VARK website
The learning-styles view has acquired great influence within the education field, and is frequently encountered at levels ranging from kindergarten to graduate school. There is a thriving industry devoted to publishing learning-styles tests and guidebooks for teachers, and many organizations offer professional development workshops for teachers and educators built around the concept of learning styles. Our review of the literature disclosed ample evidence that children and adults will, if asked, express preferences about how they prefer information to be presented to them. There is also plentiful evidence arguing that people differ in the degree to which they have some fairly specific aptitudes for different kinds of thinking and for processing different types of information. However, we found virtually no evidence for the interaction pattern mentioned above, which was judged to be a precondition for validating the educational applications of learning styles.
La Jolla Study.

Pashler et.al.

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Just so, says Pashler – most of it is "weak."
Snippy, snippy

Science Samples.

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Why do some experts say that knowing your learning style does not contribute to improved learning?

The statement is true, in the same way that knowing you have a disease does not cure the disease or weighing yourself does not fix obesity. It is the next step that is important - when people make changes to their learning, based on their VARK preferences, their learning will be enhanced. They do this by using strategies that align with their preferences. It is what you do after you learn your preferences that has the potential to make a difference.
Vark response.

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Willingham Video.

Willingham video.
Willingham Video.

People have different capabilities:
Willingham Video.

Willingham video.

People have different capabilities:
  visual,
Willingham Video.

Willingham video.

People have different capabilities:
  visual, auditory,
Willingham Video.

People have different capabilities:
  visual, auditory, kinesthetic.
Willingham Video.

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Information is inherent content:
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Information is inherent content: visual,
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Information is inherent content: visual, hearing,
Willingham Video.

People have different capabilities: visual, auditory, kinesthetic.

Information is inherent content: visual, hearing, meaning.
What do students mean...

“Professor should use more figures, because I am a visual leaner.”
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“Professor should use more figures, because I am a visual leaner.”

In computer science, inherent content may lend itself to pictures, or different views to understanding.
“Professor should use more figures, because I am a visual learner.”

In computer science, inherent content may lend itself to pictures, or different views to understanding.

Example:
Recursion:
  code
What do students mean...

“Professor should use more figures, because I am a visual leaner.”

In computer science, inherent content may lend itself to pictures, or different views to understanding.

Example:
Recursion:
   code
   stack
“Professor should use more figures, because I am a visual learner.”

In computer science, inherent content may lend itself to pictures, or different views to understanding.

Example:
Recursion:
  code
  stack
  tree
What do students mean...

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In computer science, inherent content may lend itself to pictures, or different views to understanding.

Example:
Recursion:
  code
  stack
  tree
  inductive proof.
What do students mean...

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Example:
Recursion:
  code
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In computer science, inherent content may lend itself to pictures, or different views to understanding.

Example:
Recursion:
  code
  stack
  tree
  inductive proof.
  environment diagram

Connect views even.
Example: coloring.

Thm: Any graph with degree at most $d$ can be $d + 1$ colored.
Example: coloring.

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DasGupta, Papadimitriou, Vazirani, “Algorithms”
Example: coloring.
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.
Example: coloring.

Thm: Any graph with degree at most $d$ can be $d+1$ colored.


“Greedily consistently color nodes with one of $d+1$ colors. At least one color is available when a node is colored.”
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“Greedily consistently color nodes with one of $d + 1$ colors. At least one color is available when a node is colored.”

Available = Green, Red, Yellow, Blue
Example: coloring.

Thm: Any graph with degree at most $d$ can be $d + 1$ colored.


“Greedily consistently color nodes with one of $d + 1$ colors. At least one color is available when a node is colored.”

Available = Green, Red, Yellow, Blue
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Example: coloring.
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Available = Green, Red, Yellow, Blue
Available = Green, Yellow, Blue
Available = Yellow, Blue
Available = Blue
Example: coloring.

Thm: Any graph with degree at most \( d \) can be \( d+1 \) colored.


“Greedily consistently color nodes with one of \( d+1 \) colors. At least one color is available when a node is colored.”

Available = Green, Red, Yellow, Blue
Available = Green, Yellow, Blue
Available = Yellow, Blue
Available = Blue

...so 4 is enough!
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.
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Graph: $G = (V, E)$. 
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$. 
The MIT-way...(CLRS)... 

Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.
For $u \in V$:
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.
For $u \in V$:
  color[u] = False
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.

For $u \in V$:
- $\text{color}[u] = \text{False}$

For $u \in V$:
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.

For $u \in V$:
- $\text{color}[u] = \text{False}$

For $u \in V$:
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Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.

