61A Lecture 5

Wednesday, September 5

Office Hours: You Should Go!

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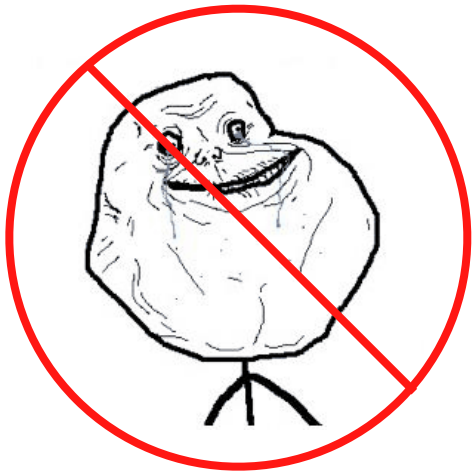

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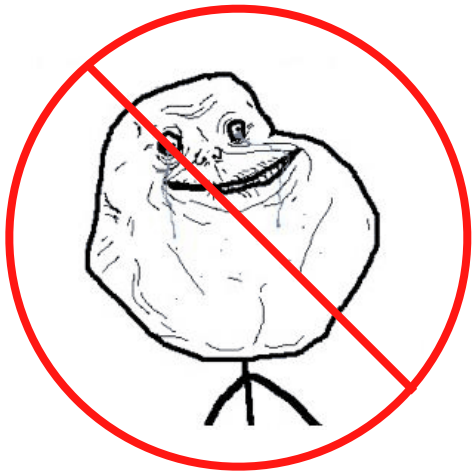

http://inst.eecs.berkeley.edu/~cs61a/fa12/staff.html

The Game of Hog

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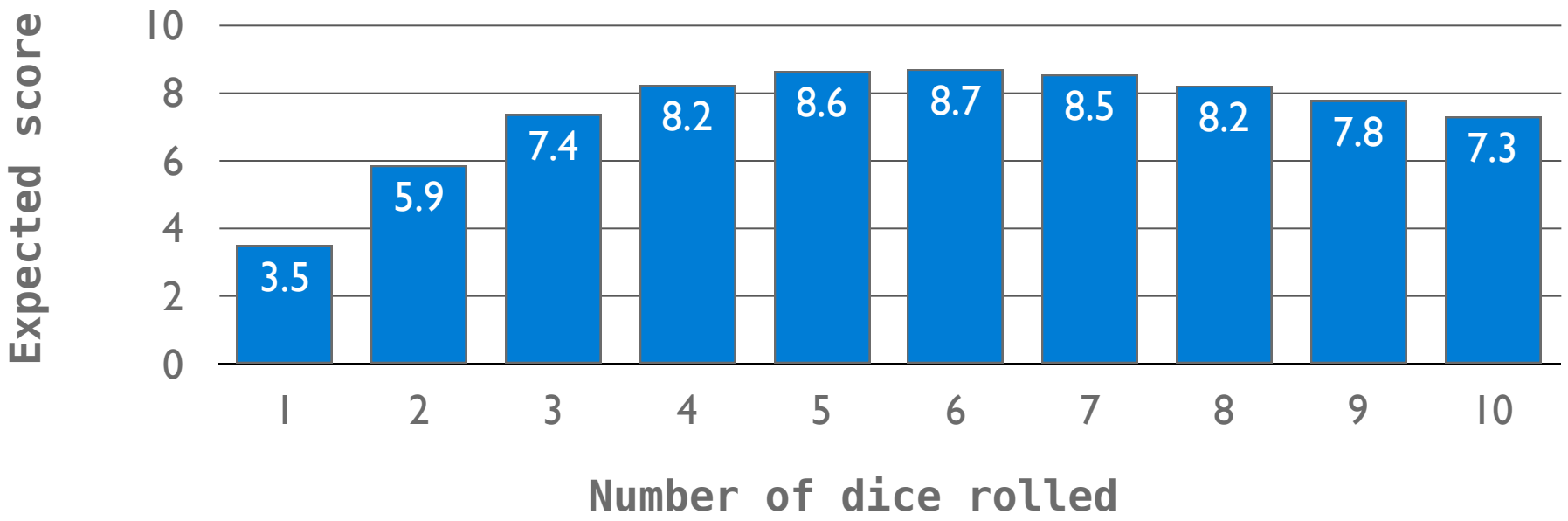

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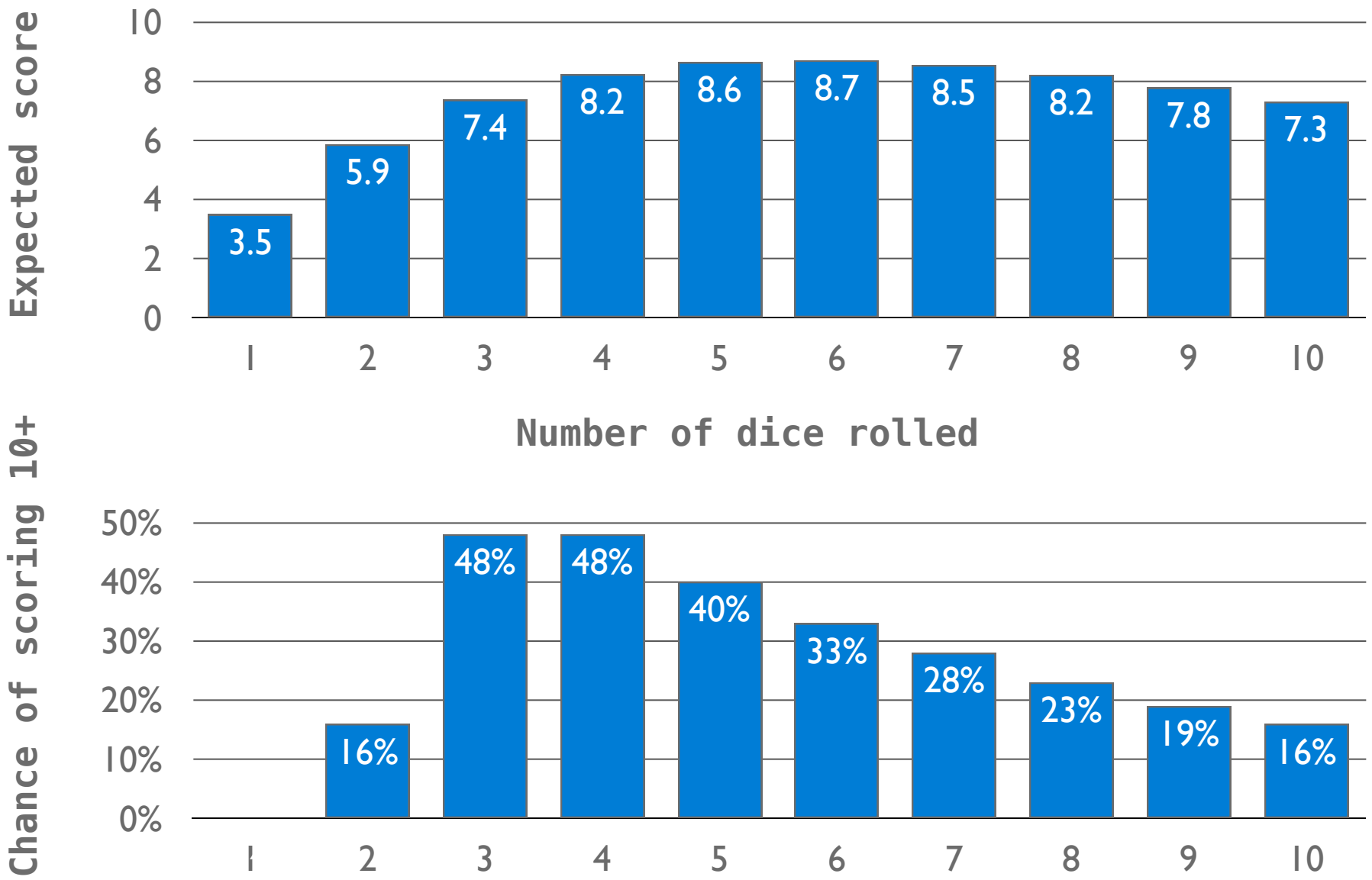

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(demo)

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1 def apply_twice(f, x):
2     return f(f(x))
3
4 def square(x):
5     return x * x
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7 result = apply_twice(square, 2)
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Example: http://goo.gl/Gbtc5

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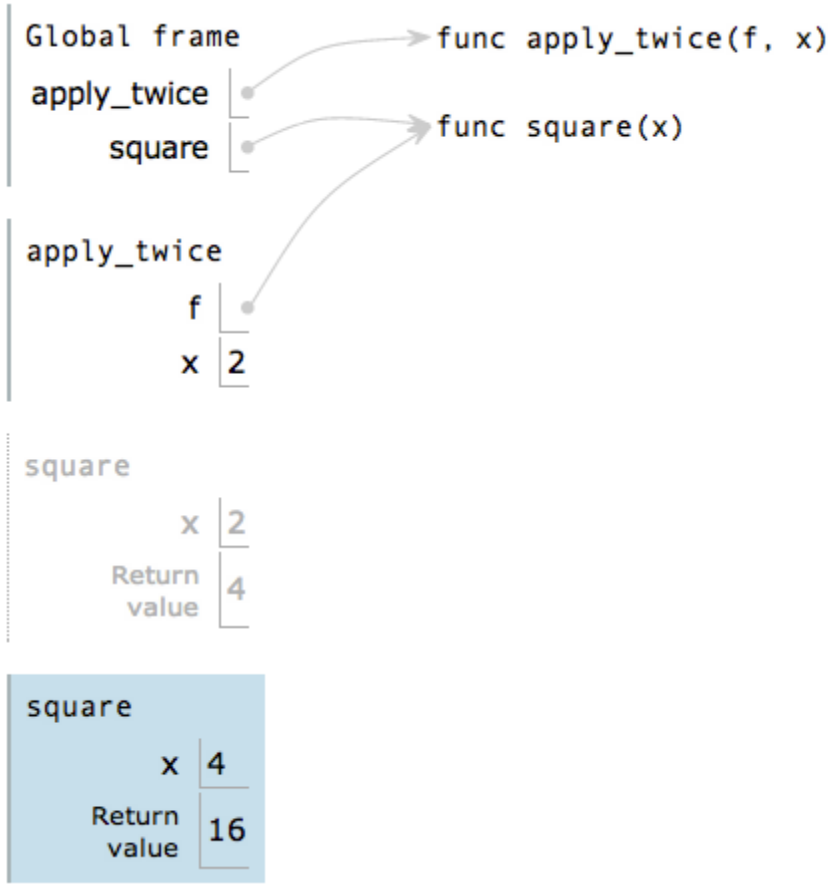

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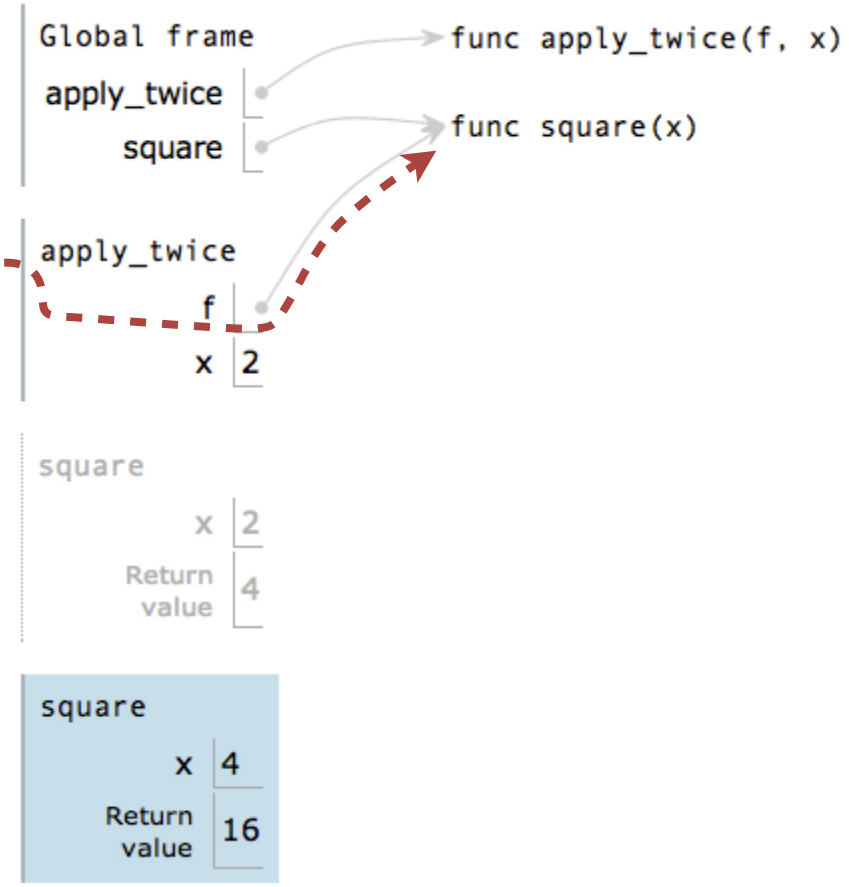

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Non-Nested Functions Calls Have One Local Frame

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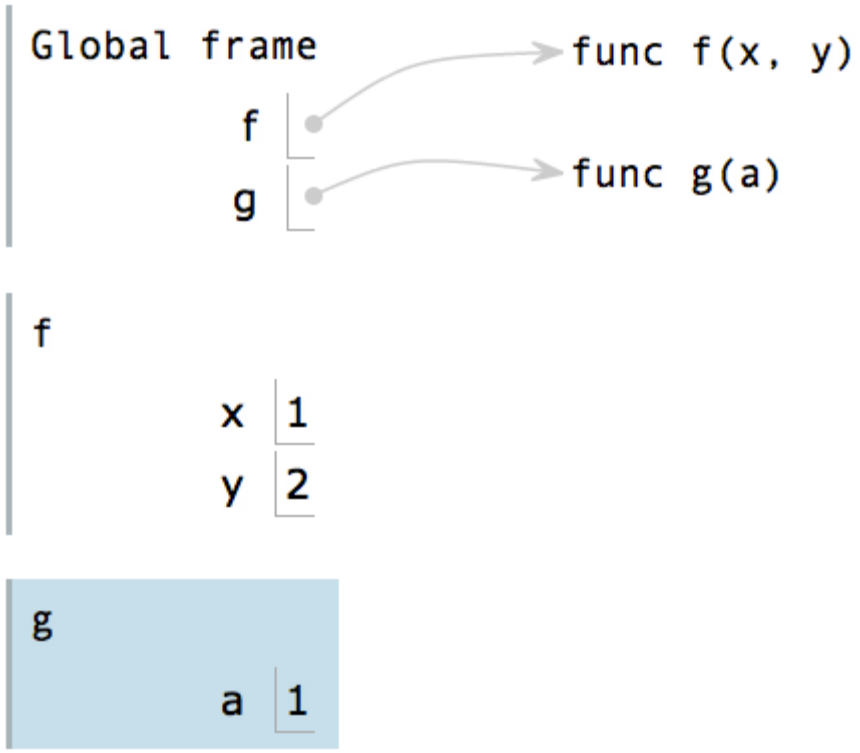

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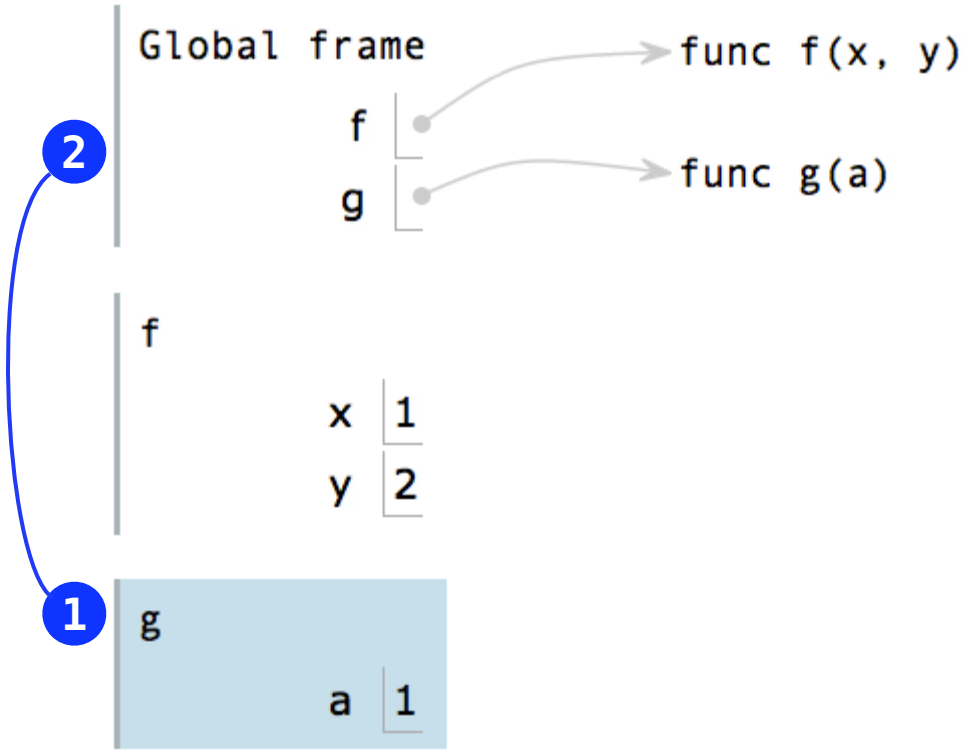

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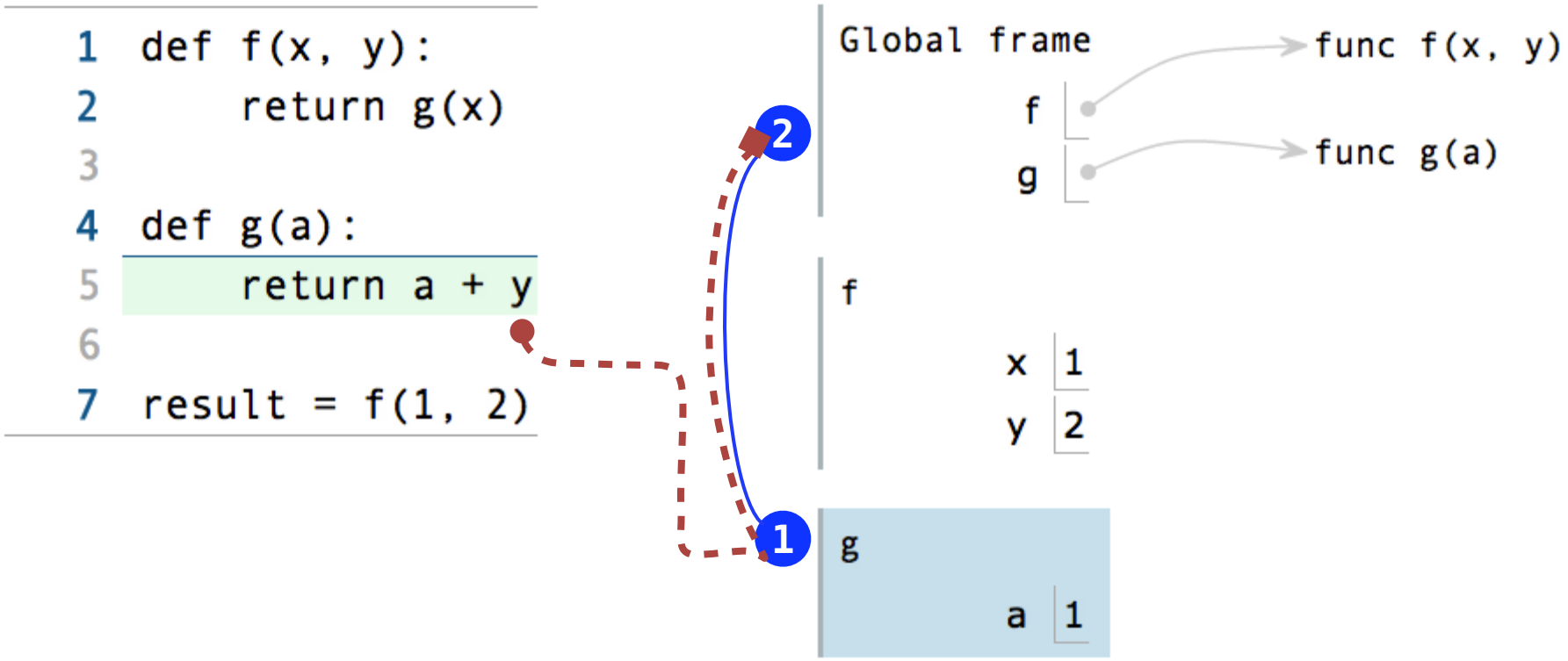

