Structure-from-Motion (SfM) and Multi-View Stereo (MVS)

A lot of slides borrowed from Noah Snavely + Shree Nayar's YT series: First principals of Computer Vision

CS194: Intro to Computer Vision and Comp. Photo
Kanazawa and Efros, UC Berkeley, Fall 2022

Recall: Camera calibration & triangulation

- Suppose we know **3D points** and their **matches** in an image
  - How can we compute the **camera parameters**?

- Suppose we know **camera parameters** for multiple cameras, each observing a point
  - How can we compute the **3D location** of that point?

Structure from motion

- SfM solves both of these problems *at once*
- A kind of chicken-and-egg problem
  - (but solvable)

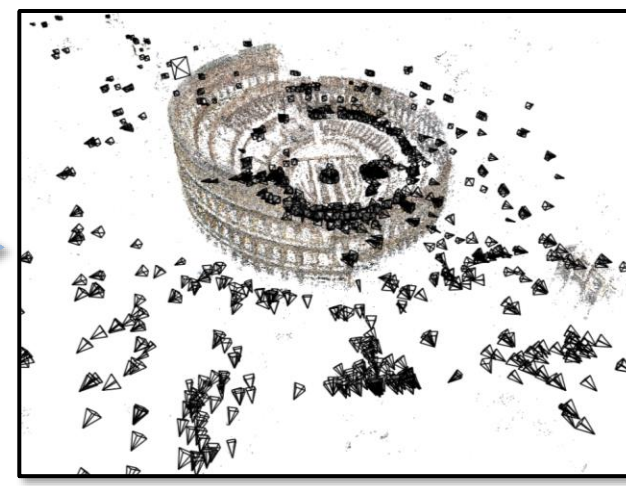
Photo Tourism

Noah Snavely, Steven M. Seitz, Richard Szeliski, "Photo tourism: Exploring photo collections in 3D," SIGGRAPH 2006

https://youtu.be/mTBPGuPLI5Y

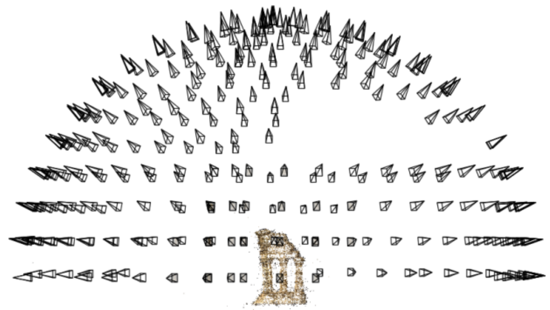
Structure from Motion (SfM)

- Given many images, how can we
  a) figure out where they were all taken from?
  b) build a 3D model of the scene?

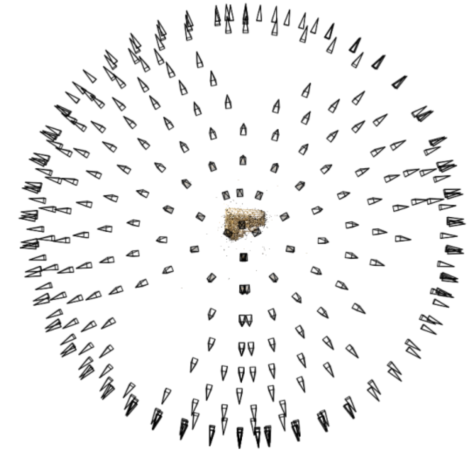


This is (roughly) the **structure from motion** problem

Structure from motion



Reconstruction (side)



(top)

- Input: images with points in correspondence
  $p_{i,j} = (u_{i,j}, v_{i,j})$

- Output
  - structure: 3D location $\mathbf{x}_i$ for each point p_i
  - motion: camera parameters $\mathbf{R}_j$, $\mathbf{t}_j$ possibly $\mathbf{K}_j$

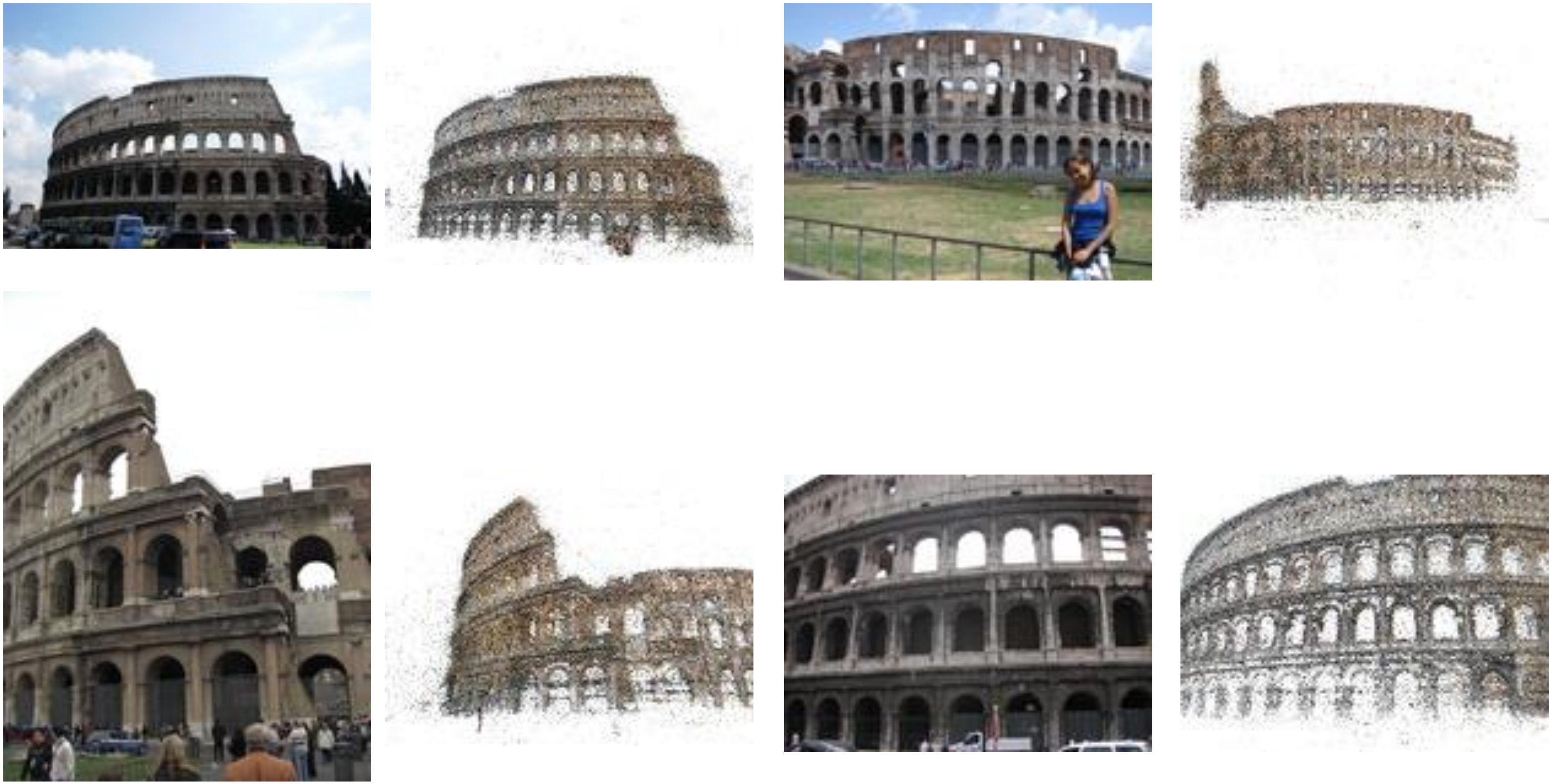
- Objective function: minimize *reprojection error*

Large-scale structure from motion

Dubrovnik, Croatia. 4,619 images (out of an initial 57,845).
Total reconstruction time: 23 hours
Number of cores: 352

Building Rome in a Day, Agarwal et al. ICCV 2009

Large-scale structure from motion



Rome’s Colosseum

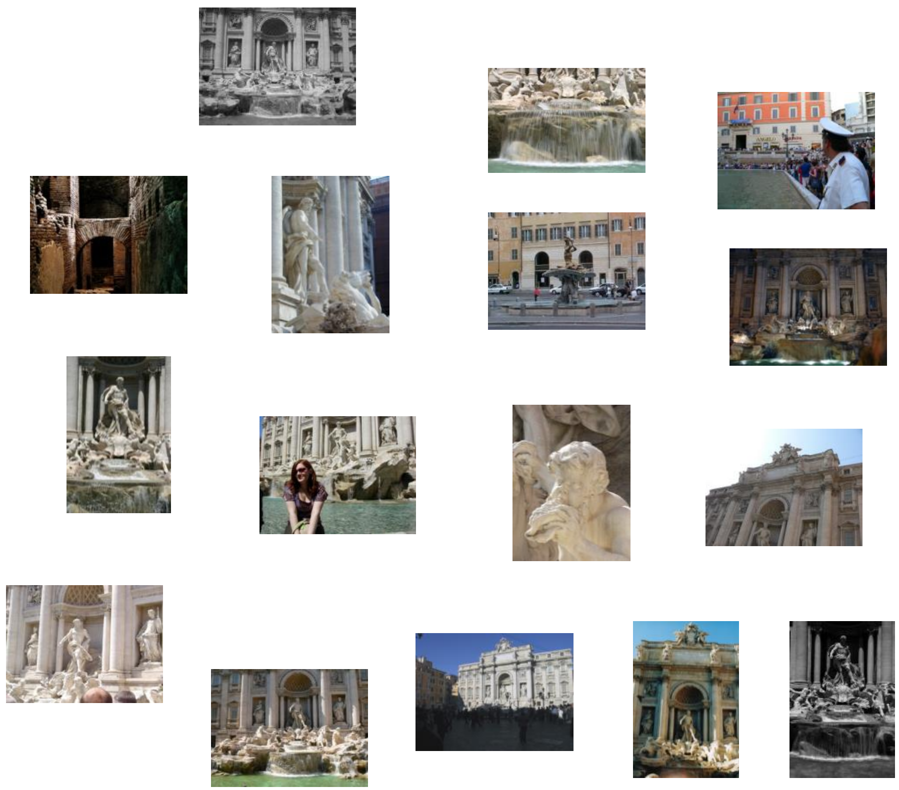
Building Rome in a Day, Agarwal et al. ICCV 2009

