Kinematic Synthesis

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Classifying Mechanisms

Several dichotomies

Serial and Parallel

Few DOFS and Many DOFS

Planar/Spherical and Spatial

Rigid and Compliant
# Mechanism Trade-offs

<table>
<thead>
<tr>
<th>Workspace</th>
<th>Rigidity</th>
<th>Designing Kinematics</th>
<th>No. of Actuators</th>
<th>Flexibility of Motion</th>
<th>Complexity of Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial</td>
<td>Large</td>
<td>Low</td>
<td>Simple</td>
<td>Depends</td>
<td>Depends</td>
</tr>
<tr>
<td>Parallel</td>
<td>Small</td>
<td>High</td>
<td>Complex</td>
<td>Depends</td>
<td>Depends</td>
</tr>
<tr>
<td>Few DOF</td>
<td>Small</td>
<td>Depends</td>
<td>Complex</td>
<td>Few</td>
<td>Little</td>
</tr>
<tr>
<td>Many DOF</td>
<td>Large</td>
<td>Depends</td>
<td>Simple</td>
<td>Many</td>
<td>A lot</td>
</tr>
</tbody>
</table>

Serial, Many DOF  
Parallel, Many DOF  
Parallel, Few DOF  
Serial, Few DOF
Problems in Kinematics

**Dimensions**

**Joint Parameters**

**End Effector Coordinates**

**Forward Kinematics**
Known: Dimensions, Joint Parameters
Solve for: End Effector Coordinates

**Inverse Kinematics**
Known: Dimensions, End Effector Coordinates
Solve for: Joint Parameters

**Synthesis**
Known: End Effector Coordinates
Solve for: Dimensions, Joint Parameters
Challenges in Kinematics

• Using sweeping generalizations, how difficult is it to solve
  – forward kinematics
  – inverse kinematics
  – synthesis

over different types of mechanisms?
• Ranked on a scale of 1 to 4 with 4 being the most difficult:

<table>
<thead>
<tr>
<th></th>
<th>Forward Kinematics</th>
<th>Inverse Kinematics</th>
<th>Synthesis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Serial</td>
<td>Parallel</td>
<td>Serial</td>
</tr>
<tr>
<td>Planar</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Spherical</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Spatial</td>
<td>1.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Planar
Planar
Spherical
Spherical
Spatial
Synthesis Approaches

• Synthesis equations are hard to solve because almost nothing is known about the mechanism beforehand

Some Methods for Synthesis

• Graphical constructions – 1 soln per construction
• Use atlases (libraries) (see http://www.saltire.com/LinkageAtlas/)
• Evolutionary algorithms – multiple solutions
• Optimization – 1 soln, good starting approximation required
• Sampling potential pivot locations
• Resultant elimination methods – all solutions, limited to simpler systems
• Groebner Bases – all solutions, limited to simpler systems
• Interval analysis – all solutions within a box of useful geometric parameters
• Homotopy – all solutions, can handle degrees in the millions and possibly greater with very recent developments
Terminology:

**Circuits** - not dependent on input link specification

**Branches** - dependent on input link specification

Circuit and branches can lead to linkage defects.
Types of Synthesis Problems

a) Function generation: set of input angles and output angles;
b) Motion generation: set of positions and orientations of a workpiece;
c) Path generation: set of points along a trajectory in the workpiece.

Above are examples of function, motion, and path generation for planar six-bar linkages. Analogous problems exist for spherical and spatial linkages of all bars.
Examples of Function Generation

Measurements from stroke survivors

Summation

Mechanism Generated torque
The Bird Example Technique

• Spatial chains are constrained by six-bar function generators

Spatial chain
4 DOF

Function generators to control joint angles

A single DOF constrained spatial chain

Goal: achieve accurate biomimetic motion
Examples of Motion Generation and Path Generation
Kinematics and Polynomials

- Kinematics are intimately linked with polynomials because they are composed of revolute and prismatic joints which describe circles and lines in space, which are algebraic curves.
- These lines and circles combine to describe more complex algebraic surfaces.

The Plane  Sphere  Circular Cylinder  Elliptical Cylinder
PPS  TS  CS  PRS

Hyperboloid  Right Torus  Torus
RPS  RRS  RRS
Polynomials and Complexity

- Linkages can always be expressed as **polynomials**
- When new links are added, the complexity of synthesis rapidly increases

### Synthesis Problems

<table>
<thead>
<tr>
<th></th>
<th>Max Number of Positions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Function</td>
</tr>
<tr>
<td>Four-bar</td>
<td>5</td>
</tr>
<tr>
<td>Stephenson I</td>
<td>5</td>
</tr>
<tr>
<td>Stephenson II</td>
<td>11</td>
</tr>
<tr>
<td>Stephenson III</td>
<td>11</td>
</tr>
</tbody>
</table>

The degree of polynomial synthesis equations increases rapidly when links are added.
Ways to Model Kinematics

- **Planar**
  - Rotation matrices, homogeneous transforms, vectors
  - Planar quaternions
  - Complex numbers

- **Spherical**
  - Rotation matrices
  - Quaternions

- **Spatial**
  - Rotation matrices, homogeneous transforms, vectors
  - Dual quaternions

- All methods create equivalent systems, although they might look different. Different conveniences are made available by how kinematics are modelled.
Planar Kinematics With Complex Numbers

\[
\begin{align*}
\begin{bmatrix} a_x \\ a_y \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \end{bmatrix} &= \begin{bmatrix} a_x + b_x \\ a_y + b_y \end{bmatrix} \\
(a_x + ia_y) + (b_x + ib_y) &= (a_x + b_x) + i(a_y + b_y)
\end{align*}
\]

\[
\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} a_x \cos \theta - a_y \sin \theta \\ a_x \sin \theta + a_y \cos \theta \end{bmatrix}
\]

\[
e^{i\theta}(a_x + ia_y) = (\cos \theta + i\sin \theta)(a_x + ia_y) = (a_x\cos \theta - a_y\sin \theta) + i(a_x\sin \theta + a_y\cos \theta)
\]