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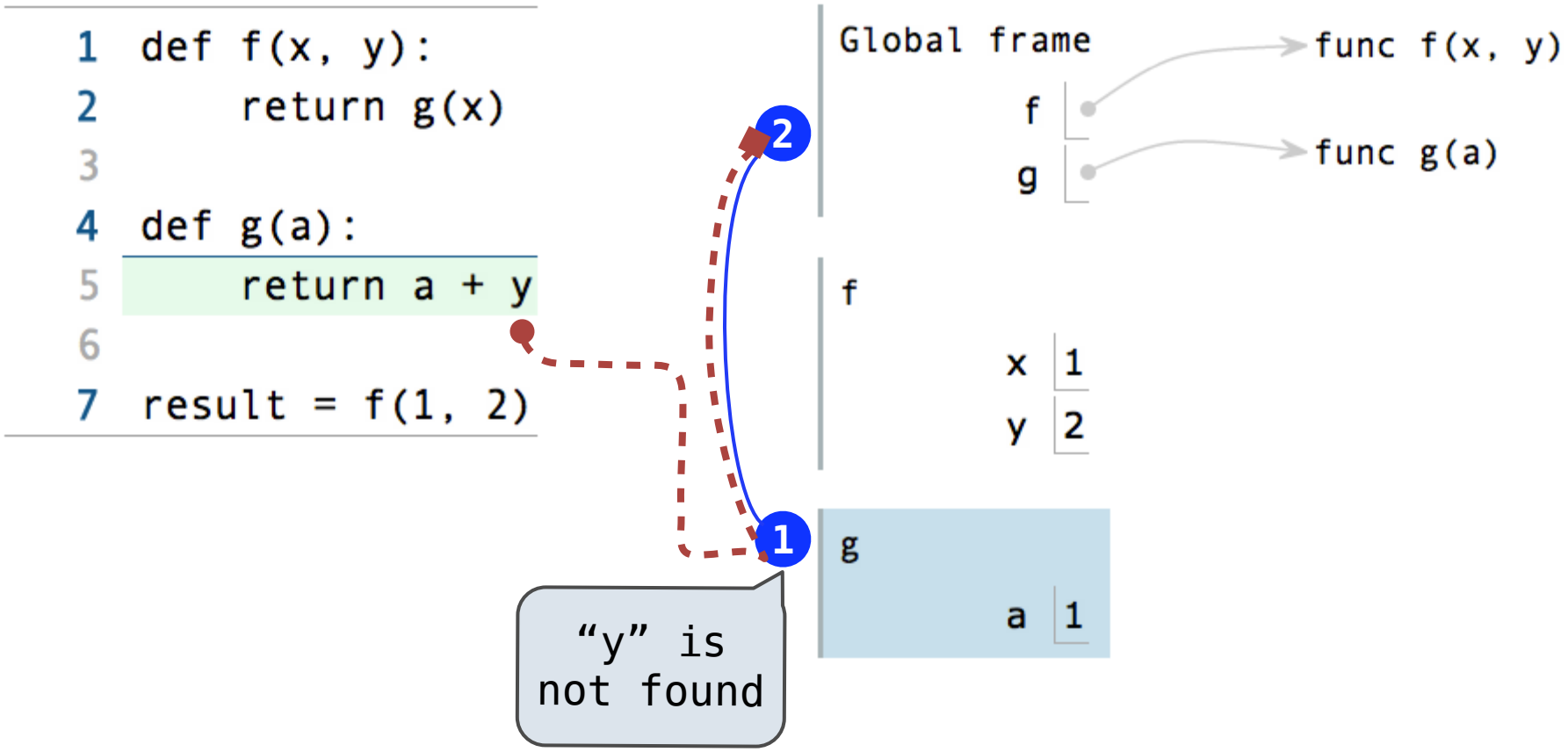

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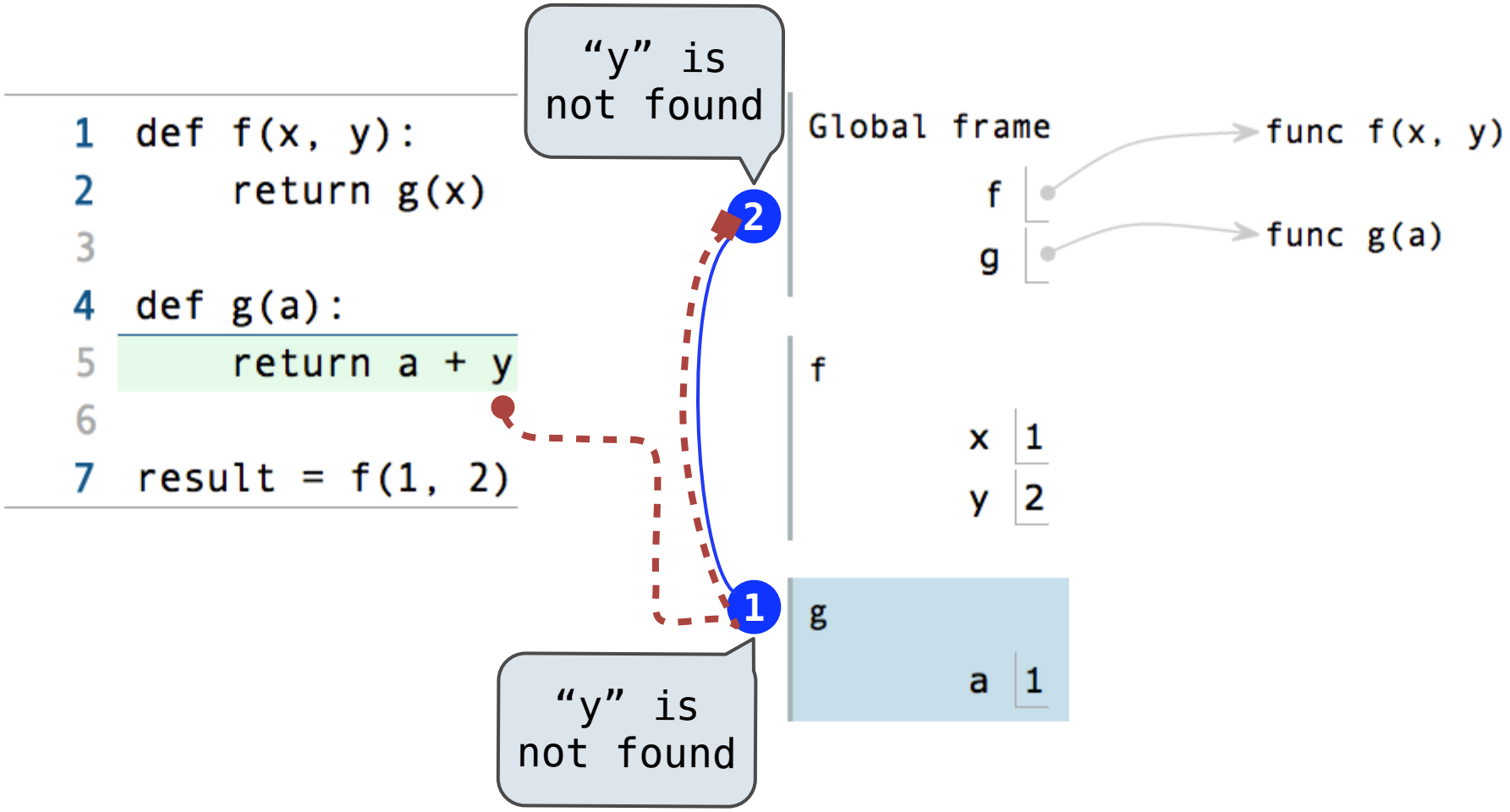

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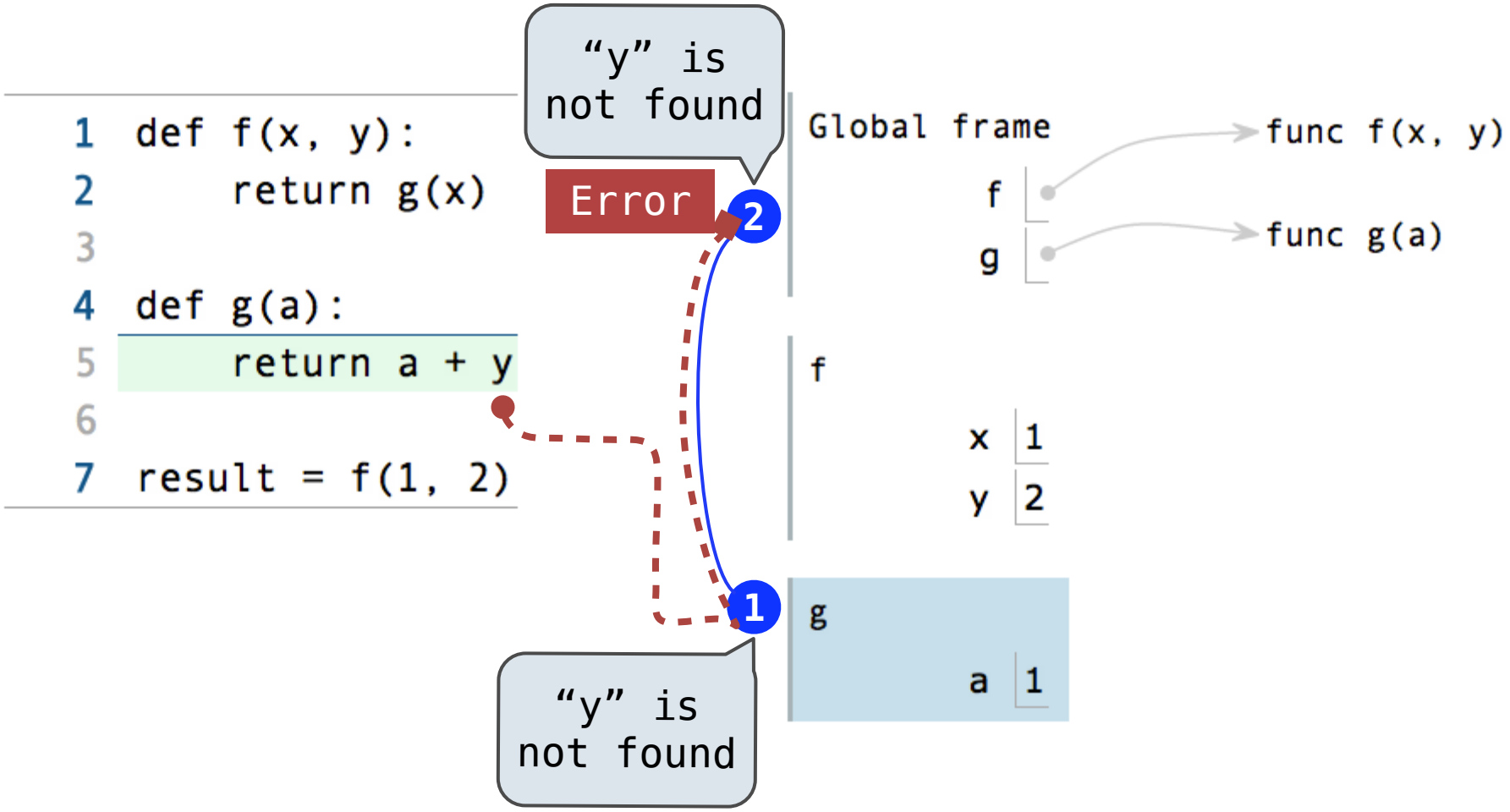

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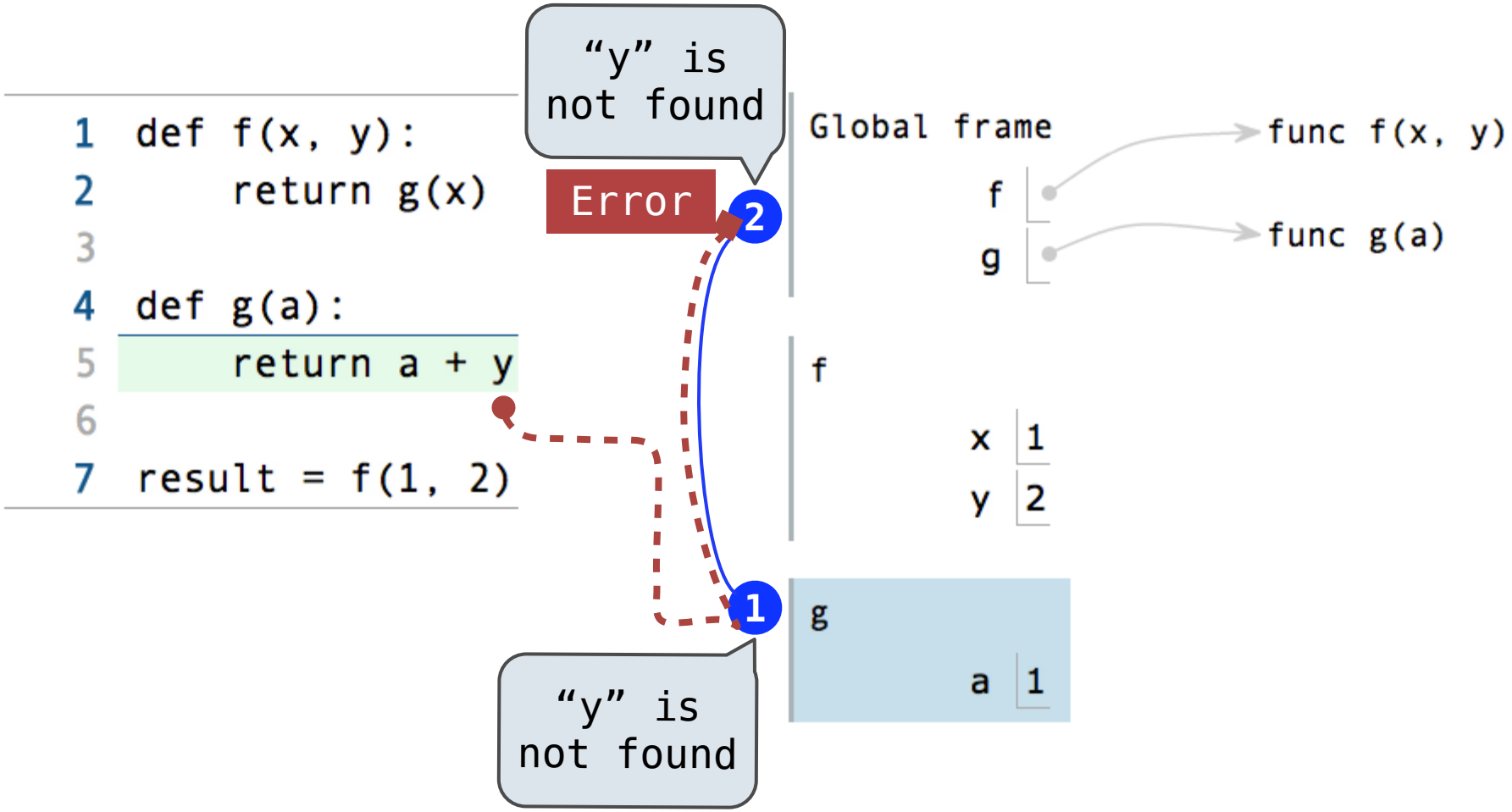

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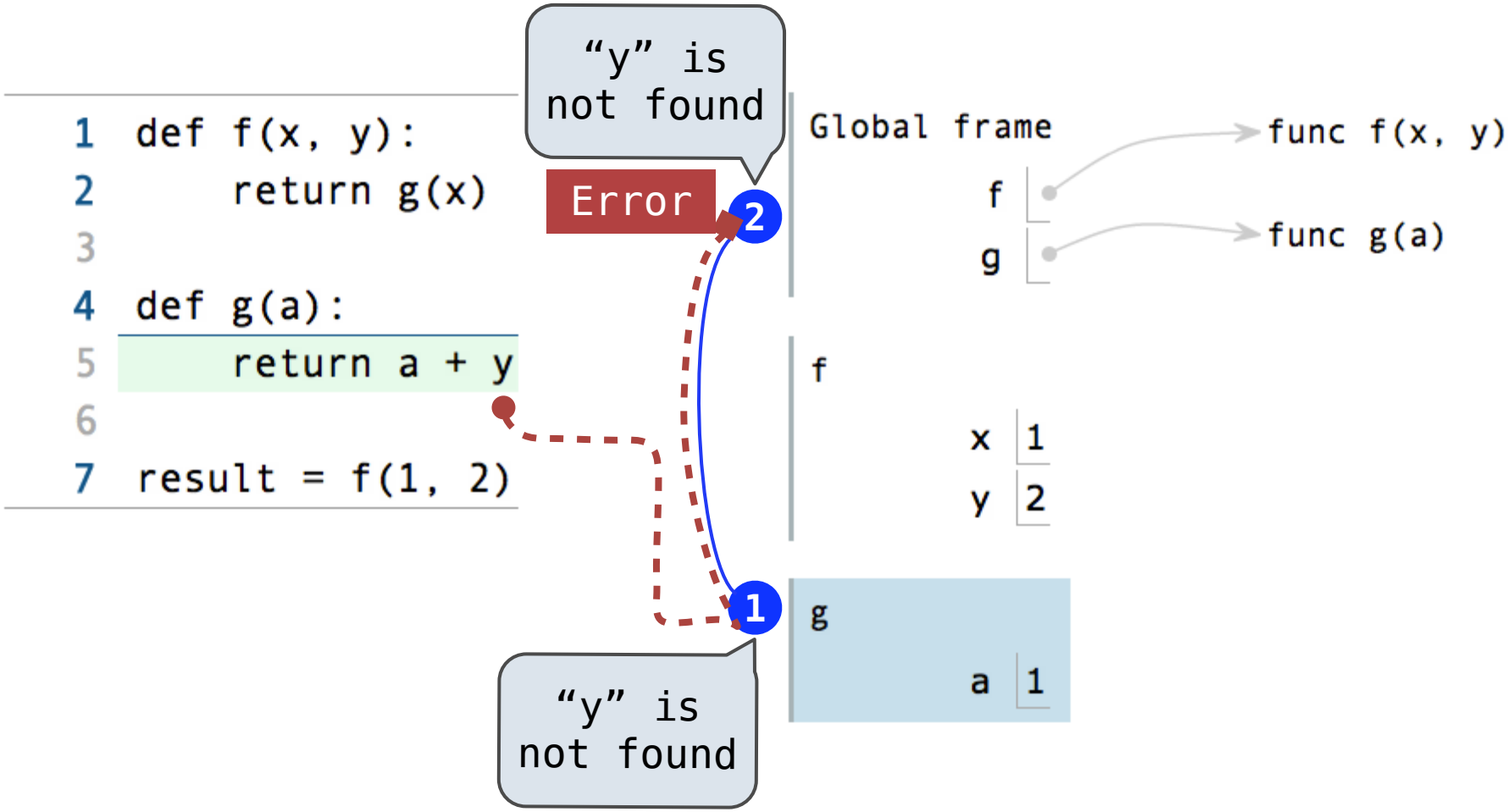

- An environment is a sequence of frames
- An environment for a non-nested function (no def within def) consists of one local frame, followed by the global frame

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Environment Diagrams for Nested Def Statements

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1 def make_adder(n):
2     def adder(k):
3         return k + n
4     return adder
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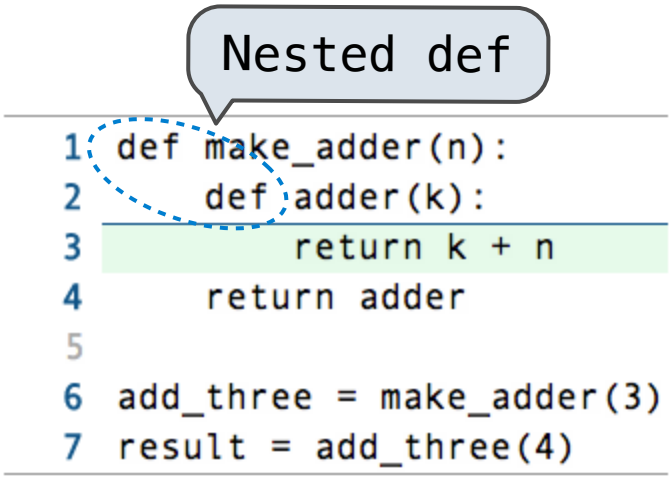

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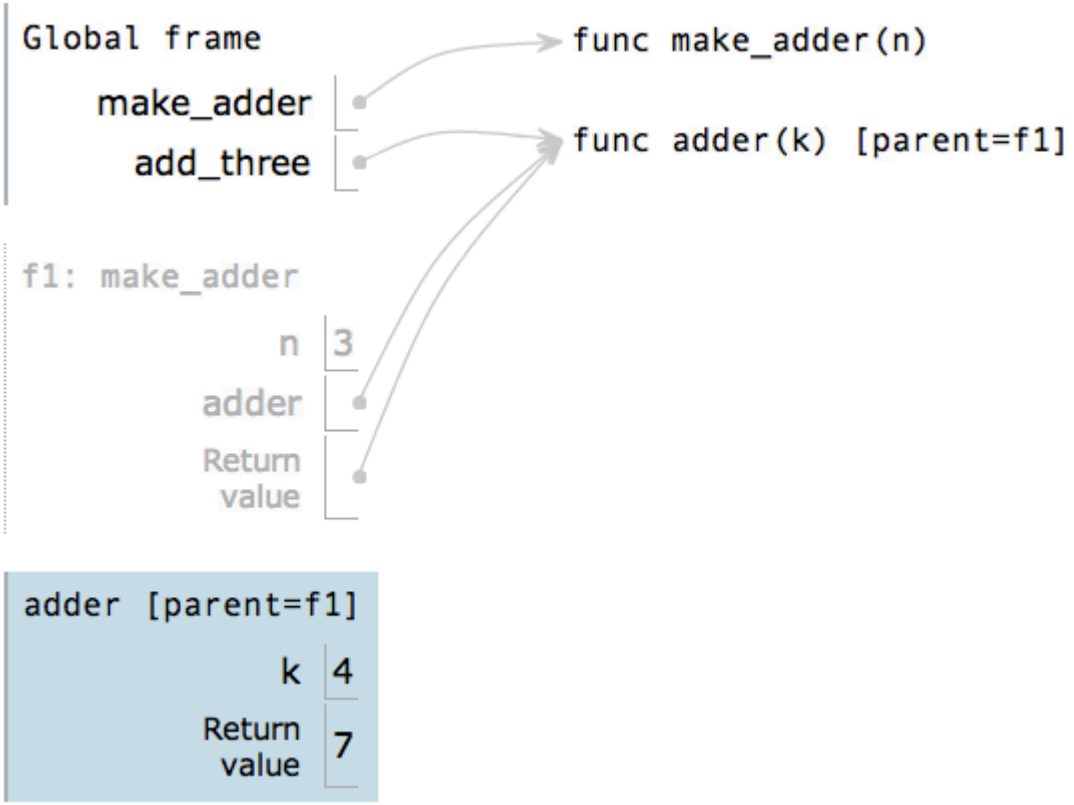