First step: Correspondence

- Feature detection and matching

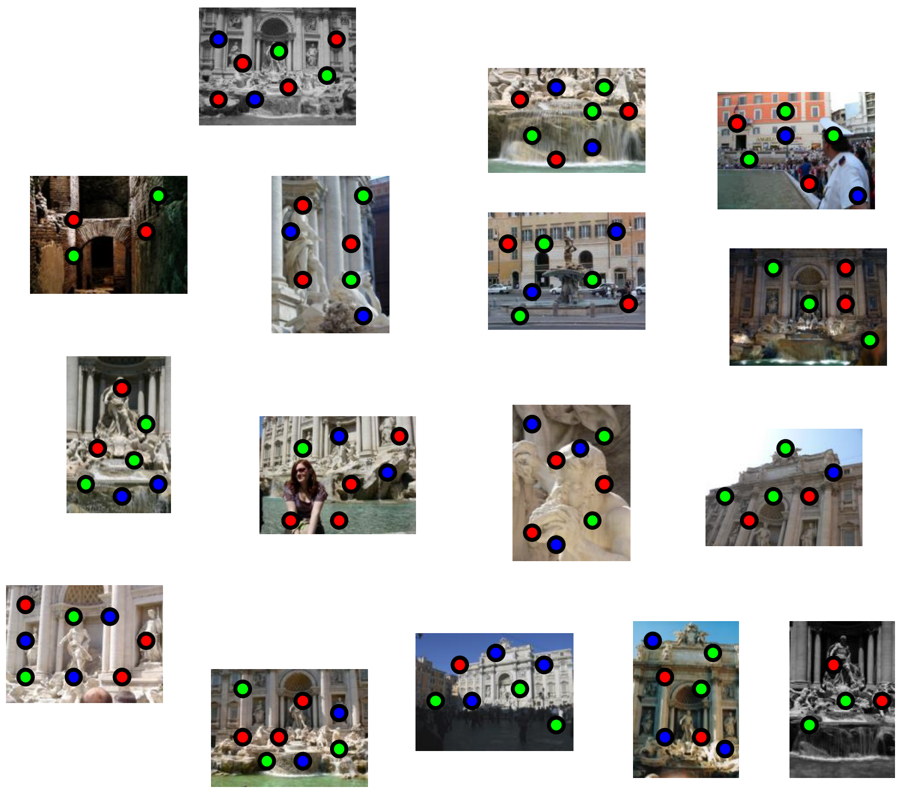
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



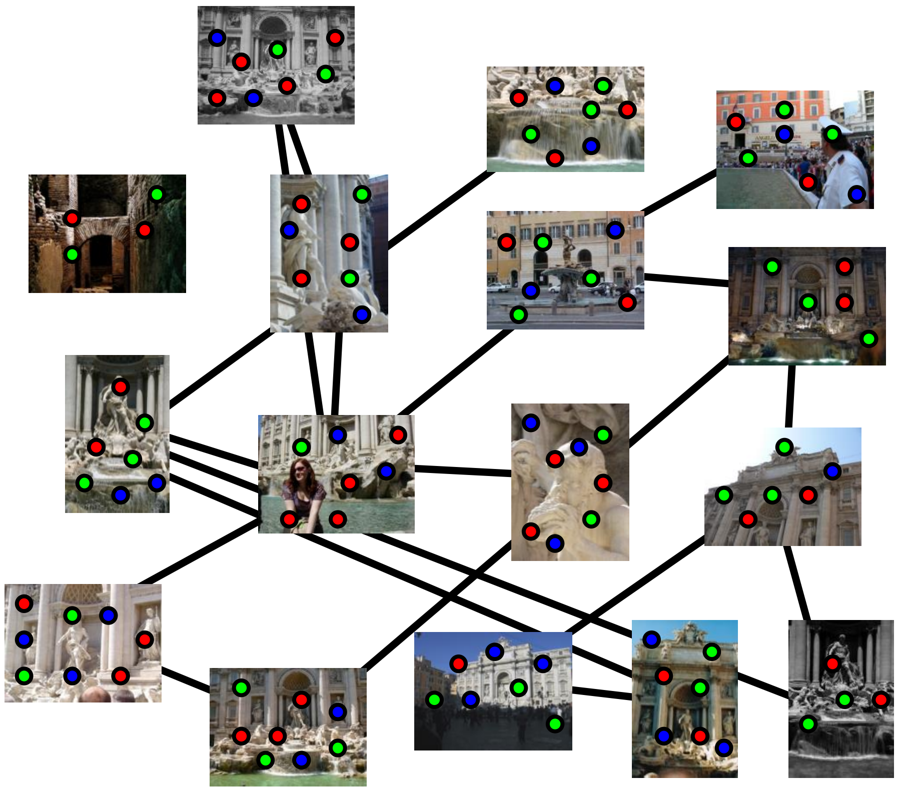
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



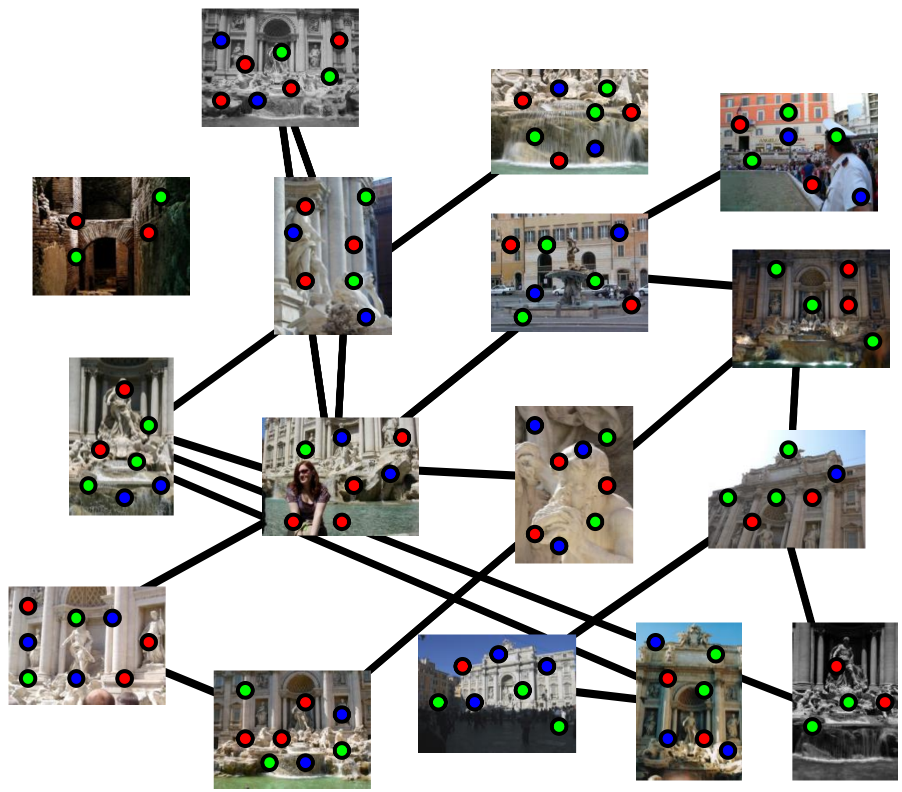
Feature matching

Match features between each pair of images



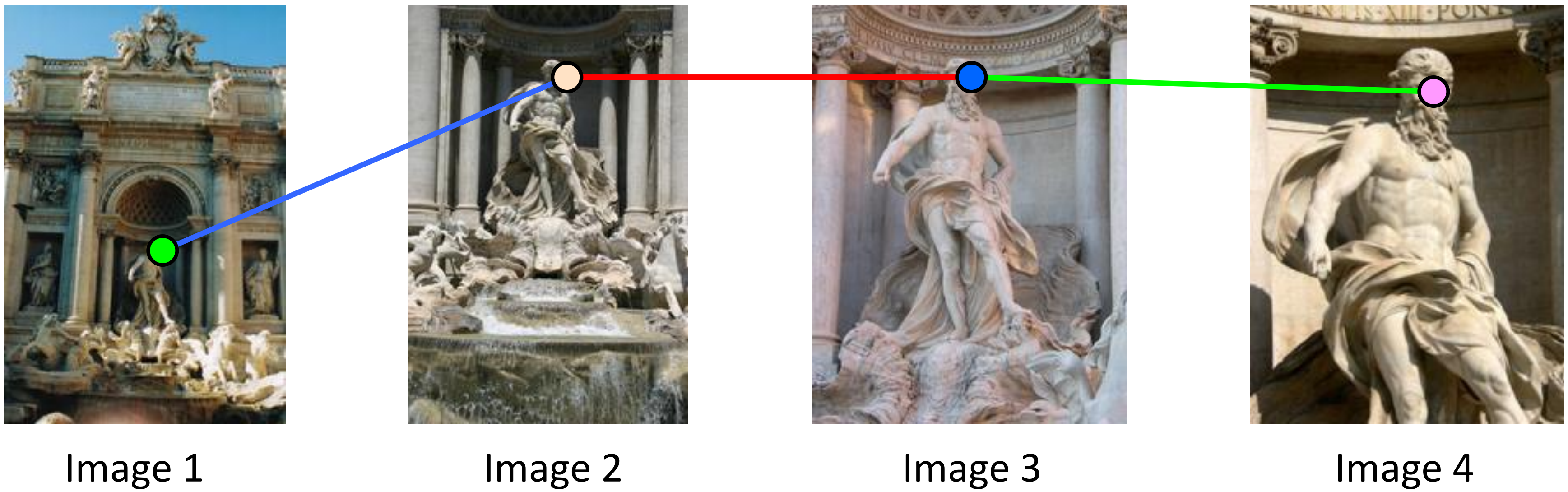
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

