Private Decision Trees

CS 261 Fall 2018

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Fully Homomorphic Encryption

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- First construction in 2009 by Craig Gentry using ideal lattices (won ACM best doctoral dissertation award)

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- Malleability isn't always desired (El Gamal, RSA)

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- Tends to be more efficient than complete FHE, especially for circuits with a small number of levels

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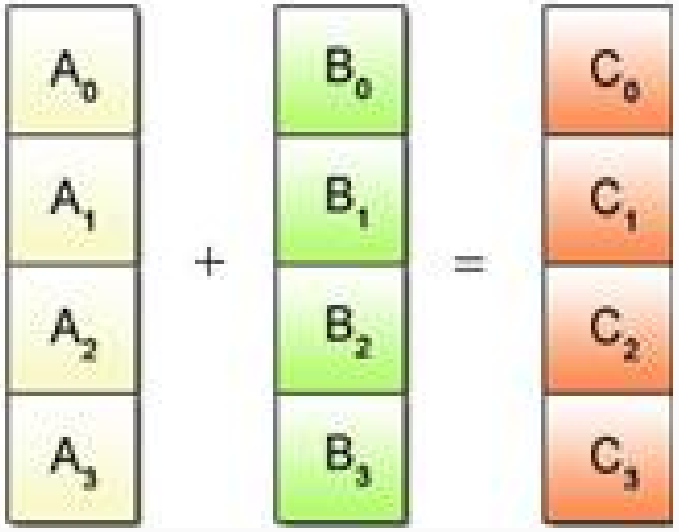

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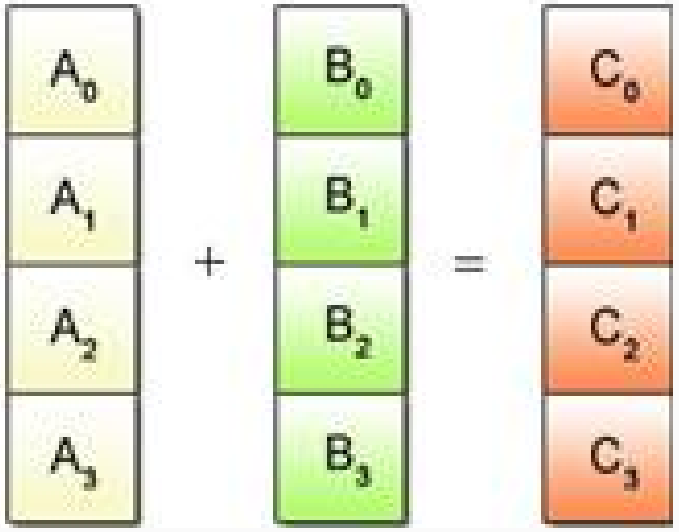

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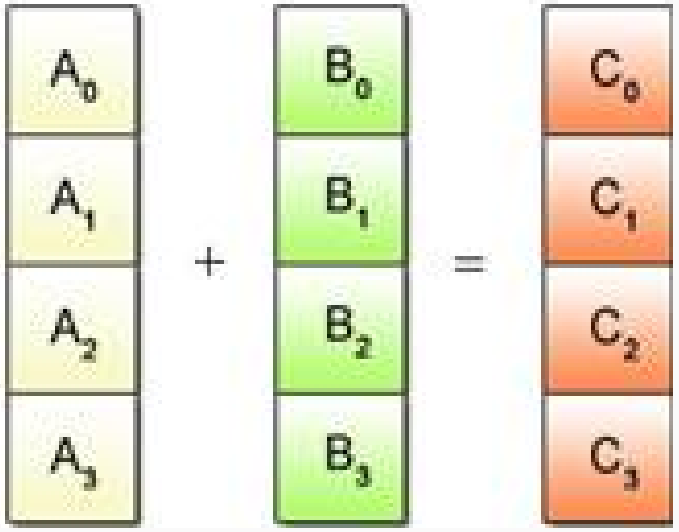

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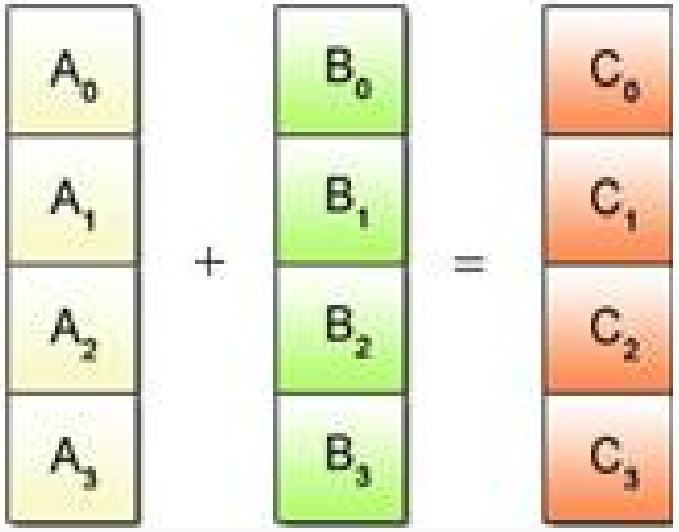

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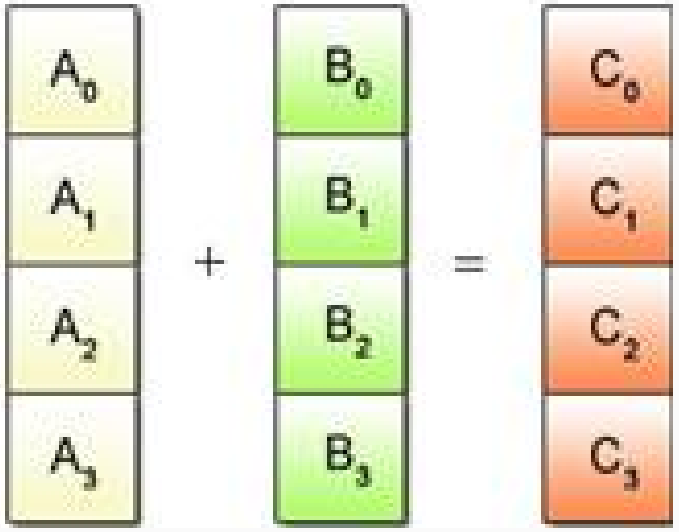

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$$c \leftarrow Enc(m_1, m_2, \ldots, m_n)$$

$$Evaluate(f, c) \rightarrow Enc(f(m_1), f(m_2), \ldots, f(m_n))$$

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In summary we'll try to:

- $\rightarrow$ Minimize the depth of our function circuit
- $\rightarrow$ Use FHE SIMD Slots to parallelize repeated instructions
- $\rightarrow$ Work in $\mathbb{F}_2$

Decision Tree Classifiers

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- Interior nodes correspond to partitioning rules
- Leaf nodes correspond to class labels

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- Perform multiplications in a tree-like manner to reduce the depth of the polynomial circuit from h_{max} to $log_2 h_{max}$
- In order to use $\mathbb{F}_2$ for FHE speedup, we encode each class as a binary string and use SIMD slots to parallelize