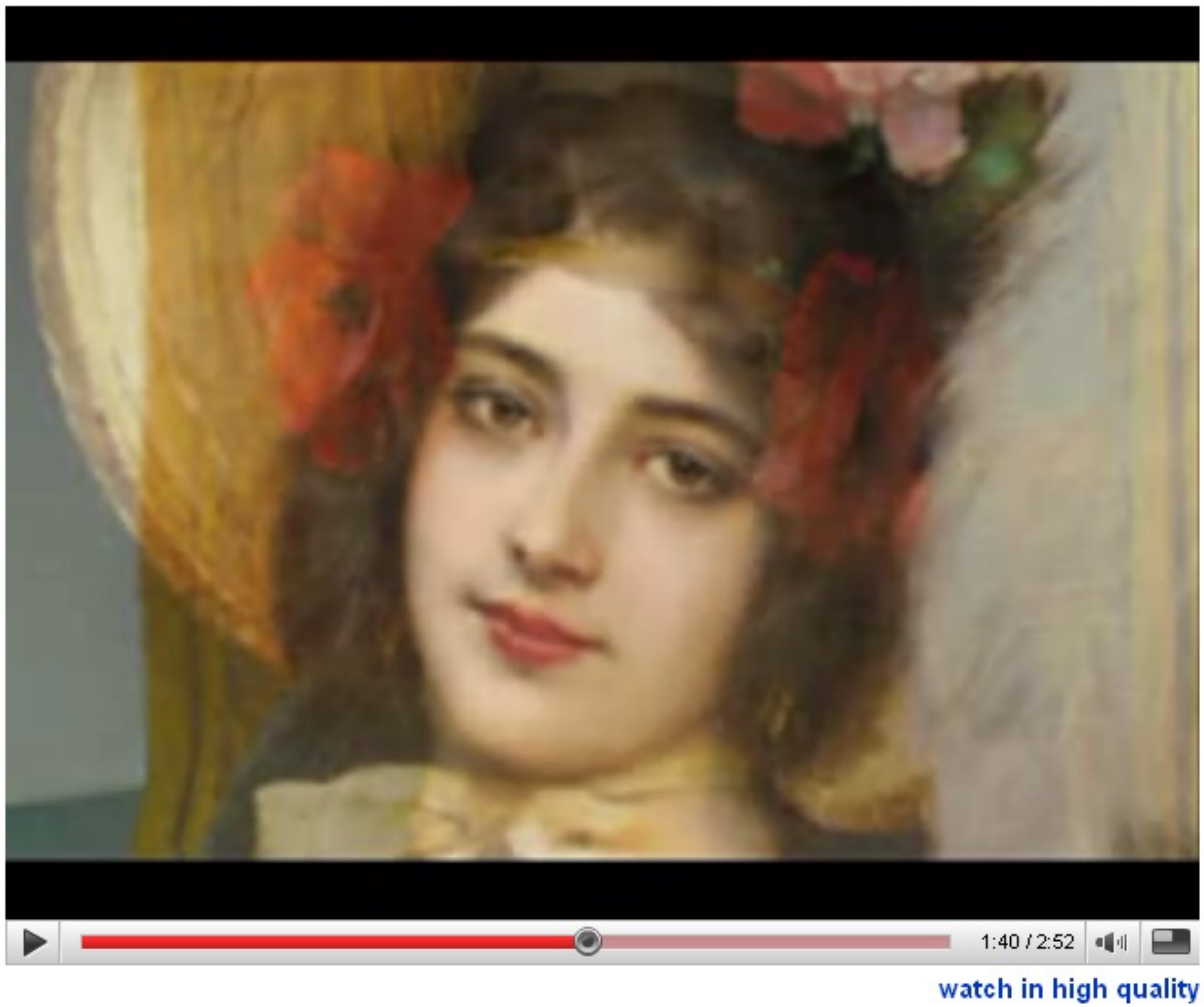
Amuse-bouche



http://youtube.com/watch?v=nUDIoN-_Hxs

http://www.youtube.com/watch?v=L0GKp-uvjO0

Image Warping and Morphing

CS194: Intro to Computer Vision and Comp. Photo

Angjoo Kanazawa & Alexei Efros, UC Berkeley, Fall 2022

Project 3 out today!!

project 2 how did it go?

project 3 is harder!

Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

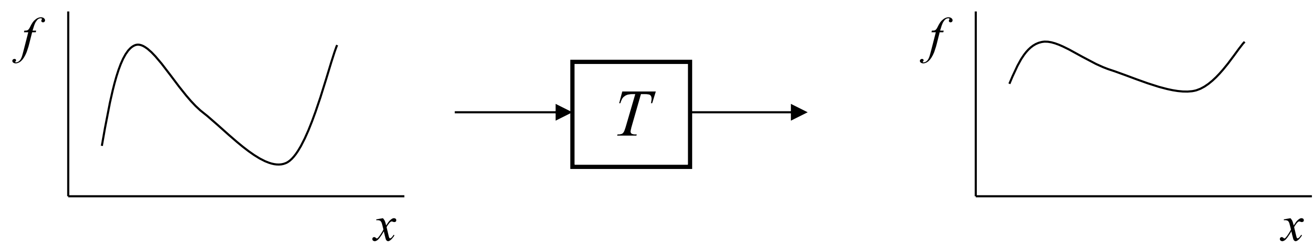


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$

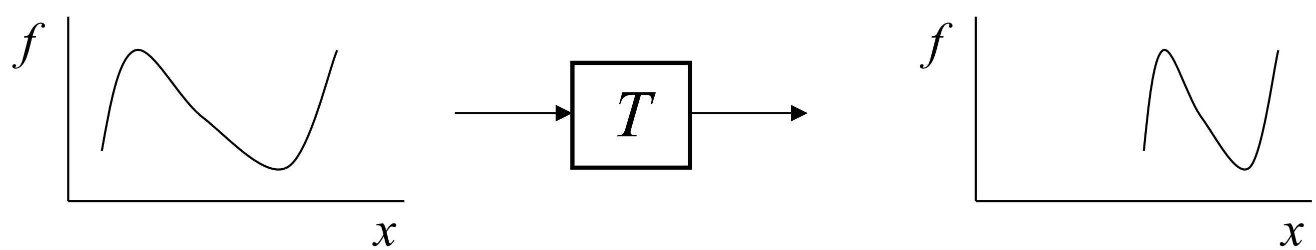


Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

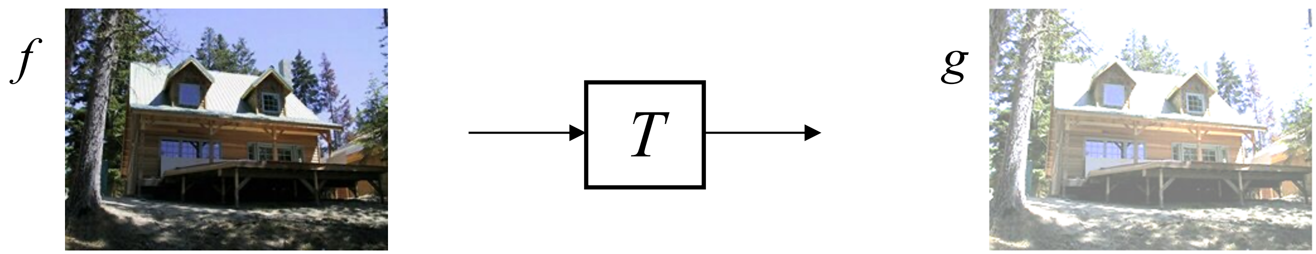
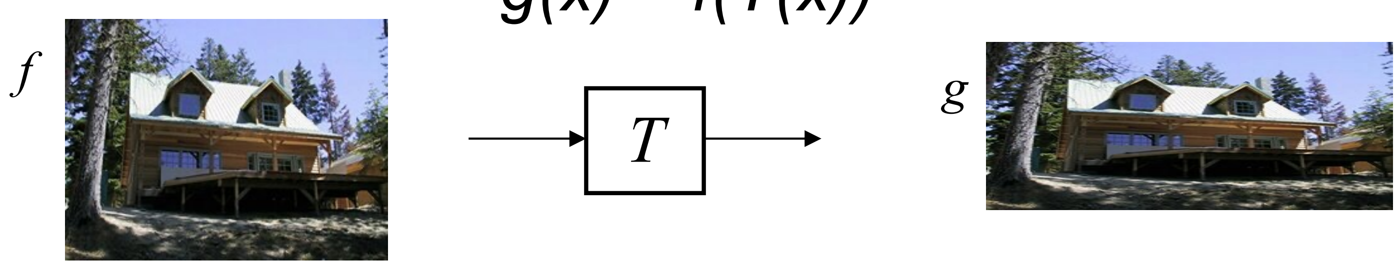


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$



All 2D Linear Transformations

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

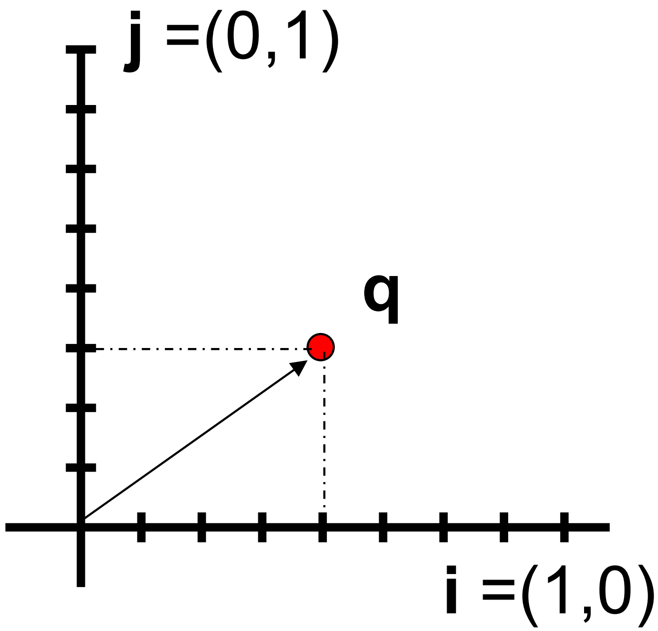
Properties of linear transformations:

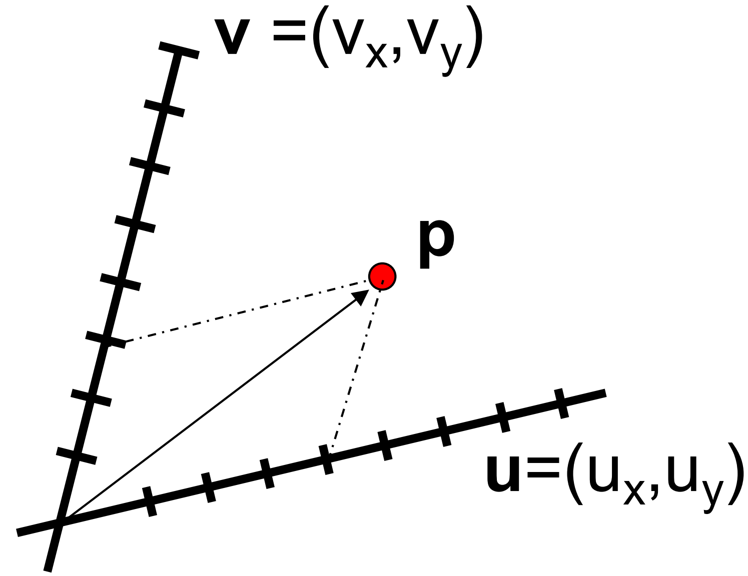
- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Consider a different Basis

