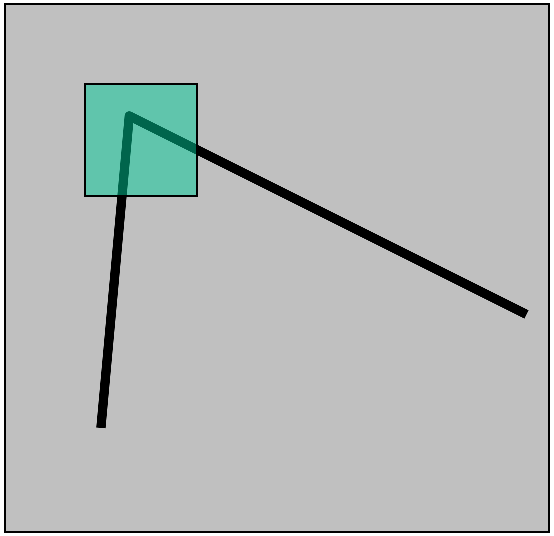
C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988



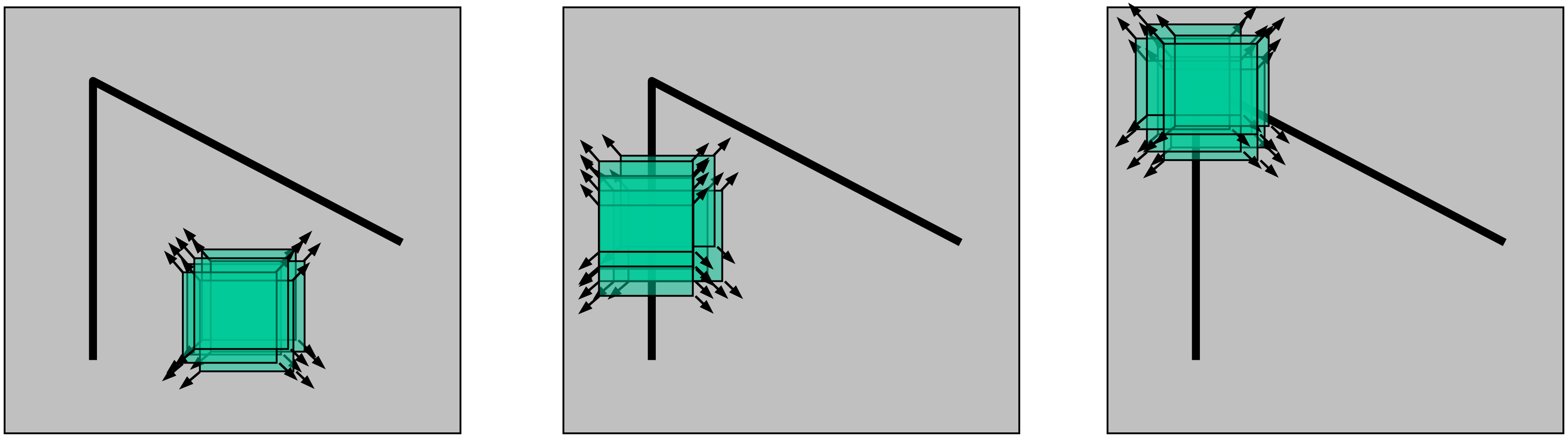
The Basic Idea

We should easily recognize the point by looking through a small window

Shifting a window in *any direction* should give *a large change* in intensity



Harris Detector: Basic Idea



"flat" region:
no change in
all directions

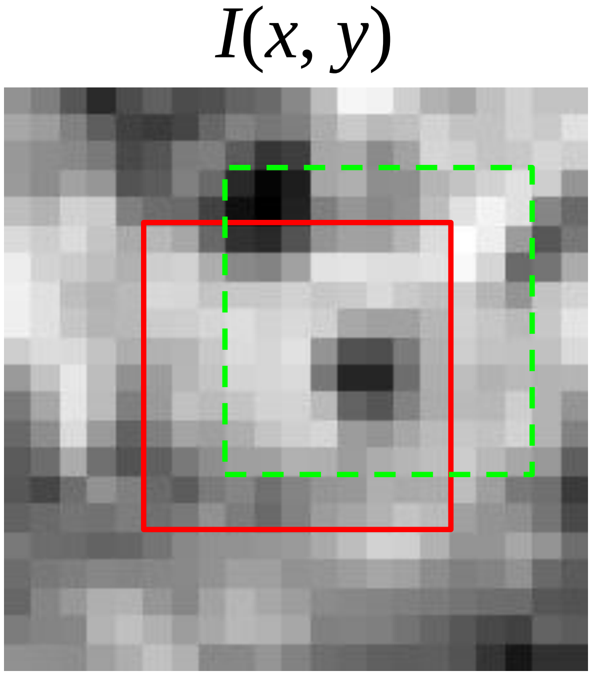
"edge":
no change along
the edge direction

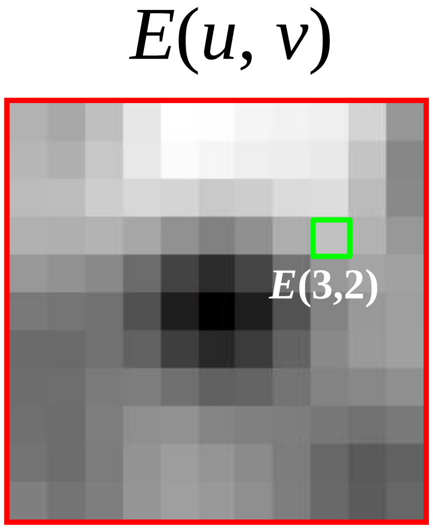
"corner":
significant change
in all directions

Corner Detection: Mathematics

Change in appearance of window W for the shift $[u,v]$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$