For $u \in V$:
- color[$u$] = False

For $u \in V$:
- Available = Colors
  for $v$ in $N(u)$:
Thm: Any graph with degree at most $d$ can be $d+1$ colored.

Graph: $G = (V, E)$.
Colors $= \{1, \ldots d+1\}$.
For $u \in V$:
  - $\text{color}[u] = \text{False}$
For $u \in V$:
  - Available $= \text{Colors}$
  - for $v$ in $N(u)$:
    - if $\text{color}[v] \neq \text{NULL}$
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = \{1, \ldots d + 1\}.

For $u \in V$:
\begin{itemize}
  \item color[u] = False
\end{itemize}

For $u \in V$:
\begin{itemize}
  \item Available = Colors
  \item for $v$ in $N(u)$:
    \begin{itemize}
      \item if color[v] != NULL
        \begin{itemize}
          \item Available = Colors - color[v]
        \end{itemize}
    \end{itemize}
\end{itemize}
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.

Colors = \{1, \ldots d + 1\}.

For $u \in V$:
- color[u] = False

For $u \in V$:
- Available = Colors
  
  for $v$ in $N(u)$:
    - if color[v] != NULL
      
      Available = Colors - color[v]

  color[u] = Available.first()
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.

Colors = $\{1, \ldots d + 1\}$.

For $u \in V$:
- $\text{color}[u] = \text{False}$

For $u \in V$:
- $\text{Available} = \text{Colors}$
  - for $v$ in $N(u)$:
    - if $\text{color}[v] \neq \text{NULL}$
      - $\text{Available} = \text{Colors} - \text{color}[v]$
  - $\text{color}[u] = \text{Available}.\text{first}()$
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors $= \{1, \ldots, d + 1\}$.
For $u \in V$:
    color[u] $= \text{False}$
For $u \in V$:
    Available $= \text{Colors}$
    for $v$ in $N(u)$:
        if color[v] $\neq \text{NULL}$
            Available $= \text{Colors} - \text{color[v]}$
    color[u] $= \text{Available.first()}$

Proof:
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors $= \{1, \ldots, d + 1\}$.
For $u \in V$:
  \[ \text{color}[u] = \text{False} \]
For $u \in V$:
  \[ \text{Available} = \text{Colors} \]
  \[ \text{for } v \text{ in } N(u): \]
  \[ \text{if color}[v] \neq \text{NULL} \]
    \[ \text{Available} = \text{Colors} - \text{color}[v] \]
  \[ \text{color}[u] = \text{Available}.first() \]

Proof: Invariant 1:
Thm: Any graph with degree at most $d$ can be $d + 1$ colored.

Graph: $G = (V, E)$.
Colors = $\{1, \ldots d + 1\}$.

For $u \in V$:
  
  color[u] = False

For $u \in V$:
  Available = Colors
  for $v$ in $N(u)$:
    if color[v] != NULL
      Available = Colors - color[v]
  color[u] = Available.first() 

Todo: graph coloring.

Make lecture active!
Todo: graph coloring.

Make lecture active!
Make into discussion activity!
Todo: graph coloring.

Make lecture active!
Make into discussion activity!
Example: Use idea in problems:
Todo: graph coloring.

Make lecture active!

Make into discussion activity!

Example: Use idea in problems:
  Graph and Complement can be colored with $n + 1$ colors.
Every planar graph has a node of degree 6.
Just for fun..

Every planar graph has a node of degree 6.
6 color a planar graph!
Completing the square

Version 1:
Completing the square

Version 1: “Just the facts, Ma’am”
Completing the square.

Solve $x^2 + 16x - 57 = 0$. 

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$.

Is $x^2 + 16x - 57$ a perfect square?

$2a = 16 \rightarrow a = 8$ and $a^2 = 64 \neq -57$.

Not a perfect square.

$x^2 + 16x = 57$.

$x^2 + 16x + 64 = 57 + 64$.

$(x + 8)^2 = 121$.

$x + 8 = \pm 11$.

$x = -19$ or $x = 3$.

Move 57 over. Make left hand side a perfect square! Can take square root!

$2ax = 16x \rightarrow a = \frac{16x}{2} = 8x$, $a^2 = (8x)^2 = 64x^2$.
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

If perfect square can take square root.
Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?
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Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

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Is \( x^2 + 16x - 57 \) a perfect square?

\[
2a = 16
\]
Completing the square.

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Is \( x^2 + 16x - 57 \) a perfect square?

\[
2a = 16 \quad \rightarrow \quad a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64
\]
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

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Is \( x^2 + 16x - 57 \) a perfect square?

\[ 2a = 16 \rightarrow a = \frac{16}{2} = 8 \text{ and } a^2 = 64 \neq -57! \]
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

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2a = 16 \Rightarrow a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64 \neq -57!
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Not a perfect square.
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

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Is \( x^2 + 16x - 57 \) a perfect square?