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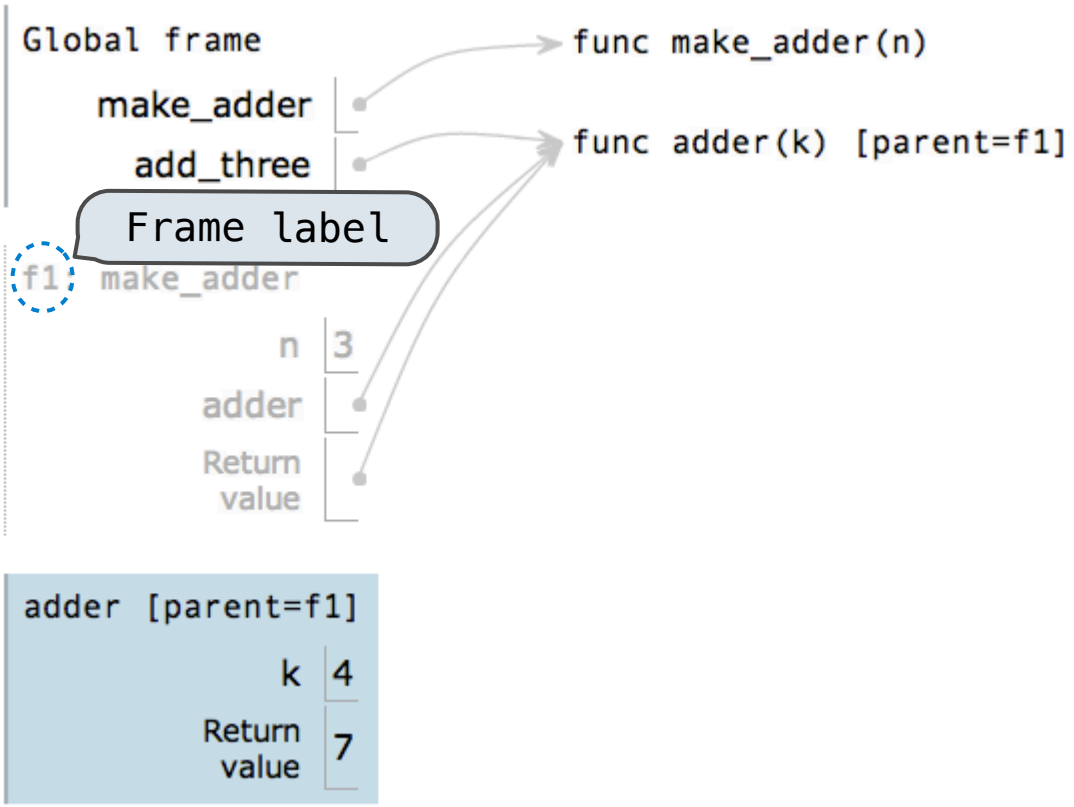

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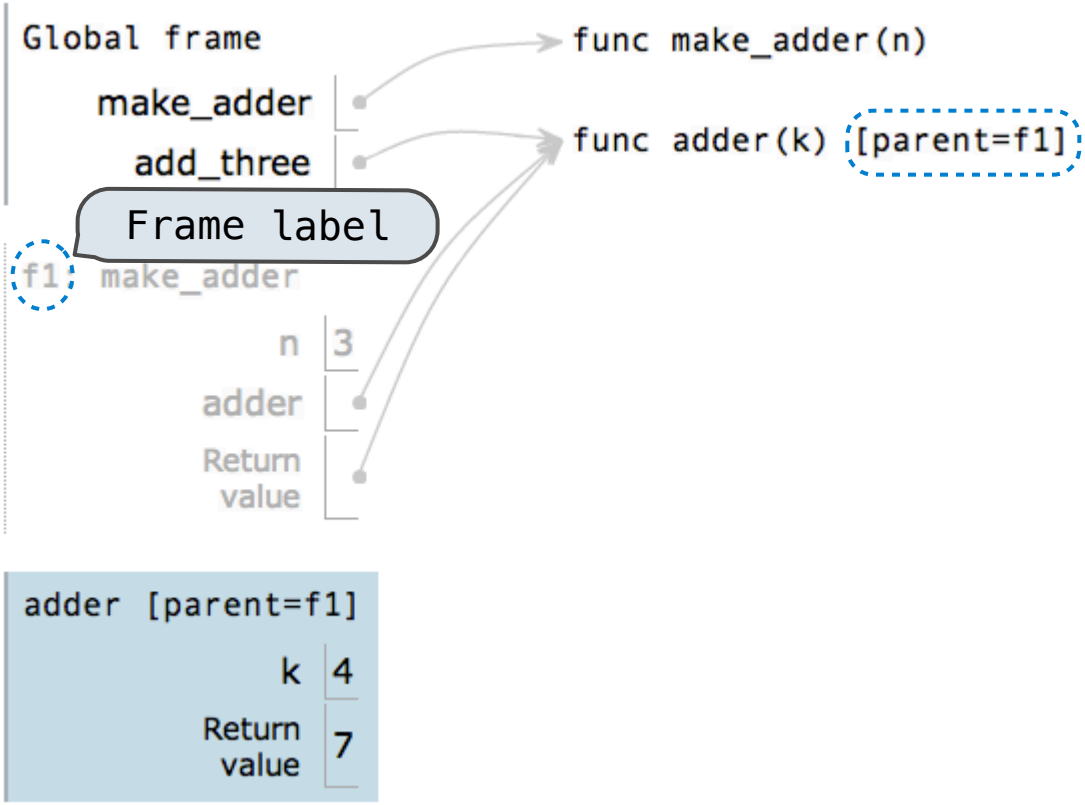

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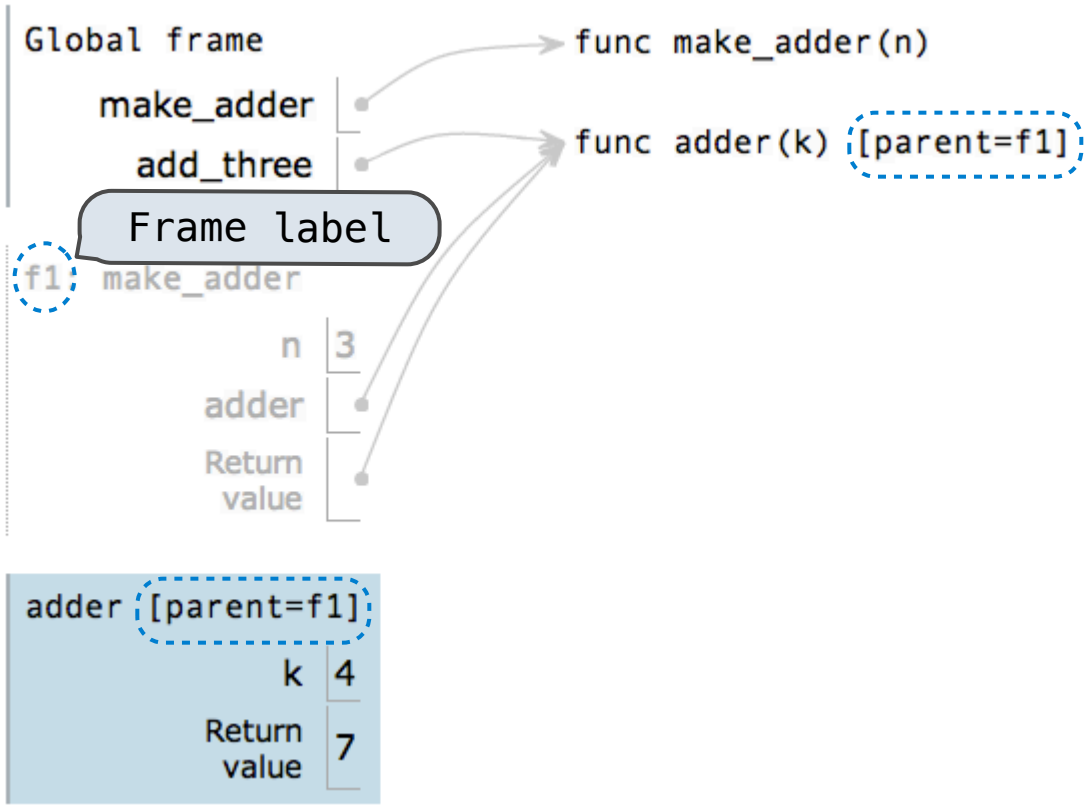

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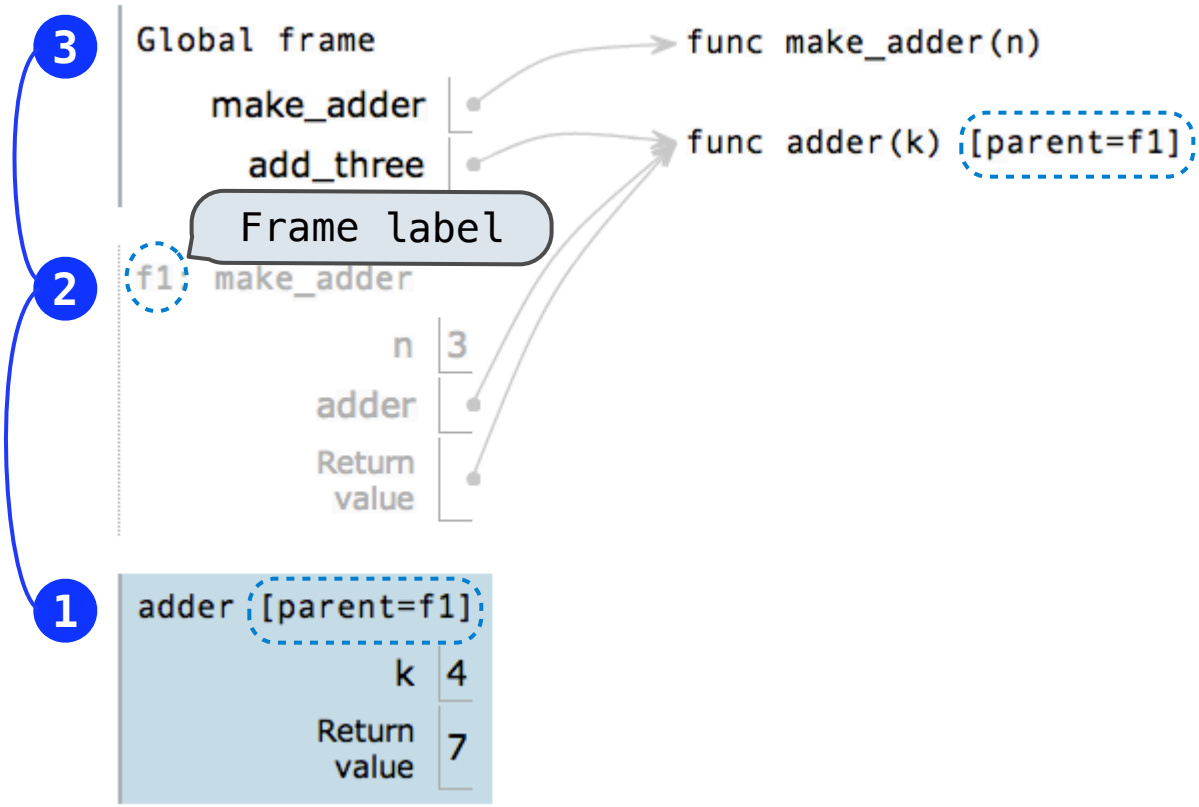

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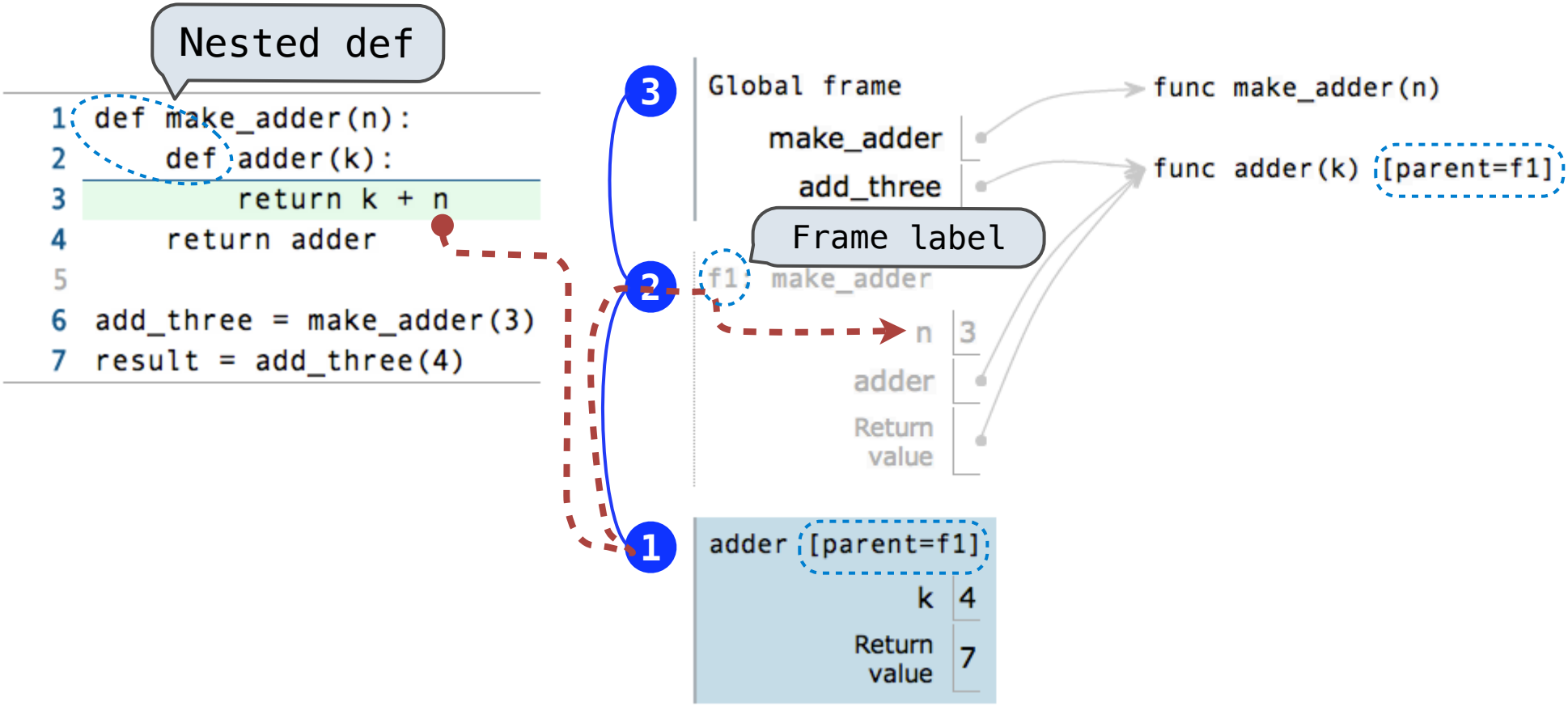

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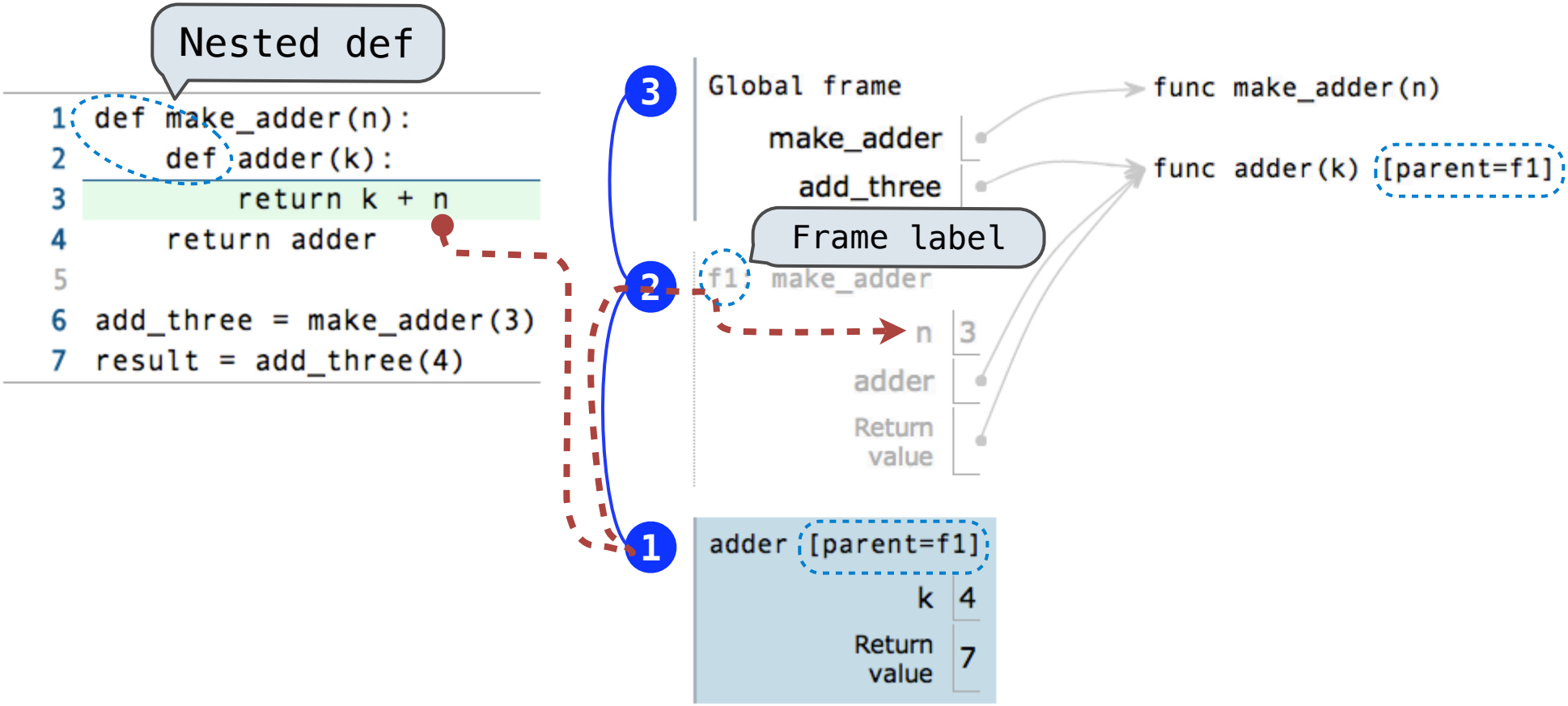