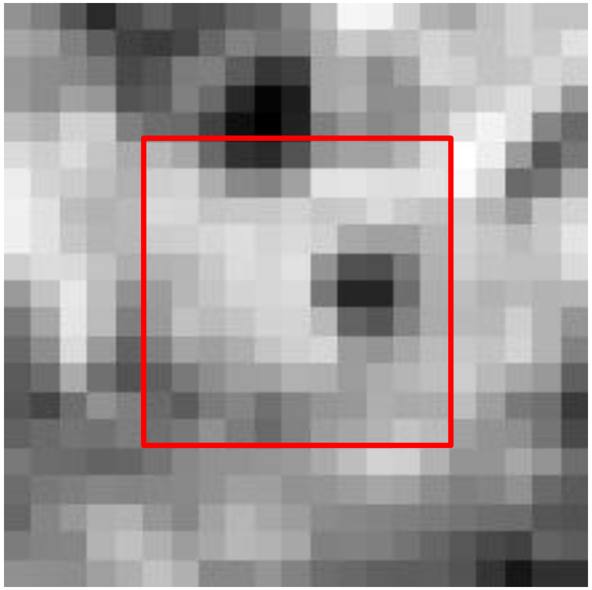


Corner Detection: Mathematics

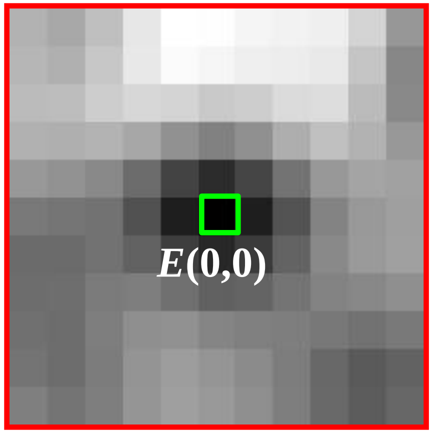
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$



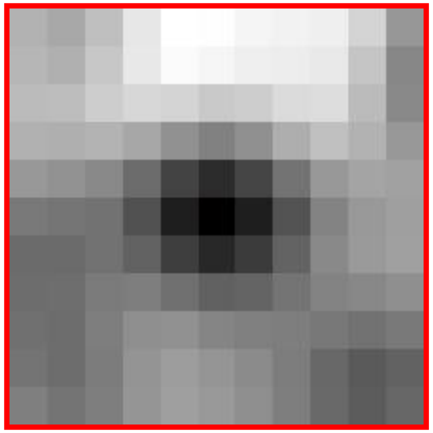
Corner Detection: Mathematics

Change in appearance of window W for the shift $[u,v]$:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$\begin{aligned} I(x+u, y+v) &= I(x, y) + I_x u + I_y v + \text{higher order terms} \\ &\approx I(x, y) + I_x u + I_y v \\ &= I(x, y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

- Let's plug this into

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

Corner Detection: Mathematics

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$$\approx \sum_{(x,y)\in W} [I(x,y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - I(x,y)]^2$$

$$= \sum_{(x,y)\in W} \left(\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)^2$$

$$= \sum_{(x,y)\in W} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

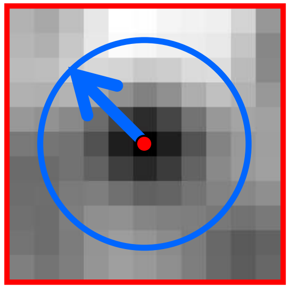
where *M* is a *second moment matrix* computed from image derivatives:

$$M = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Interpreting the second moment matrix

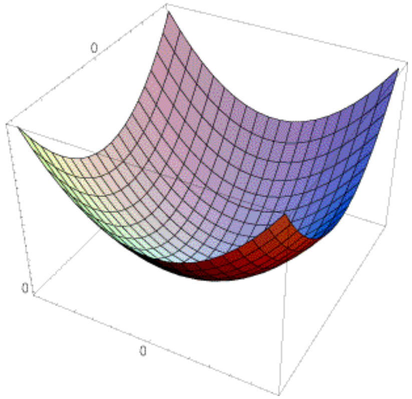
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$E(u, v)$



$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

First, consider the axis-aligned case (gradients are either horizontal or vertical)

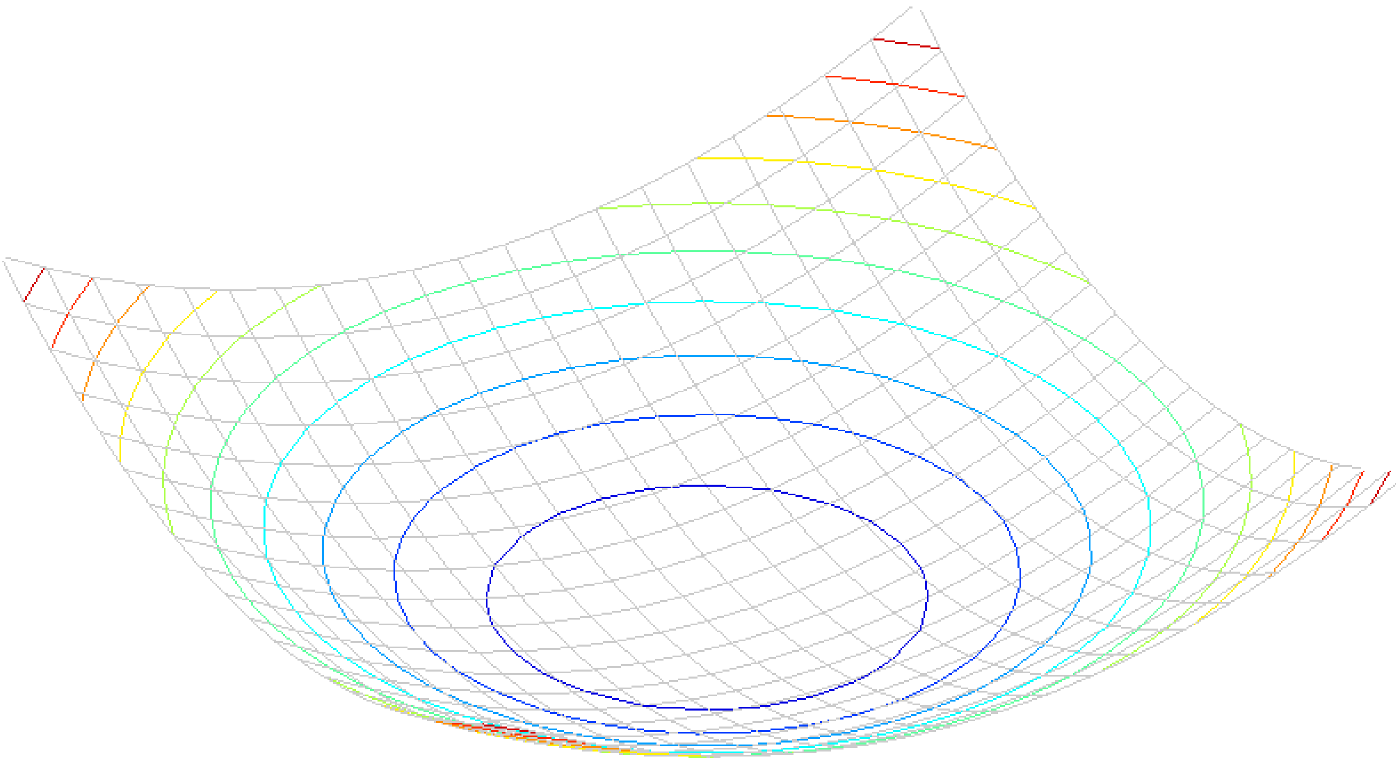
$$M = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