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2a = 16 \rightarrow a = \frac{16}{2} = 8 \text{ and } a^2 = 64 \neq -57! \\
\text{Not a perfect square.}
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Completing the square.

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Solve $x^2 + 16x - 57 = 0$.

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Is $x^2 + 16x - 57$ a perfect square?

$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57$!

Not a perfect square.

$x^2 + 16x = 57$ \quad Move 57 over.
Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?

$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57$

Not a perfect square.

$x^2 + 16x = 57$ \hspace{1cm} Move 57 over.

$x^2 + 2ax + a^2 = 57 + a^2$ \hspace{1cm} Make left hand side a perfect square!
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

\[
2a = 16 \rightarrow a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64 \neq -57!
\]

Not a perfect square.

\[
\begin{align*}
  x^2 + 16x &= 57 \\
  x^2 + 2ax + a^2 &= 57 + a^2
\end{align*}
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

If perfect square can take square root.

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\]

Not a perfect square.

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x^2 + 16x = 57 \\
x^2 + 2ax + a^2 = 57 + a^2
\]

Move 57 over.

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$x^2 + 16x = 57$

$x^2 + 2ax + a^2 = 57 + a^2$

Move 57 over.

Make left hand side a perfect square!

Can take square root!

$2ax = 16x \rightarrow a$
Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

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$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57$!

Not a perfect square.

$x^2 + 16x = 57$

Move 57 over.

$x^2 + 2ax + a^2 = 57 + a^2$

Make left hand side a perfect square!

Can take square root!

$2ax = 16x \rightarrow a = \frac{16x}{2x}$, \hspace{1cm} .
Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?  

$2a = 16 \Rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57$!  

Not a perfect square.

$x^2 + 16x - 57 = 0$  

Move 57 over.

Make left hand side a perfect square!

Can take square root!

$2ax = 16x \Rightarrow a = \frac{16x}{2x} = 8$. 

Doh!
Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?

$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57$!

Not a perfect square.

$x^2 + 16x = 57$

$x^2 + 2ax + a^2 = 57 + a^2$

Move 57 over.

Make left hand side a perfect square!

Can take square root!

$2ax = 16x \rightarrow a = \frac{16x}{2x} = 8$, $a^2 = 8^2 = 64$. 

Doh!
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

If perfect square can take square root.

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2a = 16 \implies a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64 \neq -57!
\]

Not a perfect square.

\[
x^2 + 16x = 57
\]
\[
x^2 + 2ax + a^2 = 57 + a^2
\]
\[
x^2 + 16x + 64 = 57 + 64
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!

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2ax = 16x \implies a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.
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Completing the square.

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

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\]

Not a perfect square.

\[
x^2 + 16x = 57
\]

Move 57 over.

\[
x^2 + 2ax + a^2 = 57 + a^2
\]

Make left hand side a perfect square!

\[
x^2 + 16x + 64 = 57 + 64
\]

Can take square root!

\[
x^2 + 16x + 64 = 121
\]

\[
2ax = 16x \quad \rightarrow \quad a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.
\]
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \( (x + a)^2 = x^2 + 2ax + a^2 \)

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

\[
2a = 16 \quad \Rightarrow \quad a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64 \neq -57!
\]

Not a perfect square.

\[
x^2 + 16x = 57 \\
x^2 + 2ax + a^2 = 57 + a^2 \\
x^2 + 16x + 64 = 57 + 64 \\
x^2 + 16x + 64 = 121 \\
(x + 8)^2 = 121
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!

\[
2ax = 16x \quad \Rightarrow \quad a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.
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Completing the square.

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\[2a = 16 \rightarrow a = \frac{16}{2} = 8\] and \(a^2 = 64 \neq -57!\)

Not a perfect square.

\[x^2 + 16x = 57\]
\[x^2 + 2ax + a^2 = 57 + a^2\]
\[x^2 + 16x + 64 = 57 + 64\]
\[x^2 + 16x + 64 = 121\]
\[(x + 8)^2 = 121\]
\[(x + 8) = 11\]

Move 57 over.
Make left hand side a perfect square!
Can take square root!

\[2ax = 16x \rightarrow a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.\]
Completing the square.