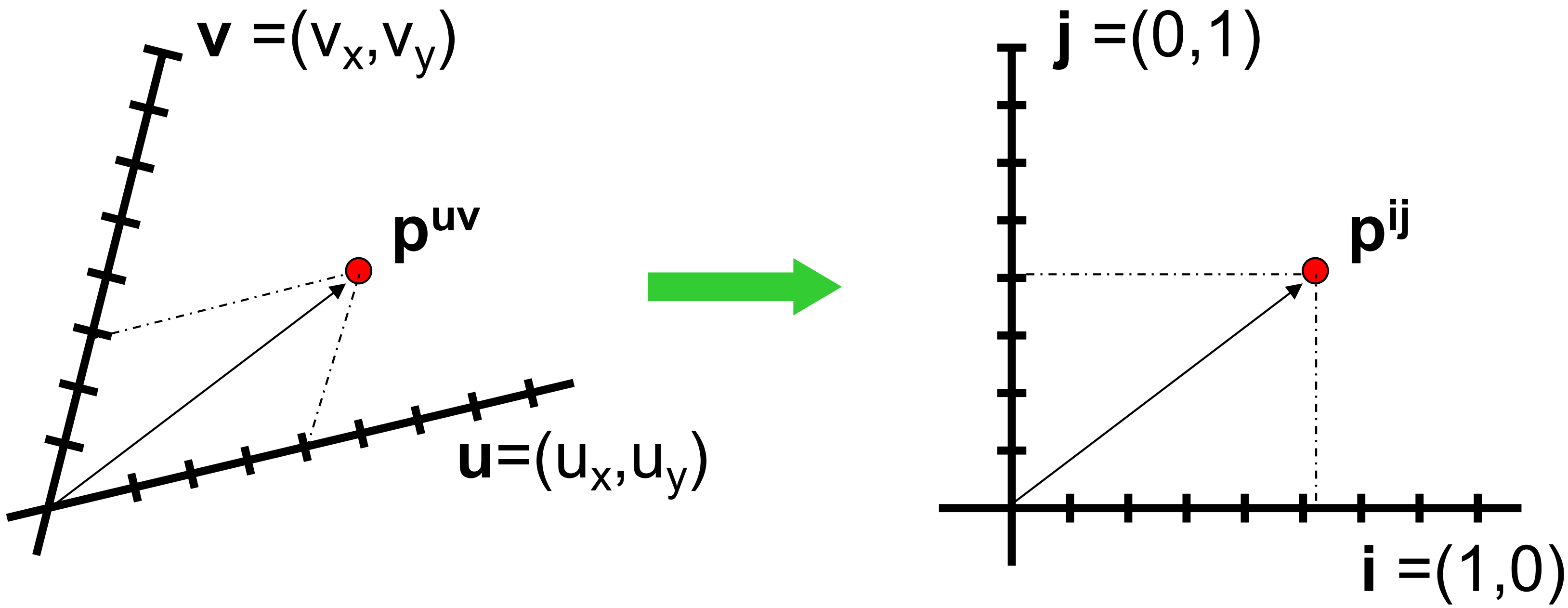




$\mathbf{q} = 4\mathbf{i} + 3\mathbf{j} = (4,3)$

$\mathbf{p} = 4\mathbf{u} + 3\mathbf{v}$

Linear Transformations as Change of Basis



$\mathbf{p}^{uv} = (4,3)$

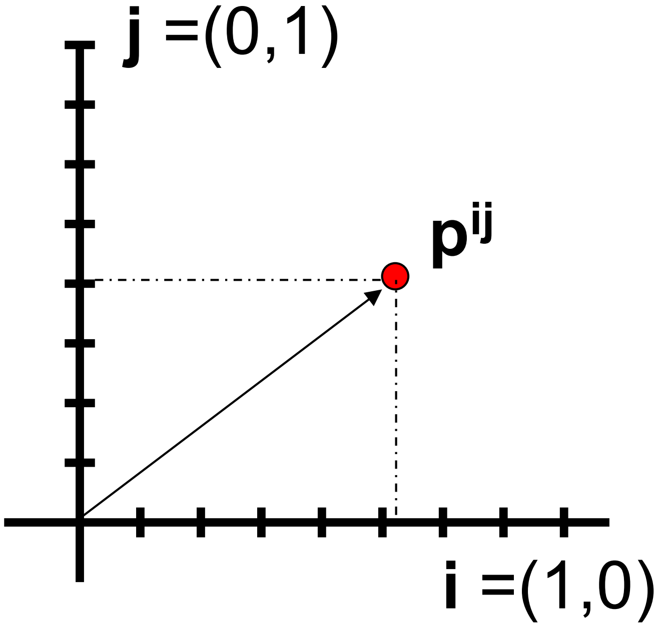
$\mathbf{p}^{ij} = ?\mathbf{i} + ?\mathbf{j}$

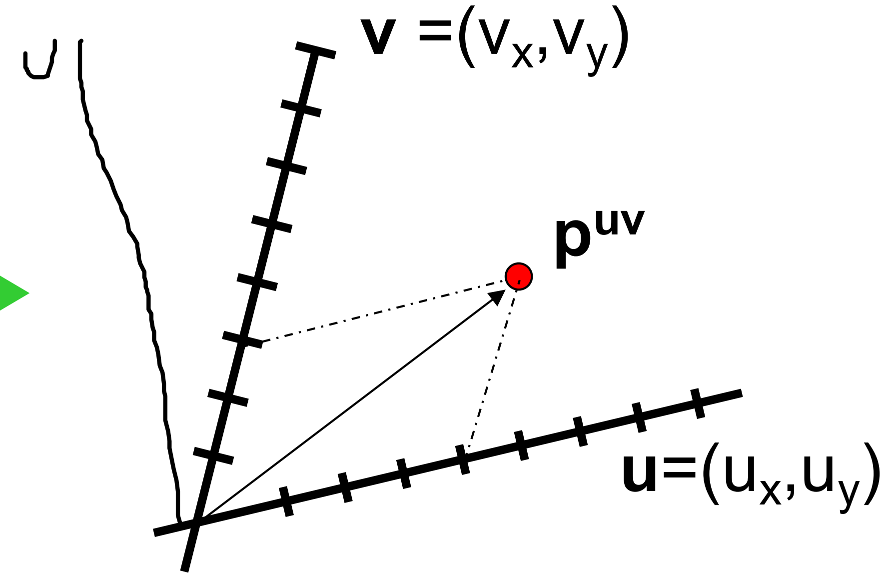
$p_x = 4u_x + 3v_x$
$p_y = 4u_y + 3v_y$

$$\mathbf{p}^{ij} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix} \mathbf{p}^{uv}$$

Any linear transformation is a basis!!!

What's the inverse transform?





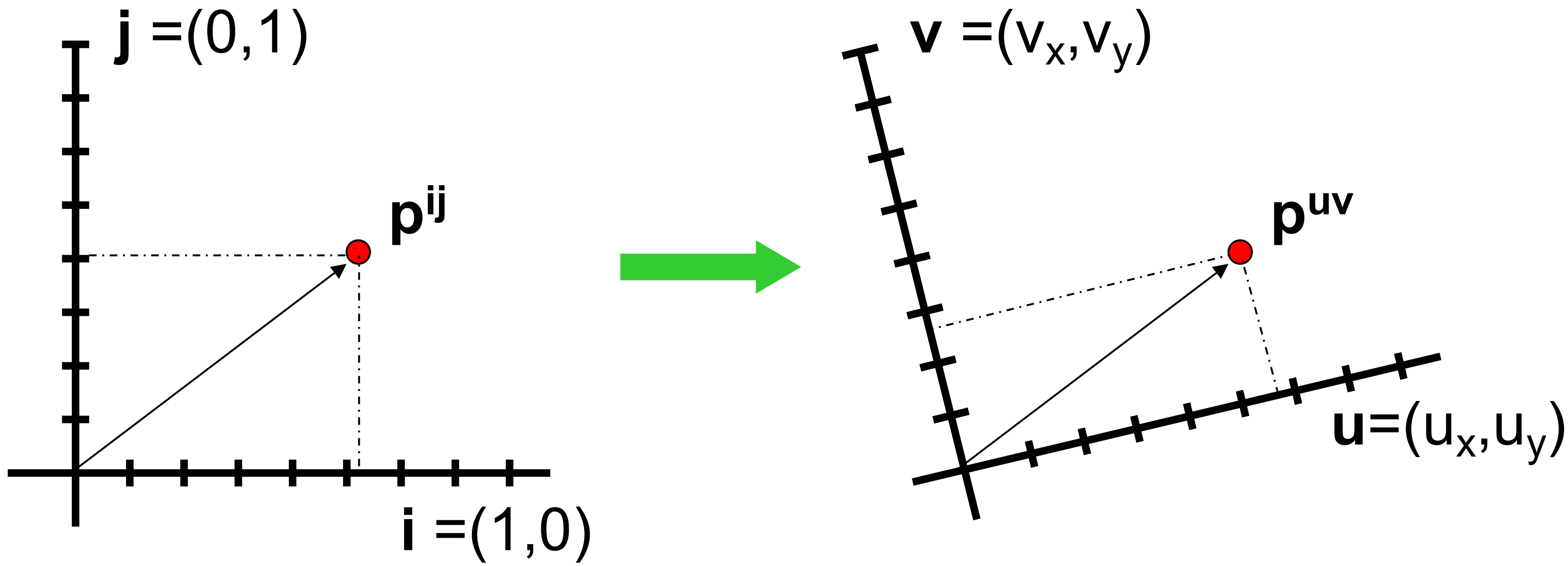
$\mathbf{p}^{\mathbf{ij}} = (5,4) = p_x\mathbf{u} + p_y\mathbf{v}$

$\mathbf{p}^{\mathbf{uv}} = (p_x, p_y) = ?$

$$\mathbf{p}^{\mathbf{uv}} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix}^{-1} \mathbf{p}^{\mathbf{ij}}$$

- How can we change from any basis to any basis?
- What if the basis are orthogonal?

Projection onto orthogonal basis



$\mathbf{p}^{\mathbf{ij}} = (5,4)$

$\mathbf{p}^{\mathbf{uv}} = (\mathbf{u}\cdot\mathbf{p}^{\mathbf{ij}}, \mathbf{v}\cdot\mathbf{p}^{\mathbf{ij}})$

$$\mathbf{p}^{\mathbf{uv}} = \begin{bmatrix} u_x & u_x \\ v_y & v_y \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \mathbf{p}^{\mathbf{ij}}$$

Homogeneous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

Homogeneous Coordinates

Homogeneous coordinates

- represent coordinates in 2 dimensions with a 3-vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

A: Using the rightmost column:

$$\mathbf{T}ranslation = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Composition

Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\mathbf{p}' \quad = \quad T(t_x,t_y) \qquad R(\Theta) \qquad S(s_x,s_y) \qquad \mathbf{p}$$

Does the order of multiplication matter?

Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Models change of basis

Will the last coordinate *w* always be 1?

Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

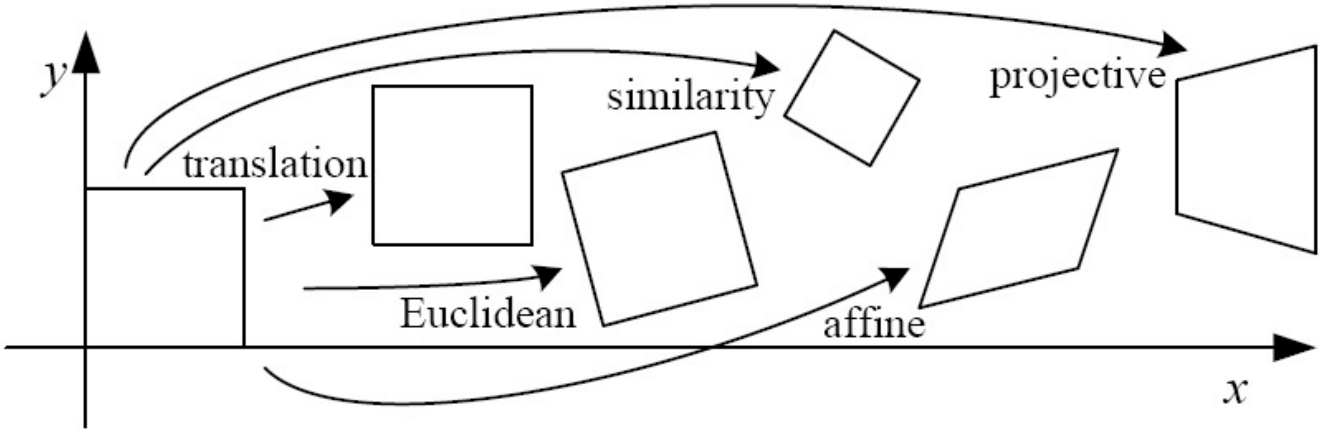
Projective transformations …

- Affine transformations, and
- Projective warps

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$			
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$			

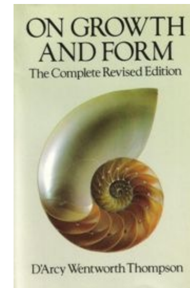
These transformations are a nested set of groups

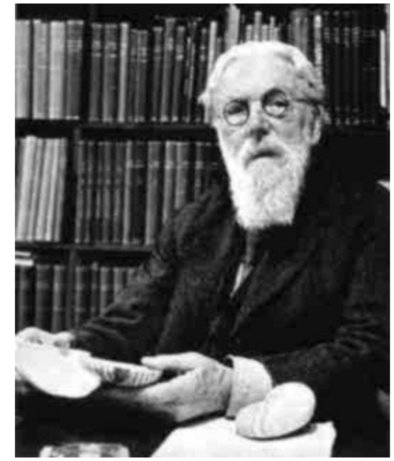
- Closed under composition and inverse is a member