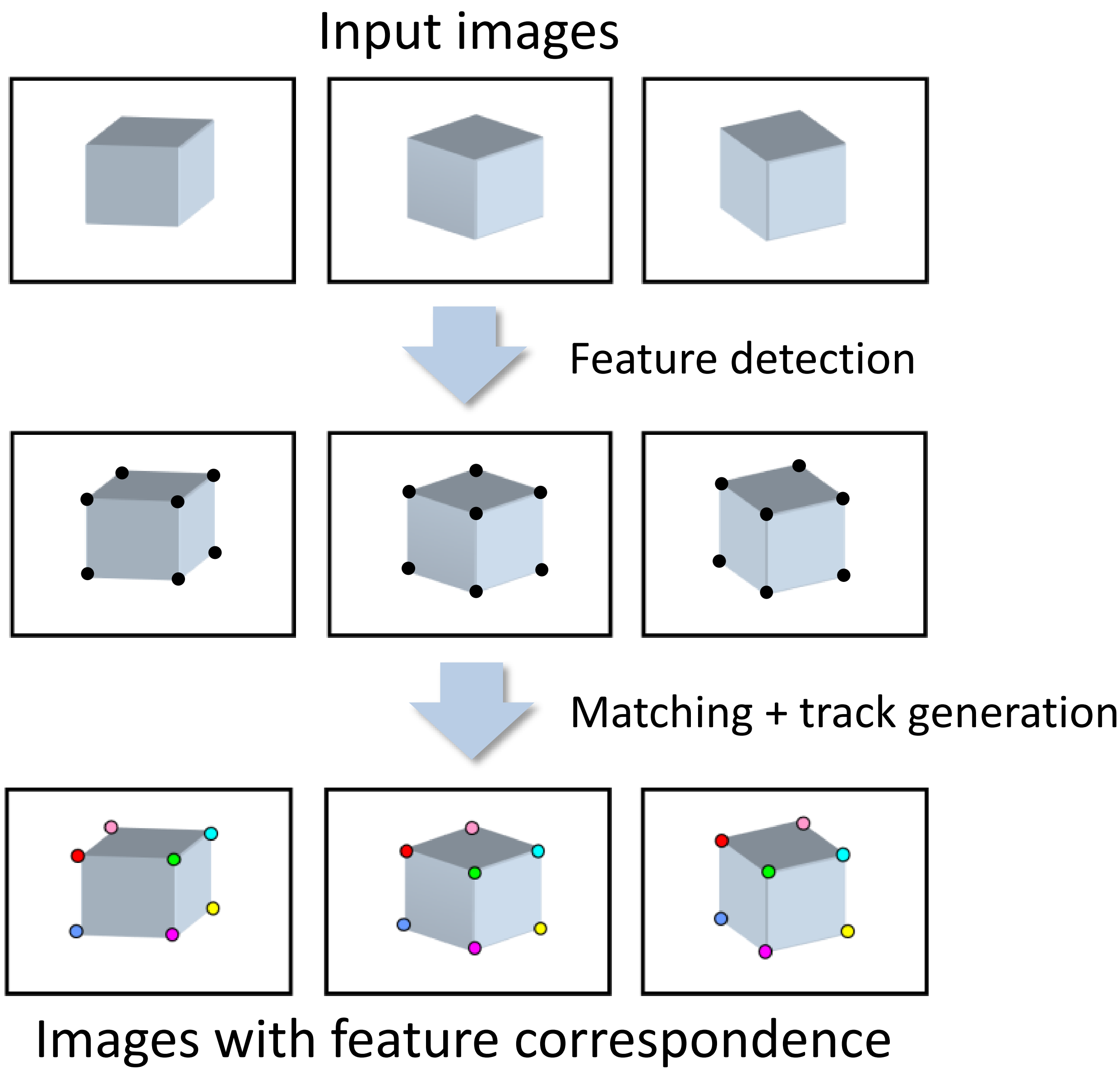


Correspondence estimation

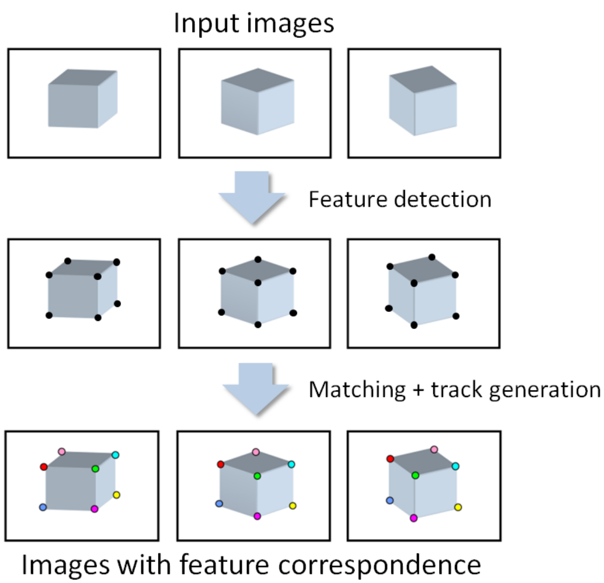
- Link up pairwise matches to form connected components of matches across several images



The story so far…



The story so far…

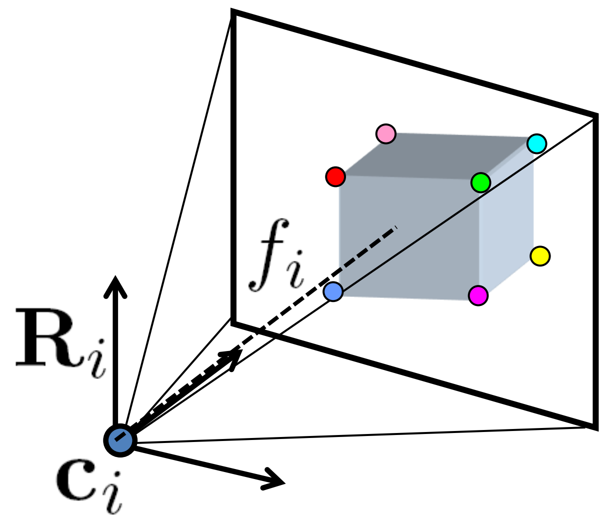


- Next step:
  - Use structure from motion to solve for geometry (cameras and points)

- First: what are cameras and points?

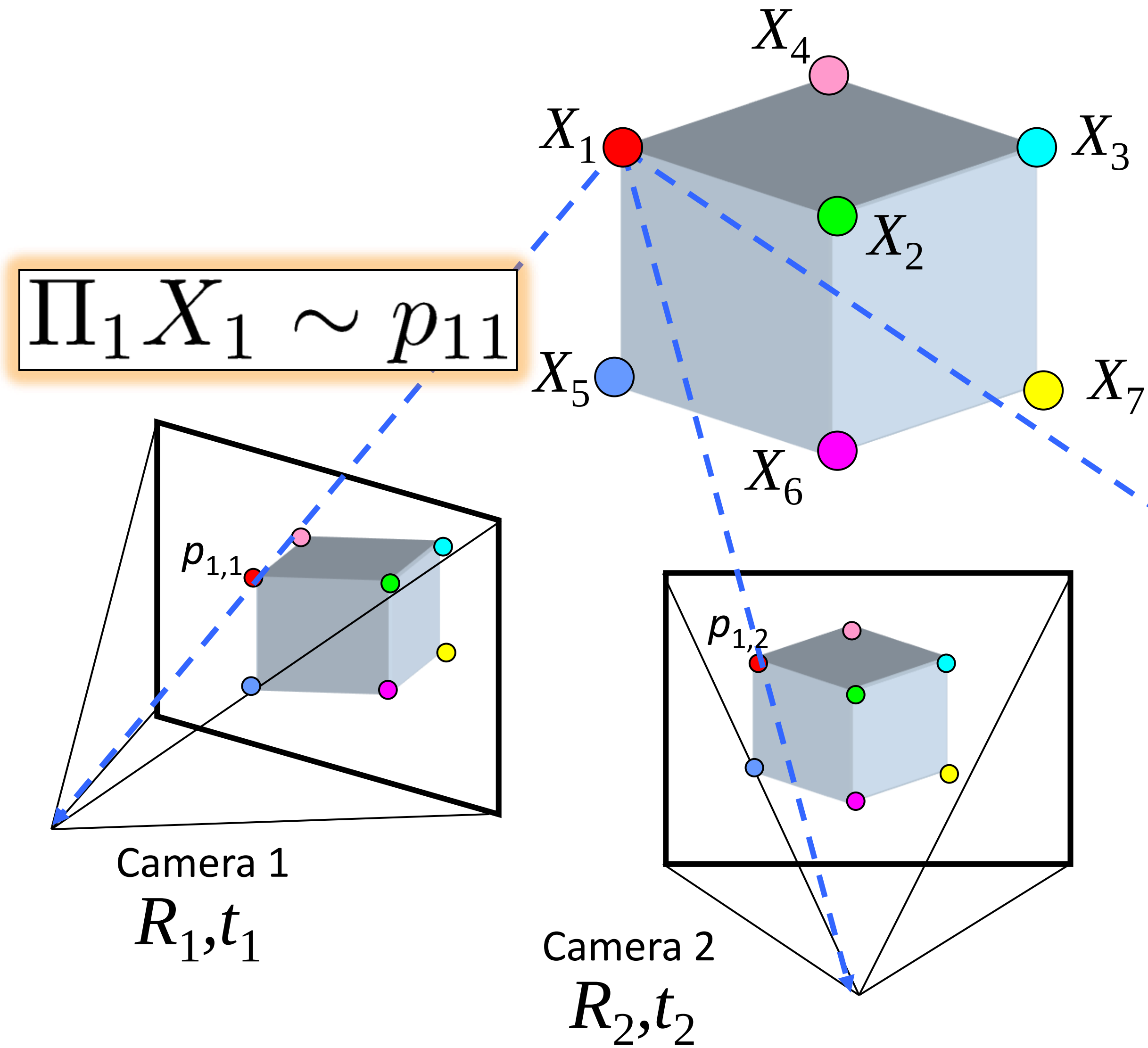
Review: Points and cameras

- Point: 3D position in space ($\mathbf{X}_j$)



- Camera (C_i):
  - A 3D position ($\mathbf{c}_i$)
  - A 3D orientation ($\mathbf{R}_i$)
  - Intrinsic parameters (**focal length**, aspect ratio, …)
  - 7 parameters (3+3+1) in total

Structure from motion



minimize

$$g(\mathbf{R}, \mathbf{T}, \mathbf{X})$$

non-linear least squares

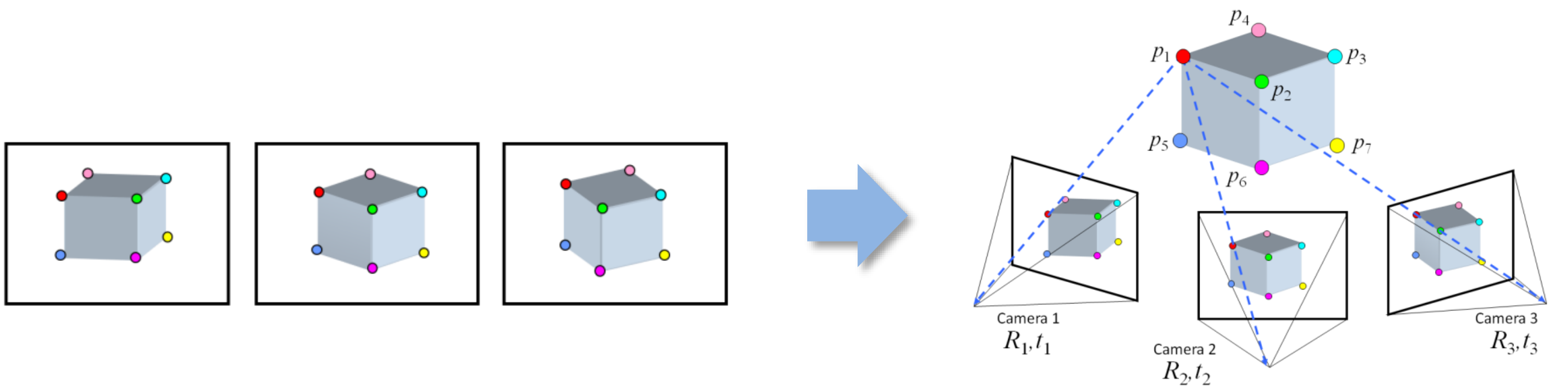
Structure from motion

- Minimize sum of squared reprojection errors:

$$g(\mathbf{X},\mathbf{R},\mathbf{T}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \underbrace{w_{ij}}_{\substack{\textit{indicator variable}: \\ \text{is point } i \text{ visible in image } j\ ?}} \cdot \Bigg\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\substack{\textit{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\textit{observed} \\ \text{image location}}} \Bigg\|^2$$

- Minimizing this function is called *bundle adjustment*
  - Optimized using non-linear least squares, e.g. Levenberg-Marquardt