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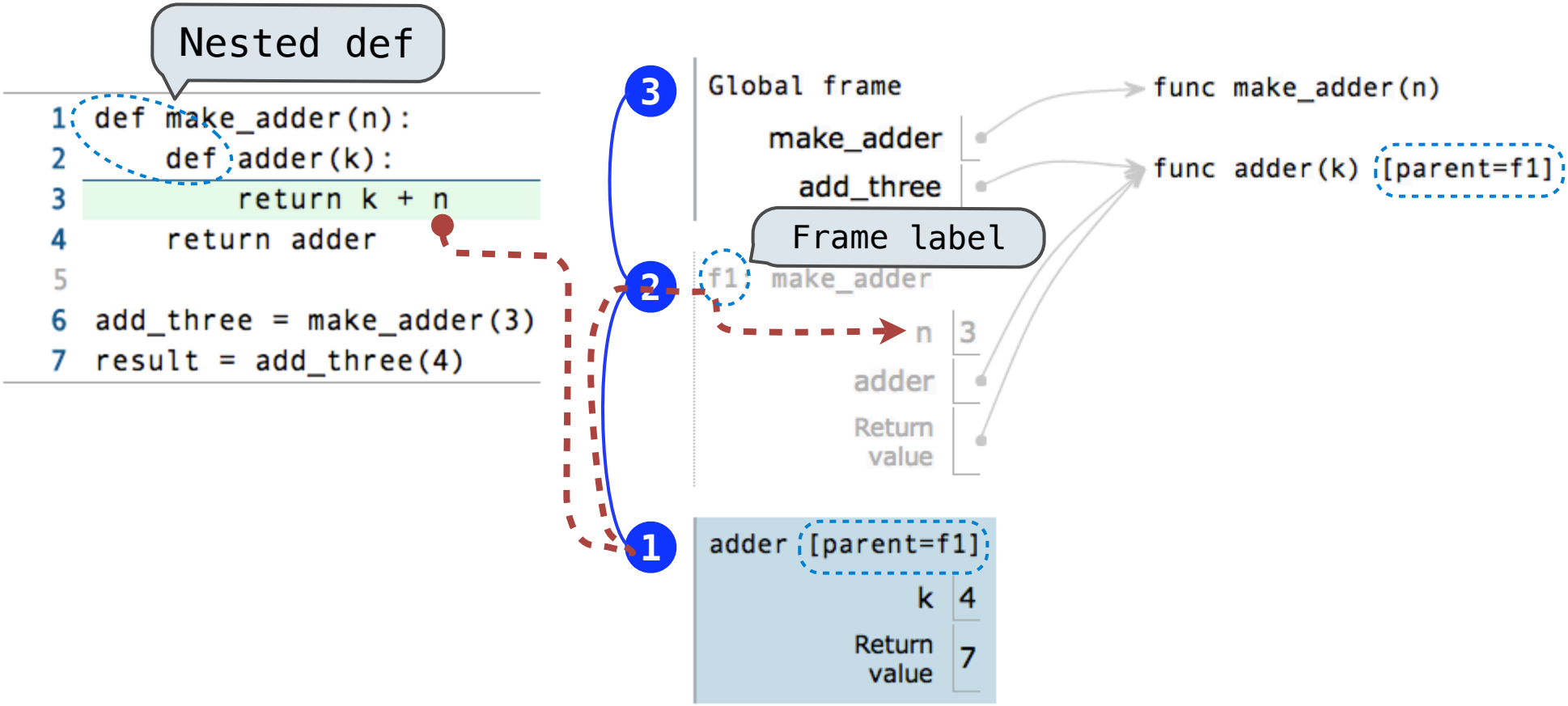

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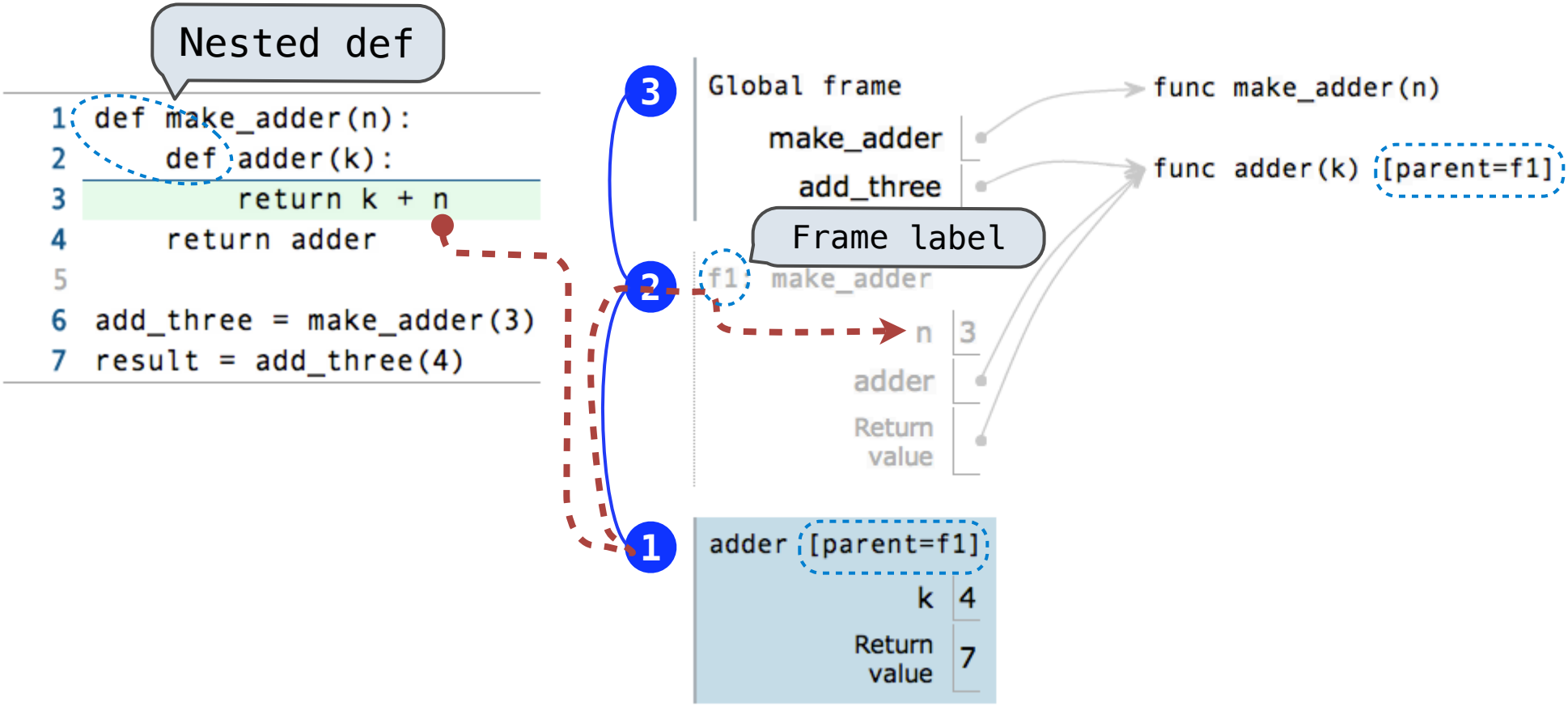

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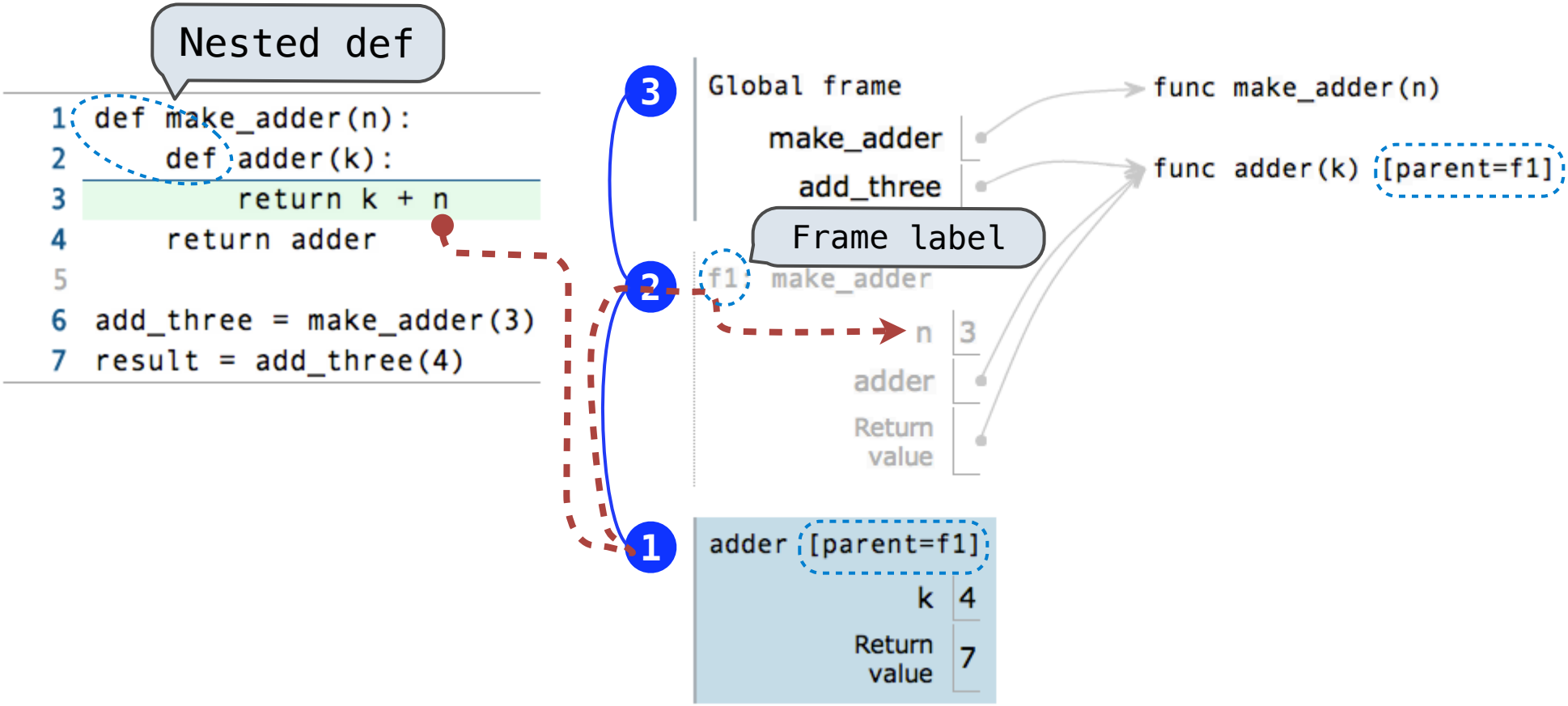

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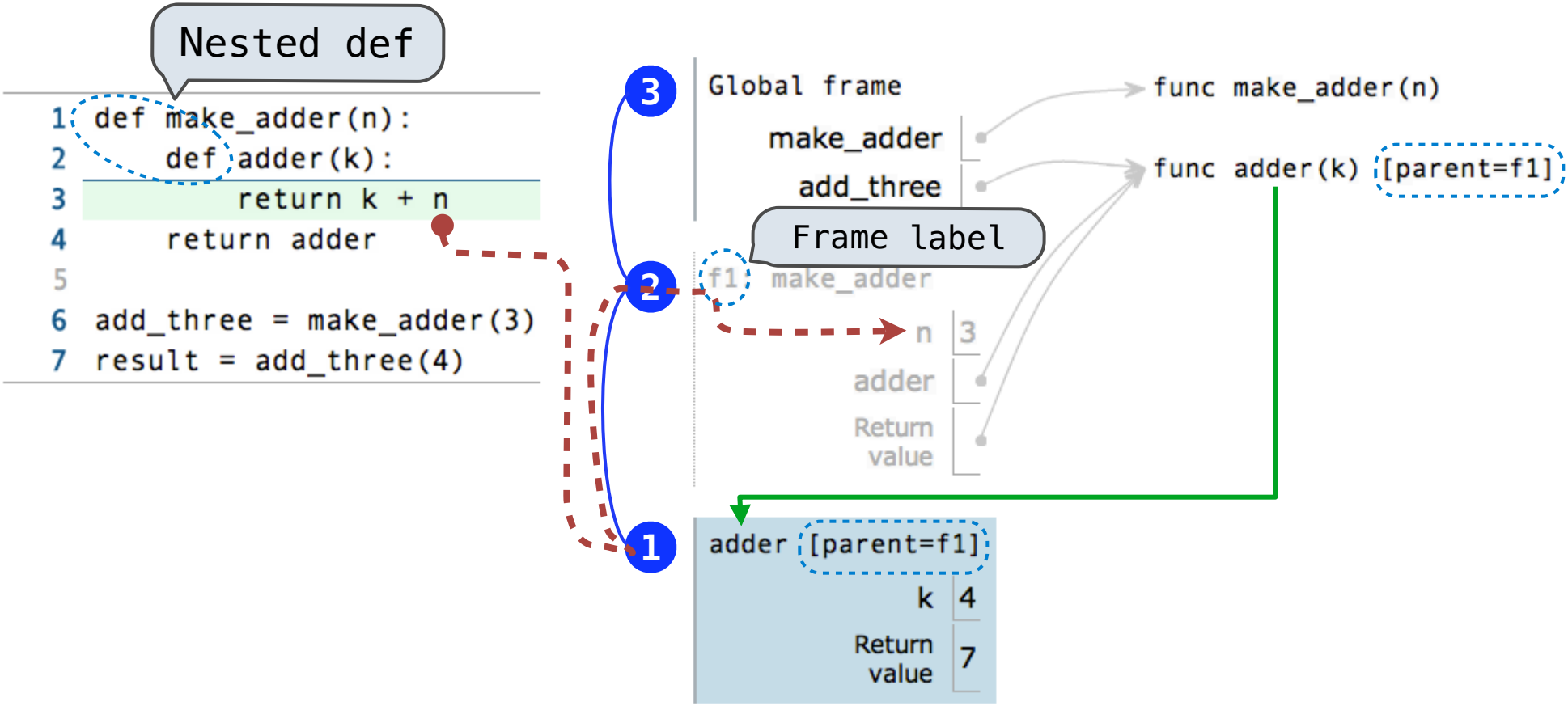

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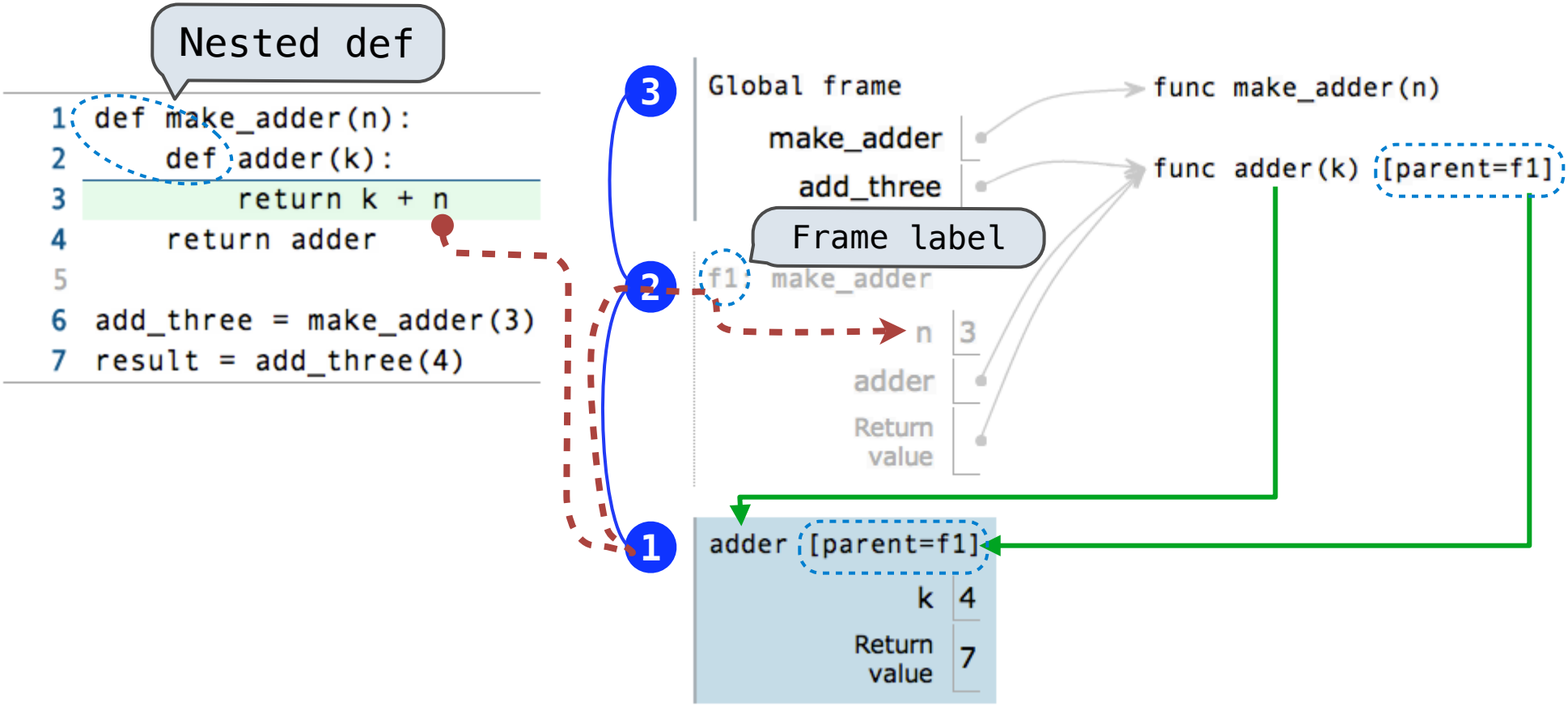

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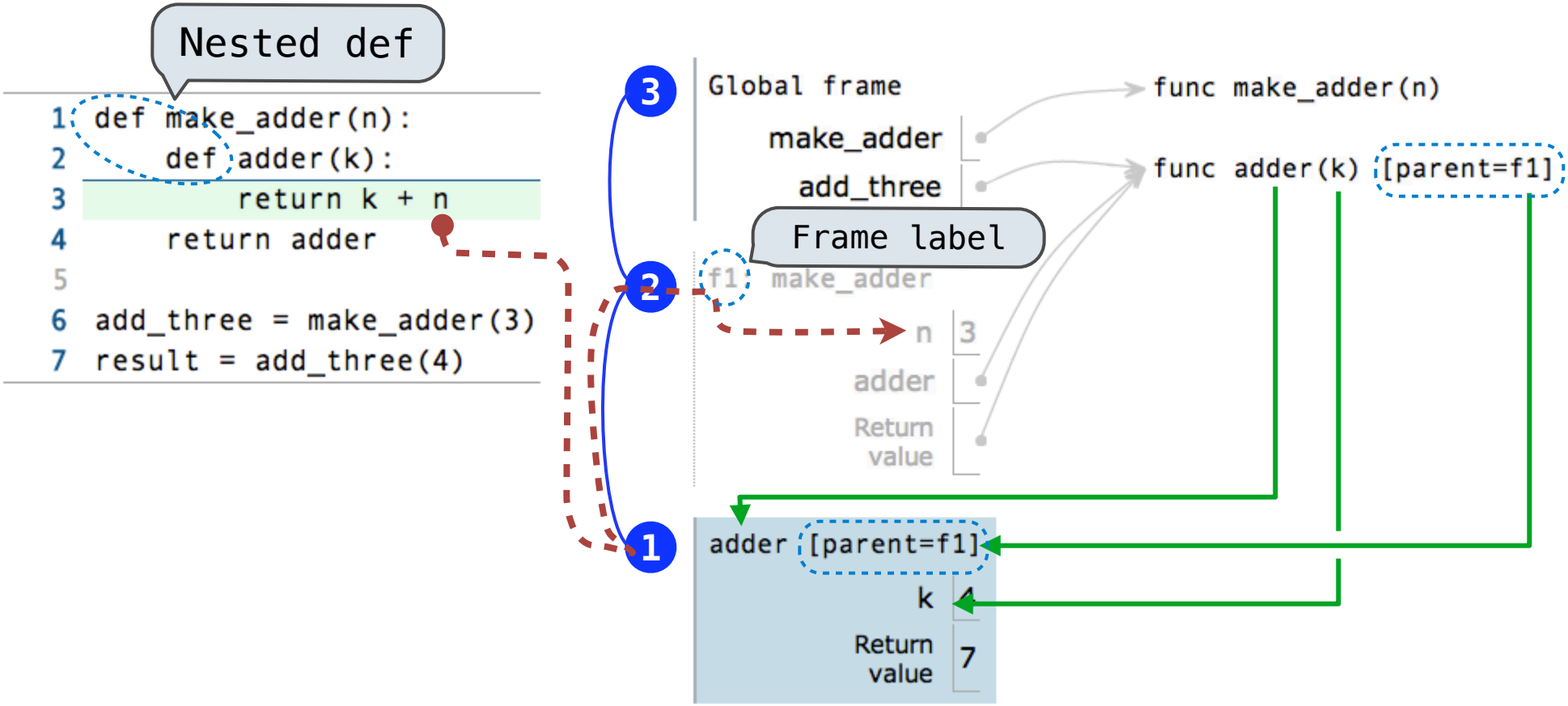

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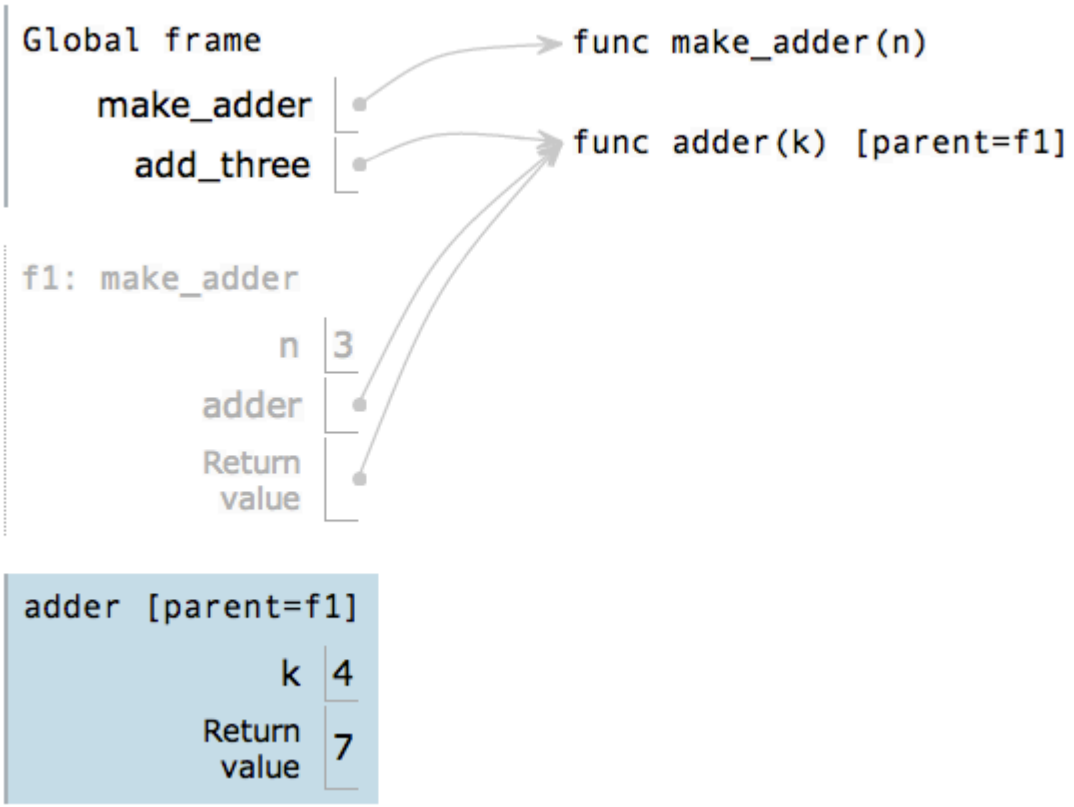