Interpreting the second moment matrix

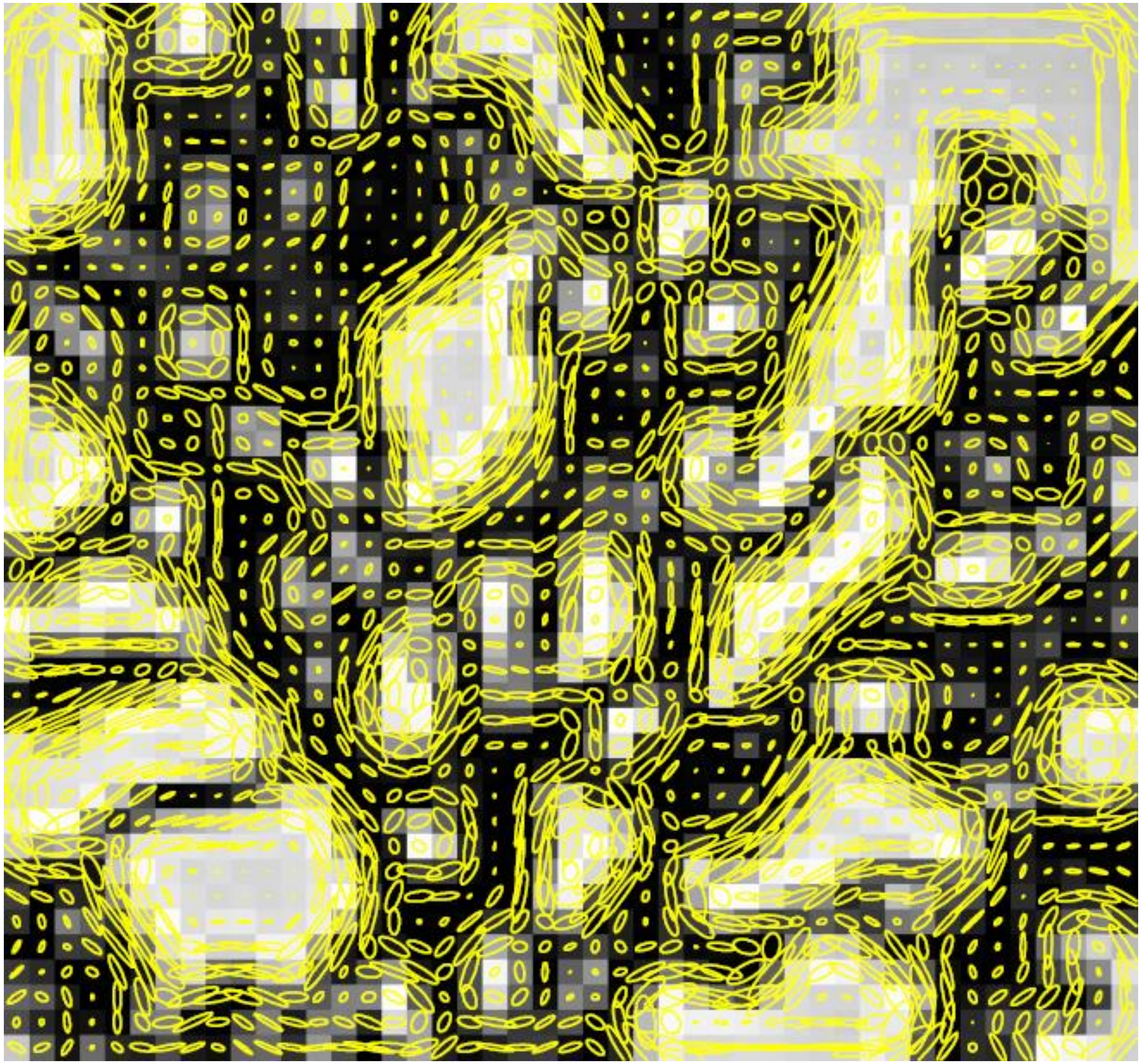
Consider a horizontal "slice" of $E(u, v)$: $$[u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

This is the equation of an ellipse.



Visualization of second moment matrices

Visualization of second moment matrices



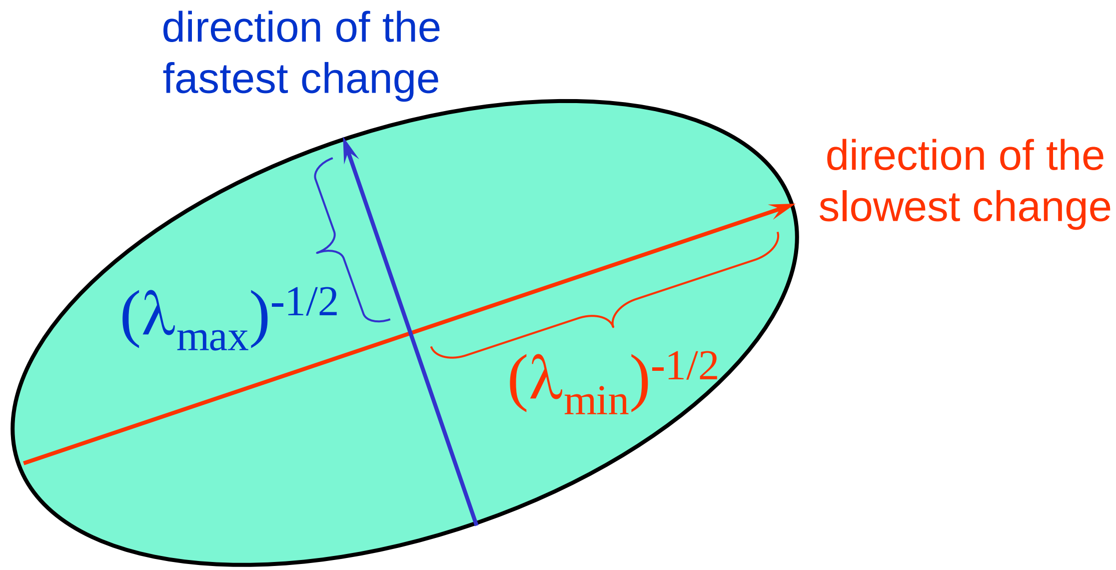
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\quad [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