Solving structure from motion

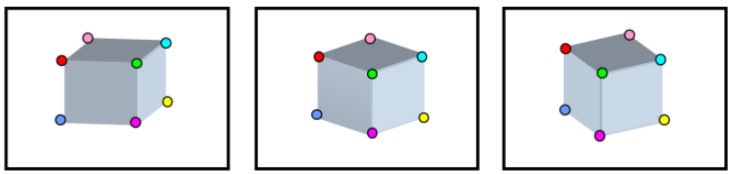


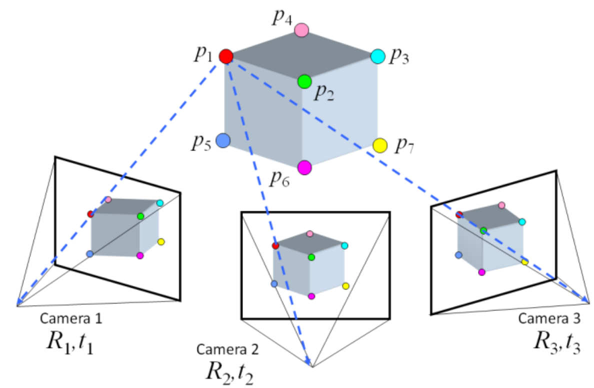
Inputs: feature tracks

Outputs: 3D cameras and points

- Challenges:
  - Large number of parameters (1000's of cameras, millions of points)
  - Very non-linear objective function

Solving structure from motion



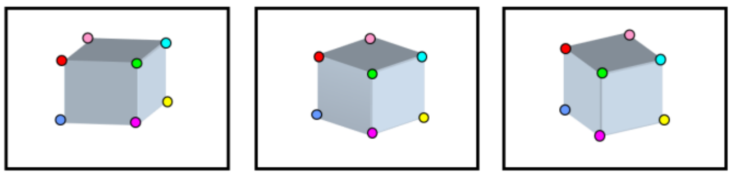


Inputs: feature tracks

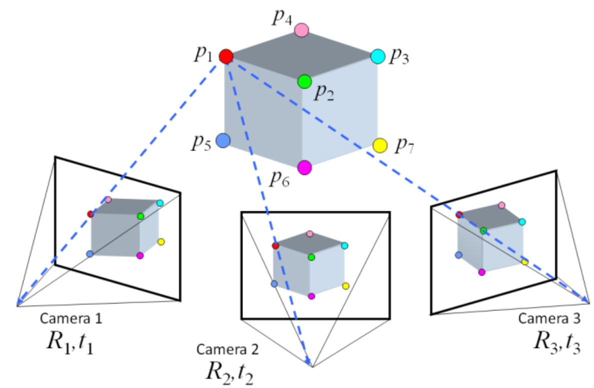
Outputs: 3D cameras and points

- Important tool: Bundle Adjustment [Triggs *et al.* ’00]
  - Joint non-linear optimization of both cameras and points
  - Very powerful, elegant tool
- The bad news:
  - Starting from a random initialization is very likely to give the wrong answer
  - Difficult to initialize all the cameras at once

Solving structure from motion



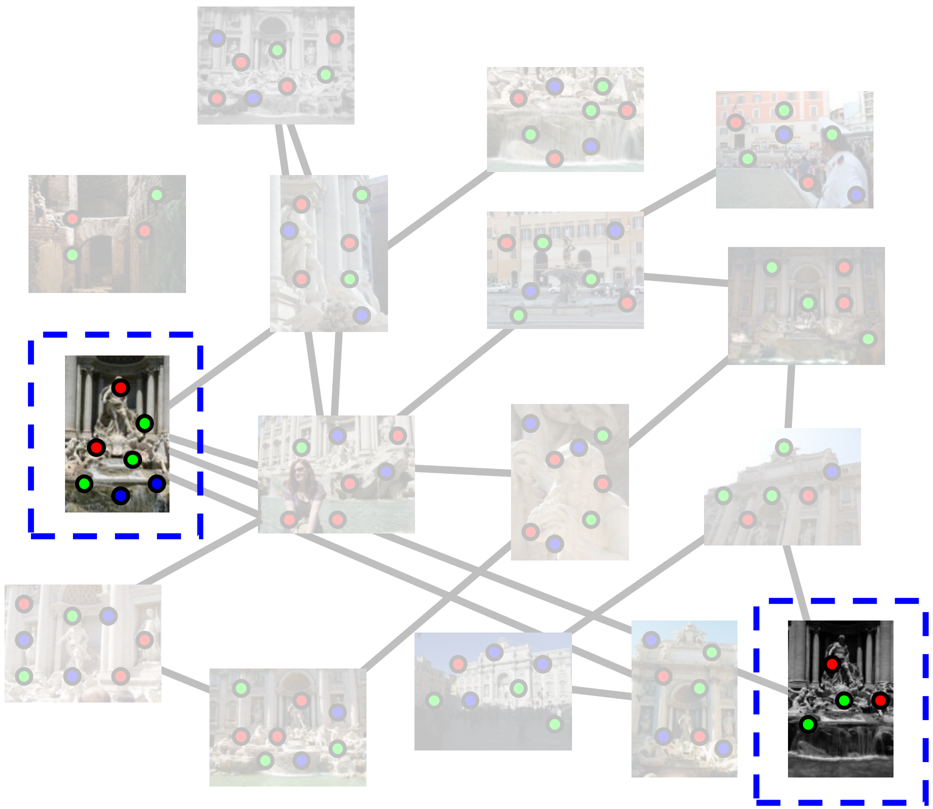
Inputs: feature tracks



Outputs: 3D cameras and points

- The good news:
  - Structure from motion with two cameras is (relatively) easy
  - Once we have an initial model, it's easy to add new cameras
- Idea:
  - Start with a small seed reconstruction, and grow

Incremental SfM



- Automatically select an initial pair of images

1. Picking the initial pair

- We want a pair with many matches, but which has as large a baseline as possible

✓ lots of matches
✗ small baseline

✓ large baseline
✗ very few matches

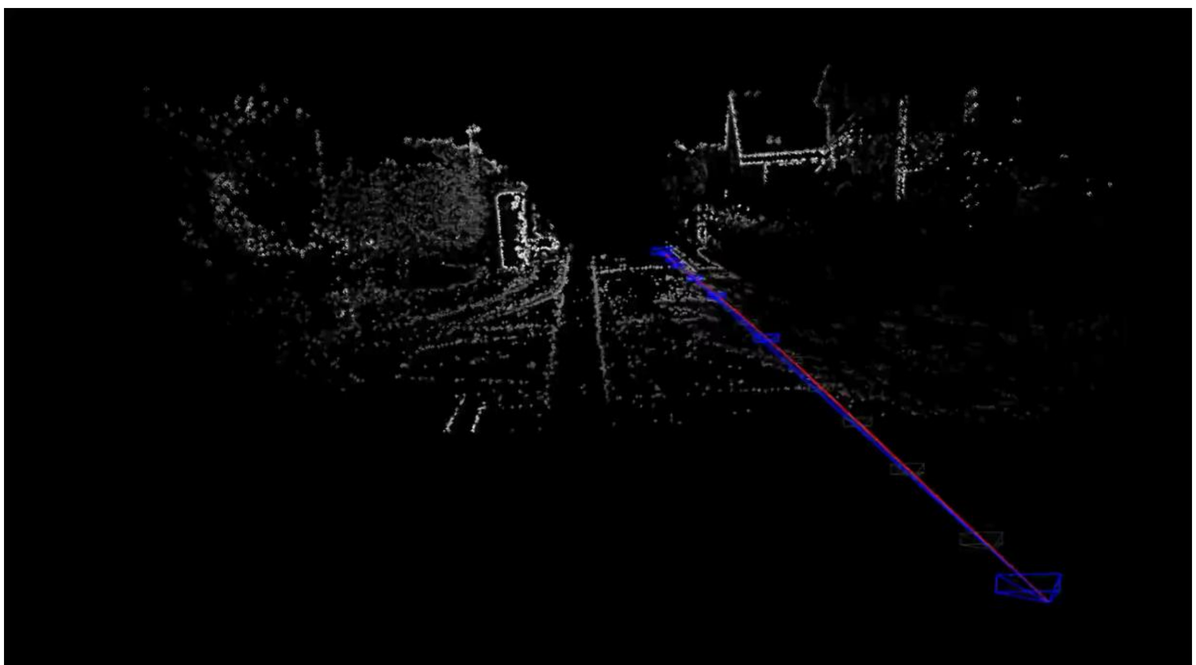
✓ large baseline
✓ lots of matches

Incremental SfM: Algorithm

1. Pick a strong initial pair of images
2. Initialize the model using two-frame SfM
3. While there are connected images remaining:
   a. Pick the image which sees the most existing 3D points
   b. Estimate the pose of that camera
   c. Triangulate any new points
   d. Run bundle adjustment

Visual Simultaneous Localization and Mapping (V-SLAM)

- Main differences with SfM:
  - Continuous visual input from sensor(s) over time
  - Gives rise to problems such as loop closure
  - Often the goal is to be online / real-time