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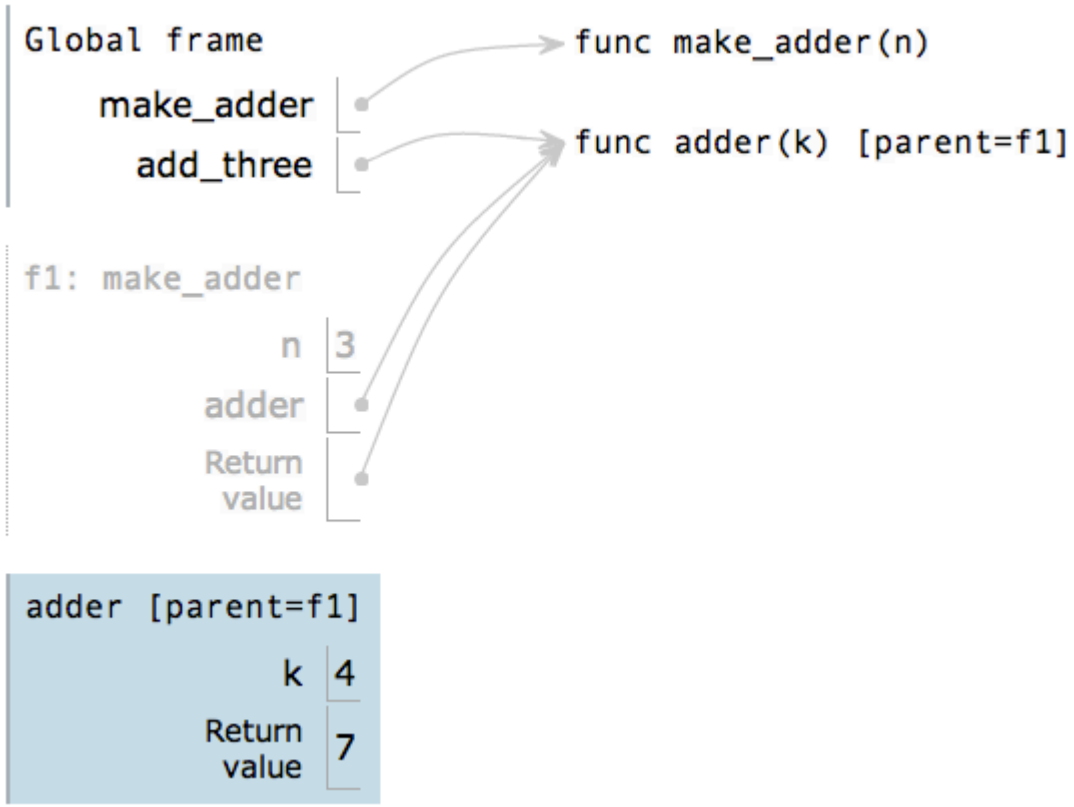

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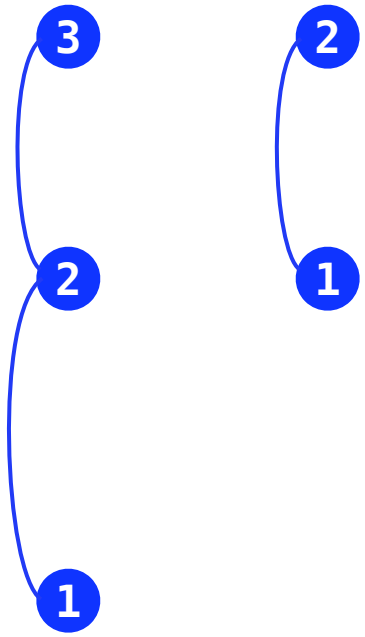

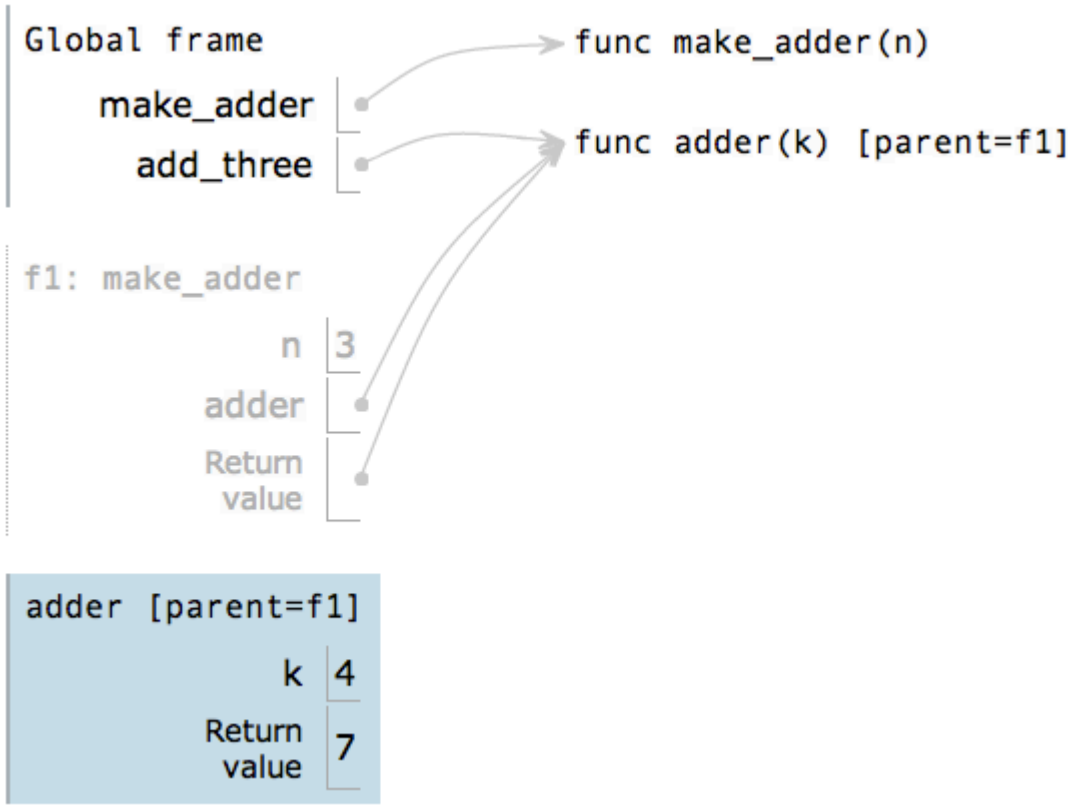

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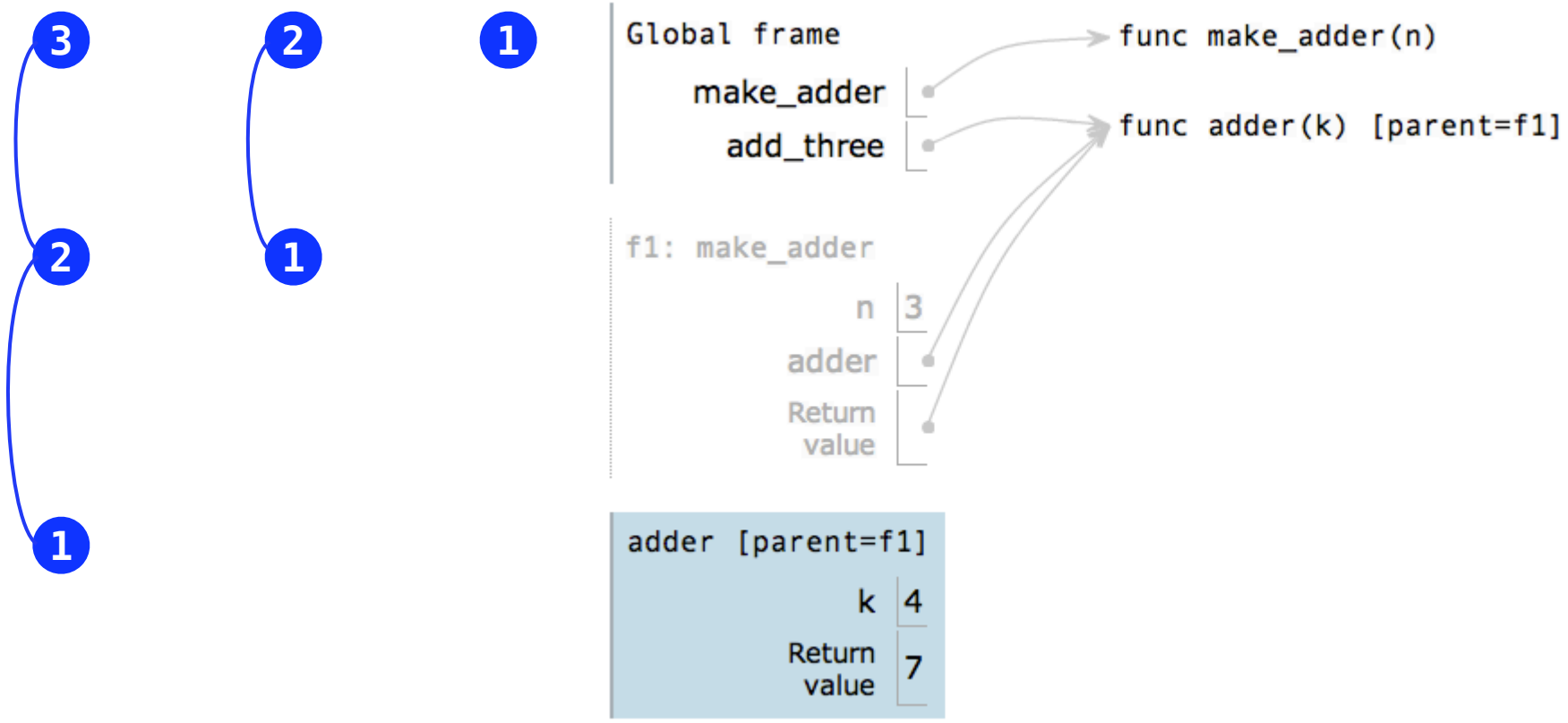

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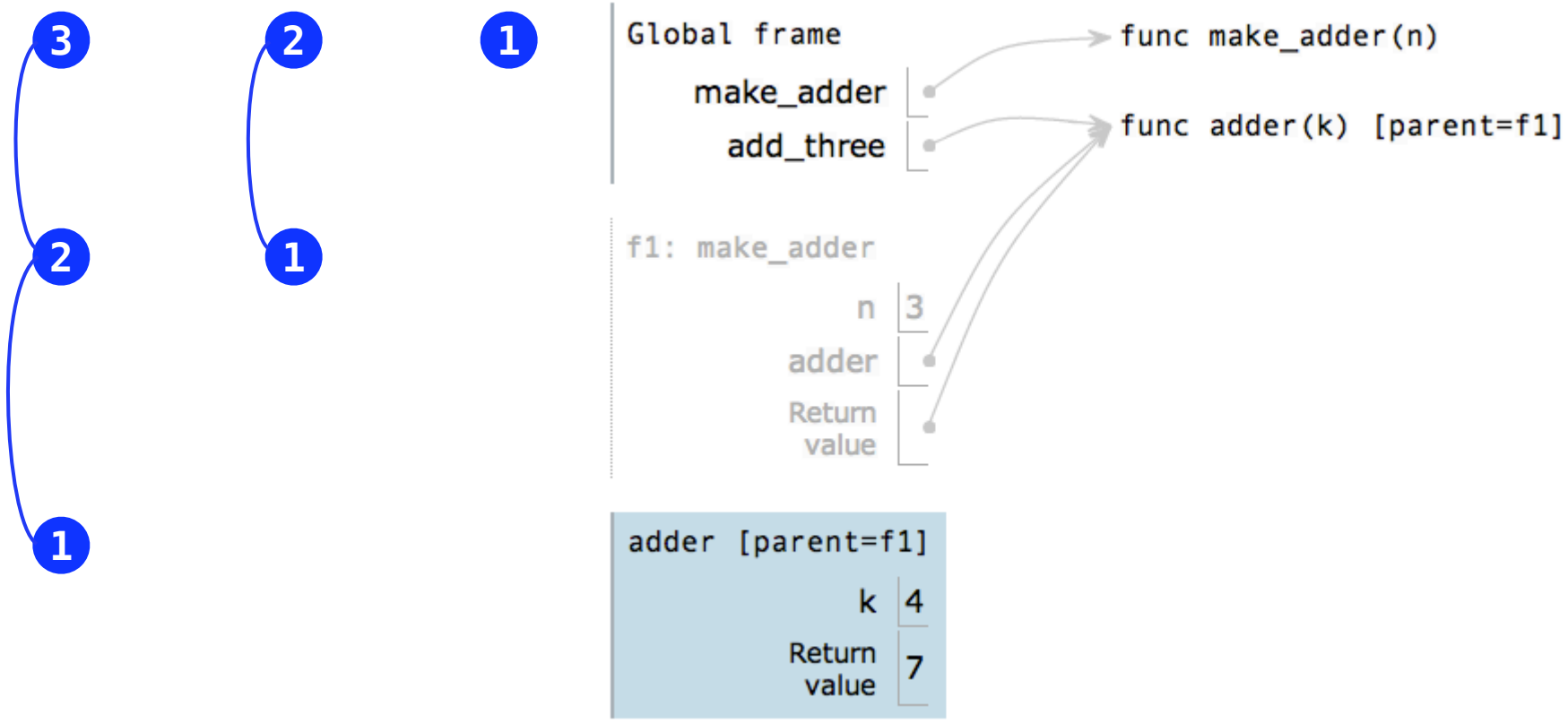

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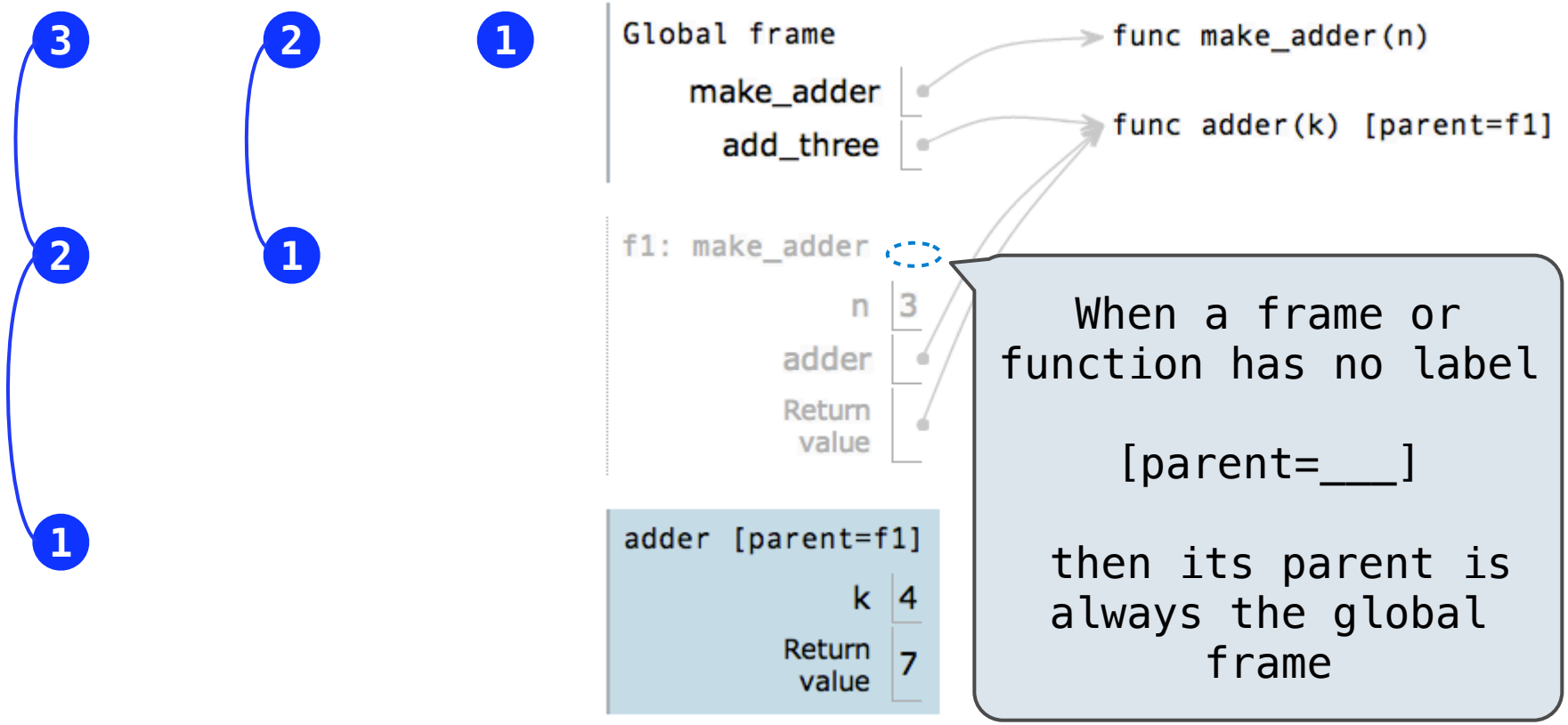

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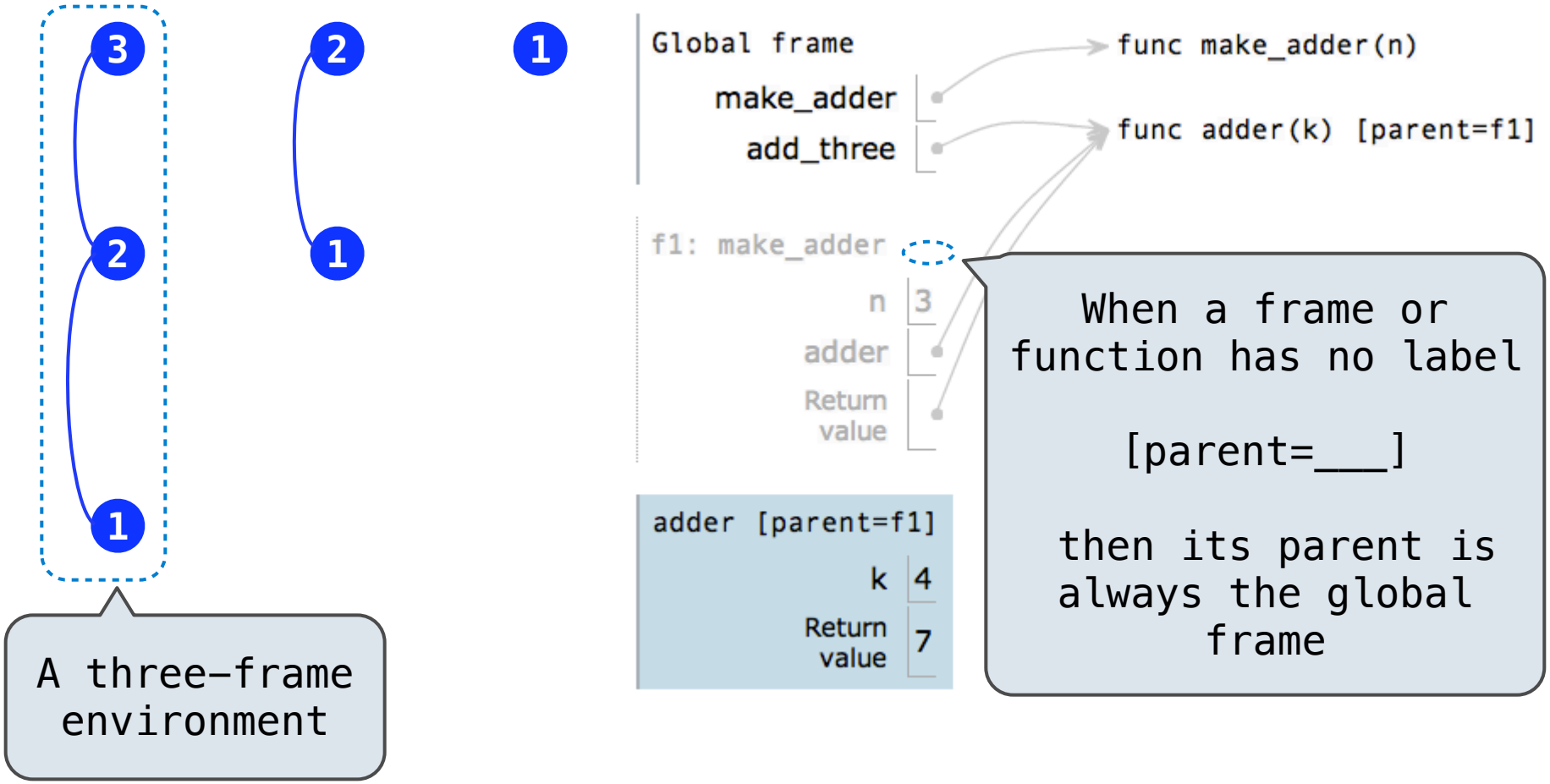