Image Warping in Biology

D'Arcy Thompson

- http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html
- http://en.wikipedia.org/wiki/D'Arcy_Thompson





Importance of shape and structure in evolution

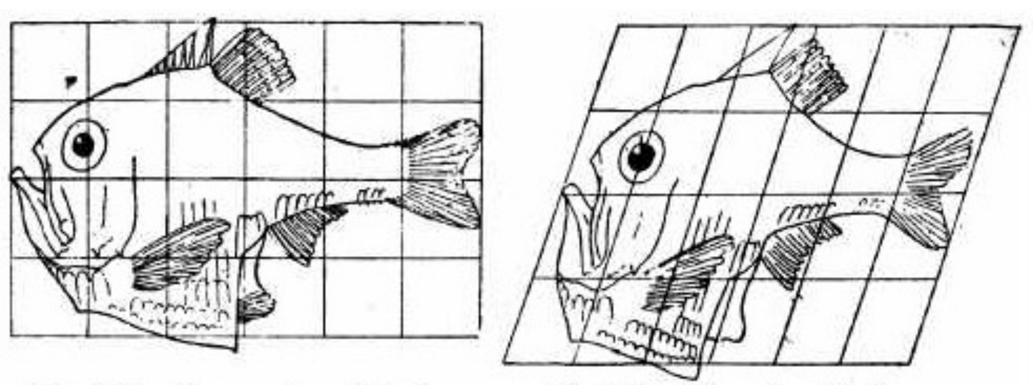
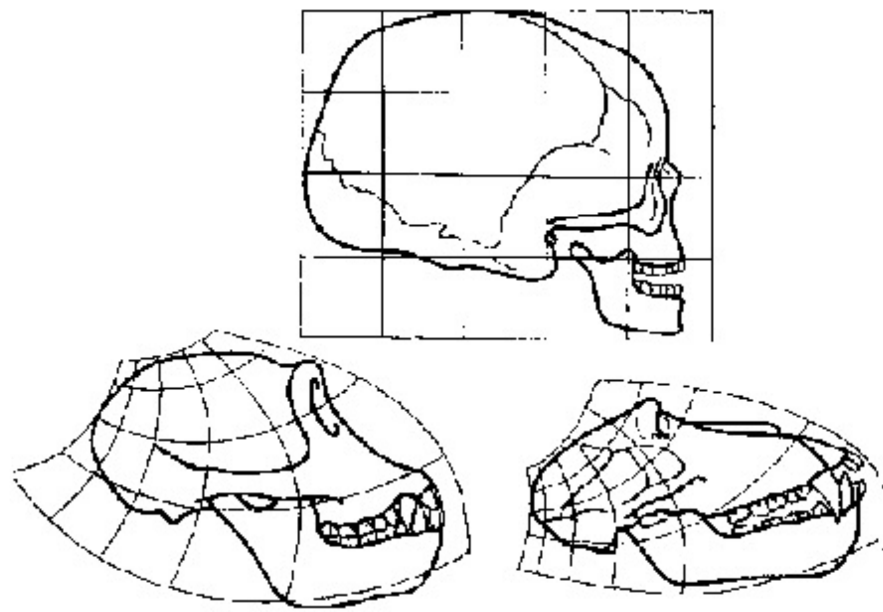


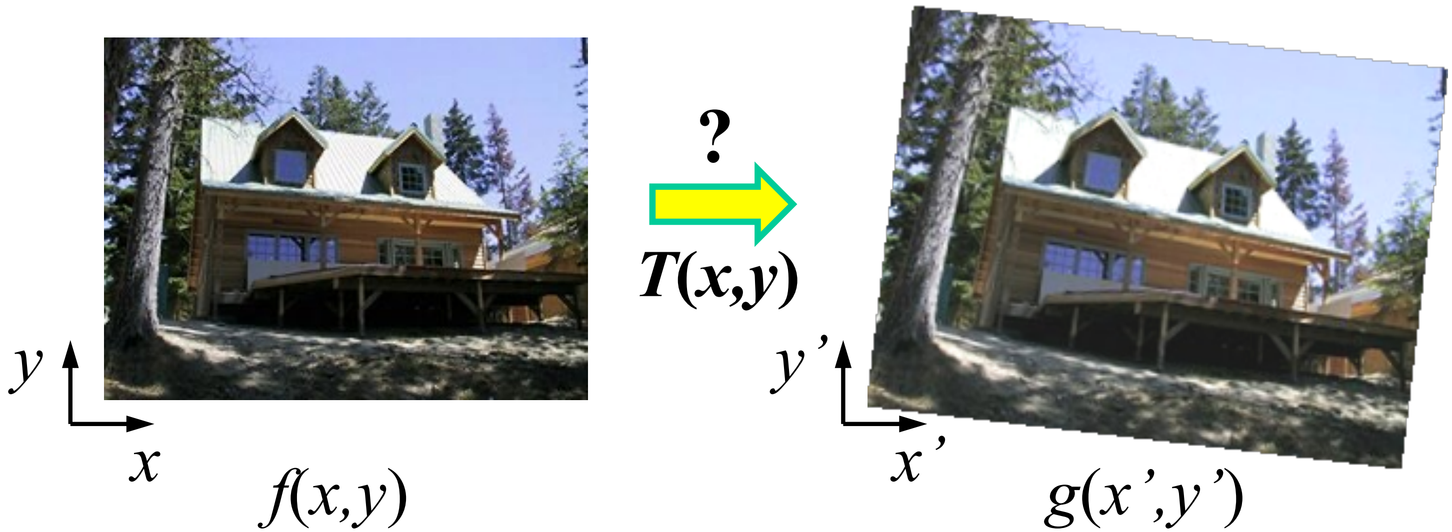
Fig. 517. *Argyropelecus Olfersi*. Fig. 518. *Sternoptyx diaphana*.



Skulls of a human, a chimpanzee and a baboon and transformations between them

Slide by Durand and Freeman

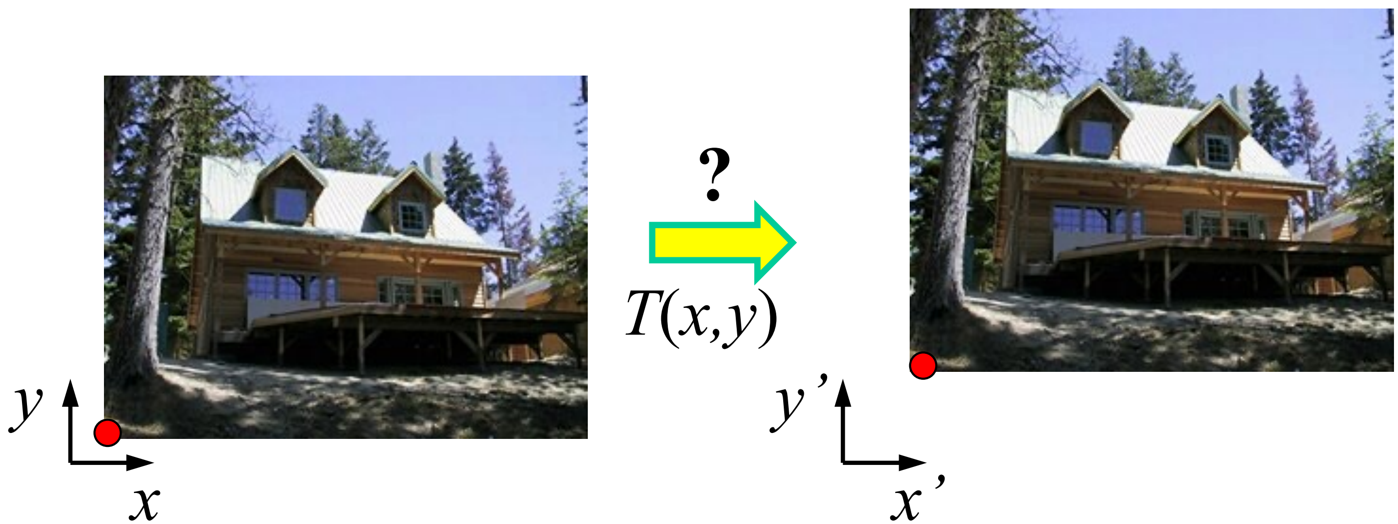
Recovering Transformations



What if we know *f* and *g* and want to recover the transform T?

- e.g. better align images from Project 1
- willing to let user provide correspondences
  - How many do we need?

Translation: # correspondences?



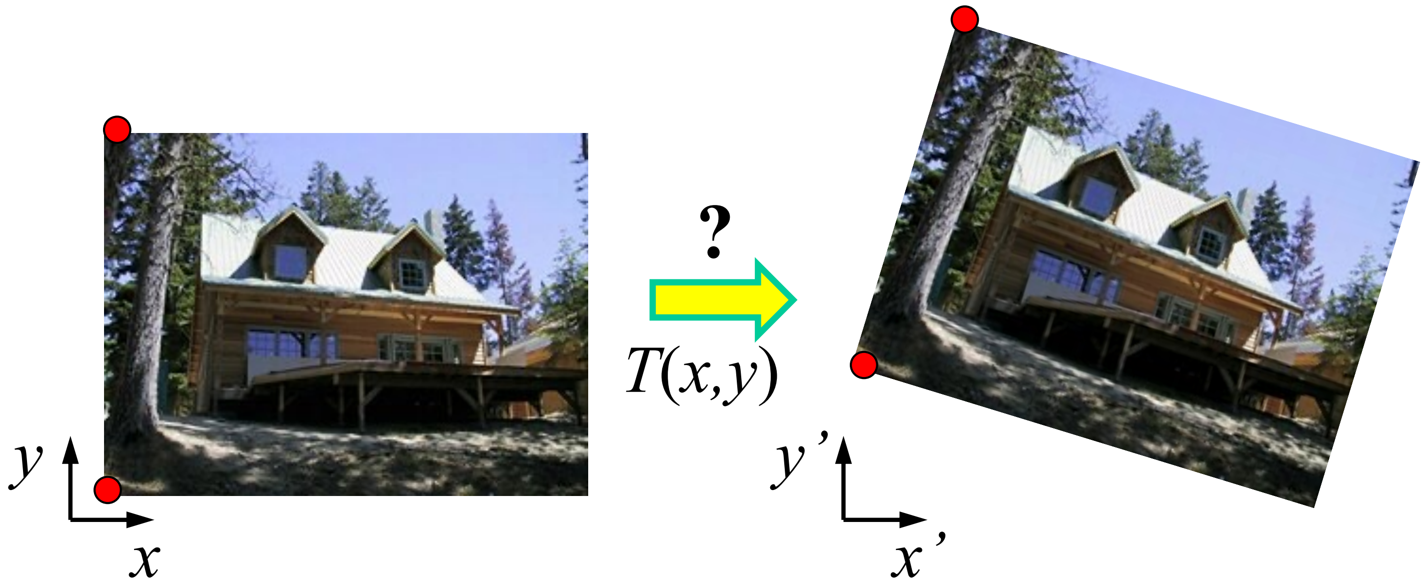
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

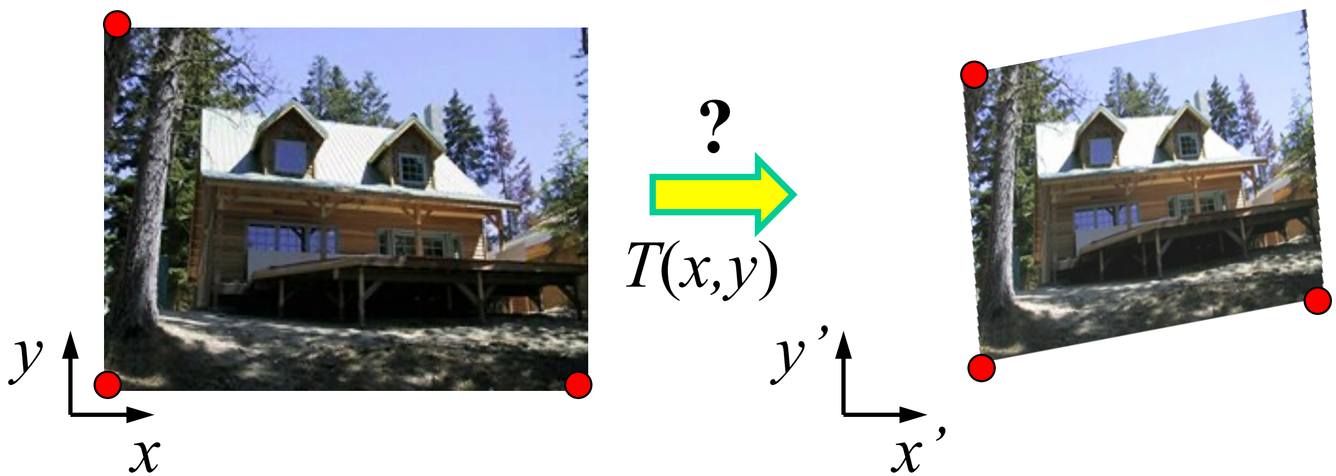
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