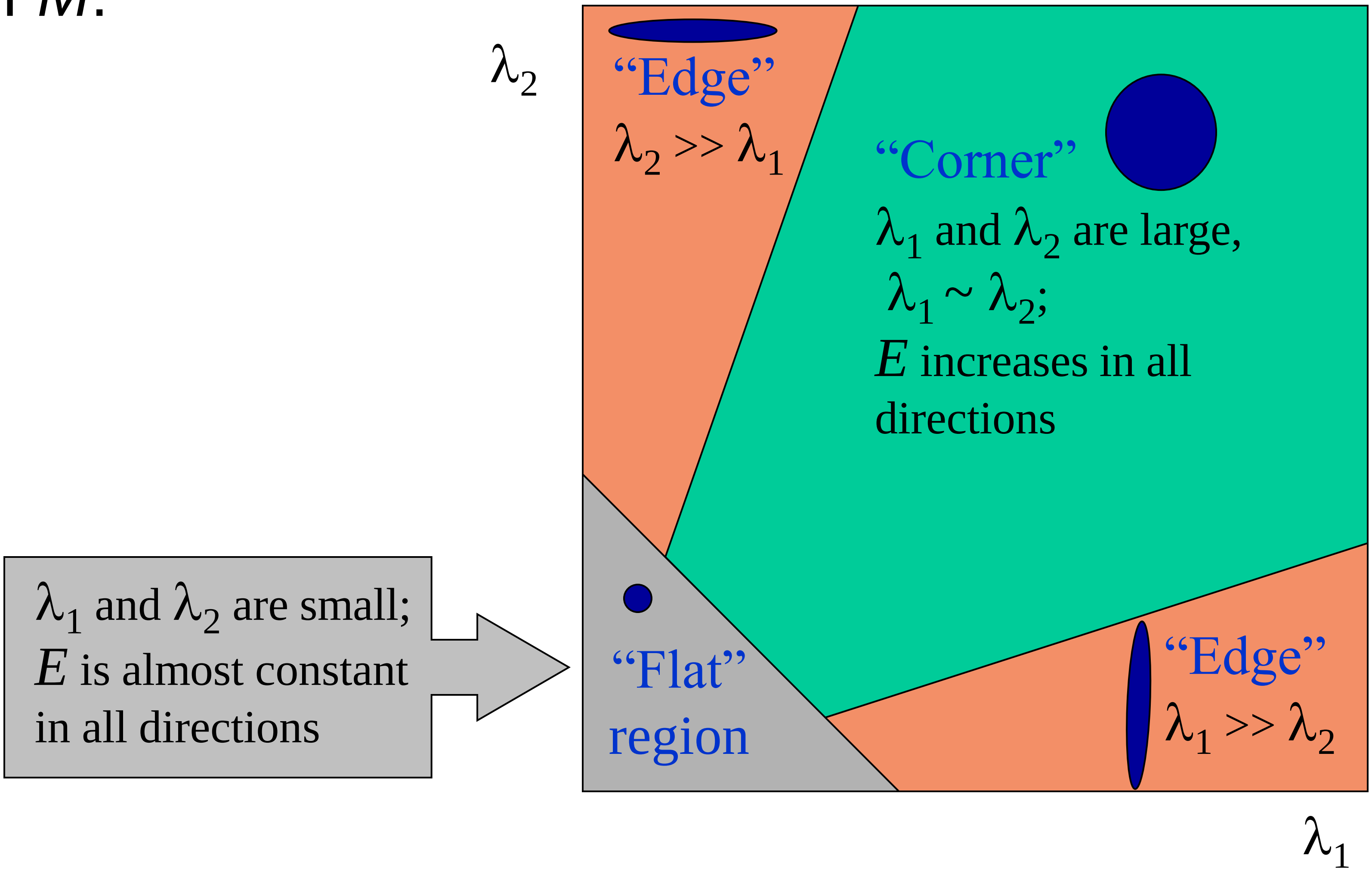
Diagonalization of M: $\quad M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:



Harris Detector: Mathematics

Measure of corner response:

$$R = \frac{\det M}{\operatorname{Trace} M}$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

Harris detector: Steps

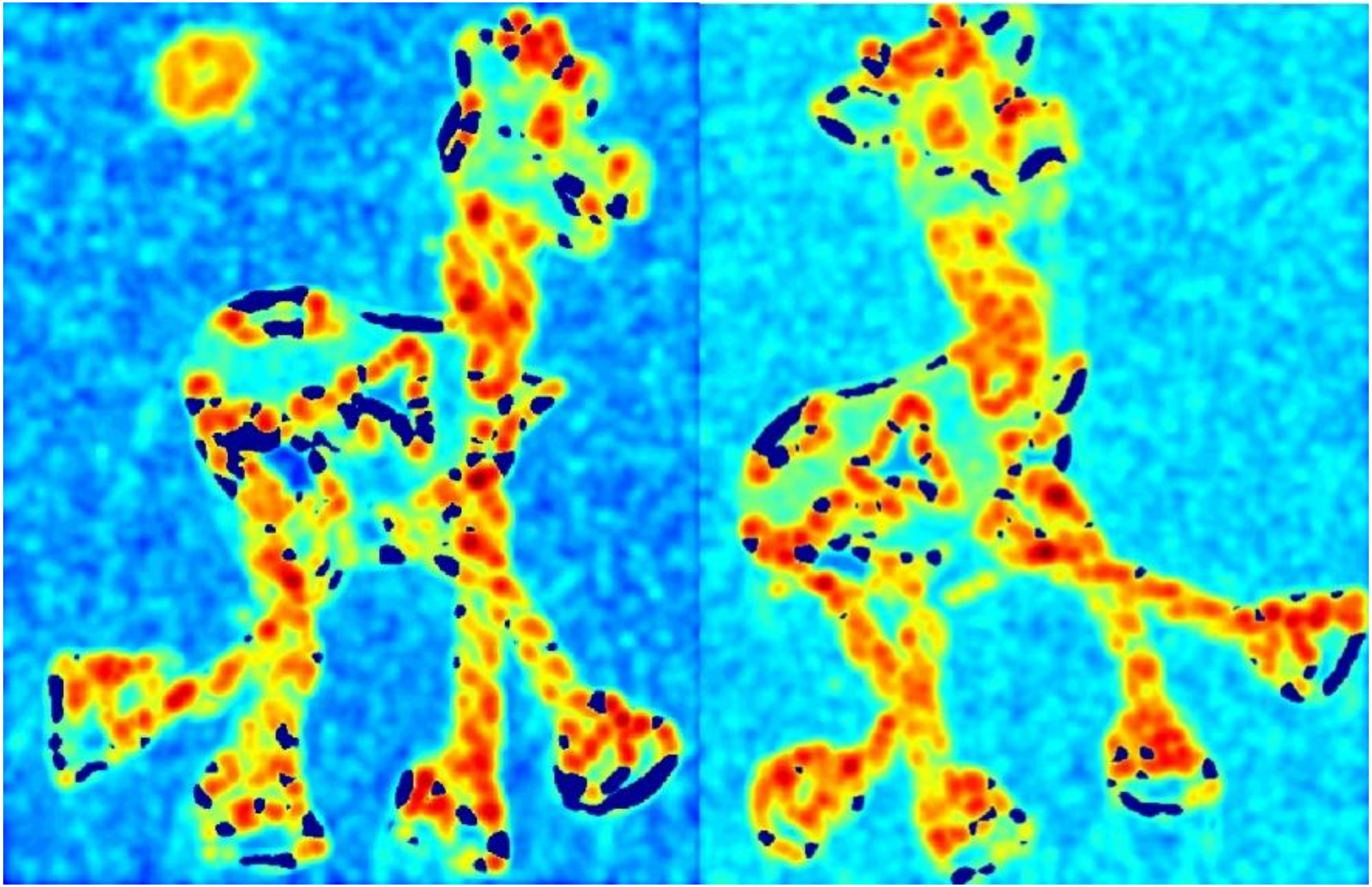
1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. [“A Combined Corner and Edge Detector.”] ***Proceedings of the 4th Alvey Vision Conference*****: pages 147—151, 1988.**