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

\[ 2a = 16 \rightarrow a = \frac{16}{2} = 8 \text{ and } a^2 = 64 \neq -57! \]

Not a perfect square.

\[
\begin{align*}
x^2 + 16x &= 57 \\
x^2 + 2ax + a^2 &= 57 + a^2 \\
x^2 + 16x + 64 &= 57 + 64 \\
x^2 + 16x + 64 &= 121 \\
(x + 8)^2 &= 121 \\
(x + 8) &= \pm 11
\end{align*}
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!

\[ 2ax = 16x \rightarrow a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64. \]
Completing the square.

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\[x^2 + 16x = 57\]
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\[x^2 + 16x + 64 = 121\]
\[(x + 8)^2 = 121\]
\[(x + 8) = \pm 11\]
\[x = \pm 11 - 8\]

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x^2 + 16x + 64 = 121 \\
(x + 8)^2 = 121 \\
(x + 8) = \pm 11 \\
x = \pm 11 - 8 \\
\rightarrow x = 19 \text{ Doh!}
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&x = \pm 11 - 8 \\
&\rightarrow x = -19
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Move 57 over.
Make left hand side a perfect square!
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$$x^2 + 2ax + a^2 = 57 + a^2$$

$$x^2 + 16x + 64 = 57 + 64$$
$$x^2 + 16x + 64 = 121$$

$(x + 8)^2 = 121$

$(x + 8) = \pm 11$

$x = \pm 11 - 8$

$\rightarrow x = -19$

or $x = 11 - 8 = 3$. 

Move 57 over.

Make left hand side a perfect square!

Can take square root!

$$2ax = 16x \rightarrow a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.$$
Completing the square.

V2: Engage.
Completing the square.

V2: Engage. “Learn-gage”.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$. 

Recall Perfect square: $(x + a)^2 = ?$

- (A) $x^2 + 2ax + a^2$
- (B) $x^2 + a^2$
- (C) $(x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)$

A and C.

A is simplification.

If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?

- (A) Sí
- (B) No

B. No.

$a = 16 \rightarrow a = 16 \div 2 = 8$ and $a^2 = 64 \neq -57$!

Not a perfect square.

Later in semester.

Y es? No?

Uses less space!

Students are trained.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

Recall Perfect square: $(x + a)^2 =$
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

Recall Perfect square: $(x + a)^2 = ?$

(A) $x^2 + 2ax + a^2$

Later in semester.

Uses less space!

Students are trained.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

Recall Perfect square: $(x + a)^2 = ?$

(A) $x^2 + 2ax + a^2$

(B) $x^2 + a^2$

Later in semester.
Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0 \).

Recall Perfect square: \((x + a)^2 = ?\)

(A) \( x^2 + 2ax + a^2 \)

(B) \( x^2 + a^2 \)

(C) \((x + a)x + (x + a)a\)

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

(A) Yes

(B) No

B. No.

\( a = 16 \rightarrow a = \frac{16}{2} = 8 \) and \( a^2 = 64 \neq -57 \)

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Yes? No?

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(C) $(x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)$
Completing the square: engage?

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(C) $(x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)$

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Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0 \).

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(C) \((x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)\)

A
Completing the square: engage?

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A and C.
Completing the square: engage?

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(C) \((x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2) \)

A and C. A is simplification.

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

(A) \( \text{Yes} \)

(B) \( \text{No} \)

B. No.

\( 2a = 16 \rightarrow a = 8 \) and \( a^2 = 64 \neq -57 \! \) 

Not a perfect square.

Later in semester.

Yes? No?

Uses less space!

Students are trained.
Completing the square: engage?

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If perfect square can take square root.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

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Not a perfect square.

Later in semester.

Yes?

No?

Uses less space!

Students are trained.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

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If perfect square can take square root.
Is $x^2 + 16x - 57$ a perfect square?

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Completing the square: engage?

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Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0 \).

Recall Perfect square: \((x + a)^2 = ?\)

(A) \( x^2 + 2ax + a^2 \)

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(C) \( (x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2) \)

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If perfect square can take square root.

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Solve $x^2 + 16x - 57 = 0$.

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A and C. A is simplification.

If perfect square can take square root. Is $x^2 + 16x - 57$ a perfect square?

(A) Yes

(B) No

B. No.
Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0. \)

Recall Perfect square: \((x + a)^2 = ?\)

(A) \( x^2 + 2ax + a^2 \)

(B) \( x^2 + a^2 \)

(C) \((x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)\)

A and C. A is simplification.

If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

(A) Yes

(B) No

B. No.

\(2a = 16\)
Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0 \).

Recall Perfect square: \((x + a)^2 = ?\)

(A) \( x^2 + 2ax + a^2 \)

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(C) \((x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)\)

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If perfect square can take square root.

Is \( x^2 + 16x - 57 \) a perfect square?