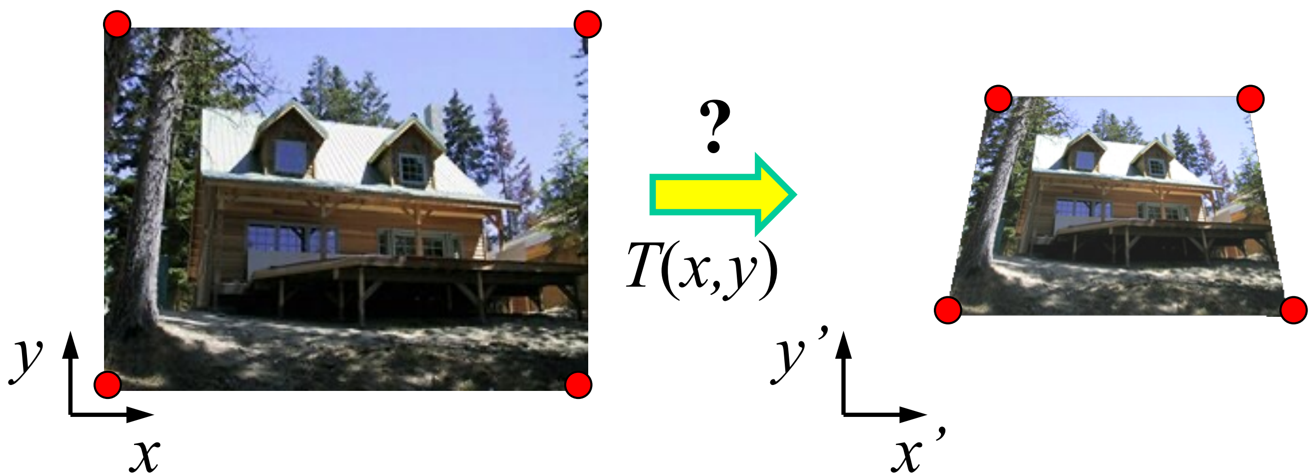
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

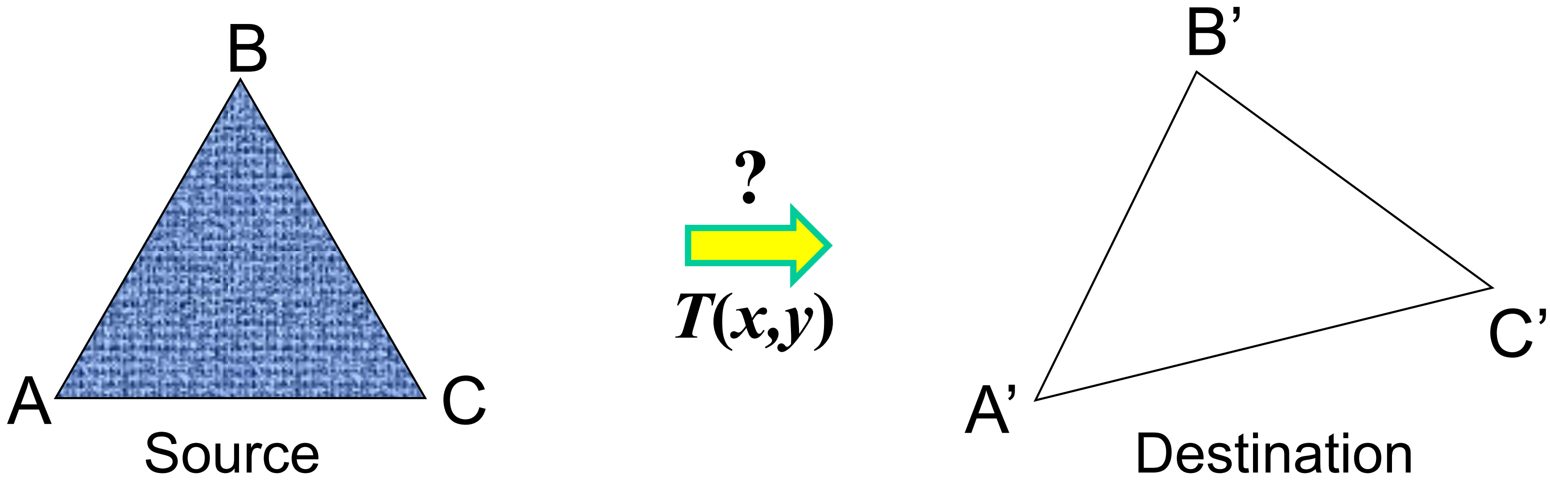
Projective: # correspondences?



How many correspondences needed for projective?

How many DOF?

Example: warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

Need to find transform T to transfer all pixels from one to the other.

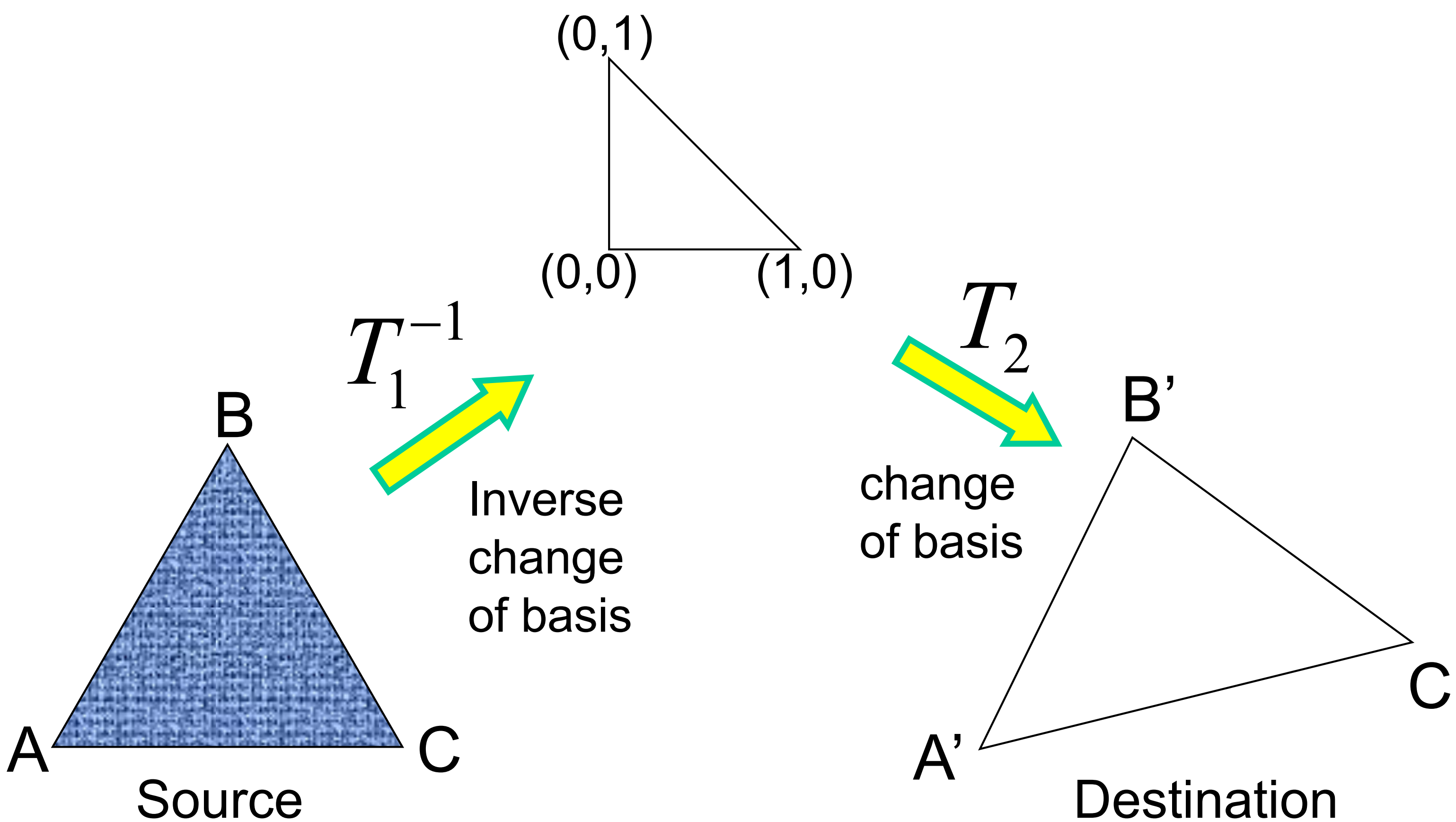
What kind of transformation is T?

How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways: Algebraic and geometric

warping triangles (Barycentric Coordinaes)



Don't forget to move the origin too!

Very useful for Project 3… (hint,hint,nudge,nudge)

Morphing = Object Averaging

The aim is to find "an average" between two objects

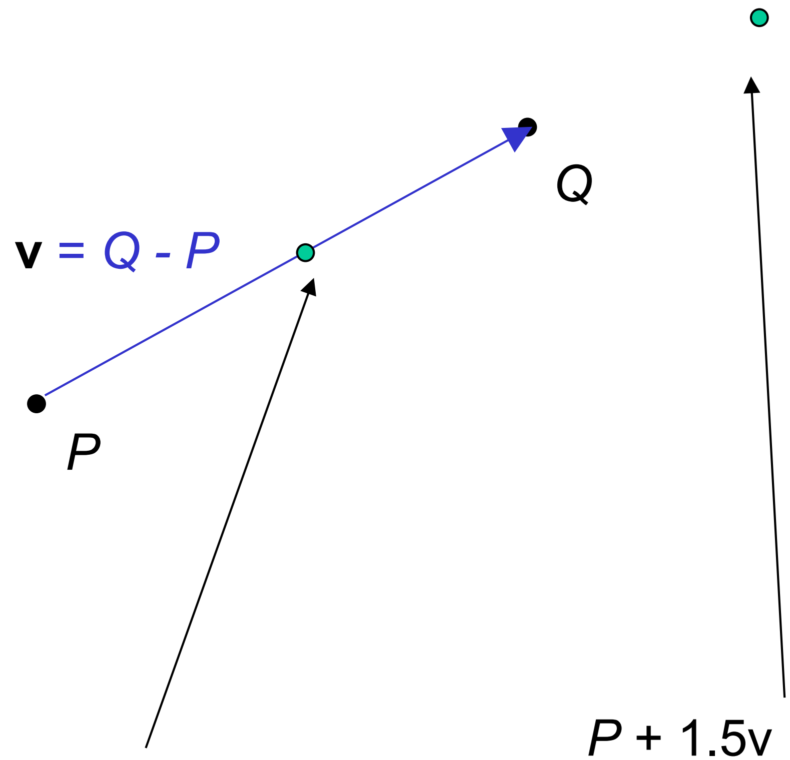
- Not an average of two images of objects…
- …but an image of the average object!
- How can we make a smooth transition in time?
  - Do a "weighted average" over time t

How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
  - Usually required user/artist input

Averaging Points

What's the average of P and Q?



$P + 0.5\mathrm{v}$
$= P + 0.5(Q - P)$
$= 0.5P + 0.5\,Q$

$P + 1.5\mathrm{v}$
$= P + 1.5(Q - P)$
$= -0.5P + 1.5\,Q$
(extrapolation)

Linear Interpolation (Affine Combination):
New point $aP + bQ$,
defined only when $a+b = 1$
So $aP+bQ = aP+(1-a)Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

Interpolate whole images:

$$Image_{halfway} = (1-t)*Image_1 + t*image_2$$

This is called **cross-dissolve** in film industry

But what is the images are not aligned?