Harris Detector: Workflow

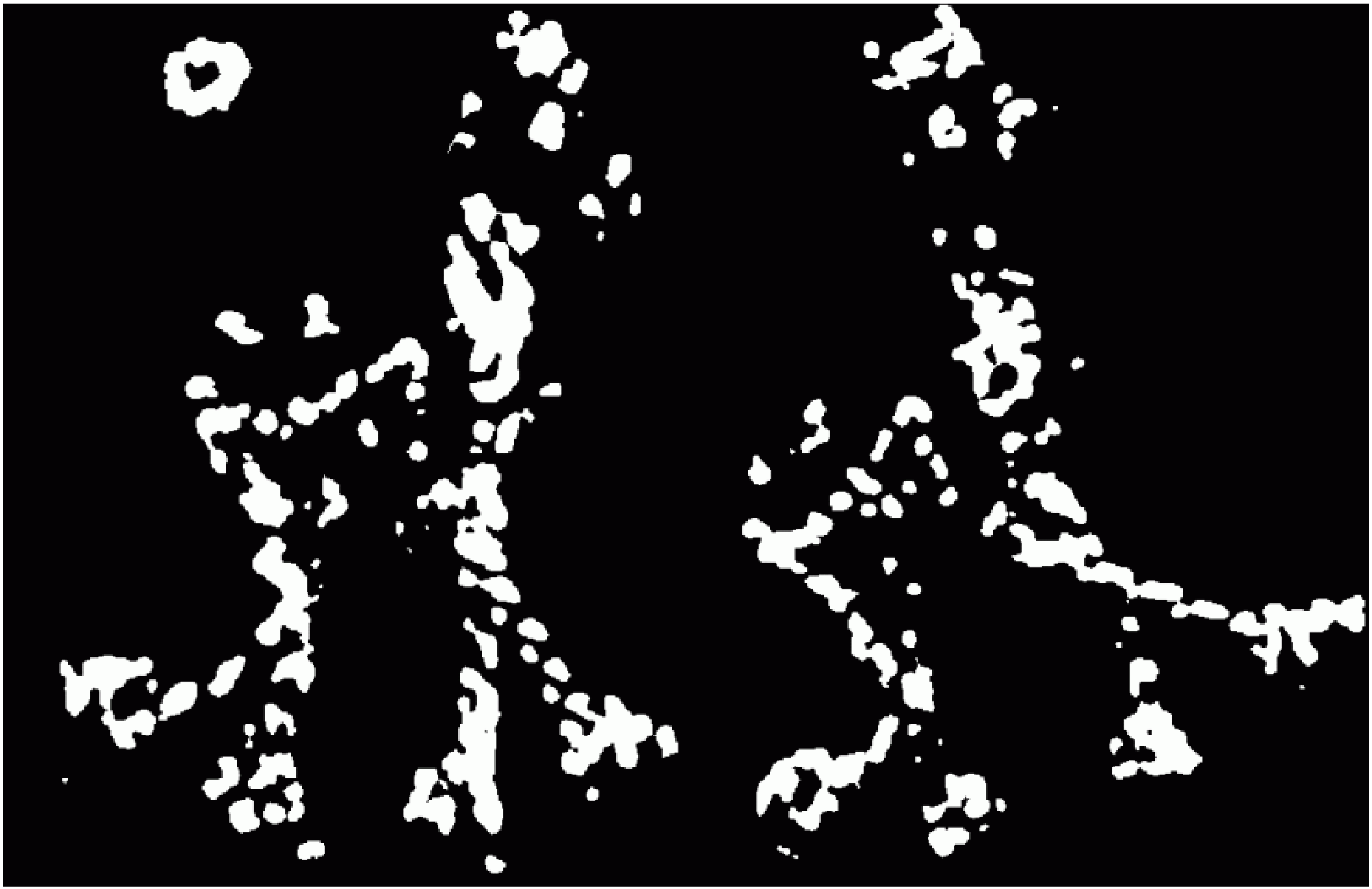
Harris Detector: Workflow

Compute corner response R



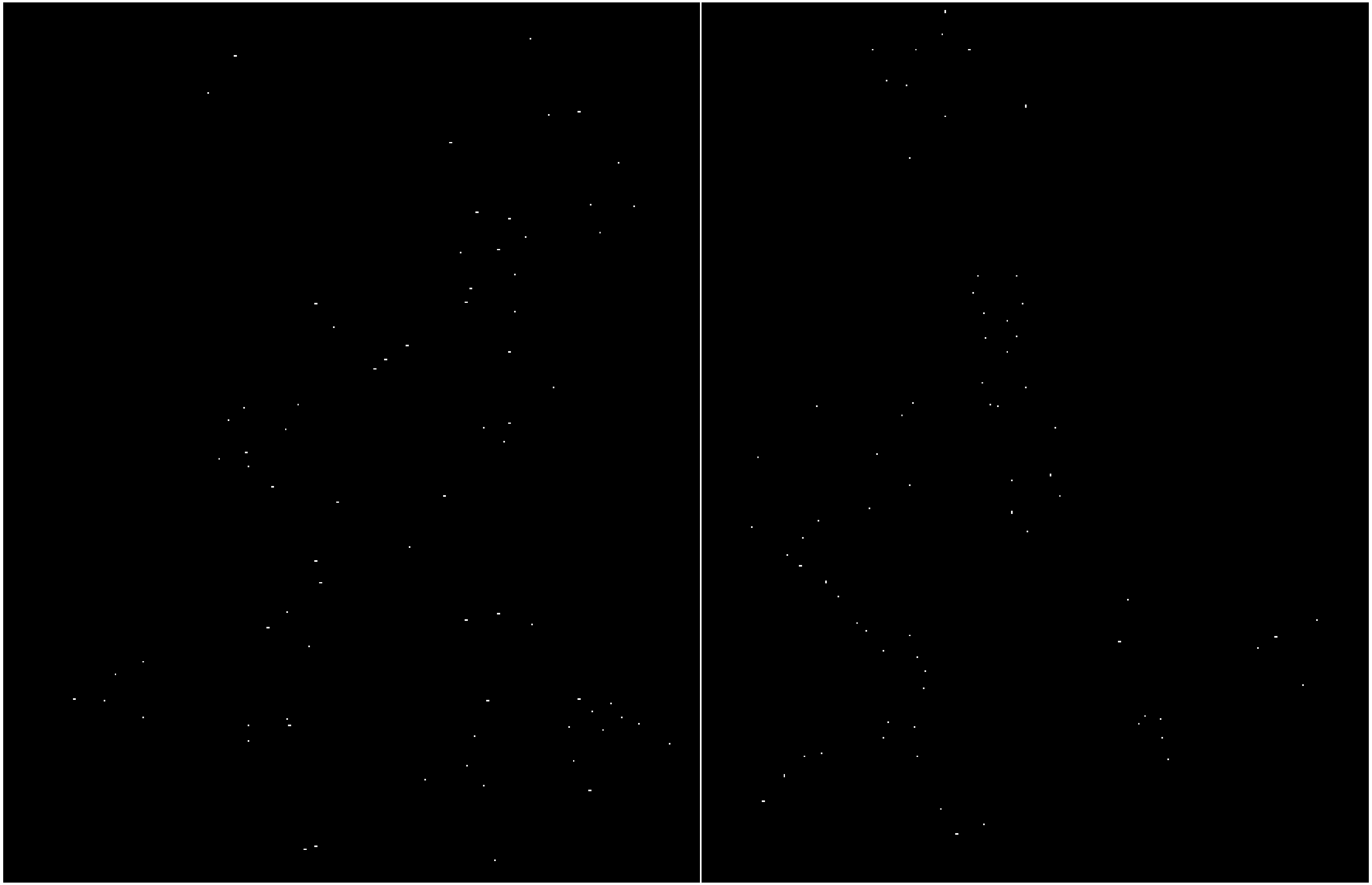
Harris Detector: Workflow

Find points with large corner response: R>threshold



Harris Detector: Workflow