(A) Yes

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B. No.

\[
2a = 16 \rightarrow a = \frac{16}{2} = 8
\]
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

Recall Perfect square: $(x + a)^2 = ?$

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Completing the square: engage?

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$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57!$

Not a perfect square.
Completing the square: engage?

Solve \( x^2 + 16x - 57 = 0 \).

Recall Perfect square: \( (x + a)^2 = ? \)

\[
\begin{align*}
\text{(A)} & \quad x^2 + 2ax + a^2 \\
\text{(B)} & \quad x^2 + a^2 \\
\text{(C)} & \quad (x + a)x + (x + a)a = (x^2 + ax) + (xa + a^2)
\end{align*}
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Is \( x^2 + 16x - 57 \) a perfect square?

\[
\begin{align*}
\text{(A)} & \quad \text{Yes} \\
\text{(B)} & \quad \text{No}
\end{align*}
\]

B. No.

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2a = 16 \implies a = \frac{16}{2} = 8 \quad \text{and} \quad a^2 = 64 \neq -57!
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Later in semester.
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

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Later in semester. Yes?
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\[ 2a = 16 \rightarrow a = \frac{16}{2} = 8 \text{ and } a^2 = 64 \neq -57! \]

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Later in semester. Yes? No?
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

Recall Perfect square: $(x + a)^2 =$ ?

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Later in semester. Yes? No? Uses less space!
Completing the square: engage?

Solve $x^2 + 16x - 57 = 0$.

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If perfect square can take square root.

Is $x^2 + 16x - 57$ a perfect square?

(A) Yes

(B) No

B. No.

$2a = 16 \rightarrow a = \frac{16}{2} = 8$ and $a^2 = 64 \neq -57!$

Not a perfect square.

Later in semester. Yes? No? Uses less space! Students are trained.
Completing the square

Solve $x^2 + 16x - 57 = 0$. 

Move 57 over. Make left hand side a perfect square! Can take square root! What should $a$ be? 16? 8?
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$  Move 57 over.

Move 57 over. Make left hand side a perfect square! Can take square root!

What should $a$ be? $16$? $8$? $2$?

$ax = 16$  $a = 8$, $a^2 = 64$. Doh!

$x = \pm 11 - 8 \rightarrow x = -19$ or $x = 11 - 8 = 3$. Move 57 over.
Completing the square

Solve \( x^2 + 16x - 57 = 0. \)

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
x^2 + 16x = 57 \\
x^2 + 2ax + a^2 = 57 + a^2
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root! What should \( a \) be? 16? 8? 2?

\[
ax = 16 \quad \rightarrow \quad a = \frac{16}{2} = 8, \quad a^2 = 64
\]

\( x \) = \( -19 \) or \( x = 11 - 8 = 3. \)
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$

Move 57 over.

Make left hand side a perfect square!

$x^2 + 2ax + a^2 = 57 + a^2$

Can take square root!

$\rightarrow x = \pm 11 - 8 \rightarrow x = -19$ or $x = 11 - 8 = 3$. Doh!
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$
$x^2 + 2ax + a^2 = 57 + a^2$

Move 57 over.
Make left hand side a perfect square!
Can take square root!

What should $a$ be?
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$  Move 57 over.
$x^2 + 2ax + a^2 = 57 + a^2$  Make left hand side a perfect square!

Can take square root!

What should $a$ be? 16?
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$
$x^2 + 2ax + a^2 = 57 + a^2$

Move 57 over.
Make left hand side a perfect square!
Can take square root!

What should $a$ be? 16? 8?

$x^2 + 16x + 64 = 57 + 64$
$(x + 8)^2 = 121$
$x + 8 = \pm 11$
$x = -19$ or $x = 3$. 

Doh!
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
\begin{align*}
\text{Move 57 over.} \\
\text{Make left hand side a perfect square!} \\
\text{Can take square root!} \\
\text{What should } a \text{ be? 16? 8? 2ax = 16x}
\end{align*}
\]
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$

$x^2 + 2ax + a^2 = 57 + a^2$

Move 57 over.
Make left hand side a perfect square!
Can take square root!

What should $a$ be? 16? 8? $2ax = 16x$

$\rightarrow a = \ldots$,

Doh!

or $x = \ldots$.
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
\begin{align*}
x^2 + 16x &= 57 \\
x^2 + 2ax + a^2 &= 57 + a^2
\end{align*}
\]

Move 57 over.
Make left hand side a perfect square!
Can take square root!