Idea #2: Align, then cross-disolve

Align first, then cross-dissolve

- Alignment using global warp – picture still valid

Image Morphing

Morphing = warping + cross-dissolving

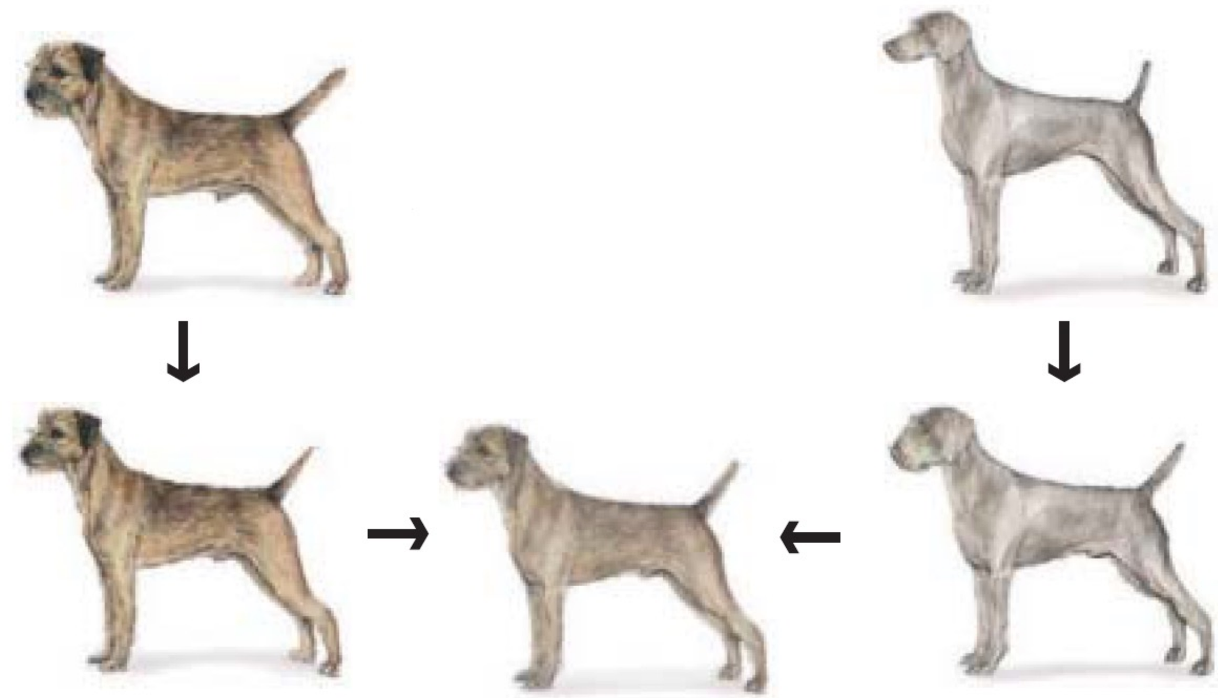
shape

(geometric)

color

(photometric)

Two-stage Morphing Procedure



Morphing procedure:

for every t,

1. Find the average shape (the “mean dog”☺)
   - warping
2. Find the average color
   - Cross-dissolve the warped images

BUT: global warp not always enough!

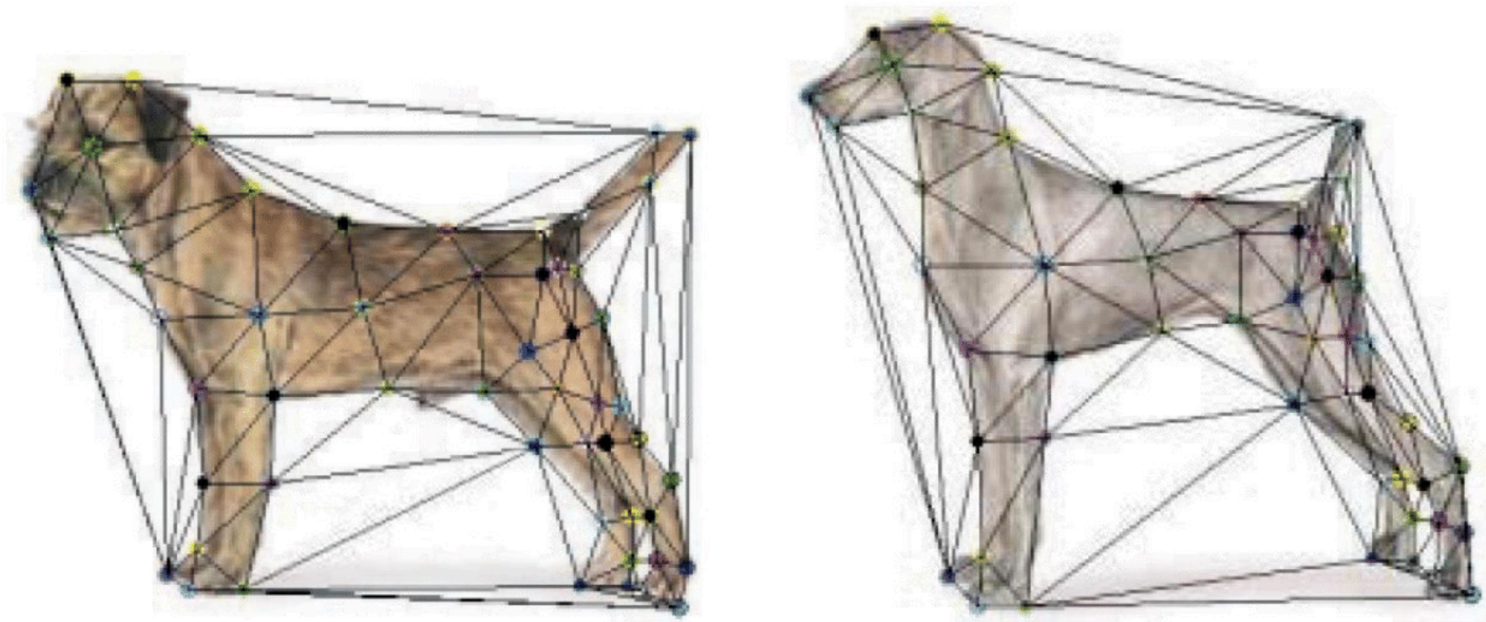
What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

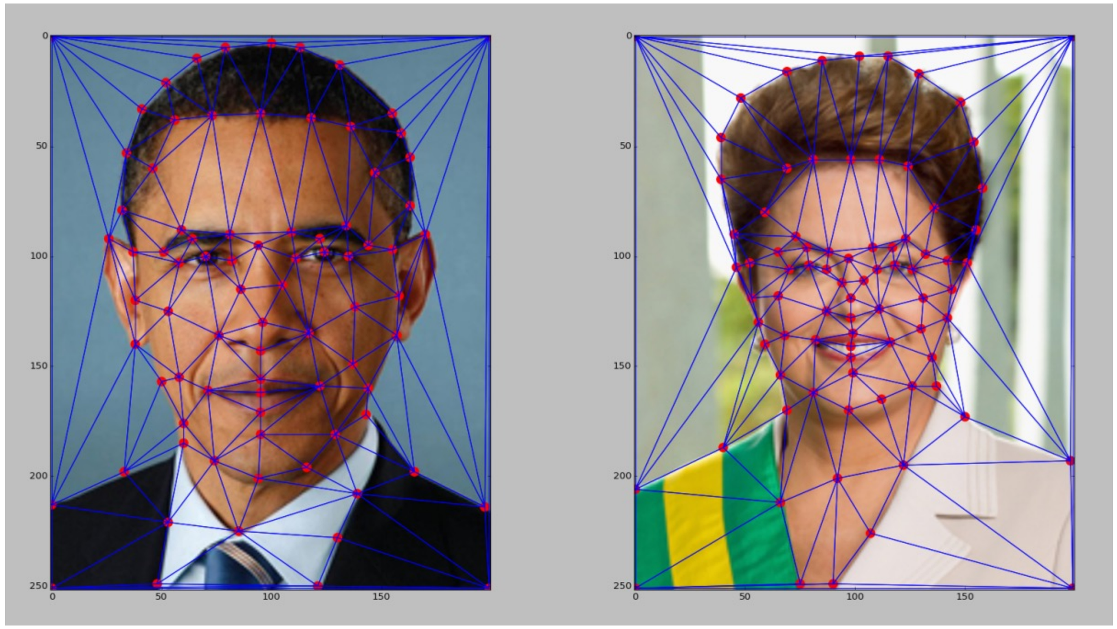
- Nose to nose, tail to tail, etc.
- But what to do with all the intermediate pixels?

Triangular Mesh



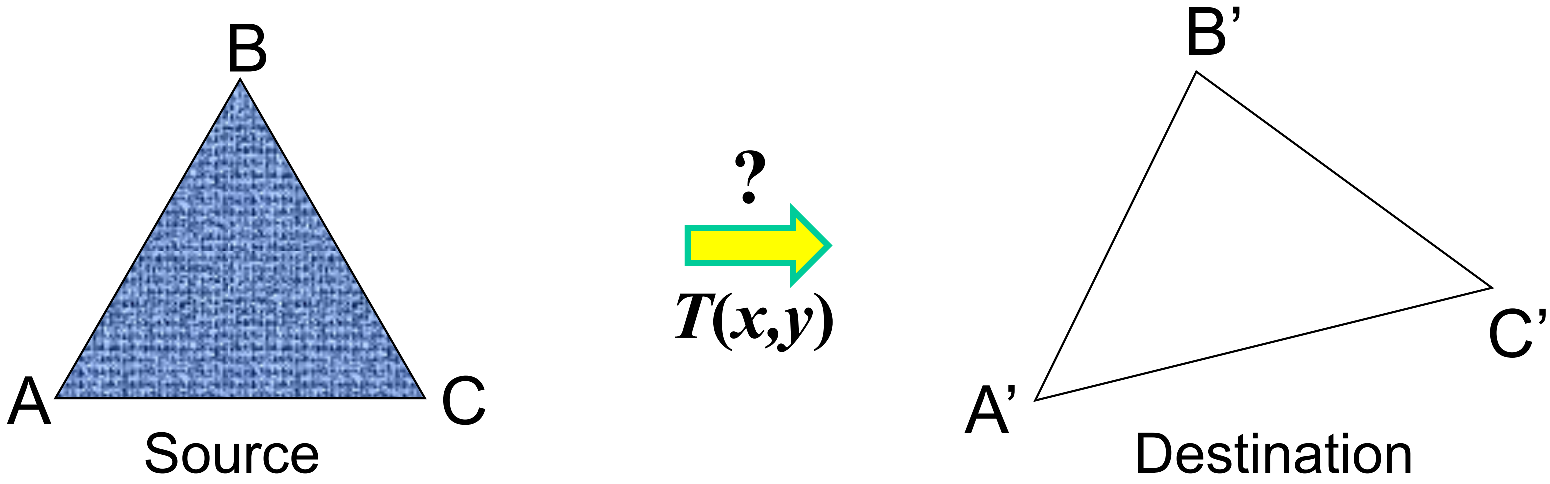
1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
   - How do we warp a triangle?

Full morphing result



(c) Ian Albuquerque Raymundo da Silva

Warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

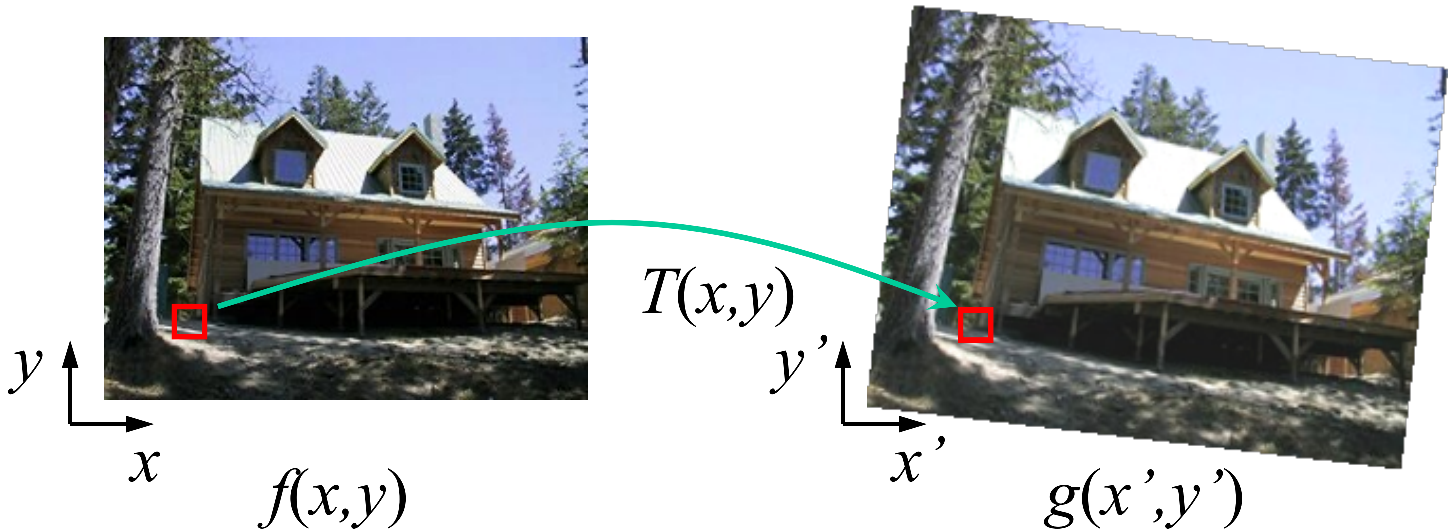
Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T?