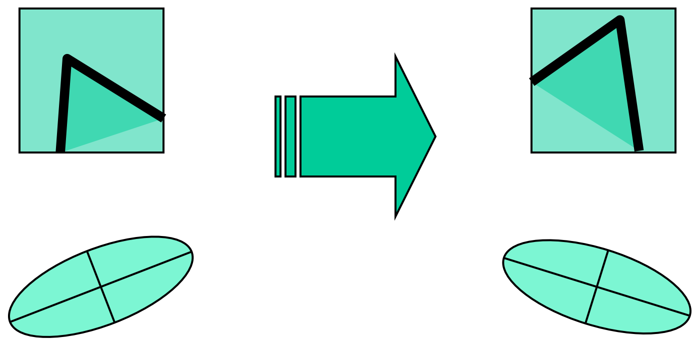
Take only the points of local maxima of R



Harris Detector: Workflow

Harris Detector: Some Properties

Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

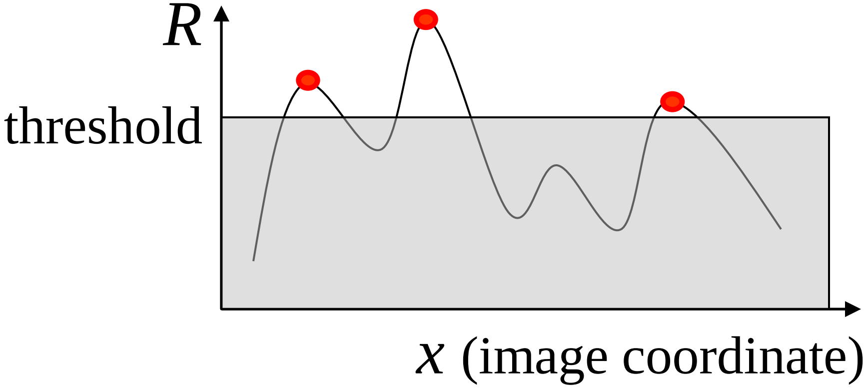
Corner response R is invariant to image rotation