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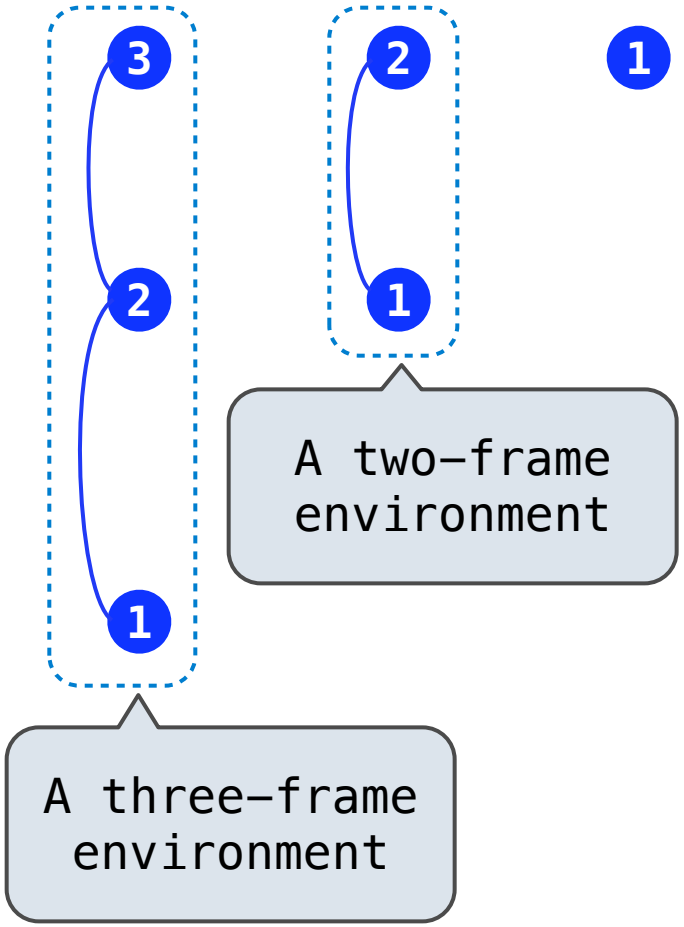

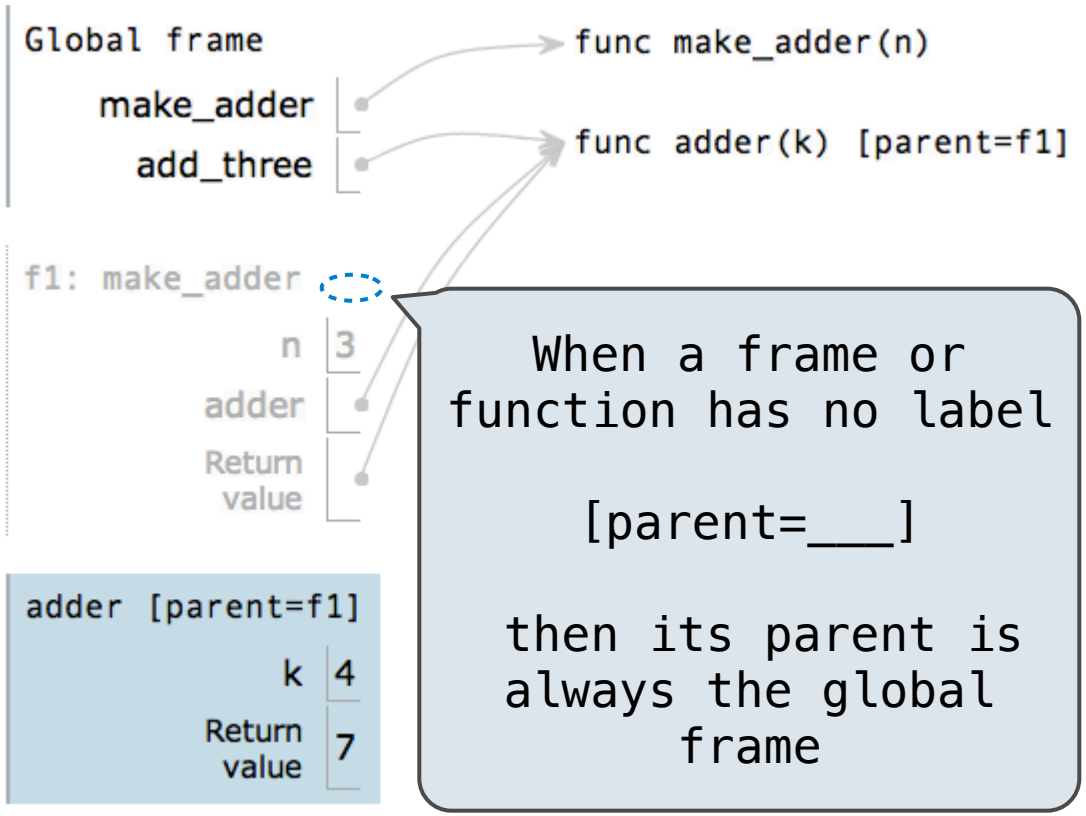

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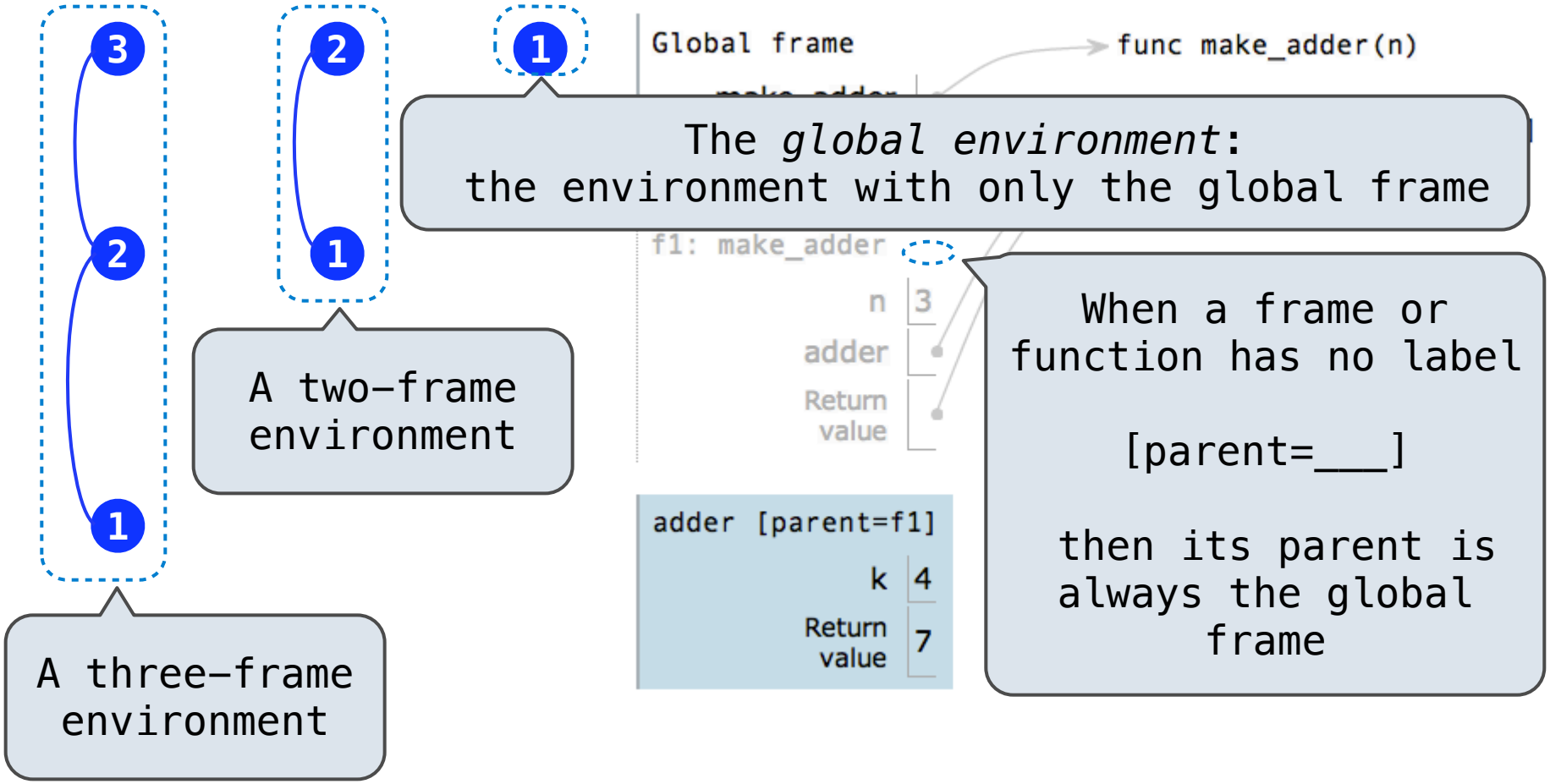

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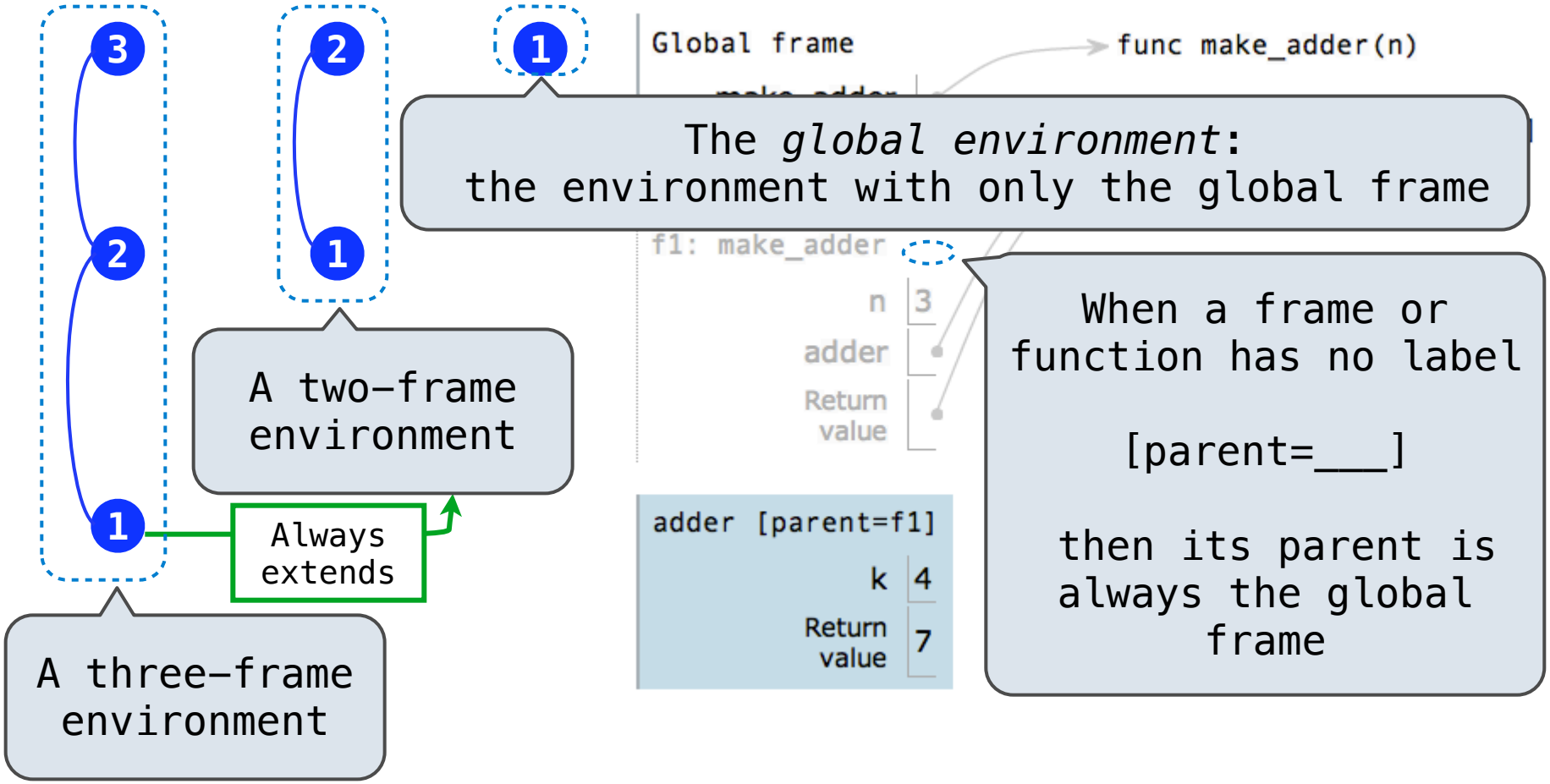

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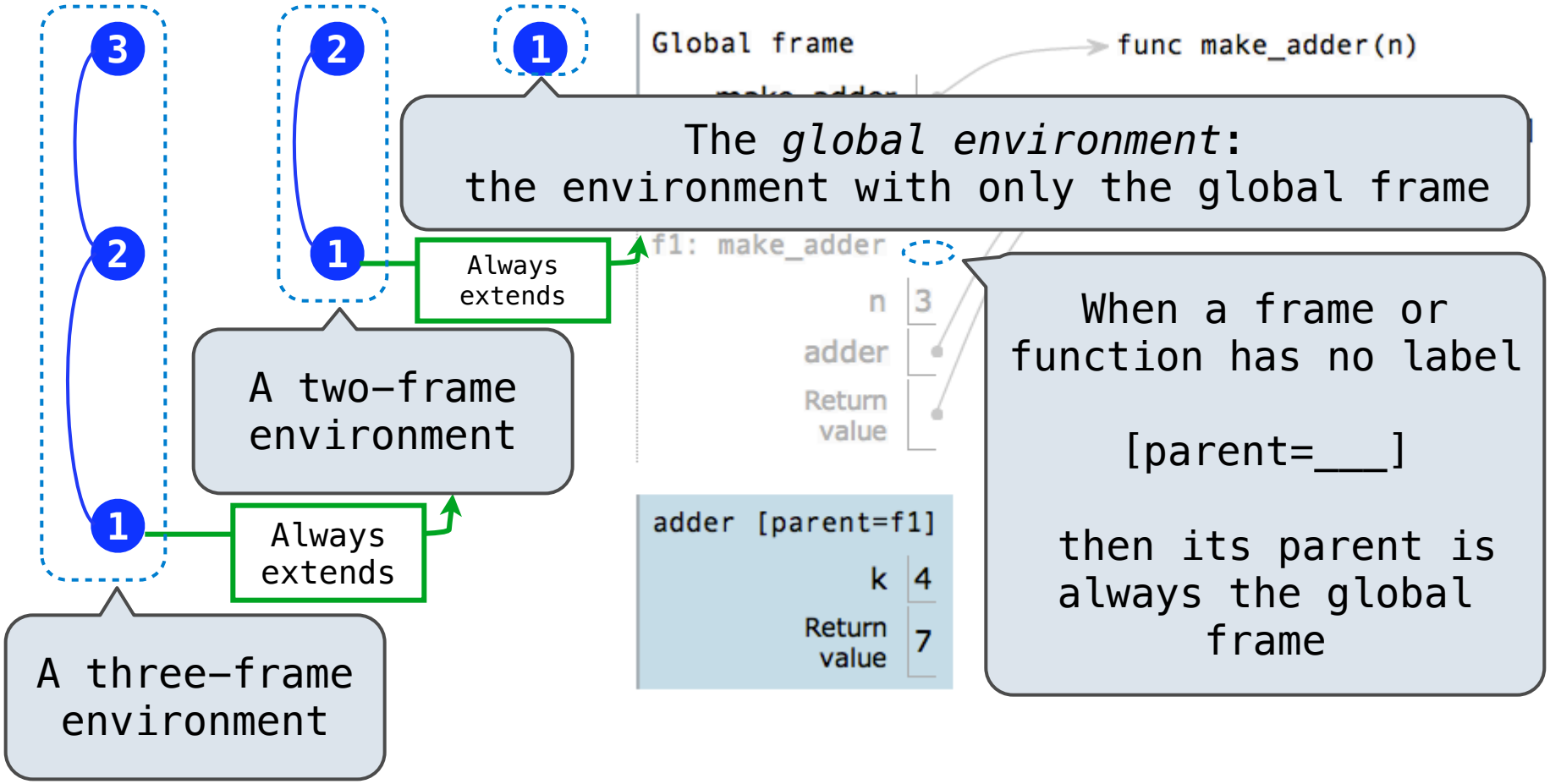

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How to Draw an Environment Diagram

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When defining a function:

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1. Create a function value with signature
   <name>(*<formal parameters>*)

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2. For nested definitions, label the parent as the first frame of the current environment

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3. Bind *<name>* to the function value in the first frame of the current environment

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When calling a function:

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2. If the function has a parent label, copy it to this frame
3. Bind the *<formal parameters>* to the arguments in this frame
4. Execute the body of the function in the environment that starts with this frame

The Environment for Function Composition

Example: http://goo.gl/2IuEO

The Environment for Function Composition

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
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13
14 compose1(square, make_adder(2))(3)
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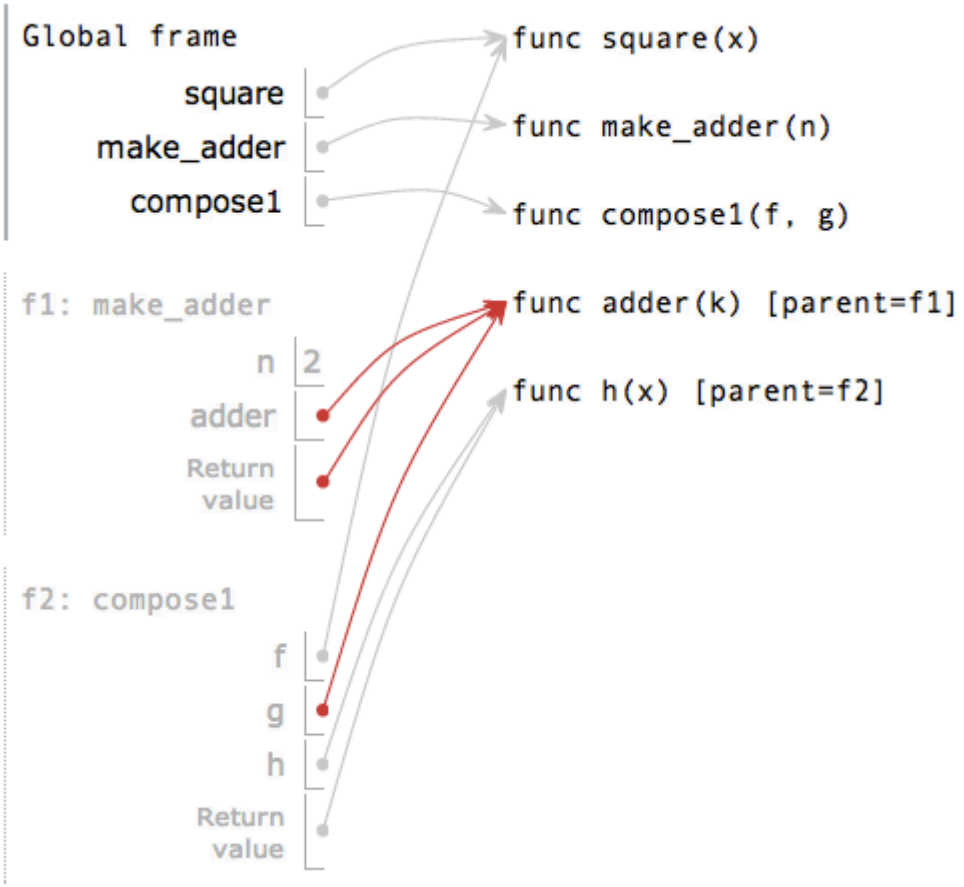