How can we compute the transformation matrix:

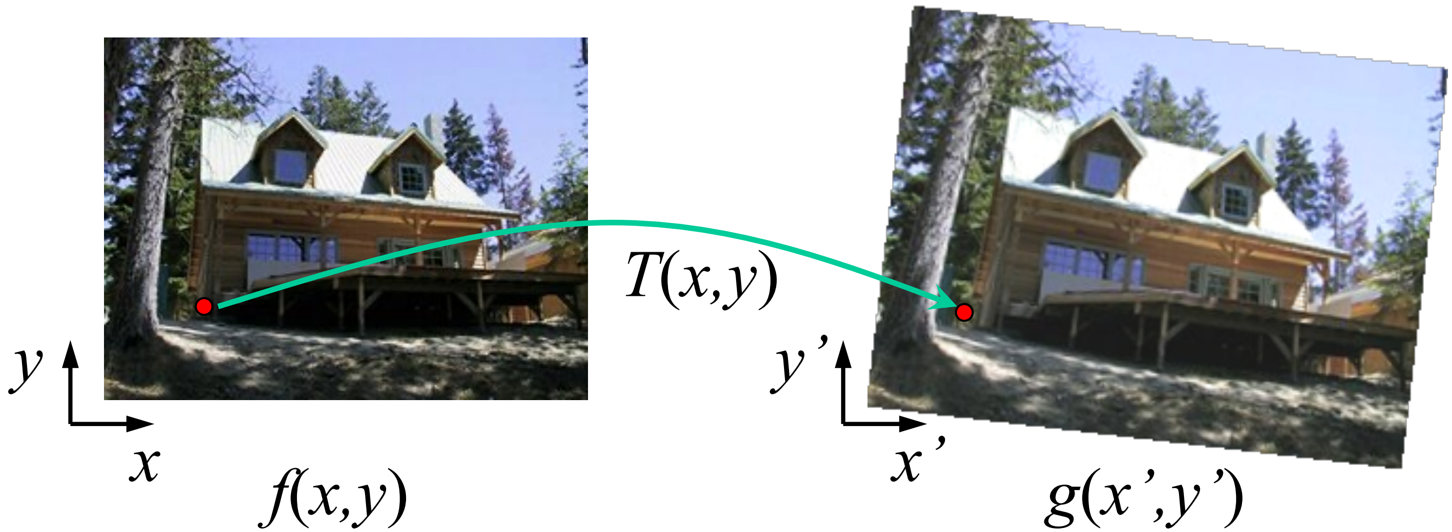
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Warping Pixels



Given a coordinate transform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

Forward warping

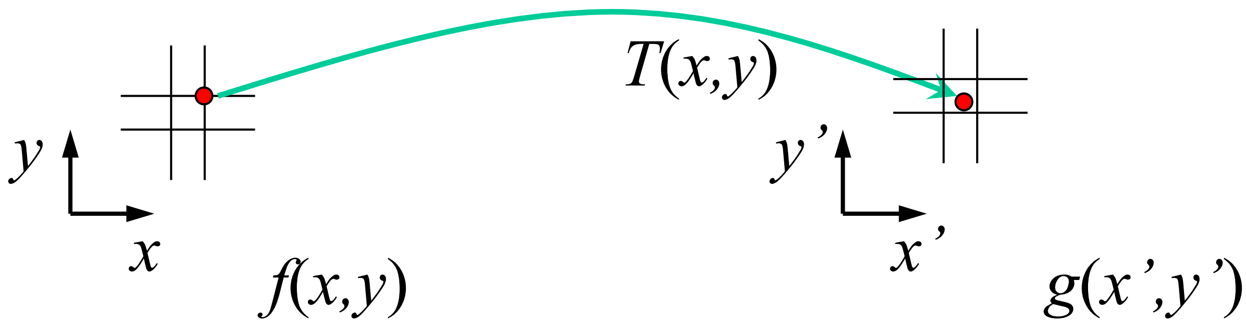


Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands “between” two pixels?

Forward warping



Send each pixel $f(x,y)$ to its corresponding location

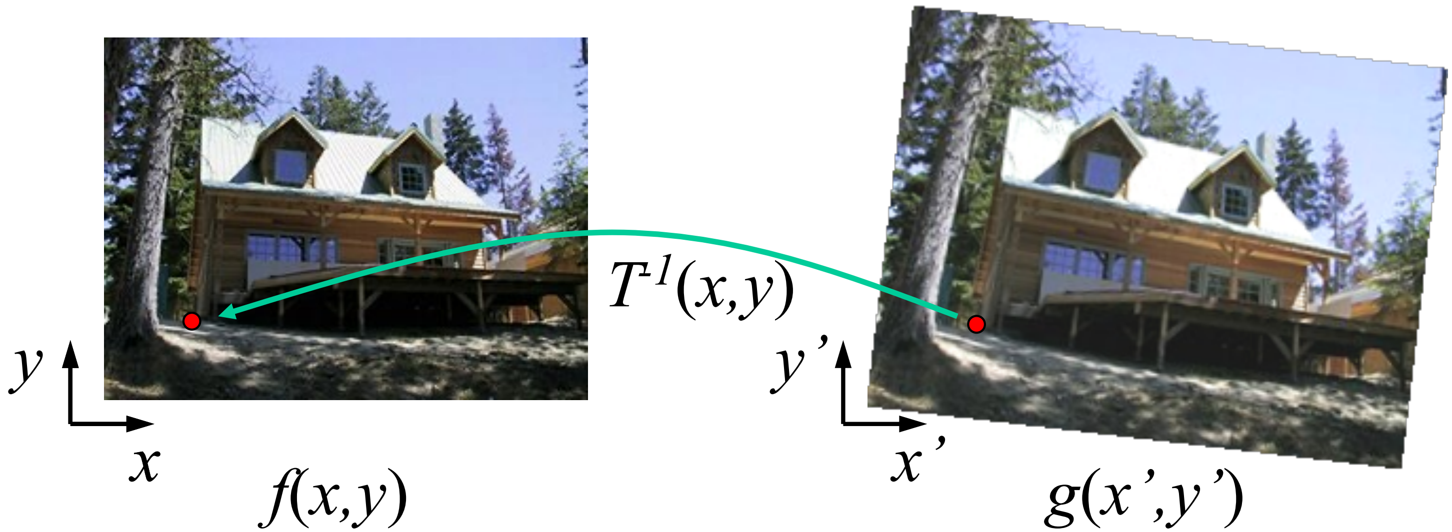
$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"
- Check out `griddata` in Matlab

Inverse warping

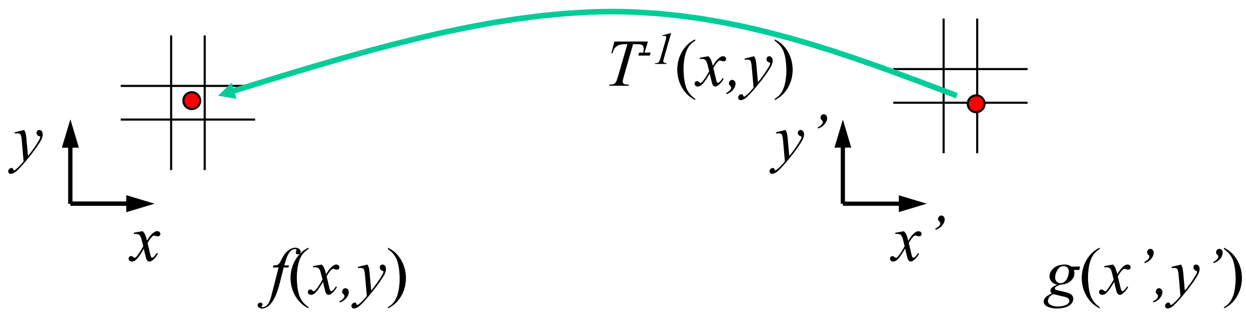


Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

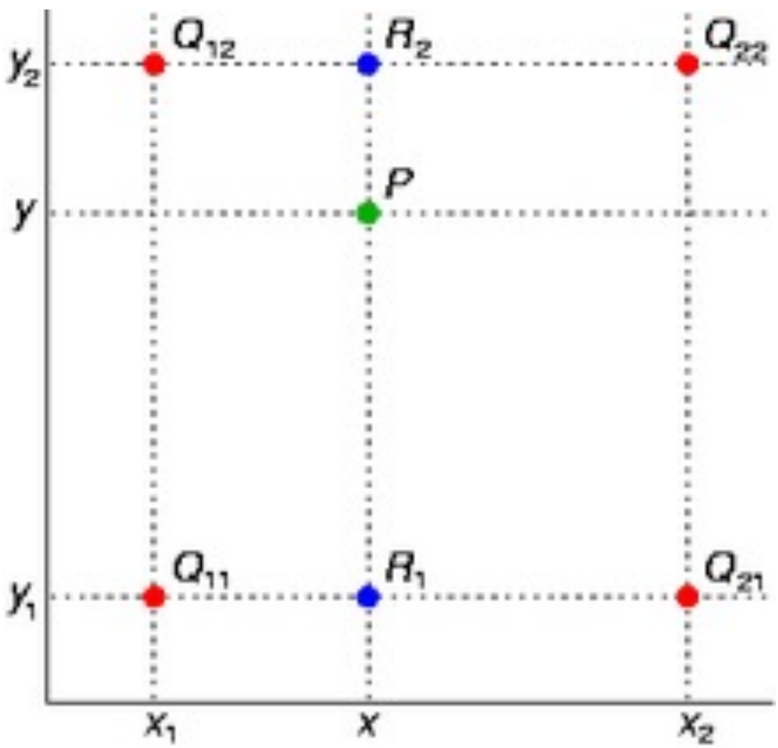
$(x,y) = T^{-1}(x',y')$ in the first image

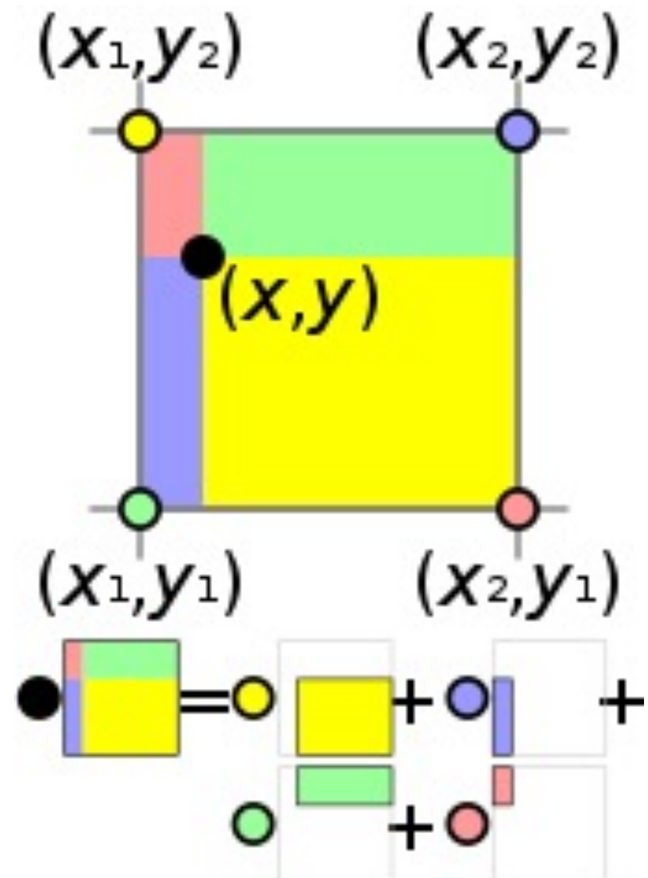
Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab / Python

Bilinear Interpolation





http://en.wikipedia.org/wiki/Bilinear_interpolation

```
Help interp2
```

Forward vs. inverse warping

Q: which is better?