Video from Daniel Cremer's Lab

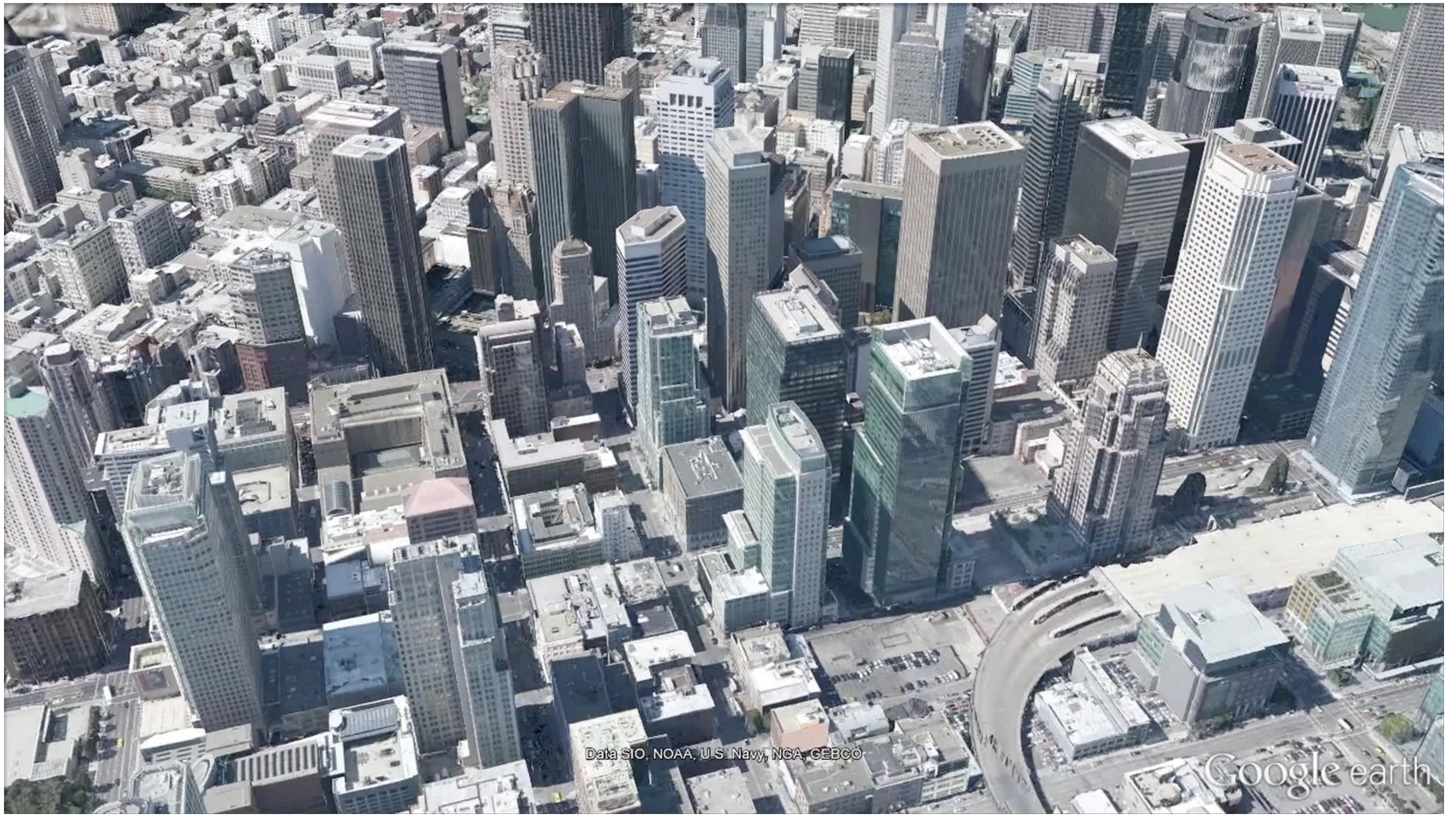
Applications: Match Moving

Or Motion tracking, solving for camera trajectory

Integral for visual effects (VFX)

Why?

What if we want solid models?

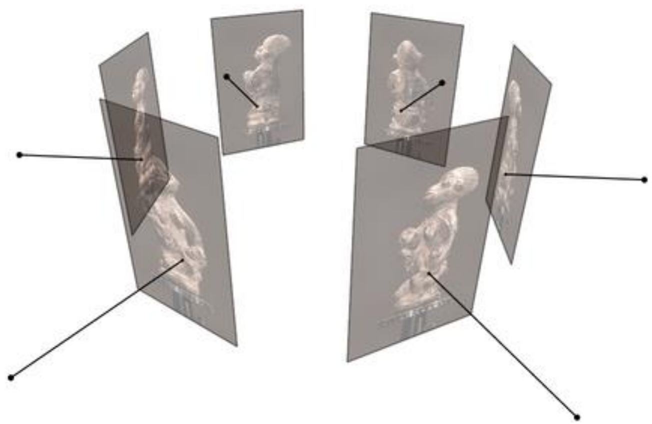


Slide credit: Noah Snavely

Multi-view Stereo (Lots of calibrated images)

Input: calibrated images from several viewpoints (known camera: intrinsics and extrinsics)

Output: 3D Model

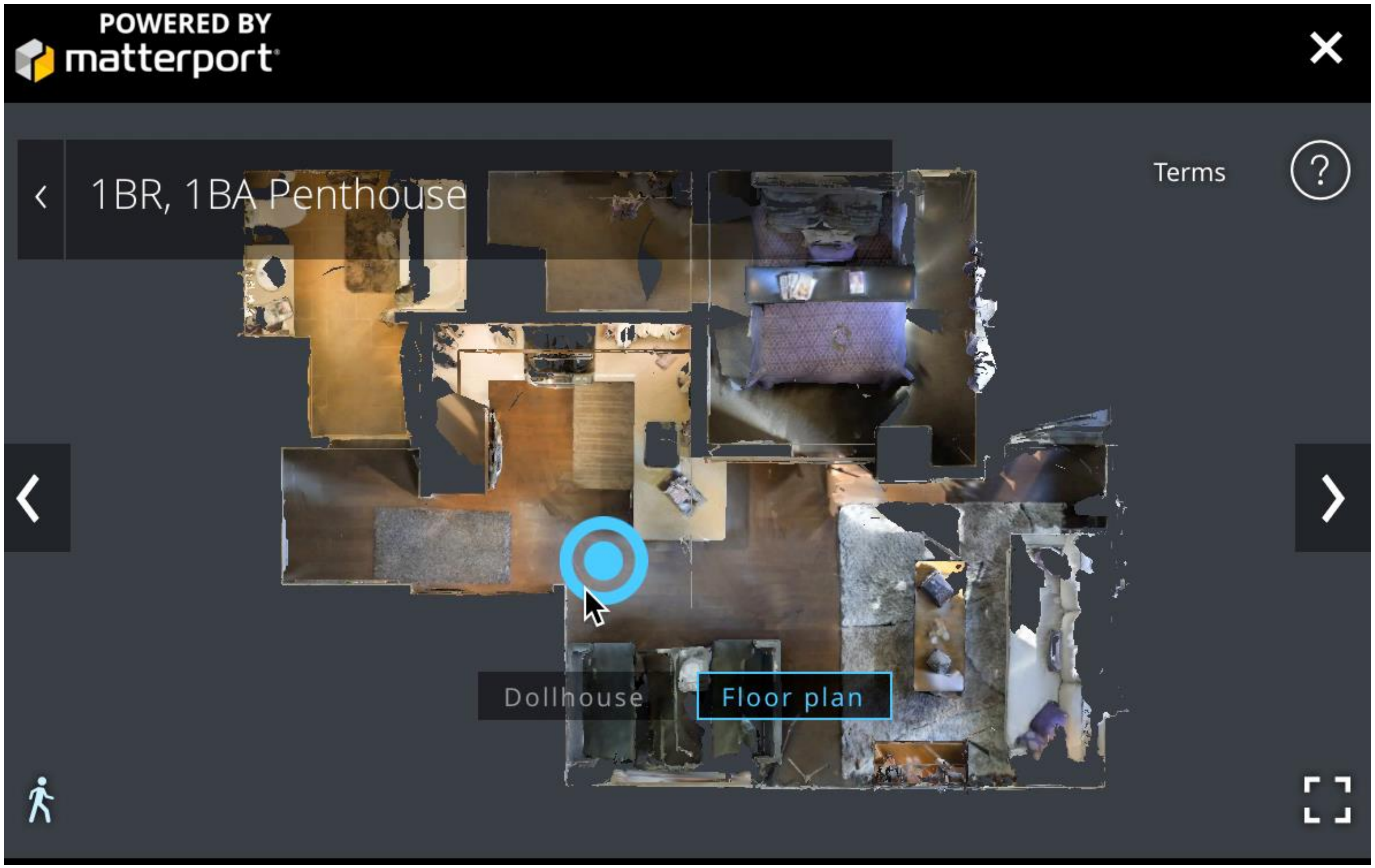


Figures by Carlos Hernandez

Slide credit: Noah Snavely

In general, conducted in a controlled environment with multi-camera setup that are all calibrated