What should \( a \) be? 16? 8? \( 2ax = 16x \)
\[
\rightarrow a = \frac{16x}{2x}, \quad \text{,}
\]

Doh!

or

\( x = \pm 11 - 8 \)
\[
\rightarrow x = \pm 3.
\]
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

\[
x^2 + 16x = 57
\]
\[
x^2 + 2ax + a^2 = 57 + a^2
\]

- Move 57 over.
- Make left hand side a perfect square!
- Can take square root!
- What should $a$ be? $16$? $8$? $2ax = 16x$

\[
\rightarrow a = \frac{16x}{2x} = 8,
\]

Doh!

or $x = 11 - 8 = 3$.
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \( (x + a)^2 = x^2 + 2ax + a^2 \)

\[
\begin{align*}
x^2 + 16x &= 57 \\
x^2 + 2ax + a^2 &= 57 + a^2
\end{align*}
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!

What should \( a \) be? \( 16? \ 8? \quad 2ax = 16x \)

\[
\begin{align*}
\rightarrow a &= \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64.
\end{align*}
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Completing the square

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Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

- Move 57 over.
- Make left hand side a perfect square!
- Can take square root!

$x^2 + 16x = 57$

$x^2 + 2ax + a^2 = 57 + a^2$

$x^2 + 16x + 64 = 57 + 64$

$x^2 + 16x + 64 = 121$

$x^2 + 16x + 64 = 121$

→ $a = \frac{16x}{2x} = 8$, $a^2 = 8^2 = 64$. 

Doh!

or $x = \frac{11}{2}$.
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
\begin{align*}
\text{Move 57 over.} & \\
\text{Make left hand side a perfect square!} & \\
\text{Can take square root!} & \\
\text{What should } a \text{ be? } 16? \ 8? & \quad 2ax = 16x \\
\rightarrow a = \frac{16x}{2x} = 8, \quad a^2 = 8^2 = 64. \\
\end{align*}
\]

\[
\begin{align*}
& x^2 + 16x = 57 \\
& x^2 + 2ax + a^2 = 57 + a^2 \\
& x^2 + 16x + 64 = 57 + 64 \\
& x^2 + 16x + 64 = 121 \\
& (x + 8)^2 = 121 \\
\end{align*}
\]
Completing the square

Solve $x^2 + 16x - 57 = 0$.

Perfect square: $(x + a)^2 = x^2 + 2ax + a^2$

$x^2 + 16x = 57$ Move 57 over.

$x^2 + 2ax + a^2 = 57 + a^2$ Make left hand side a perfect square!

$x^2 + 16x + 64 = 57 + 64$ Can take square root!

$x^2 + 16x + 64 = 121$ What should $a$ be? 16? 8? $2ax = 16x$

$(x + 8)^2 = 121$ $\rightarrow a = \frac{16x}{2x} = 8$, $a^2 = 8^2 = 64$.

$(x + 8) = 11$
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
\begin{align*}
  x^2 + 16x &= 57 \\
  x^2 + 2ax + a^2 &= 57 + a^2 \\
  x^2 + 16x + 64 &= 57 + 64 \\
  x^2 + 16x + 64 &= 121 \\
  (x + 8)^2 &= 121 \\
  (x + 8) &= \pm 11
\end{align*}
\]

Move 57 over.  
Make left hand side a perfect square!  
Can take square root!  

What should \(a\) be? \(16? \hspace{1em} 8? \hspace{1em} 2ax = 16x\)  
\(a = \frac{16x}{2x} = 8, \ a^2 = 8^2 = 64.\)
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$x^2 + 16x + 64 = 121$

$(x + 8)^2 = 121$

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$x = \pm 11 - 8$

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  x^2 + 16x & = 57 \\
  x^2 + 2ax + a^2 & = 57 + a^2 \\
  x^2 + 16x + 64 & = 57 + 64 \\
  x^2 + 16x + 64 & = 121 \\
  (x + 8)^2 & = 121 \\
  x & = \pm 11 - 8 \\
  \rightarrow x & = 19
\end{align*}
\]

Move 57 over.

Make left hand side a perfect square!

Can take square root!

What should \( a \) be? 16? 8? \( 2ax = 16x \)

\[
\begin{align*}
  \rightarrow a = \frac{16x}{2x} & = 8, \quad a^2 = 8^2 = 64.
\end{align*}
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Completing the square

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Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

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x^2 + 16x = 57
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Make left hand side a perfect square!

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What should \( a \) be? 16? 8? \( 2ax = 16x \)

\[
2a = 16 \quad \Rightarrow \quad a = \frac{16}{2} = 8, \quad a^2 = 8^2 = 64.
\]

\[
x = 8 \pm 11
\]

\[
x = 19 \quad \text{Doh!}
\]
Completing the square

Solve \( x^2 + 16x - 57 = 0 \).