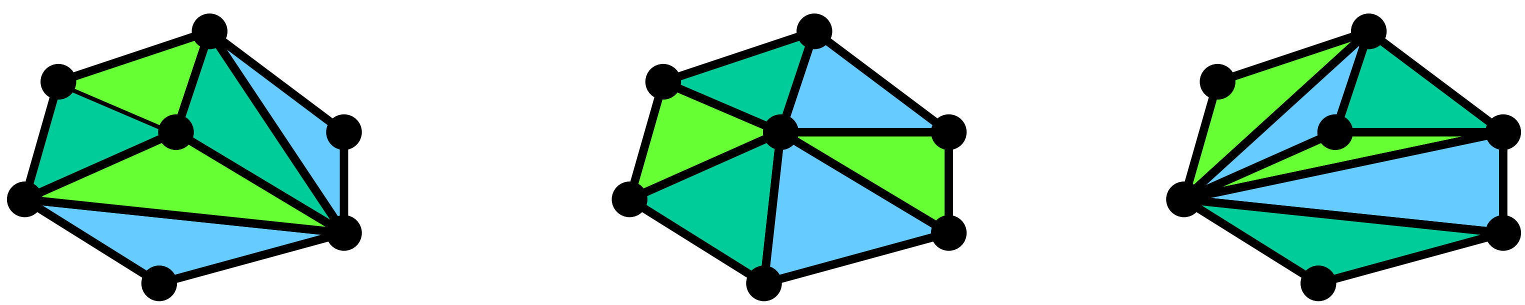
A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

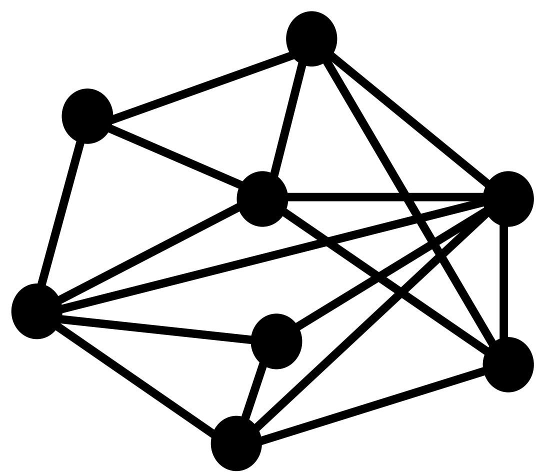
There are an exponential number of triangulations of a point set.



An O(n^3) Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



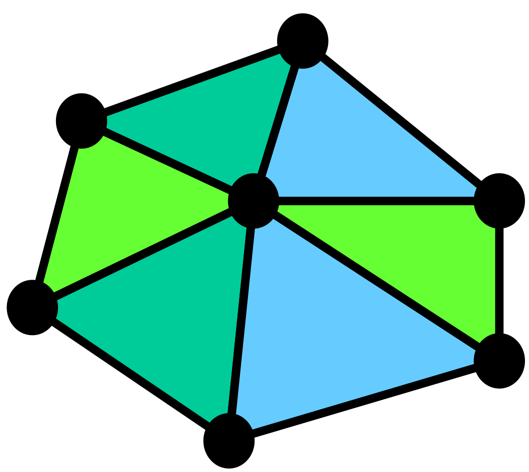
“Quality” Triangulations

Let $\alpha(T) = (\alpha_1, \alpha_2, .., \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.

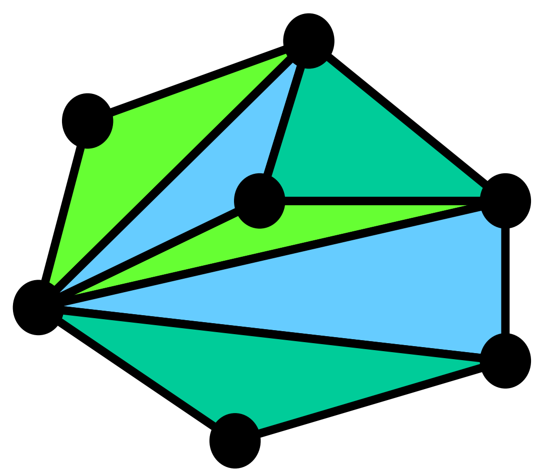
A triangulation T_1 will be “better” than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles



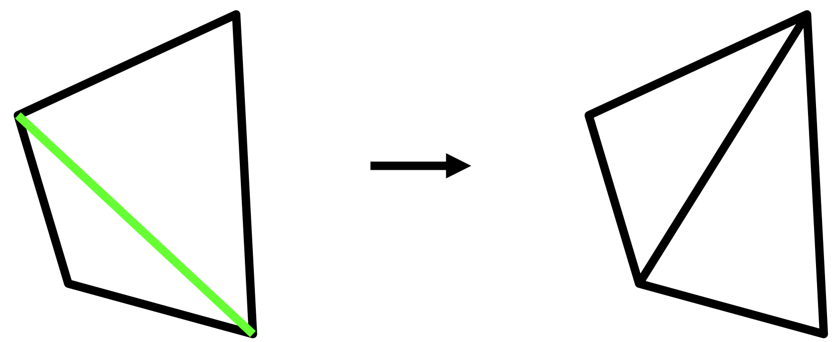
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

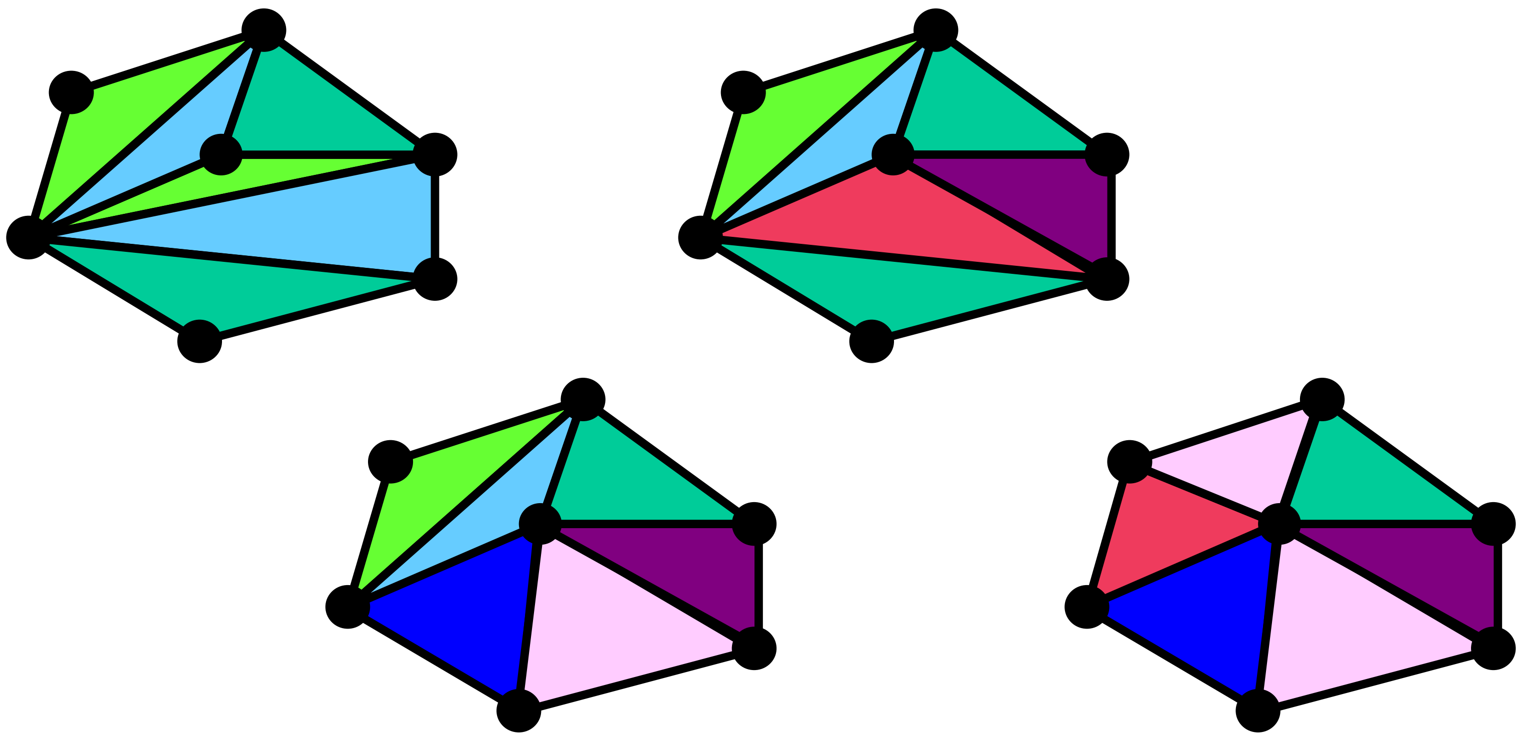


If an edge flip improves the triangulation, the first edge is called *illegal*.

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

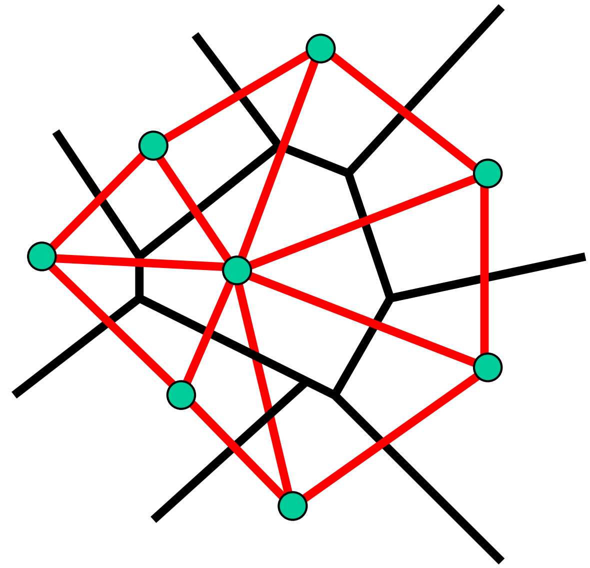
Could take a long time to terminate.



Delaunay Triangulation by Duality

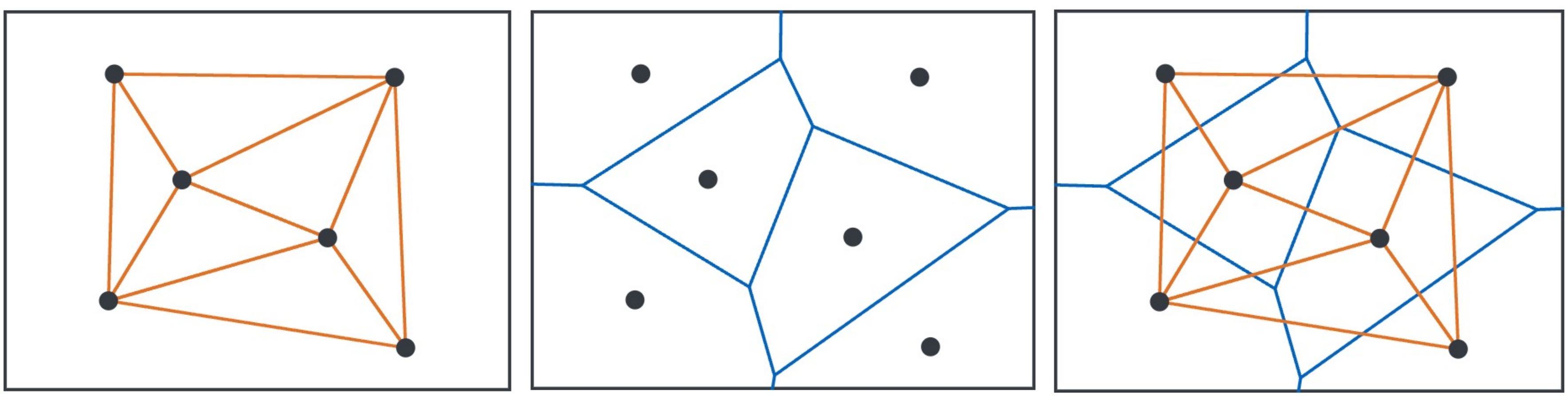
General position assumption: There are no four co-circular points.

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.



Corollary: The DT may be constructed in O($n$log$n$) time.