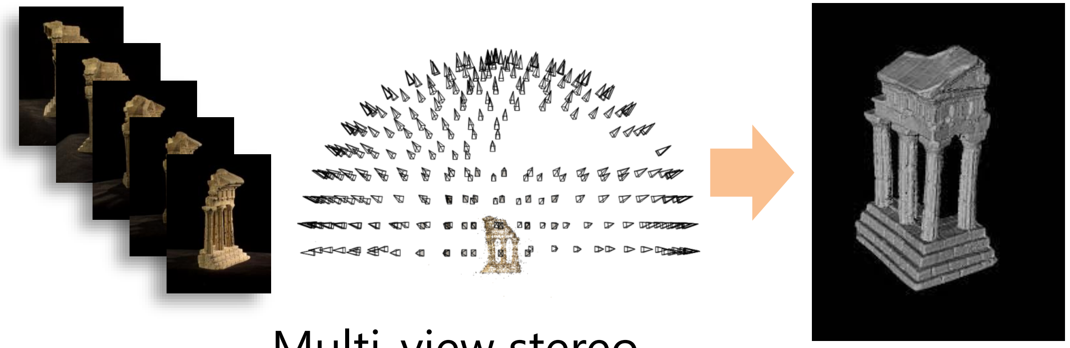
Slide credit: Noah Snavely


POWERED BY
matterport
1BR, 1BA Penthouse
Terms
Dollhouse
Floor plan

Multi-view Stereo

Problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

Binocular Stereo



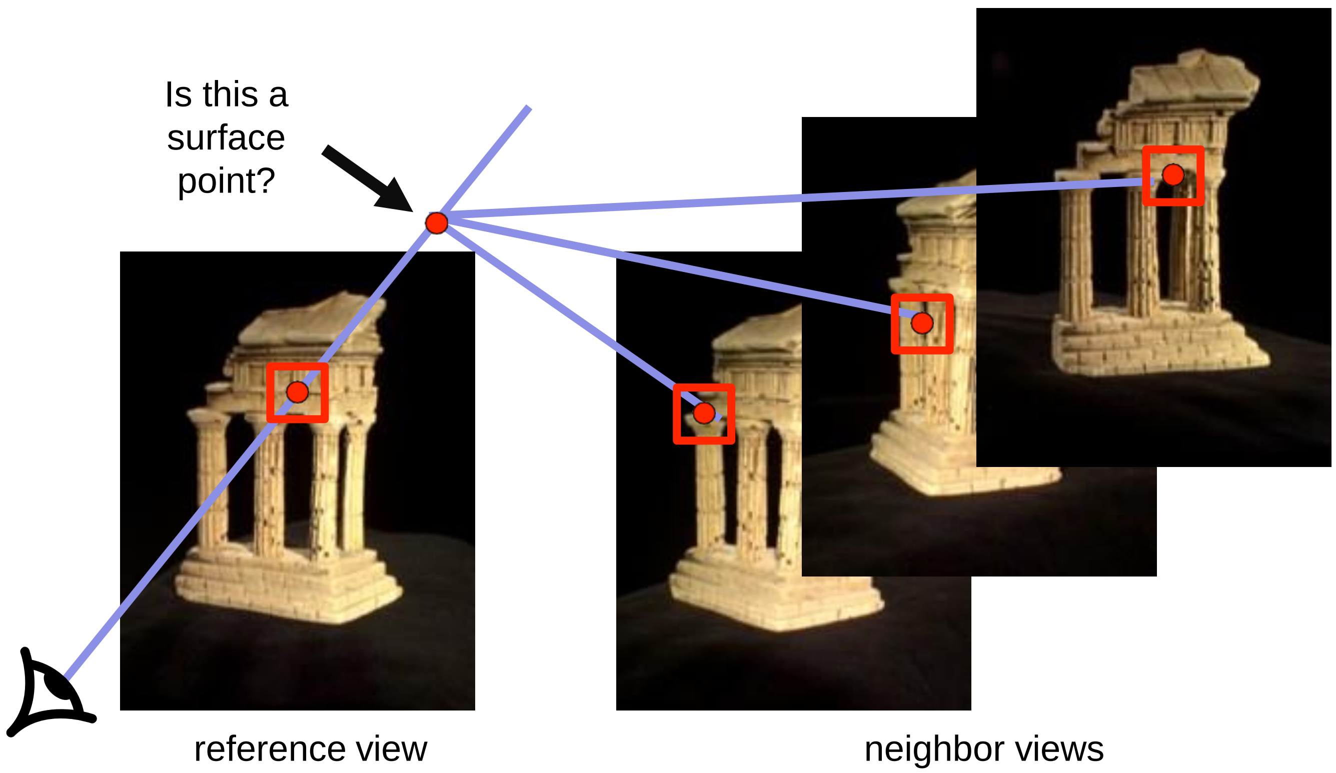
Multi-view stereo

Slide credit: Noah Snavely

Examples: Panoptic studio

http://domedb.perception.cs.cmu.edu/

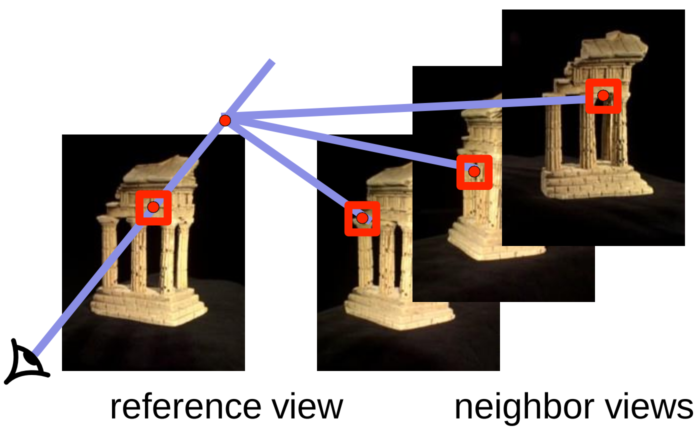
Multi-view stereo: Basic idea

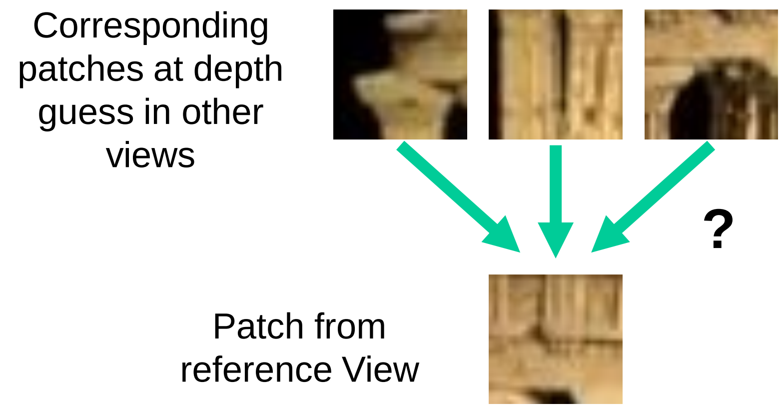


Source: Y.
Furukawa

Multi-view stereo: Basic idea

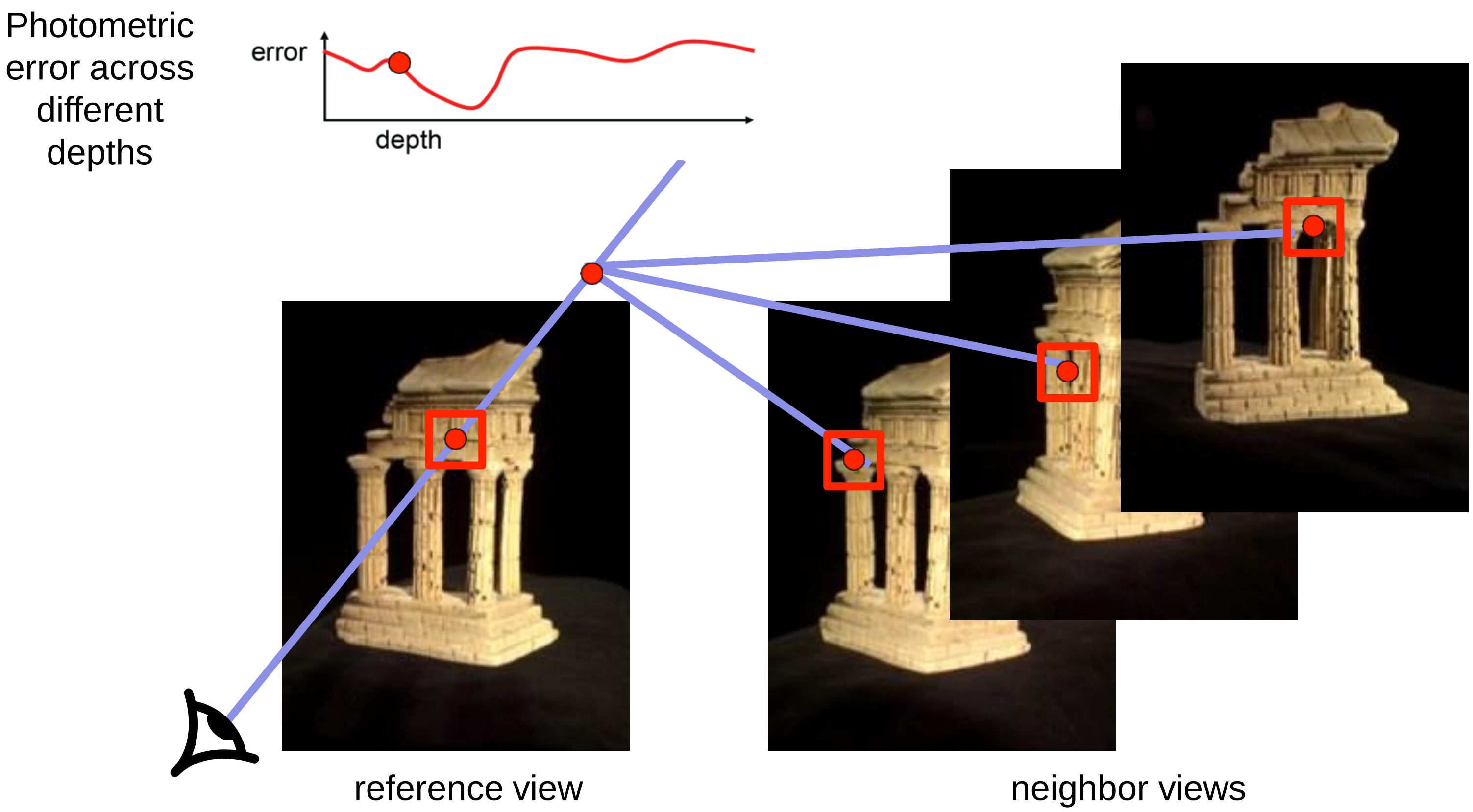
Evaluate the likelihood of geometry at a particular depth for a particular reference patch:





Source: Y. Furukawa

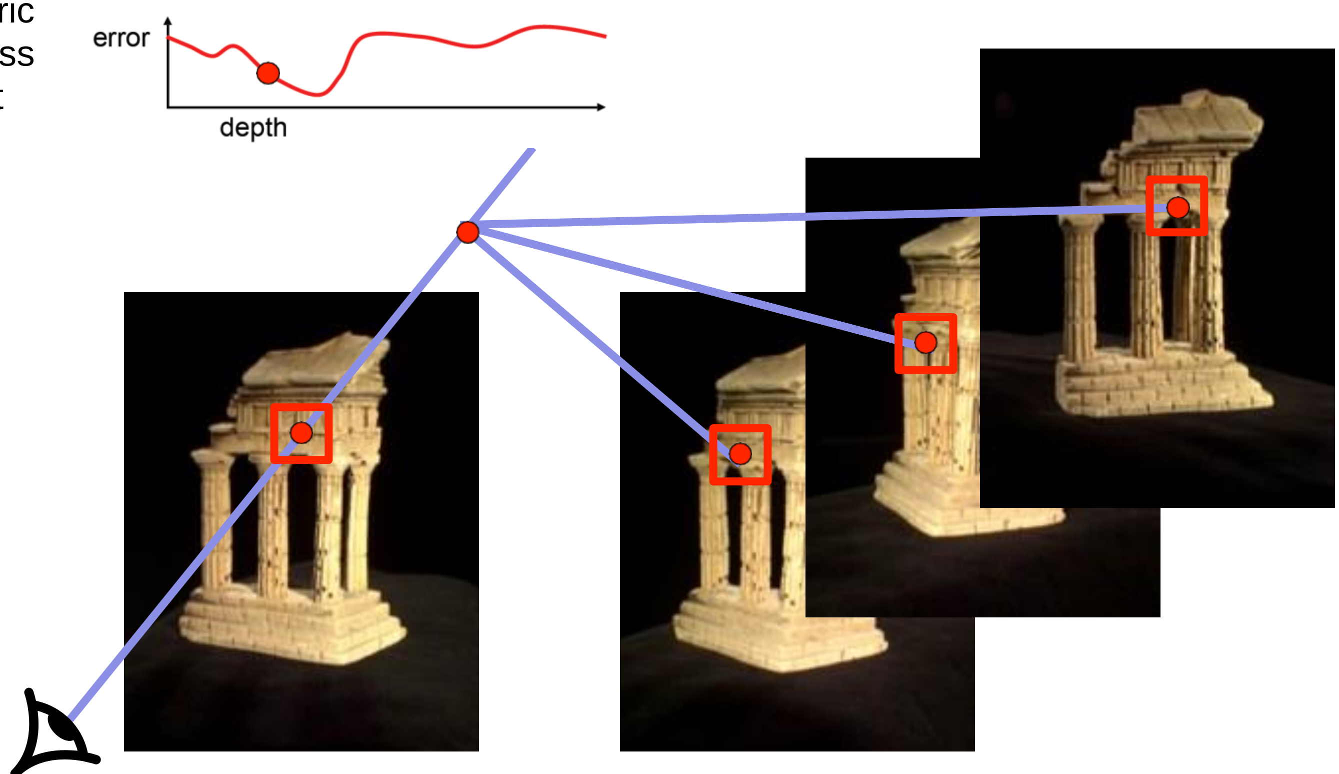
Multi-view stereo: Basic idea



Source: Y. Furukawa

Multi-view stereo: Basic idea

Photometric error across different depths



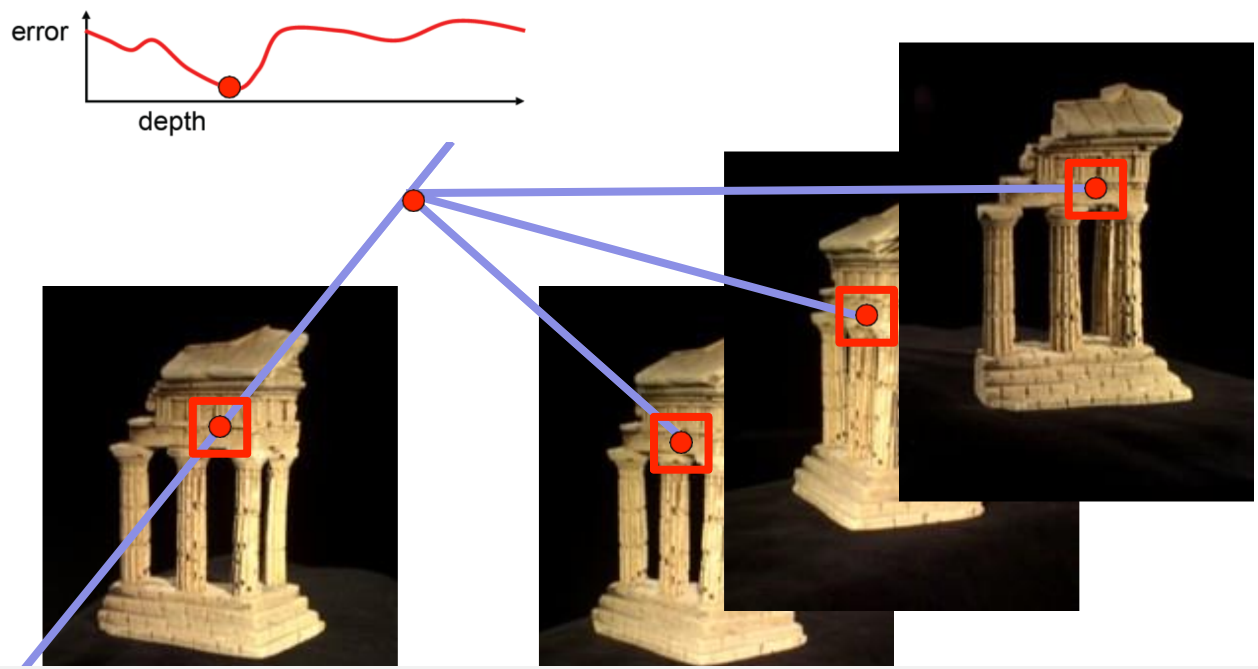
reference view

neighbor views

Source: Y. Furukawa

Multi-view stereo: Basic idea

Photometric error across different depths



In this manner, solve for a depth map over the whole reference view

Multi-view stereo: advantages

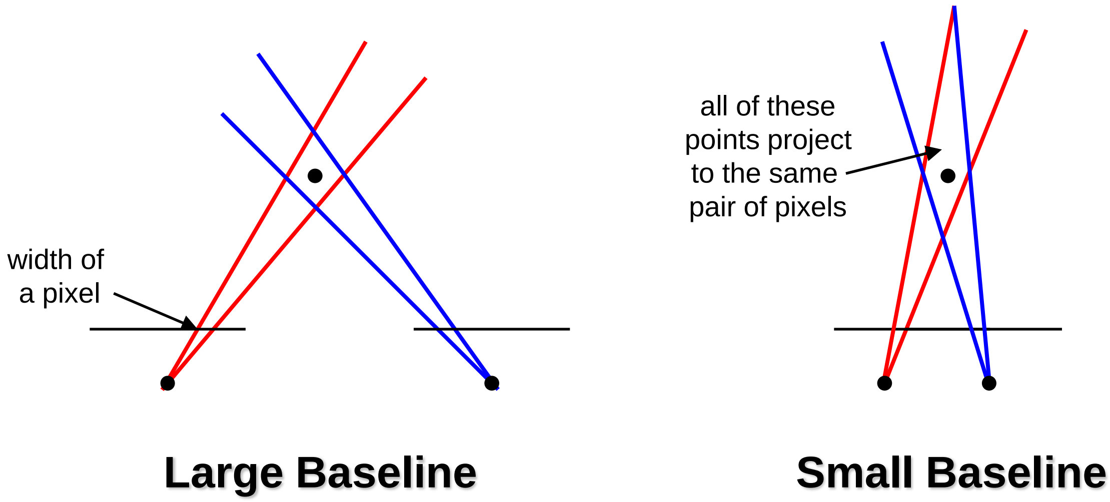
Can match windows using more than 1 other image, giving a **stronger match signal**

If you have lots of potential images, can **choose the best subset** of images to match per reference image

Can reconstruct a depth map for each reference frame, and the merge into a **complete 3D model**

Source: Y. Furukawa

Choosing the baseline

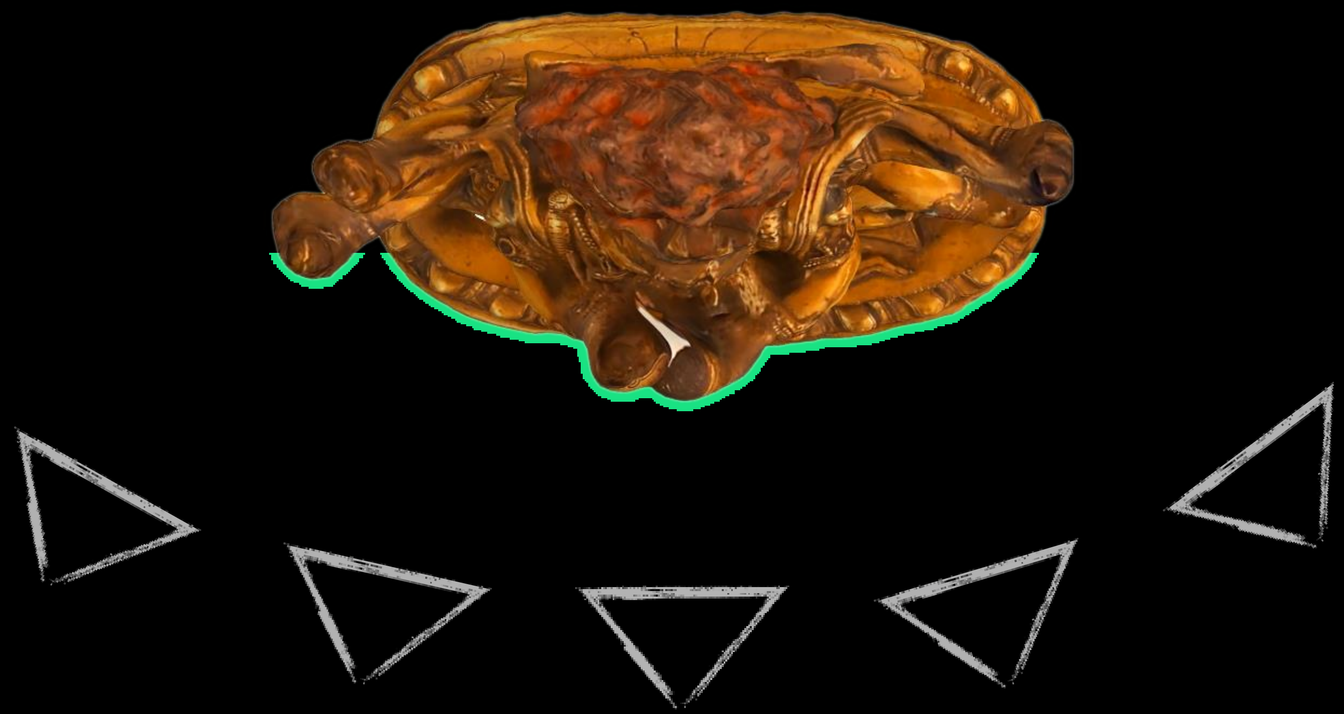


What’s the optimal baseline?

