Overview

- Moving from geometry to rendering and appearance
- Major part of course: 10 lectures
  - Includes discussion of current research topics
  - Assignments 2 and 3
- First couple of lectures recap of 184 for those in it
  - Formal illumination, reflection, global illumination
  - But quickly move to new and advanced material
- Remember Mesh Assignment due Oct 7

Radiometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional
    Reflectance Distribution Function or BRDF
  - Reflection Equation
  - Simple BRDF models
  - Environment Maps

Angles and Solid Angles

- Angle $\theta = \frac{l}{r}$
  - circle has $2\pi$ radians
- Solid angle $\Omega = \frac{A}{r^2}$
  - sphere has $4\pi$ steradians

Differential Solid Angles

$$dA = (r \, d\theta) (r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi$$
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: \( L(x, \omega) \) (W/m\(^2\) sr)
- Flux given by \( d\Phi = L(x, \omega) \cos \theta \ d\omega \ dA \)

Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[
\frac{d\Phi_1}{d\omega_1} = L_1 \ dA_1 = L_2 \ dA_2 = d\Phi_2
\]

\[
\frac{d\omega_1}{d\omega_1} = \frac{dA_1}{r^2} \quad \frac{d\omega_2}{d\omega_2} = \frac{dA_2}{r^2}
\]

\[
\therefore L_1 = L_2
\]
Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances

Consequences
- Radiance associated with rays in a ray tracer
- Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance $E$ is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle ($\cos \theta \, d\omega$)
  - Uniform illumination:
    $\text{Irradiance} = \pi$ [CW 24,25]
    - Units: W/m$^2$
- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

Directional Power Arriving at a Surface

\[ d^2\Phi_i(x,\omega) = L_i(x,\omega) \cos \theta \, dA \, d\omega \]

Irradiance from the Environment

\[ d^2\Phi_i(x,\omega) = L_i(x,\omega) \cos \theta \, dA \, d\omega \]

\[ \text{Irradiance} = \int \int L_i(x,\omega) \cos \theta \, d\omega \]

Uniform Area Source

\[ E(x) = \int \int L \cos \theta \, d\omega \]

Uniform Disk Source

Geometric Derivation

Algebraic Derivation

\[ \hat{\Omega} = \pi \sin^2 \alpha \]

\[ \hat{\Omega} = \pi \int \int \cos \theta \, d\phi \, d\cos \theta \]

\[ \hat{\Omega} = 2\pi \cos^2 \theta \]

\[ \hat{\Omega} = \pi \sin^2 \alpha \]

\[ \hat{\Omega} = \frac{r^2}{r^2 + h^2} \]
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Types of Reflection Functions

- Ideal Specular
  - Reflection Law
  - Mirror
- Ideal Diffuse
  - Lambert’s Law
  - Matte
- Specular
  - Glossy
  - Directional diffuse

Materials

From Apodaca and Gritz, Advanced RenderMan

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials
The BRDF

Bidirectional Reflectance-Distribution Function

\[ f(\omega_i, \omega_o) = \frac{\frac{dL_i(\theta, \varphi, \omega_i)}{d\Omega_i}}{\frac{dL_o(\theta, \varphi, \omega_o)}{d\Omega_o}} \]

\[ L_i(\theta, \varphi, \omega_i) \]

\[ L_o(\theta, \varphi, \omega_o) \]

\[ \Omega_i \]

\[ \Omega_o \]

BRDF

- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

\[ f(\omega_i, \omega_o) = \frac{L_i(\omega_i)}{L_o(\omega_o) \cos \theta \ d\omega_o} \]

\[ L_i(\omega_i) = L_o(\omega_o) f(\omega_i, \omega_o) \cos \theta \ d\omega_o \]

Properties of BRDF’s

1. Linearity

From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

\[ f_i(\omega_i \rightarrow \omega_o) = f_i(\omega_o \rightarrow \omega_i) \]

3. Isotropic vs. anisotropic

\[ f_i(\theta_i, \phi_i, \theta_o, \phi_o) = f_i(\theta_o, \phi_o, \theta_i, \phi_i) \]

Reciprocity and isotropy

\[ f_i(\theta_i, \phi_i, \theta_o, \phi_o) = f_i(\theta_o, \phi_o, \theta_i, \phi_i) \]

4. Energy conservation

Energy Conservation

\[ \frac{\partial \Phi_i}{\partial \Omega_i} = \int \frac{L_i(\omega_i) \cos \theta_i \ d\omega_i}{\int L_i(\omega_i) \cos \theta_i \ d\omega_i} \]

\[ \int \int f_i(\omega_i \rightarrow \omega_o) L_i(\omega_i) \cos \theta_i \ d\omega_i \]

\[ \Omega \]

\[ \Omega \]

Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction

Isotropic

Anisotropic
Reflection Equation

\[ L_r(\omega_r) = \sum L_i(\omega_i) f(\omega_i, \omega_r)(\omega_r \cdot n) \]

- Reflected Radiance (Output Image)
- Incident radiance (from light source)
- BRDF
- Cosine of Incident angle

Reflected Radiance (Output Image)
Incident radiance (from light source)
BRDF
Cosine of Incident angle

Replace sum with integral

\[ L_r(\omega_r) = \int L_i(\omega_i) f(\omega_i, \omega_r)(\cos \theta \cdot n) \, d\omega_i \]

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Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

\[ L_r(\omega_r) = \int L_i(\omega_i) \cos \theta \, d\omega_i = f_r \int L_i(\omega_i) \cos \theta \, d\omega_i \]

\[ = f_r \pi L_i \]

\[ M = \int L_i(\omega_i) \cos \theta \, d\omega_i = \int \cos \theta \, d\omega_i = \pi L_i \]

\[ \rho_r = \frac{M}{E} = \frac{\pi f_r \pi L_i}{\frac{\pi}{\omega_r}} \Rightarrow f_r = \rho_r \frac{E}{\pi} \]

Lambert’s Cosine Law

\[ M = \rho_r E = \rho_r E \cos \theta \]
One famous analytically derived BRDF is the Torrance-Sparrow model. Torrance-Sparrow is used to model specular surface, like the Phong model.
- more accurate than Phong
- has more parameters that can be set to match different materials
- derived based on assumptions of underlying geometry. (instead of ‘because it works well’)

Assume the surface is made up grooves at the microscopic level.
Assume the faces of these grooves (called microfacets) are perfect reflectors.
Take into account 3 phenomena
- Shadowing
- Masking
- Interreflection

Torrance-Sparrow Result

\[ f = \frac{F(\theta)G(\omega, \omega')D(\theta_0)}{4\cos(\theta)\cos(\theta')} \]

- Fresnel term: allows for wavelength dependency
- Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.
- Distribution: distribution function determines what percentage of micro facets are oriented to reflect in the viewer direction.
- How much of the macroscopic surface is visible to the light source
- How much of the macroscopic surface is visible to the viewer
**Other BRDF models**

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

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**Environment Maps**

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

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**Reflection Equation**

\[ \mathbf{L}_r(\omega_i, \omega_o) = \int \mathbf{L}_i(\omega_i) f(\omega_i, \omega_o, \omega_o \cdot \mathbf{n}) \, d\omega_i \]

Reflected Radiance (Output Image)

Environment map (continuous)

Cosine of Incident angle

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**Demo**