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CHECKMATE!

A Brief Introduction
to Game Theory

The World



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Game Theory: Economic or Combinatorial?

- Economic

- ◇ von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*
- ◇ Matrix games
- ◇ Prisoner's dilemma
- ◇ Incomplete info, simultaneous moves
- ◇ Goal: Maximize payoff

- Combinatorial

- ◇ Sprague and Grundy's 1939 *Mathematics and Games*
- ◇ Board (table) games
- ◇ Nim, Domineering
- ◇ Complete info, alternating moves
- ◇ Goal: Last move



Combinatorial Game Theory

History

- Early Play
 - ◇ Egyptian wall painting of Senat (c. 3000 BC)
- Theory
 - ◇ C. L. Bouton's analysis of Nim [1902]
 - ◇ Sprague [1936] and Grundy [1939] Impartial games and Nim
 - ◇ Knuth *Surreal Numbers* [1974]
 - ◇ Conway *On Numbers and Games* [1976]
 - ◇ Prof. Elwyn Berlekamp (UCB), Conway, & Guy *Winning Ways* [1982]



What is a combinatorial game?

- Two players (Left & Right) move alternately
- No chance, such as dice or shuffled cards
- Both players have perfect information
 - ◊ No hidden information, as in Stratego & Magic
- The game is finite – it must eventually end
- There are no draws or ties
- **Normal Play: Last to move wins!**



What games are out, what are in?

- Out

- ◊ All card games

- ◊ All dice games



- In

- ◊ Nim, Domineering, Dots-and-Boxes, Go, etc.

- ◊ 1,2,...,10 , Kayles, Toads & Frogs, Snake, Tactix, Poison

- In, but not normal play

- ◊ Chess, Checkers, Othello, Tic-Tac-Toe, etc.

“Computational” Game Theory (for non-normal play games)

- **Large games**
 - ◇ Can theorize strategies, build AI systems to play
 - ◇ Can study endgames, smaller version of original
 - Examples: Quick Chess, 9x9 Go, 6x6 Checkers, etc.
- **Small-to-medium games**
 - ◇ Can have computer solve and teach us strategy
 - ◇ **GAMESMAN does exactly this**
 - It can solve BOTH normal and non-normal play games



Computational Game Theory

- Simplify games / value

- ◇ Store turn in position

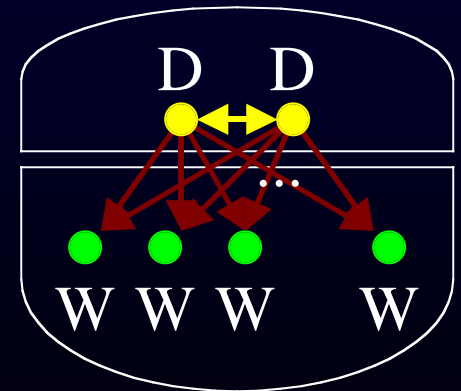
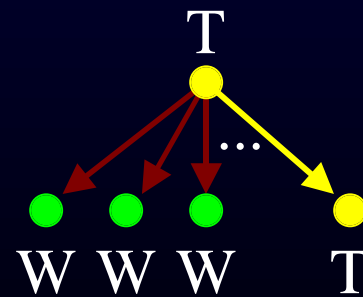
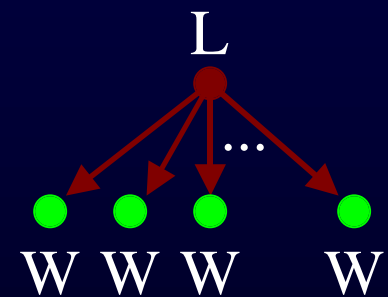
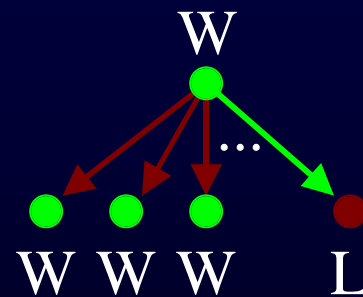
- ◇ Each position is (for player whose turn it is)

- Winning (\exists losing child)

- Losing (All children winning)

- Tieing ($\neg \exists$ losing child, but \exists tieing child)

- Drawing (can't force a win or be forced to lose)



Exciting Game Theory Research at Berkeley

- Combinatorial Game Theory Workshop
 - ◇ MSRI July 24-28th, 2000: Son of Games of No Chance
 - ◇ 1994 Workshop book: Games of No Chance
- Prof. Elwyn Berlekamp
 - ◇ Dots & Boxes, Go endgames
 - ◇ Economist's View of Combinatorial Games
- **Dr. Dan Garcia**
 - ◇ **Undergraduate Game Theory Research Group**
<http://www.cs.berkeley.edu/~ddgarcia/research/gametheory/current/>