Perfect square: \((x + a)^2 = x^2 + 2ax + a^2\)

\[
\begin{align*}
x^2 + 16x &\quad = 57 \\
x^2 + 2ax + a^2 &\quad = 57 + a^2 \\
x^2 + 16x + 64 &\quad = 57 + 64 \\
x^2 + 16x + 64 &\quad = 121 \\
(x + 8)^2 &\quad = 121 \\
(x + 8) &\quad = \pm 11 \\
x &\quad = \pm 11 - 8 \\
\rightarrow x &\quad = -19
\end{align*}
\]

Move 57 over.
Make left hand side a perfect square!
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  x^2 + 16x &= 57 \\
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  x^2 + 16x + 64 &= 57 + 64 \\
  x^2 + 16x + 64 &= 121 \\
  (x + 8)^2 &= 121 \\
  (x + 8) &= \pm 11 \\
  x &= \pm 11 - 8 \\
  \rightarrow x &= -19 \\
  \text{or } x &= 11 - 8 = 3.
\end{align*}$
Completing the square.

Discover.
Completing the square: discover.

Find $x$ when $16x = 4$?

Here it is!

How?

Divide both sides by 16.

Generally: inverse of multiply by 16.

$x = \frac{4}{16} = \frac{1}{4}$.

Find $x$ when $x^2 = 9$?

$x = \pm 3$.

Find $x$ when $x^2 = 10$?

$x = \sqrt{10}$ or $x = -\sqrt{10}$.

Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?

$x^2 = 8x - 16$,

Uh oh.

What to do?

$x^2 - 8x + 16 = (x - 4)^2 = 0$

So $x = 4$!!

Find $x$ when $x^2 - 8x + 16 = 10$?

Does $x = \sqrt{10}$?

$x = 4 \pm \sqrt{10}$

$(x - 4)^2 = 10$ → $(x - 4) = \pm \sqrt{10}$ → $x = 4 \pm \sqrt{10}$

If left hand side is perfect square...can solve!

Can we always make it so?
Completing the square: discover.

Find $x$ when $16x = 4$?  Here it is!

How?


$x = \frac{4}{16} = \frac{1}{4}$.

Find $x$ when $x^2 = 9$?

$x = \pm 3$.

Find $x$ when $x^2 = 10$?

$x = \sqrt{10}$ or $x = -\sqrt{10}$.

Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?

$x^2 = 8x - 16$, $x = \sqrt{8x - 16}$

Uh oh.

What to do?

$x^2 - 8x + 16 = (x - 4)^2 = 0$  

So $x = 4$!!

Find $x$ when $x^2 - 8x + 16 = 10$?

Does $x = \sqrt{10}$?

$= \pm \sqrt{10}$?

$= 4 \pm \sqrt{10}$?

$(x - 4)^2 = 10 \rightarrow (x - 4) = \pm \sqrt{10} \rightarrow x = 4 \pm \sqrt{10}$.

If left hand side is perfect square...can solve!

Can we always make it so?
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$x^2 = 8x - 16$,
$x = \sqrt{8x - 16}$
Uh oh.

$x^2 - 8x + 16 = (x - 4)^2 = 0$ 
So $x = 4$!!

Find $x$ when $x^2 - 8x + 16 = 10$?

Does $x = \sqrt{10}$?

$= \pm \sqrt{10}$?

$= 4 \pm \sqrt{10}$

$(x - 4)^2 = 10$ 
$\rightarrow (x - 4) = \pm \sqrt{10}$ 
$\rightarrow x = 4 \pm \sqrt{10}$

If left hand side is perfect square...can solve!

Can we always make it so?
Completing the square: discover.

Find \( x \) when \( 16x = 4? \) Here it is!
How? Divide both sides by 16.
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

Completing the square: discover.

Find \( x \) when \( 16x = 4? \) Here it is!

\[ x = \frac{4}{16} = \frac{1}{4}. \]

Find \( x \) when \( x^2 = 9? \)

\[ x = \pm 3. \]

Find \( x \) when \( x^2 = 10? \)

\[ x = \sqrt{10} \text{ or } x = -\sqrt{10}. \]

Inverse operation is square root!

Find \( x \) when \( x^2 - 8x + 16 = 0? \)

\[ x^2 = 8x - 16, \quad x = \sqrt{8x - 16} \text{ Uh oh.} \]

What to do?

\[ x^2 - 8x + 16 = (x - 4)^2 = 0 \]

So \( x = 4 \)!!

Find \( x \) when \( x^2 - 8x + 16 = 10? \)

Does \( x = \sqrt{10} \)?

\[ = \pm \sqrt{10} \]

\[ = 4 \pm \sqrt{10} \]

If left hand side is perfect square...can solve!