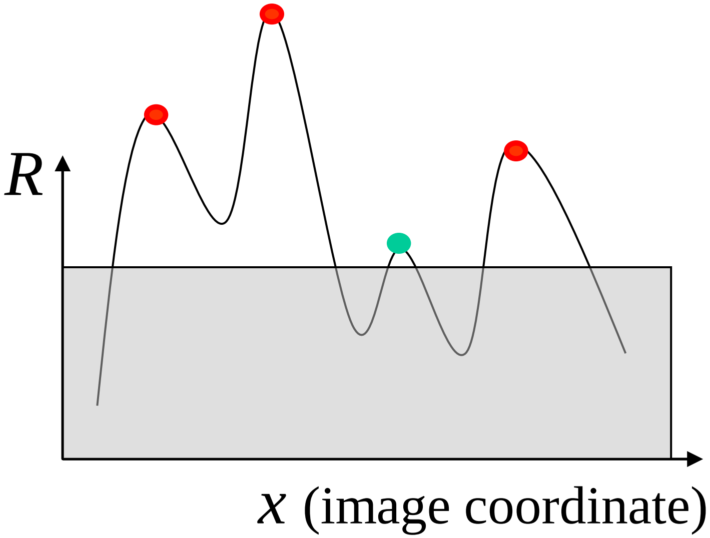
Harris Detector: Some Properties

Partial invariance to *affine intensity* change

✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

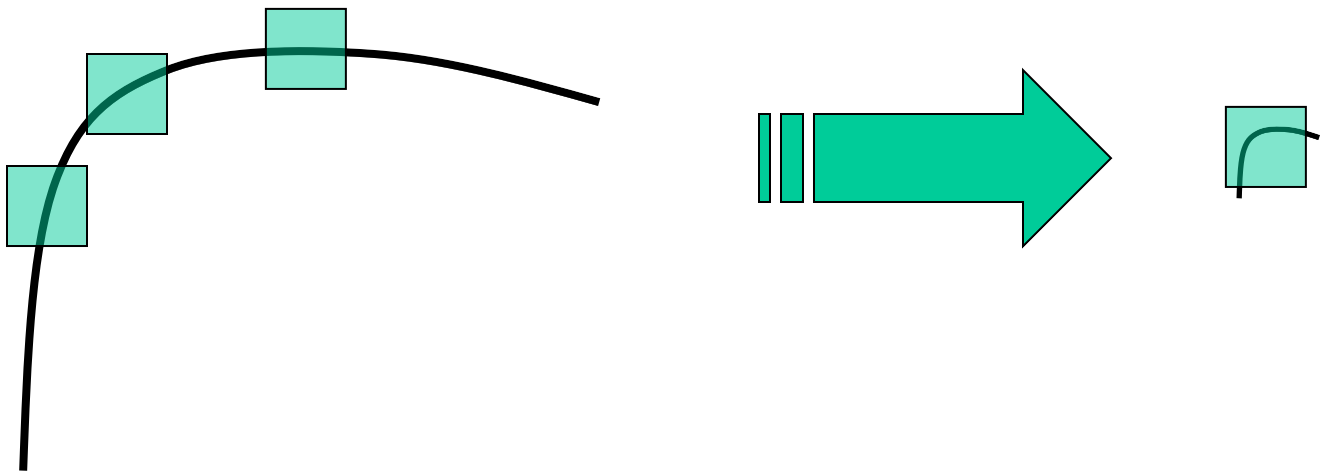
✓ Intensity scale: $I \rightarrow a\, I$





Harris Detector: Some Properties

But: non-invariant to *image scale*!

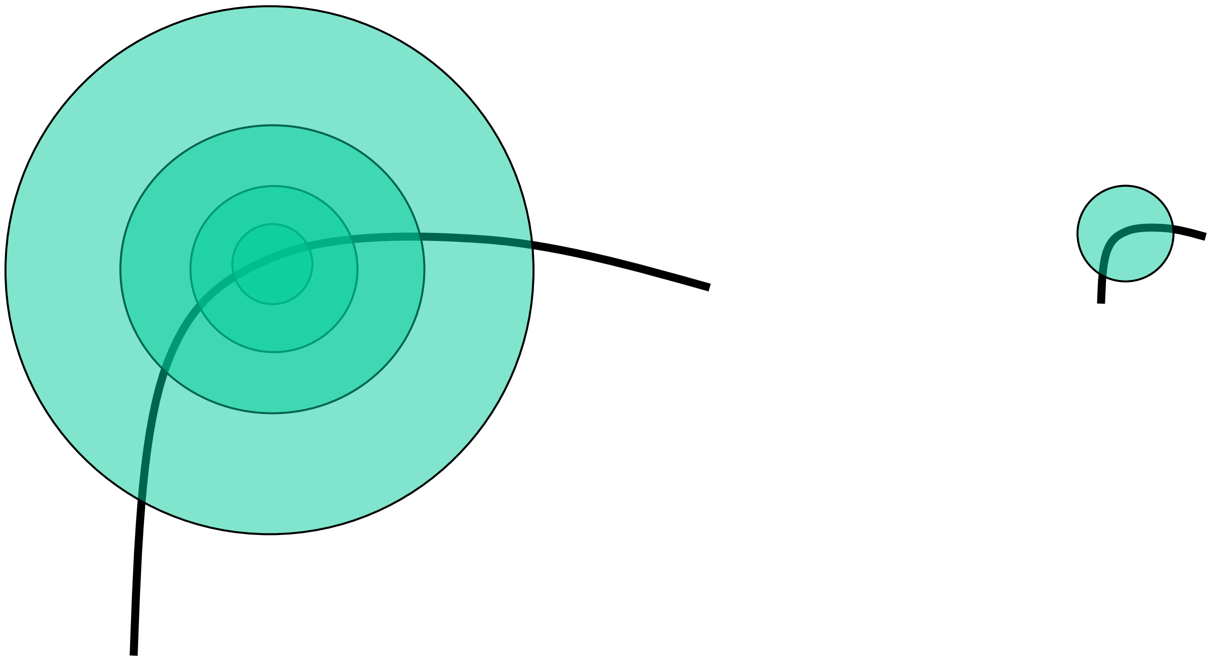


All points will be
classified as edges

Corner !