- Too small: large depth error
- Too large: difficult search problem

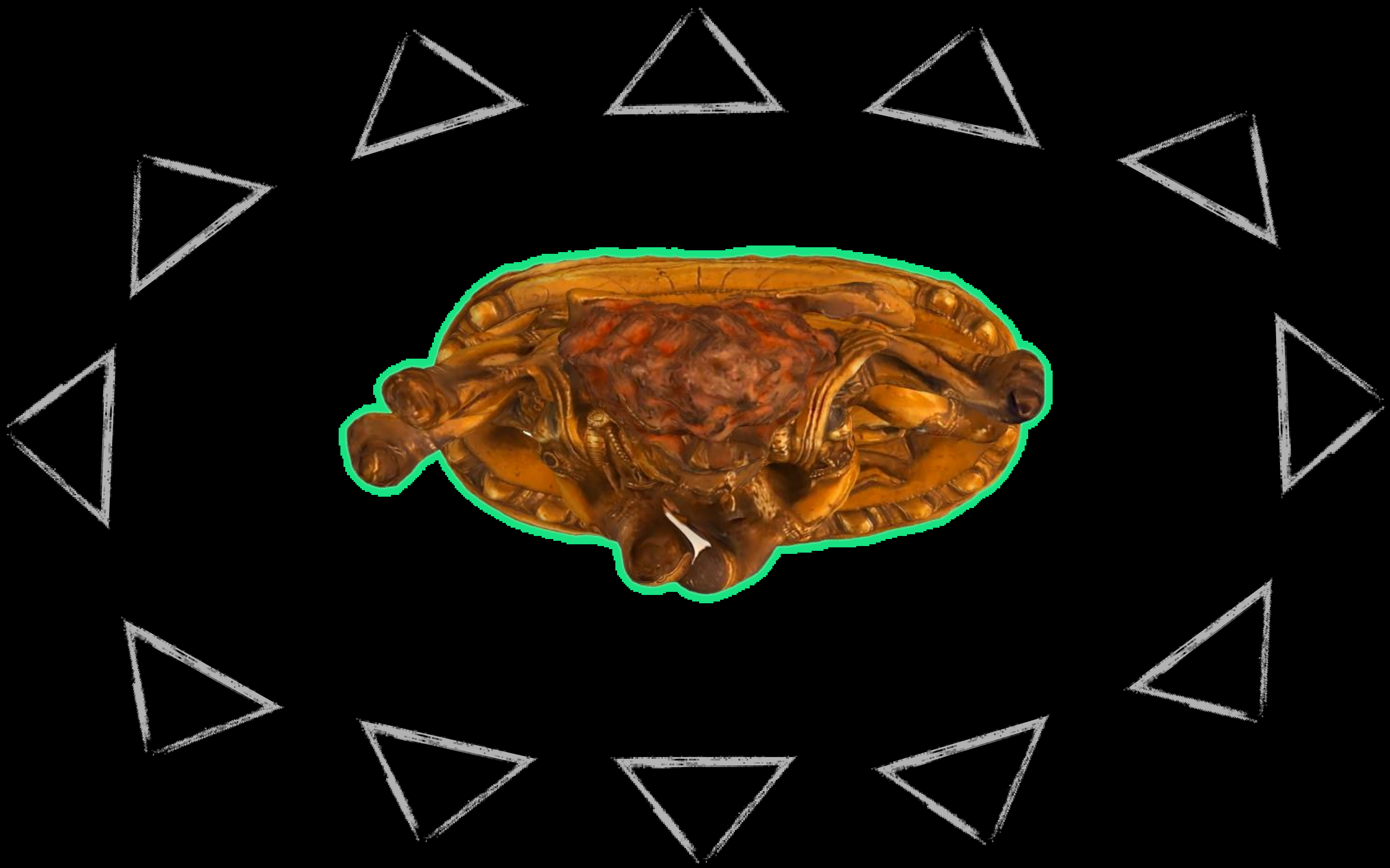
Slide credit: Noah Snavely

Single depth map often isn’t enough



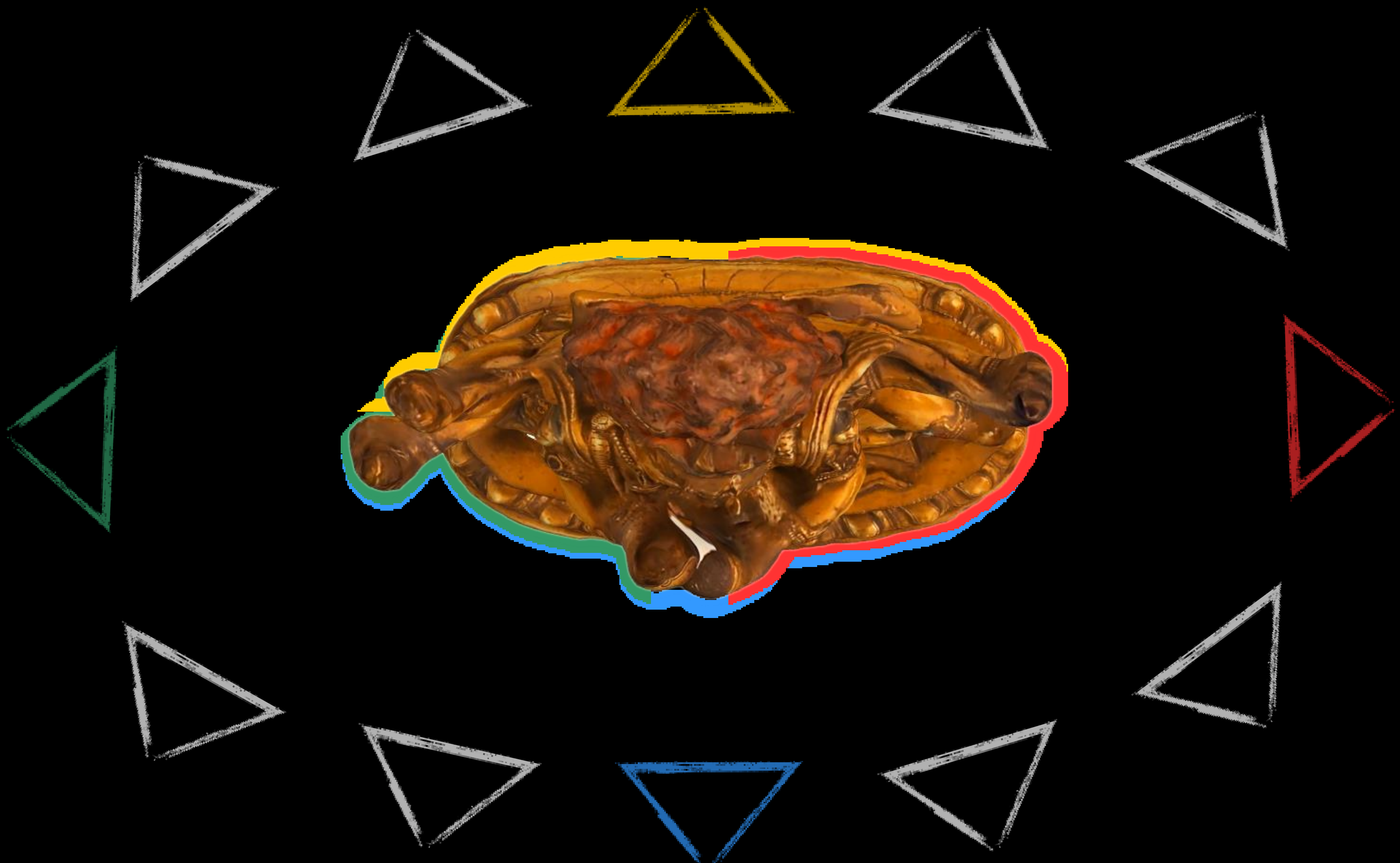
Source: N. Snavely

Really want full coverage



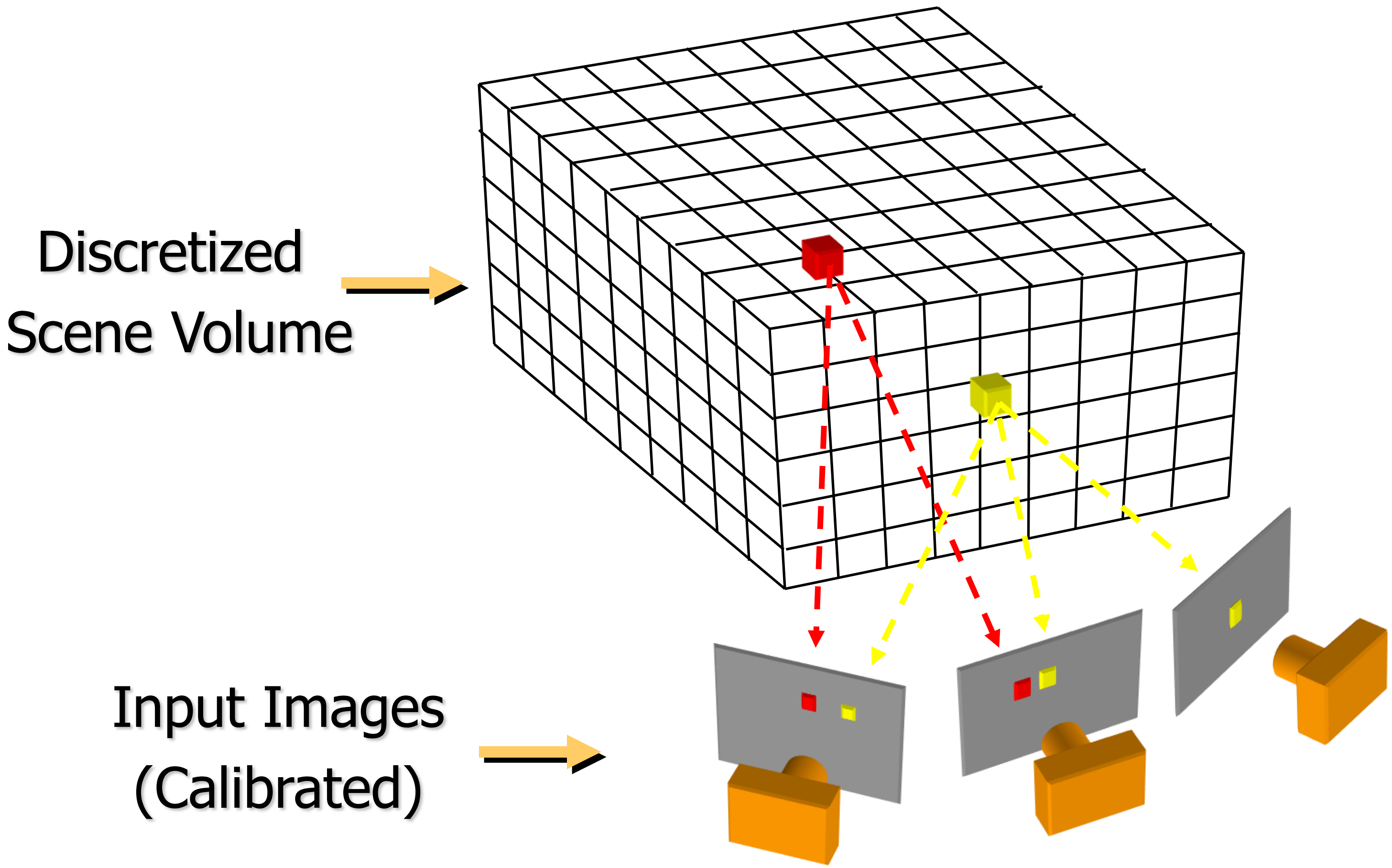
Source: N. Snavely

Idea: Combine many depth maps



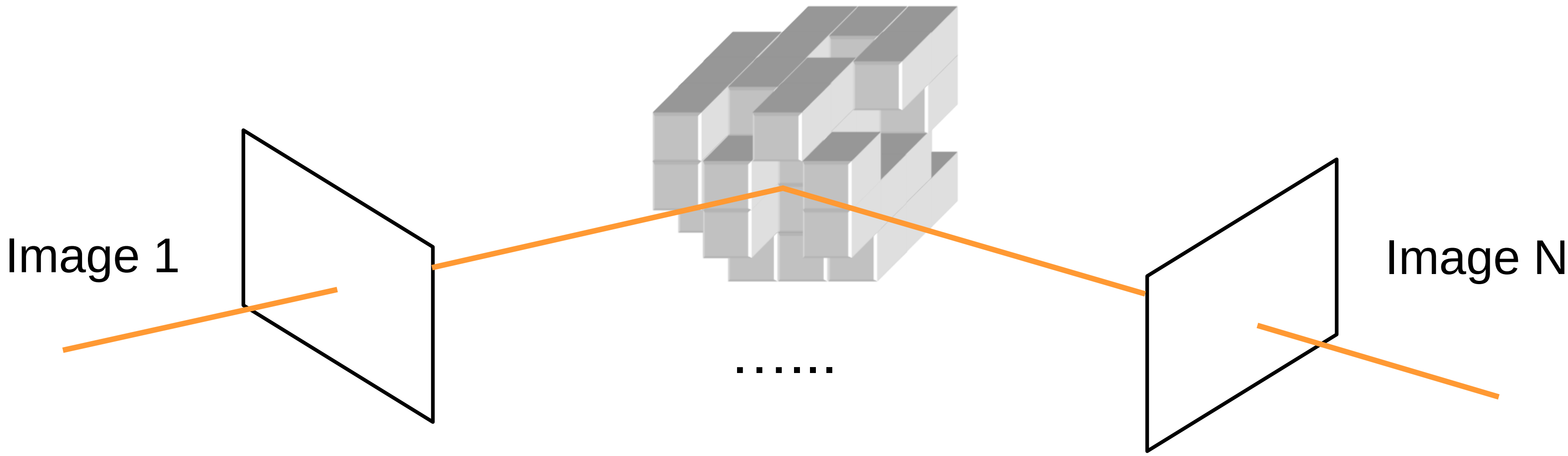
Source: N. Snavely

Volumetric stereo



Goal: **Assign RGB values to voxels in V**
photo-consistent **with images**

Space Carving



Space Carving Algorithm

- Initialize to a volume V containing the true scene
- Choose a voxel on the outside of the volume
- Project to visible input images
- Carve if not photo-consistent
- Repeat until convergence

K. N. Kutulakos and S. M. Seitz, **A Theory of Shape by Space Carving**, *ICCV* 1999

Space Carving Results

Input Image (1 of 45)

Reconstruction

Reconstruction

Reconstruction

Source: S. Seitz

Space Carving Results

Input Image
(1 of 100)

Reconstruction

Source: S. Seitz

Tool for you: COLMAP

https://github.com/colmap/colmap

General SfM + MVS pipeline