Can we always make it so?
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!
$x = 4/16$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$x = 4/16 = 1/4$. 
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$x = 4/16 = 1/4$.

Find $x$ when $x^2 = 9$?
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!
$x = \frac{4}{16} = \frac{1}{4}$.

Find $x$ when $x^2 = 9$? $x = 3$. 
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$x = 4/16 = 1/4$.

Find $x$ when $x^2 = 9$? $x = \pm 3$.

Find $x$ when $x^2 = 10$?

Inverse operation is square root!
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$x = 4/16 = 1/4$.

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Find $x$ when $x^2 = 10$? $x = \sqrt{10}$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$x = 4/16 = 1/4$.

Find $x$ when $x^2 = 9$? $x = \pm 3$.

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Completing the square: discover.

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Inverse operation is square root!
Find $x$ when $x^2 - 8x + 16 = 0$?
Completing the square: discover.

Find \( x \) when \( 16x = 4 \)? Here it is!
\[
x = \frac{4}{16} = \frac{1}{4}.
\]

Find \( x \) when \( x^2 = 9 \)? \( x = \pm 3 \).
Find \( x \) when \( x^2 = 10 \)? \( x = \sqrt{10} \) or \( x = -\sqrt{10} \).

Inverse operation is square root!

Find \( x \) when \( x^2 - 8x + 16 = 0 \)?
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x^2 = 8x - 16
\]
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!
$x = 4/16 = 1/4$.

Find $x$ when $x^2 = 9$? $x = \pm 3$.

Find $x$ when $x^2 = 10$? $x = \sqrt{10}$ or $x = -\sqrt{10}$.

Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?
$x^2 = 8x - 16$, $x = \sqrt{8x - 16}$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$$x = 4/16 = 1/4.$$ 

Find $x$ when $x^2 = 9$? $x = \pm 3$. 
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Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?

$$x^2 = 8x - 16 , \ x = \sqrt{8x - 16} \ \text{Uh oh.}$$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

$$x = \frac{4}{16} = \frac{1}{4}.$$ 

Find $x$ when $x^2 = 9$? $x = \pm 3$.

Find $x$ when $x^2 = 10$? $x = \sqrt{10}$ or $x = -\sqrt{10}$.

Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?

$$x^2 = 8x - 16, x = \sqrt{8x - 16}$$ Uh oh.

What to do?
Completing the square: discover.

Find \( x \) when \( 16x = 4 \)? Here it is!


\[
x = \frac{4}{16} = \frac{1}{4}.
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Find \( x \) when \( x^2 = 9 \)? \( x = \pm 3 \).

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Find \( x \) when \( x^2 - 8x + 16 = 0 \)?

\[
x^2 = 8x - 16, \quad x = \sqrt{8x - 16} \quad \text{Uh oh.}
\]

What to do? \( x^2 - 8x + 16 \)
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

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$x^2 = 8x - 16$, $x = \sqrt{8x - 16}$ Uh oh.

What to do? $x^2 - 8x + 16 = (x - 4)^2$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!

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Find $x$ when $16x = 4$? Here it is!

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Find $x$ when $x^2 = 9$? $x = \pm 3$.
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Find $x$ when $x^2 − 8x + 16 = 0$?

$x^2 = 8x − 16$, $x = \sqrt{8x − 16}$ Uh oh.

What to do? $x^2 − 8x + 16 = (x − 4)^2 = 0$ So $x =$
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!


$$x = \frac{4}{16} = \frac{1}{4}.$$ 

Find $x$ when $x^2 = 9$? $x = \pm 3$.

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Inverse operation is square root!

Find $x$ when $x^2 - 8x + 16 = 0$?

$$x^2 = 8x - 16 \ , \ x = \sqrt{8x - 16}$$  Uh oh.

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Find $x$ when $16x = 4$? Here it is!

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Find $x$ when $x^2 = 9$? $x = \pm 3$.
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Find $x$ when $x^2 - 8x + 16 = 0$?

$$x^2 = 8x - 16, \quad x = \sqrt{8x - 16} \text{ Uh oh.}$$

What to do? $x^2 - 8x + 16 = (x - 4)^2 = 0$ So $x = 4$ !!

Find $x$ when $x^2 - 8x + 16 = 10$?
Completing the square: discover.

Find $x$ when $16x = 4$? Here it is!


$x = 4/16 = 1/4$.

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Inverse operation is square root!

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..complete this.
Wrap up.

Learning styles.
Wrap up.

Learning styles.
Maybe?
Wrap up.

Learning styles.
Maybe?
Still...use hardware of students to get at understanding.