Scale Invariant Detection

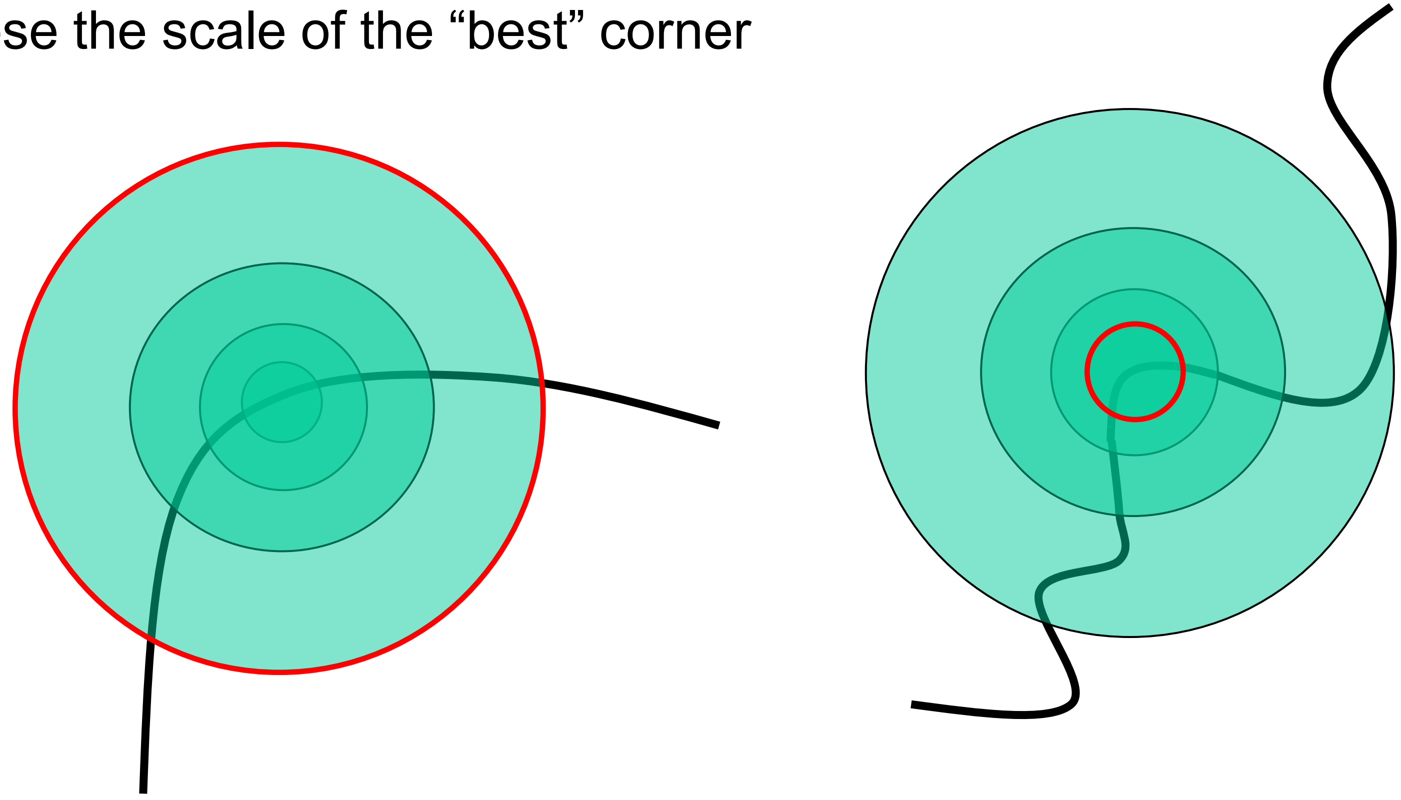
Consider regions (e.g. circles) of different sizes around a point
Regions of corresponding sizes will look the same in both images



Scale Invariant Detection

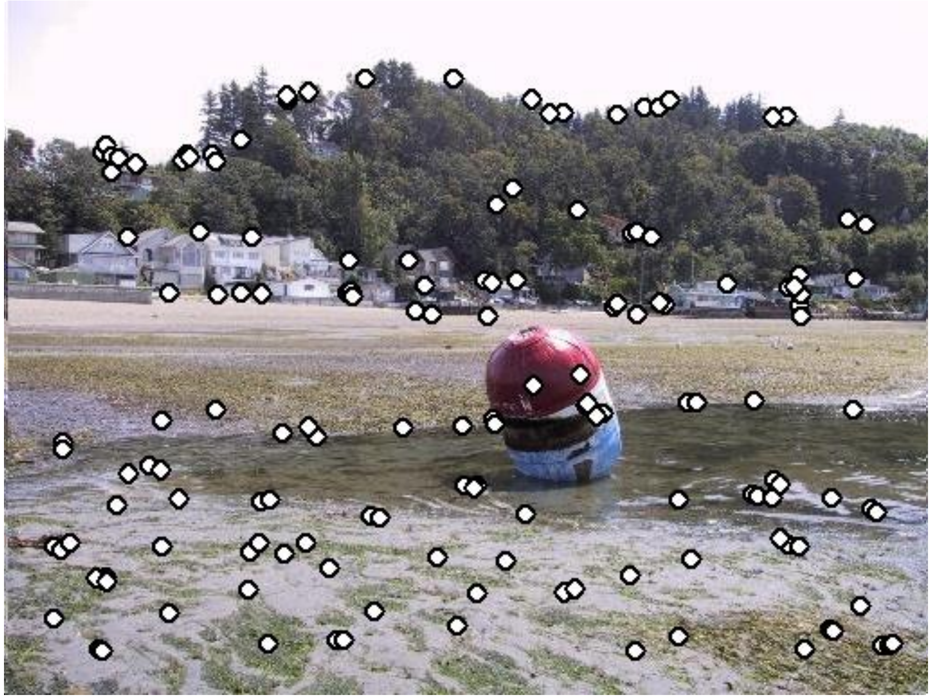
The problem: how do we choose corresponding circles *independently* in each image?

Choose the scale of the "best" corner



Feature selection

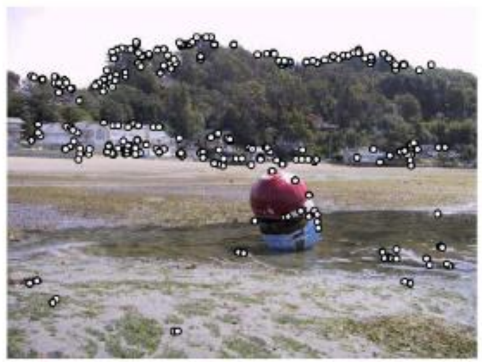
Distribute points evenly over the image



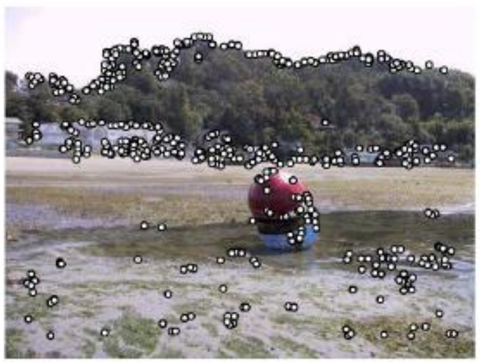
Adaptive Non-maximal Suppression

Desired: Fixed # of features per image

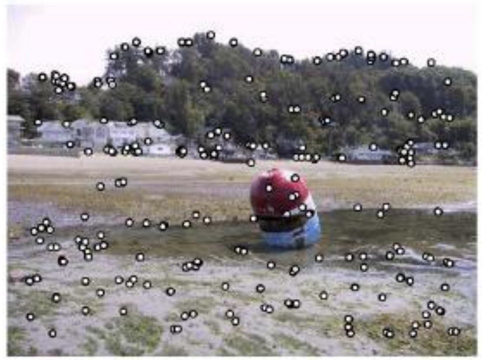
- Want evenly distributed spatially…
- Sort points by non-maximal suppression radius
  [Brown, Szeliski, Winder, CVPR'05]



(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, $r = 24$

(d) ANMS 500, $r = 16$