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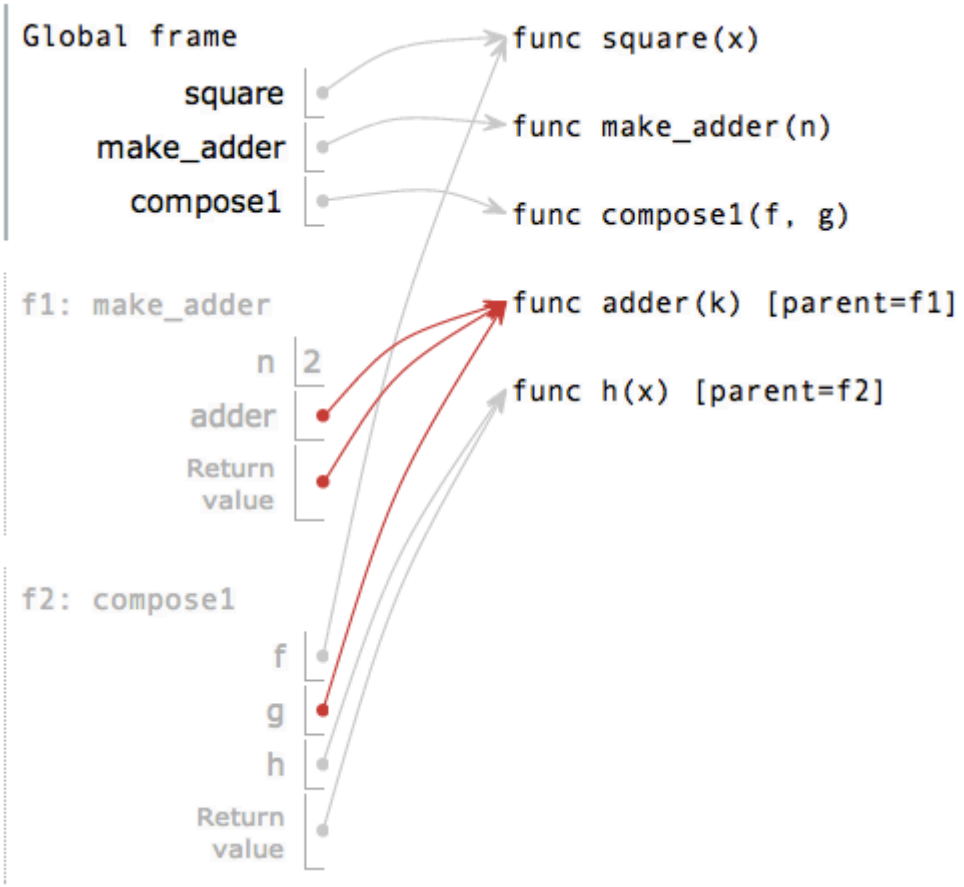

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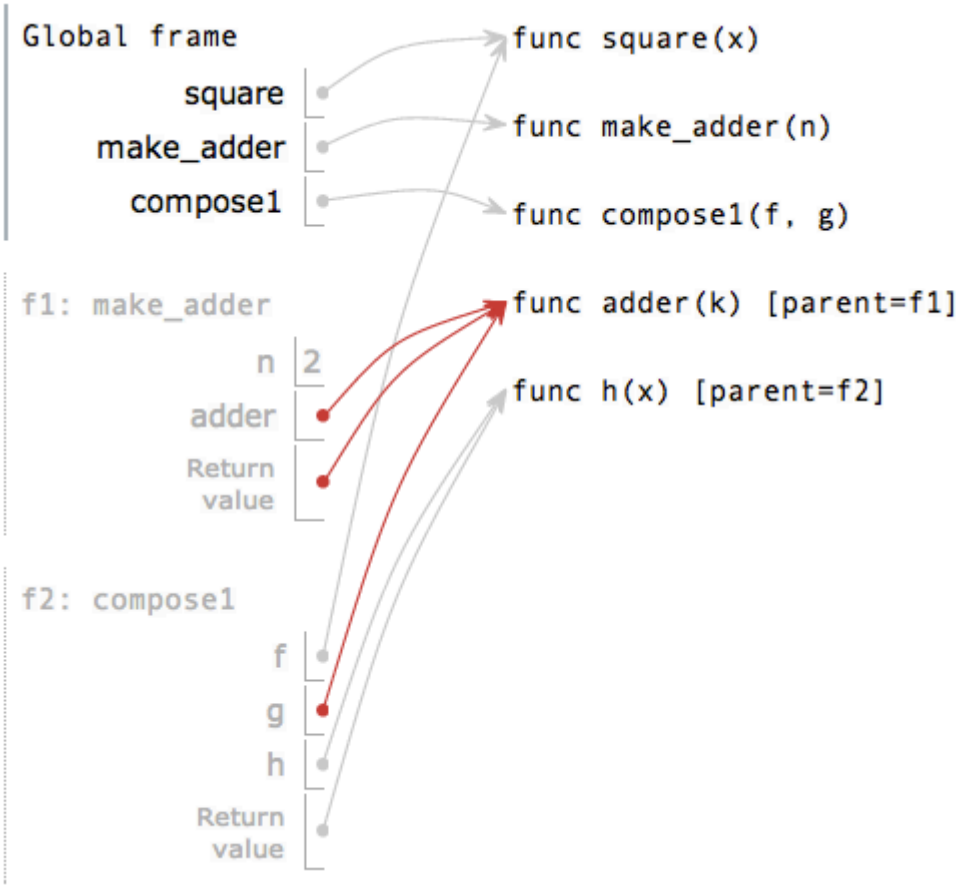

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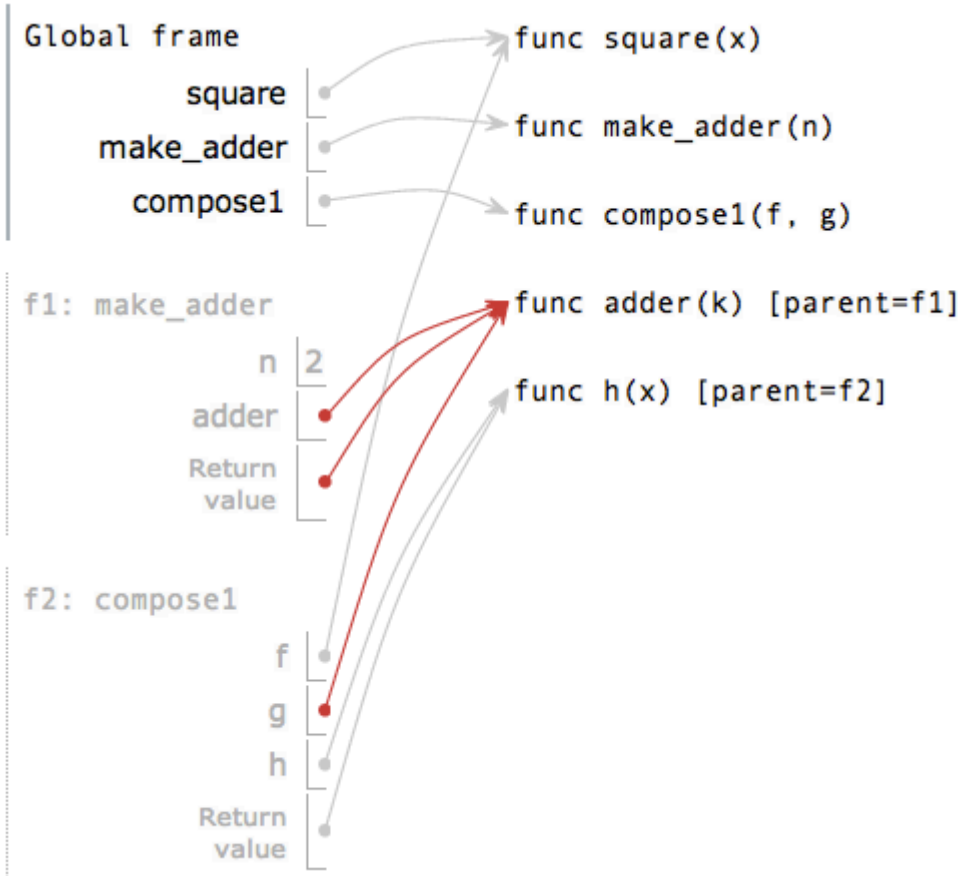
Return value of make_adder is an argument to compose1



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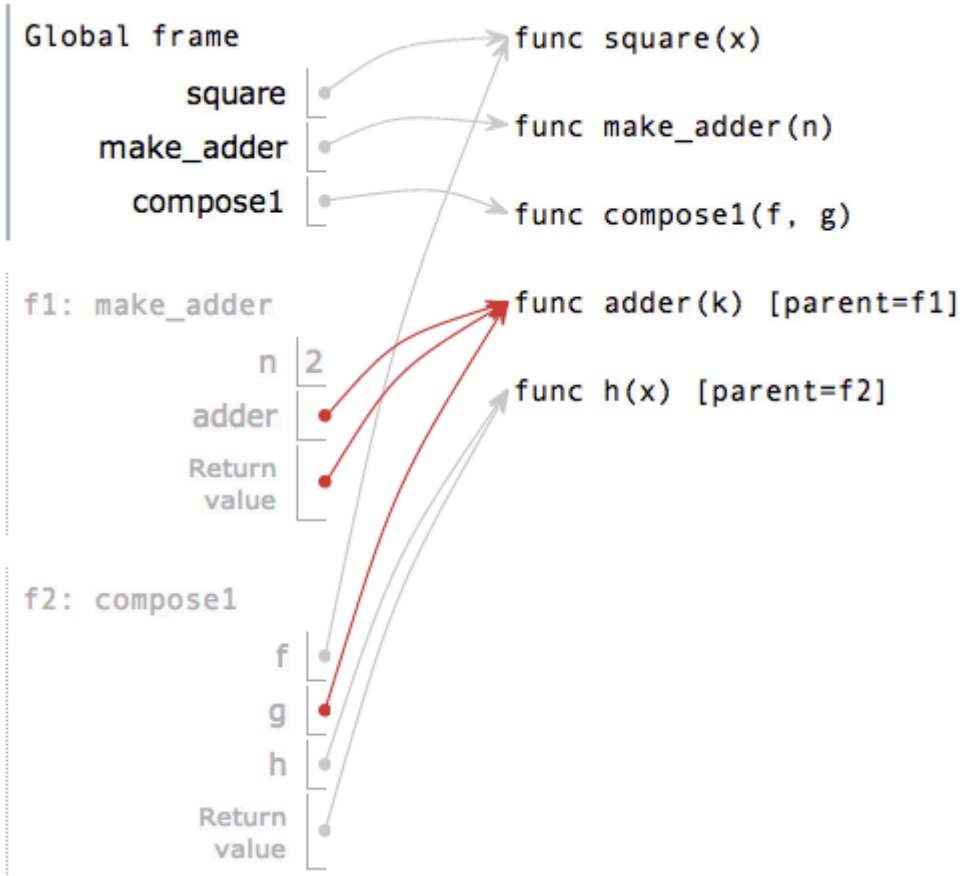

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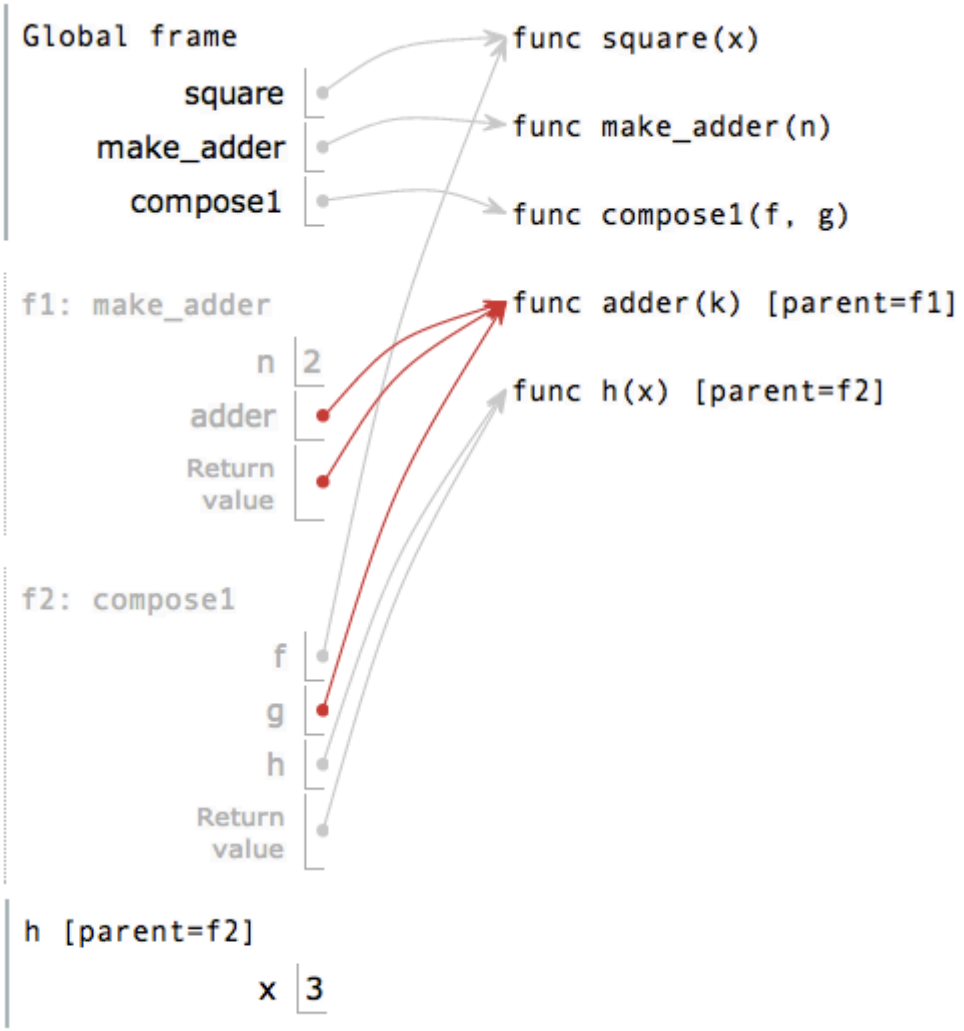

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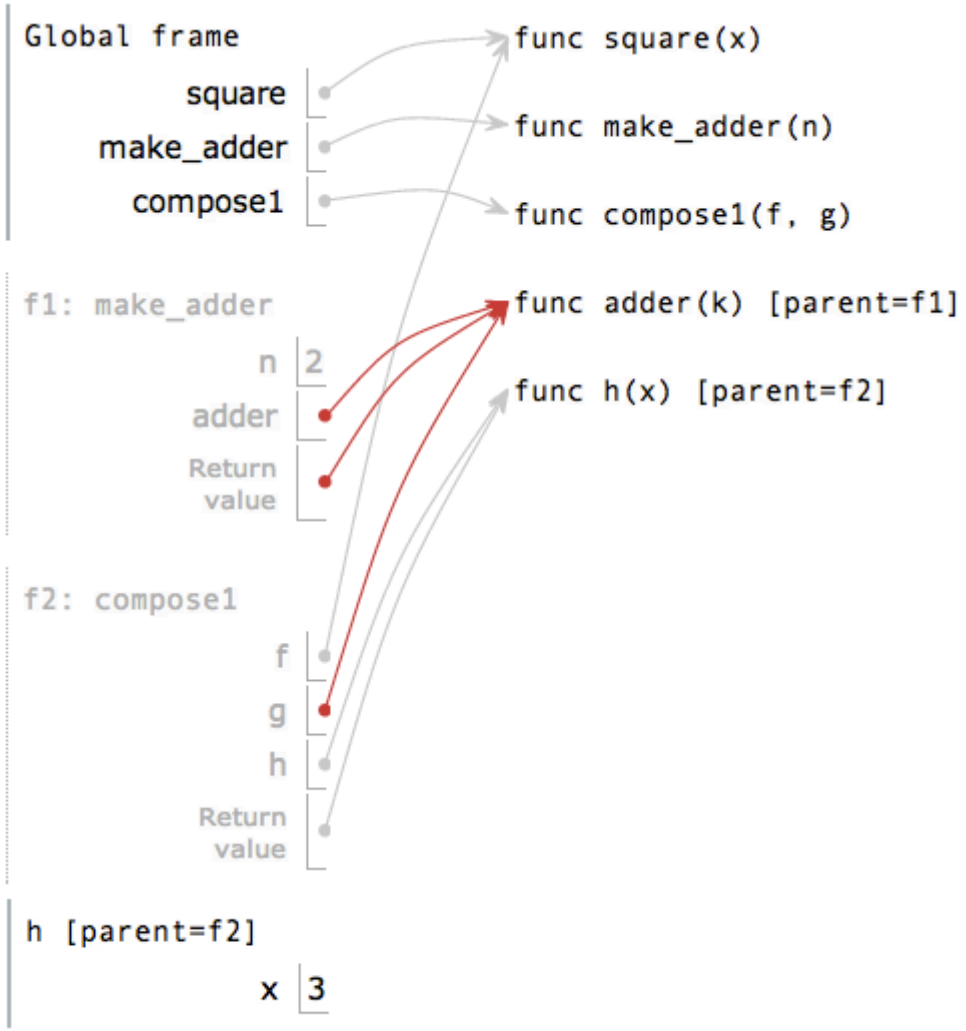

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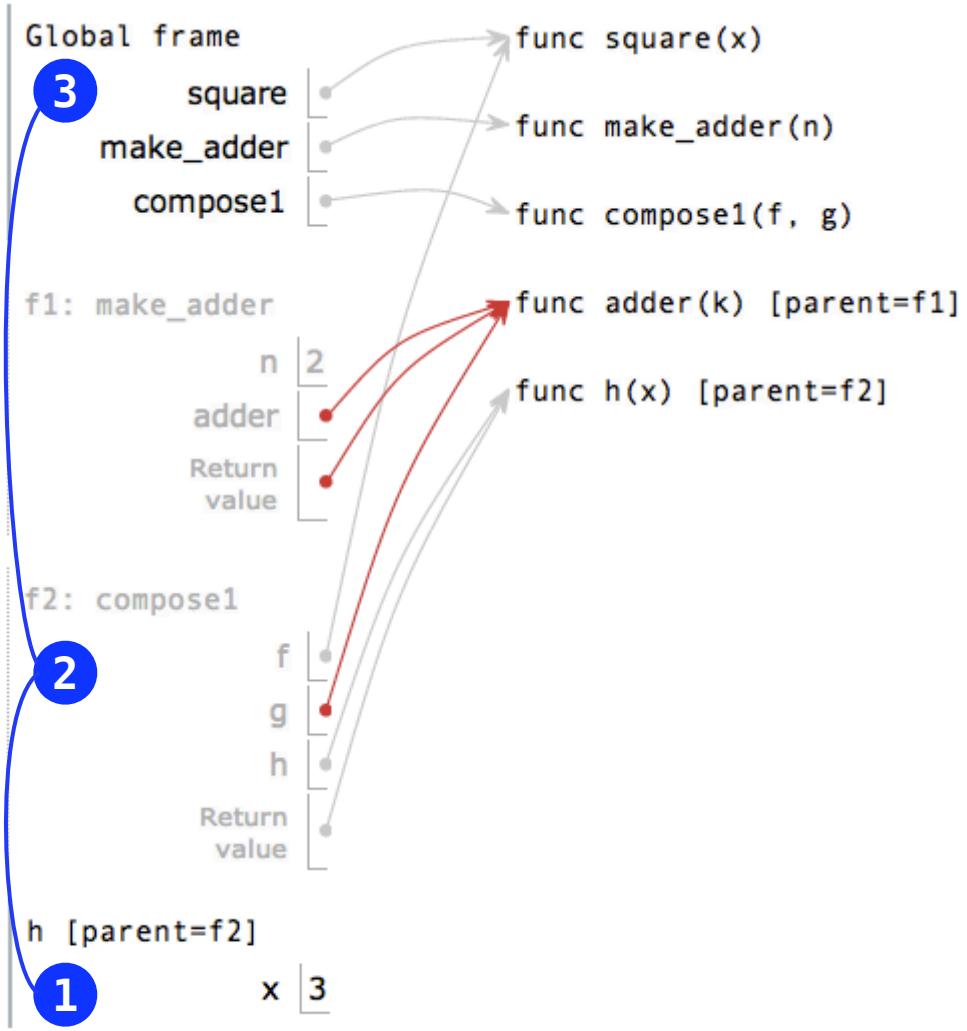