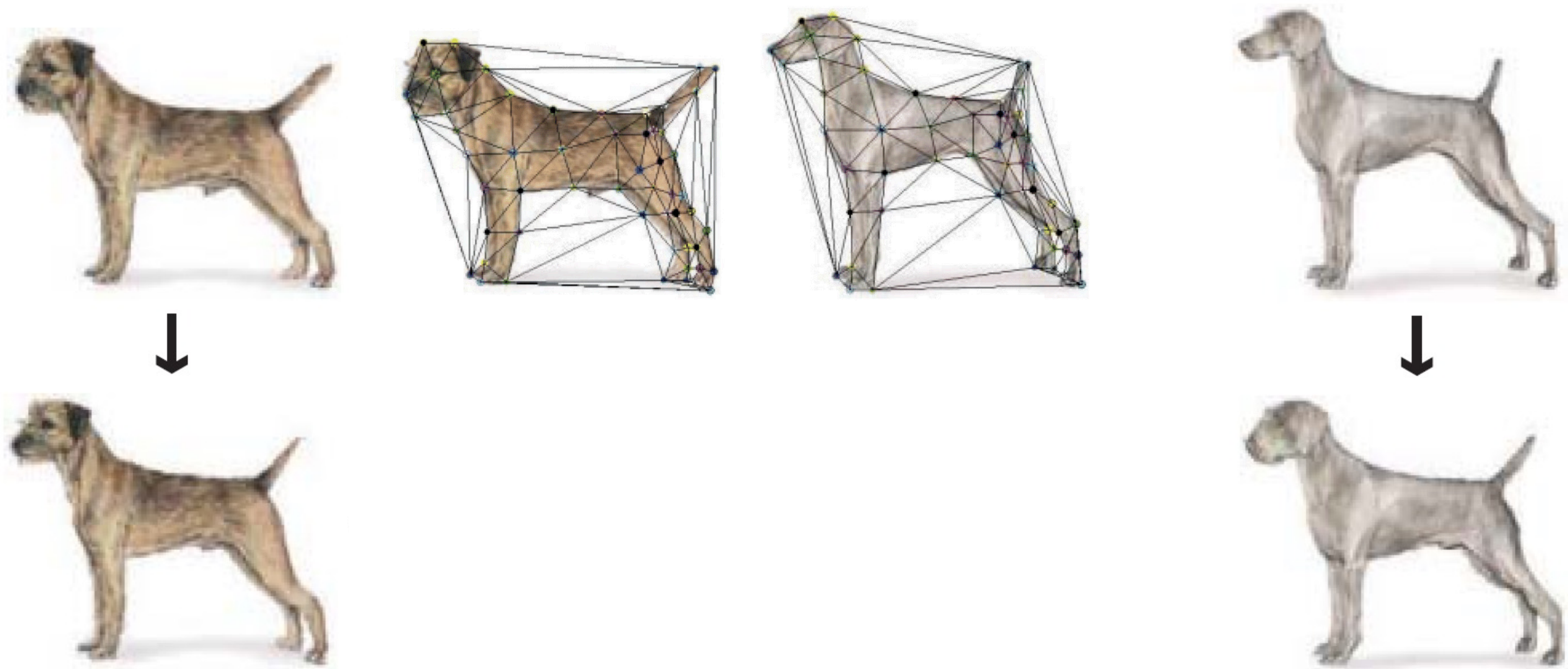
This is what Matlab's `delaunay` function uses.



1. Create Average Shape

How do we create an intermediate warp at time t?

- Assume t = [0,1]
- Simple linear interpolation of each feature pair
  - p=(x,y) -> p'(x,y)
- (1-t)*p+t*p' for corresponding features p and p'



2. Create Average Color

Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t)^{*}\text{Image} + t^{*}\text{image}'$$

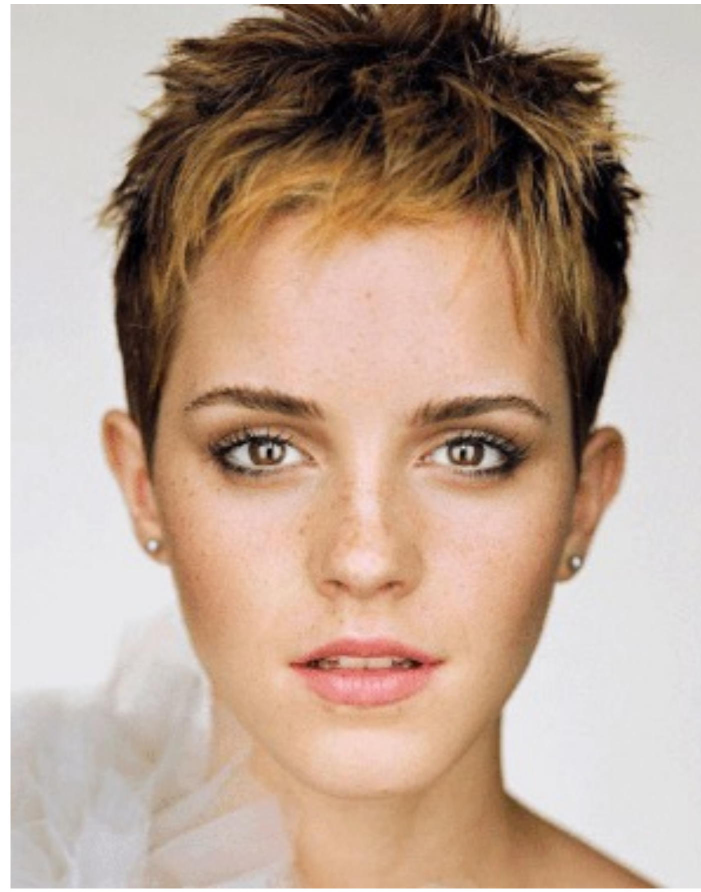
cross-dissolve!

Project #3: morphing

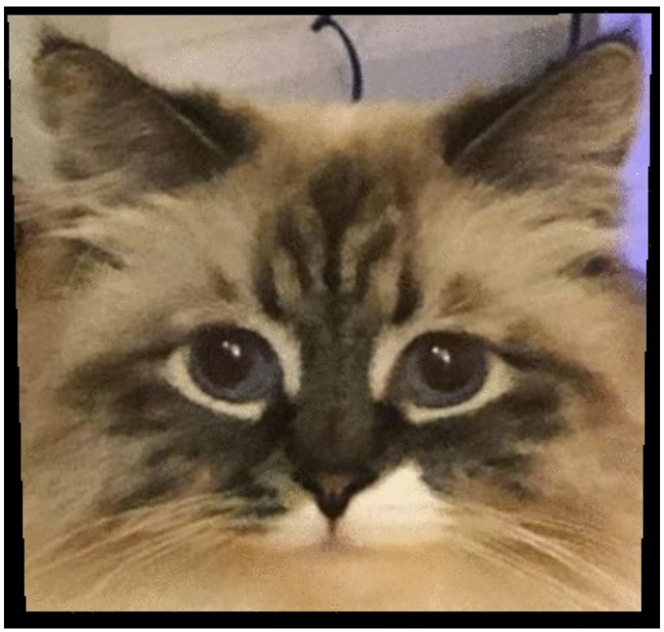
1. Define corresponding points
2. Define triangulation on points
   - Use <u>same triangulation</u> for both images
3. For each t = 0:step:1
   a. Compute the average <u>shape</u> at t (weighted average of points)
   b. For each triangle in the average shape
      – Get the affine projection to the corresponding triangles in each image
      – For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (cross-dissolve each triangle)
   c. Save the image as the next frame of the sequence

<u>Life-hack</u>: can be done with just two nested loops (for t, and for each triangle). Hint: compute warps for all pixels first, then use `interp2`

Examples



Examples from last year



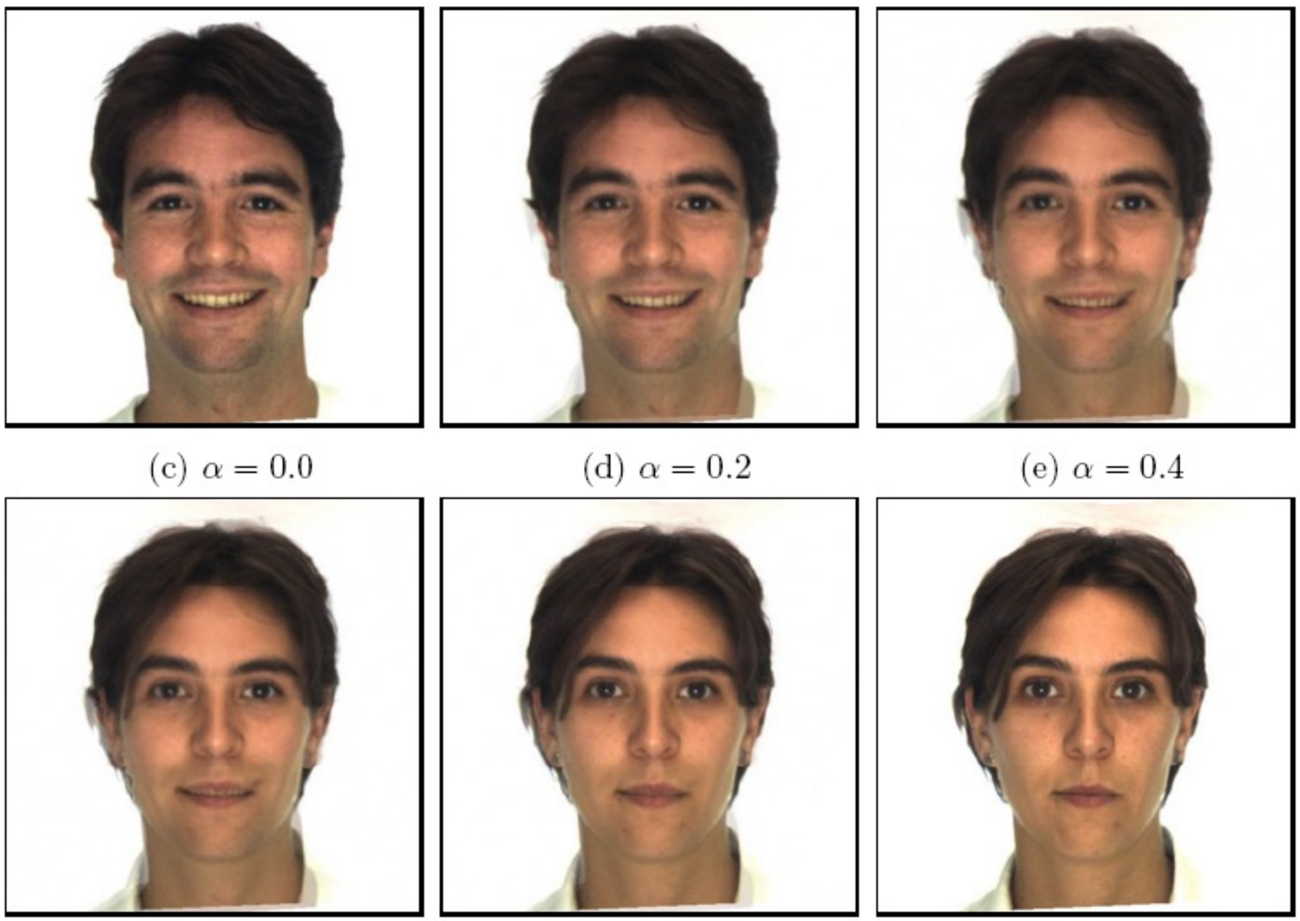
@Michael Jayasuriya

@Varun Saran

What's the difference?

Morphing & matting

Extract foreground first to avoid artifacts in the background

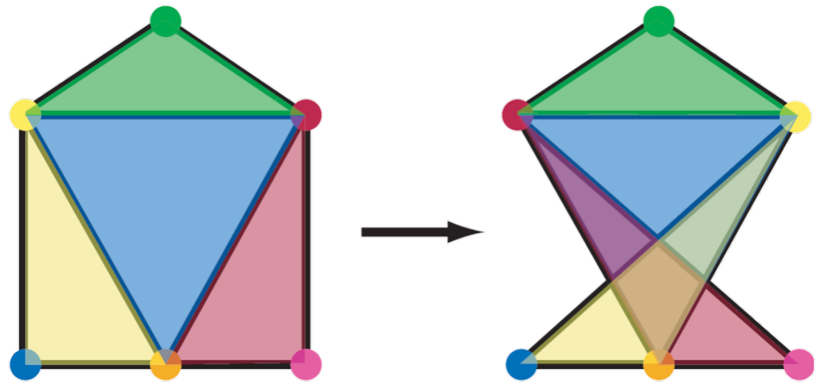


(c) $\alpha = 0.0$ (d) $\alpha = 0.2$ (e) $\alpha = 0.4$

(f) $\alpha = 0.6$ (g) $\alpha = 0.8$ (h) $\alpha = 1.0$

Slide by Durand and Freeman

Other Issues



Beware of folding

- You are probably trying to do something 3D-ish

Morphing can be generalized into 3D

- If you have 3D data, that is!

Extrapolation can sometimes produce interesting effects

- Caricatures

Dynamic Scene ("Black or White", MJ)

http://www.youtube.com/watch?v=R4kLKv5gtxc