

Homework

All these exercises are meant to be done with the methods I showed in class, i.e. reasoning about the Gauss map, drawing, etc. Q3 and Q4 need about a line of algebra. You will find that if you try to do them any other way, the questions will turn nasty awfully fast.

We will study two shading models on a surface that is smooth and compact, and has no holes; such a surface is like a deformed sphere. In particular, for any given normal vector, there is at least one point on the surface that has that normal. We assume that the albedo (reflectance) of the surface is one. In the first model, which I shall call the shadow model, the surface radiosity at the point on the surface whose parameter values are (s, t) is modeled by

$$\max\{N(s, t) \cdot S, 0\}$$

where S is some constant vector, which we shall assume to have length 1, and N is the unit normal at that point. This means that the shading is never less than zero. In the second model, which I shall call the simplified model, the surface radiosity at the point on the surface whose parameter values are (s, t) is modeled by

$$N(s, t) \cdot S$$

yielding possible negative values.

Question 1: The maximum value of radiosity under either model is 1. Show that, for almost every value of S and for both models, there are an odd number of points on the surface where the radiosity has that value, and that this number of points is greater than or equal to 1.

Question 2: We refer to points on the surface where the radiosity is at a local maximum (resp. minimum), but not equal to 1, as local maxima (resp. minima). Show that local maxima (resp. minima) occur along parabolic lines for both models.

Question 3: For the simplified model, compute

$$\int_S K dA$$

where S is the surface, I is the radiosity and K is the Gaussian curvature. Would this value change if the surface was like a torus? Now compute this value for the shadow model. Would this value change if the surface was like a torus?

Question 4: In either model, points where the normal turns away from the source (i.e. the normal and the source are perpendicular) are points on the self-shadow boundary.

Show that, if the source is at infinity --- i.e. S is constant over the surface --- and X is the tangent to the self-shadow boundary, we have that, *on the self shadow boundary*

$$H(S, X) = 0$$

At what points on the surface are X and S parallel? Do these points have special properties in renderings of the surface, in general?