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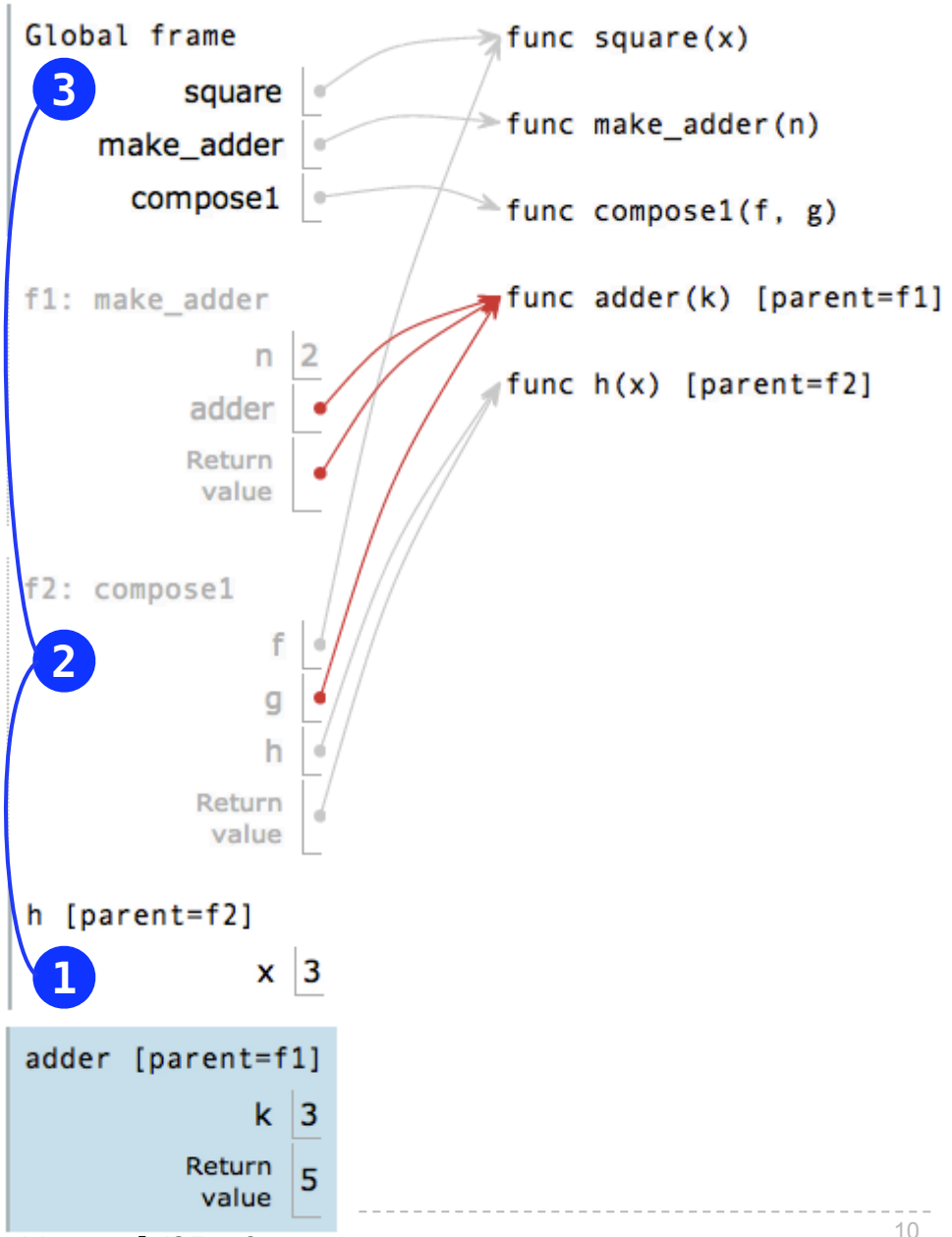

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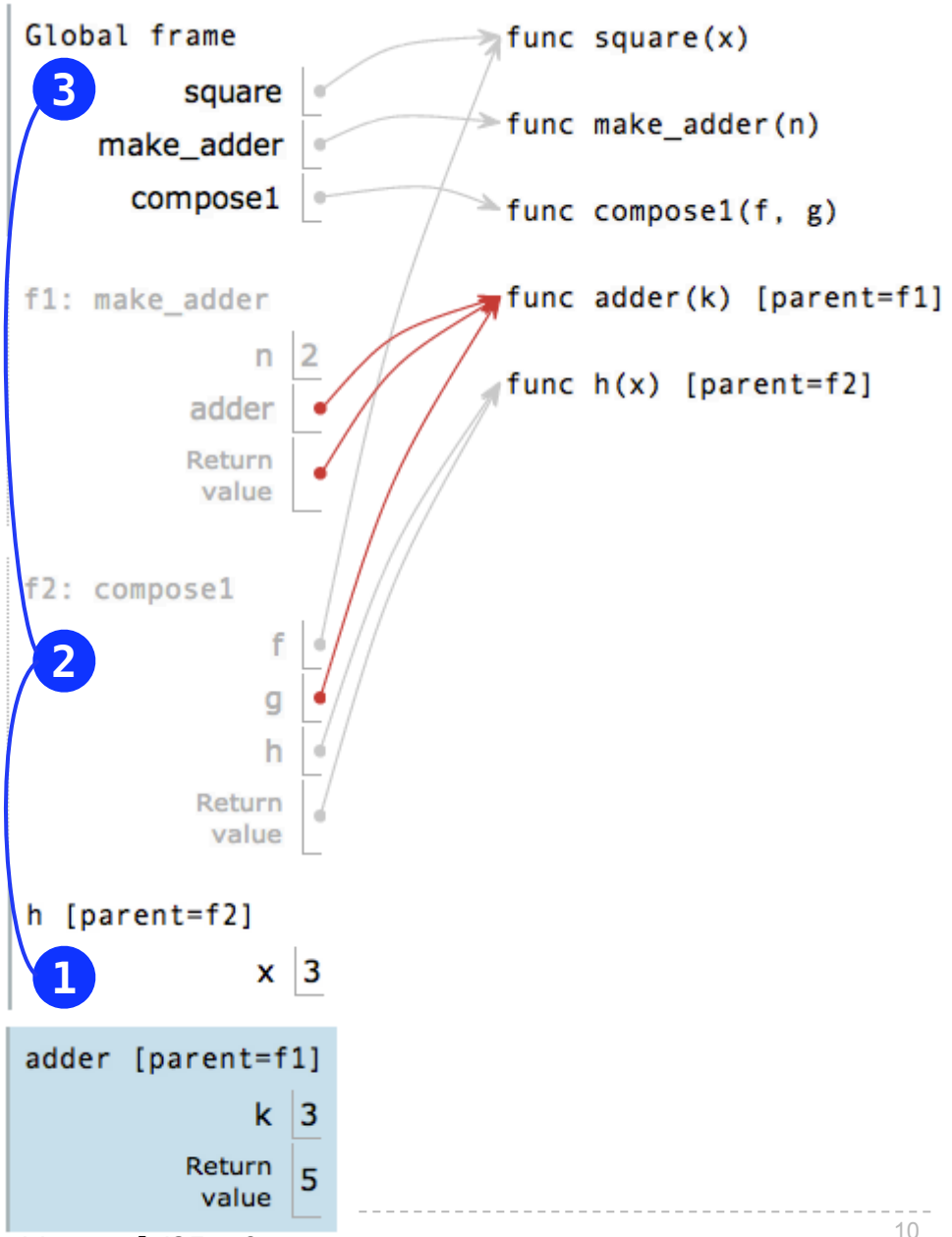

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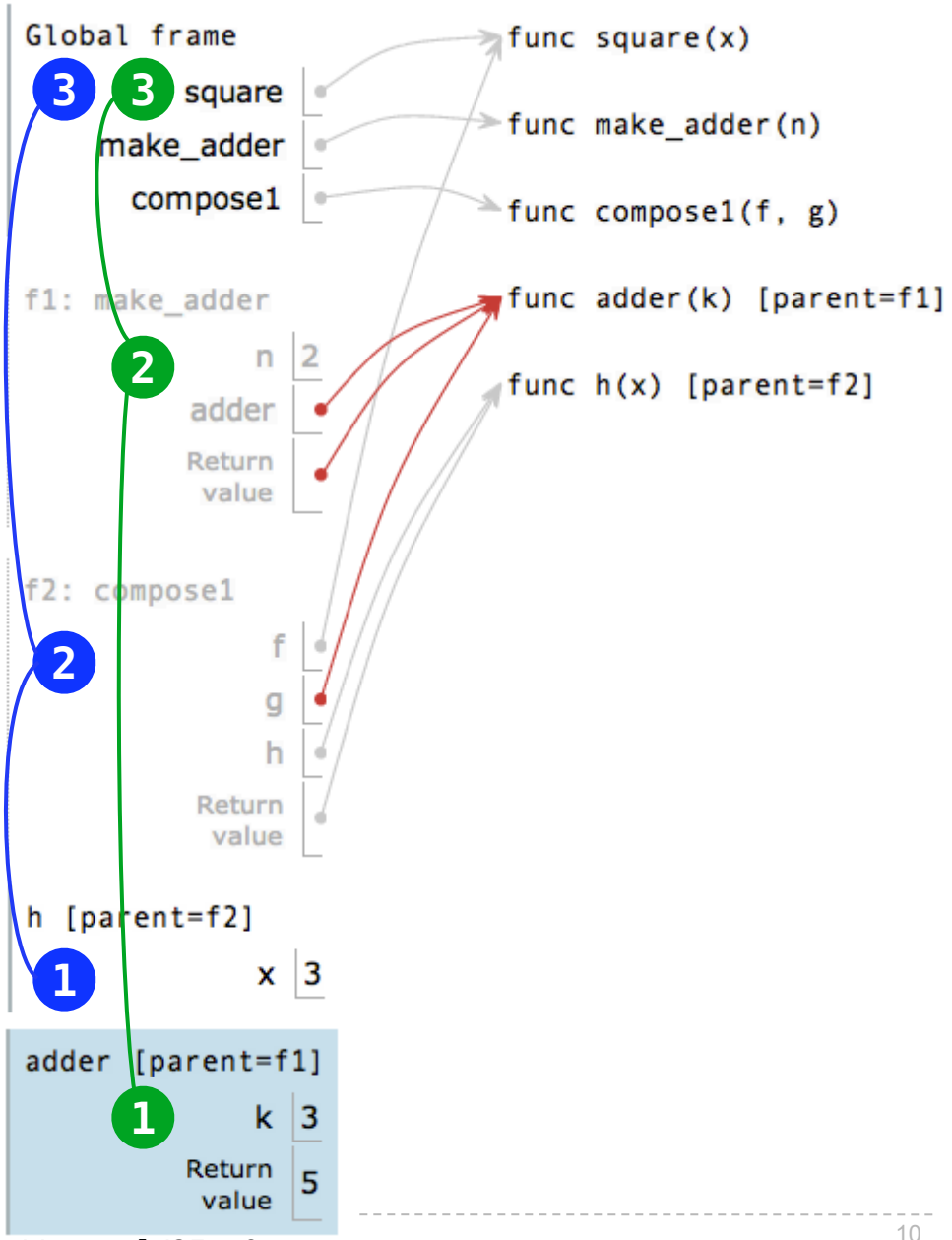

Example: http://goo.gl/2IuEO

Lambda Expressions

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>>> ten = 10
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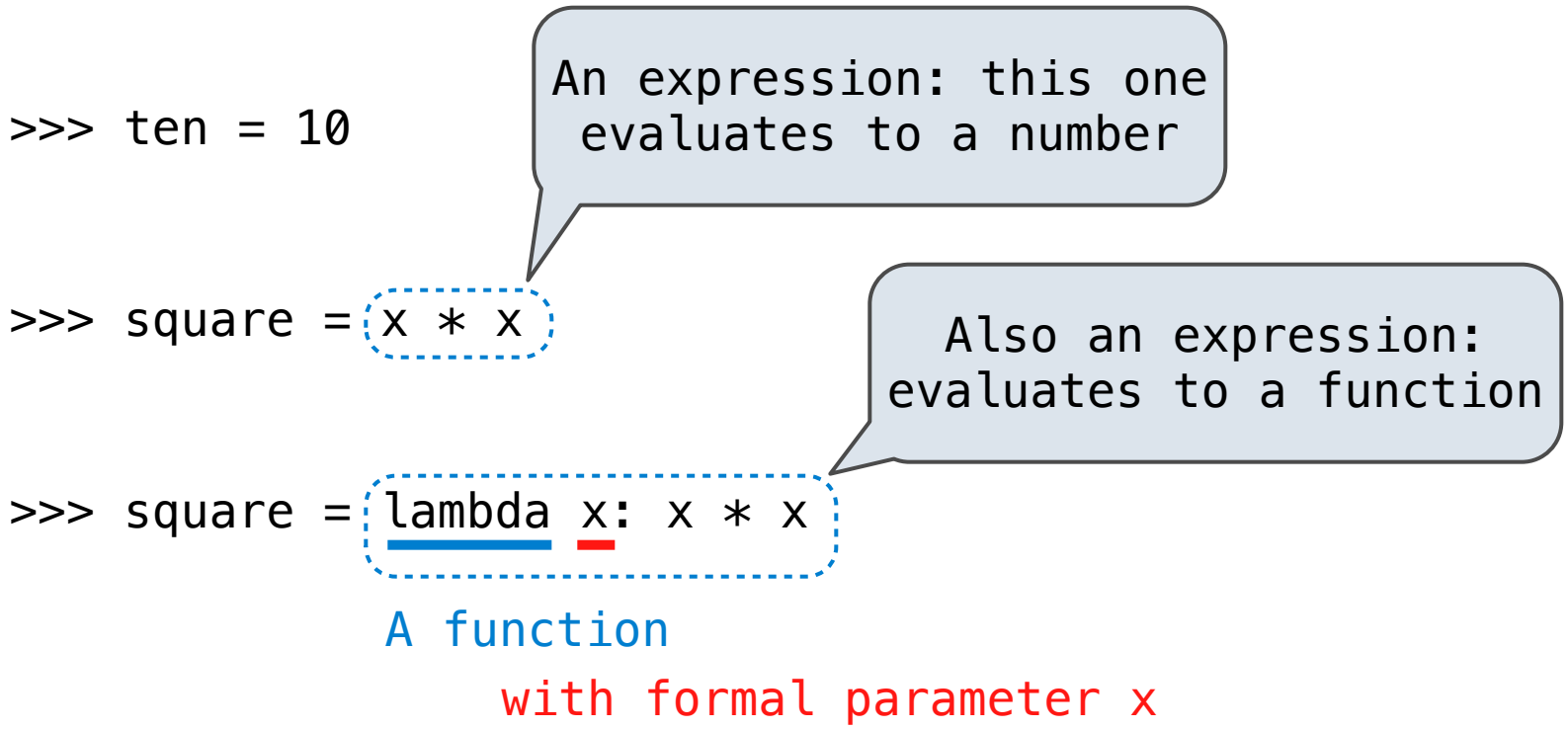
A function

Lambda Expressions



Lambda Expressions

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>>> square = x * x
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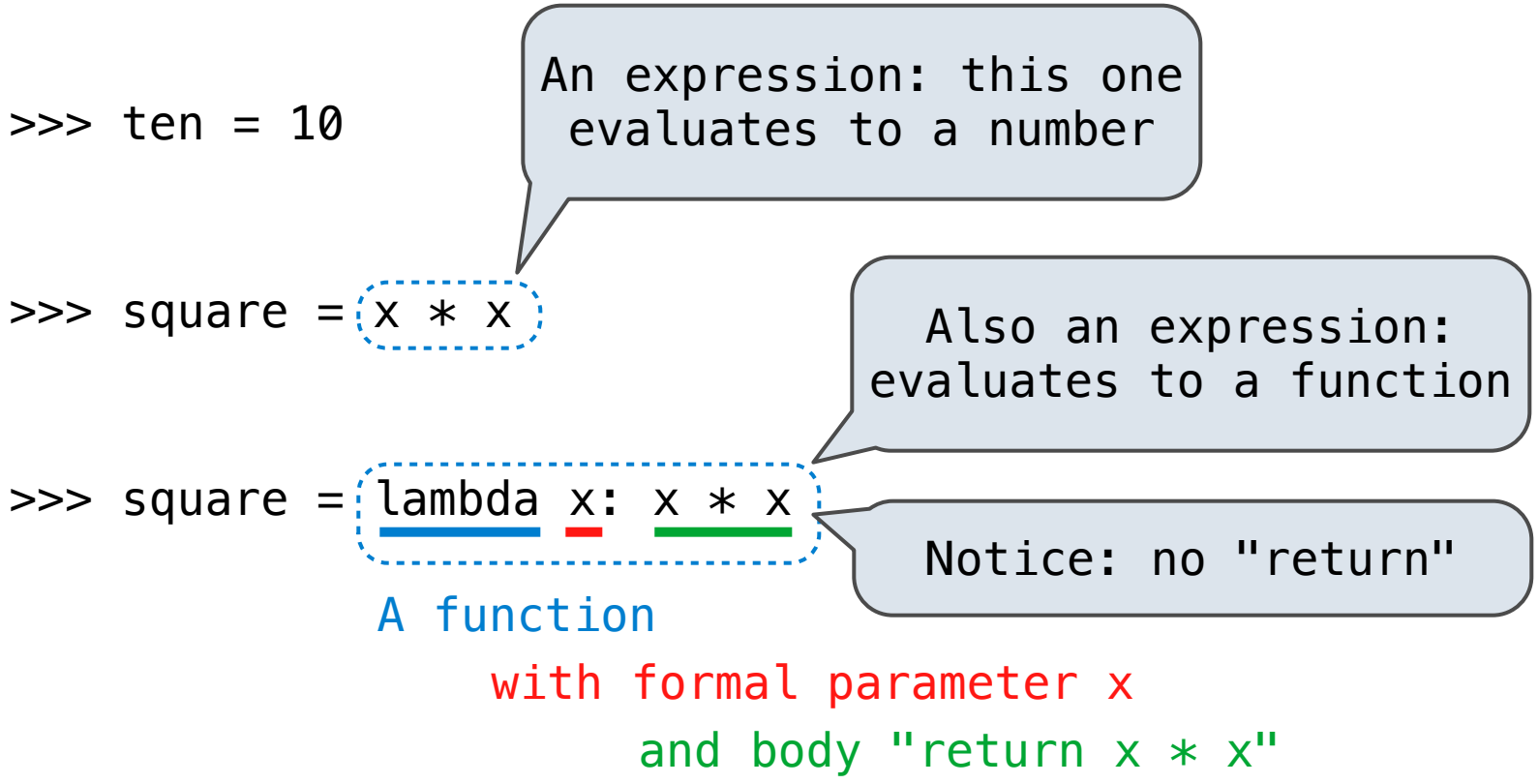
An expression: this one evaluates to a number

Also an expression: evaluates to a function

A function
with formal parameter x
and body "return x * x"

Lambda Expressions



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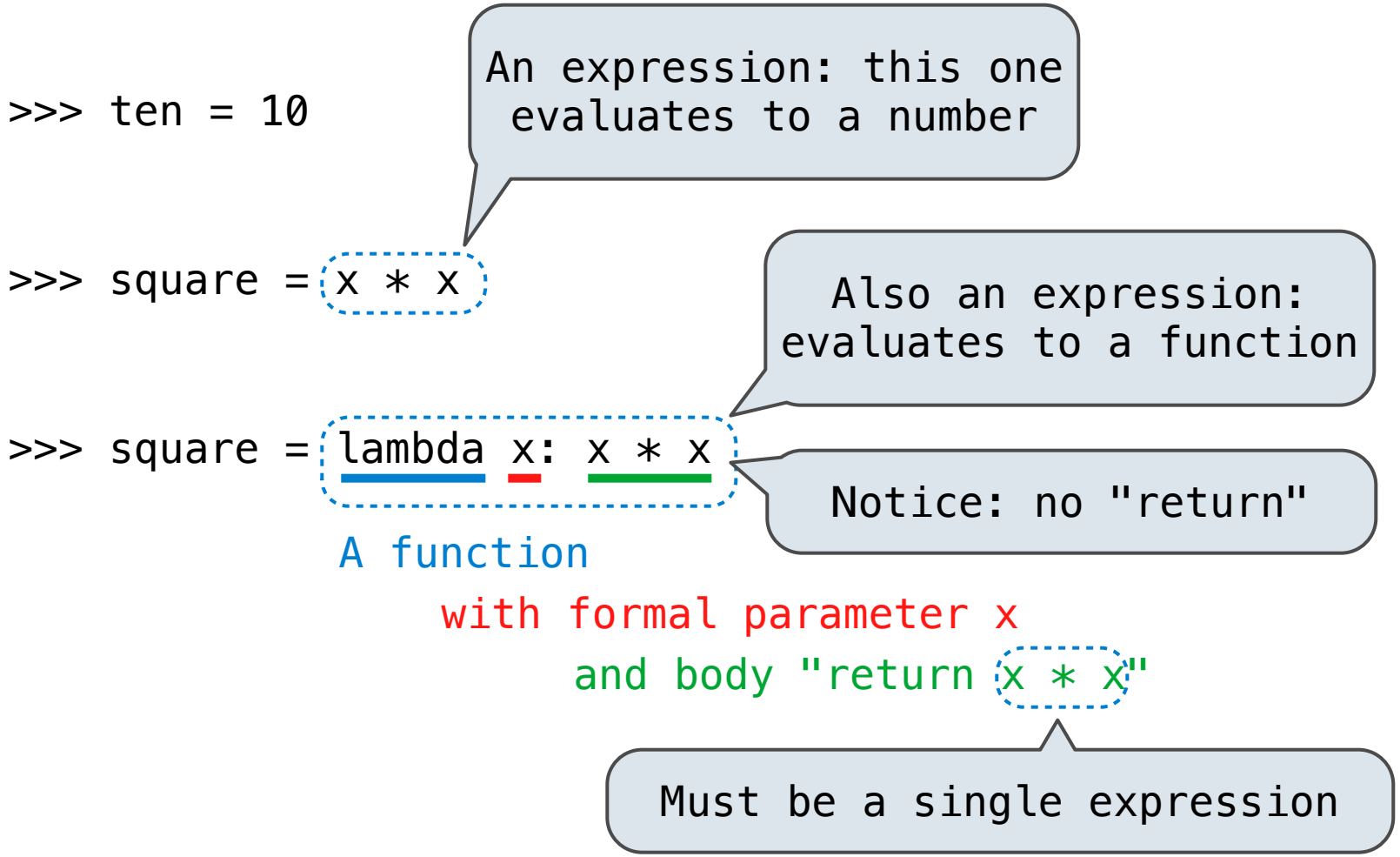

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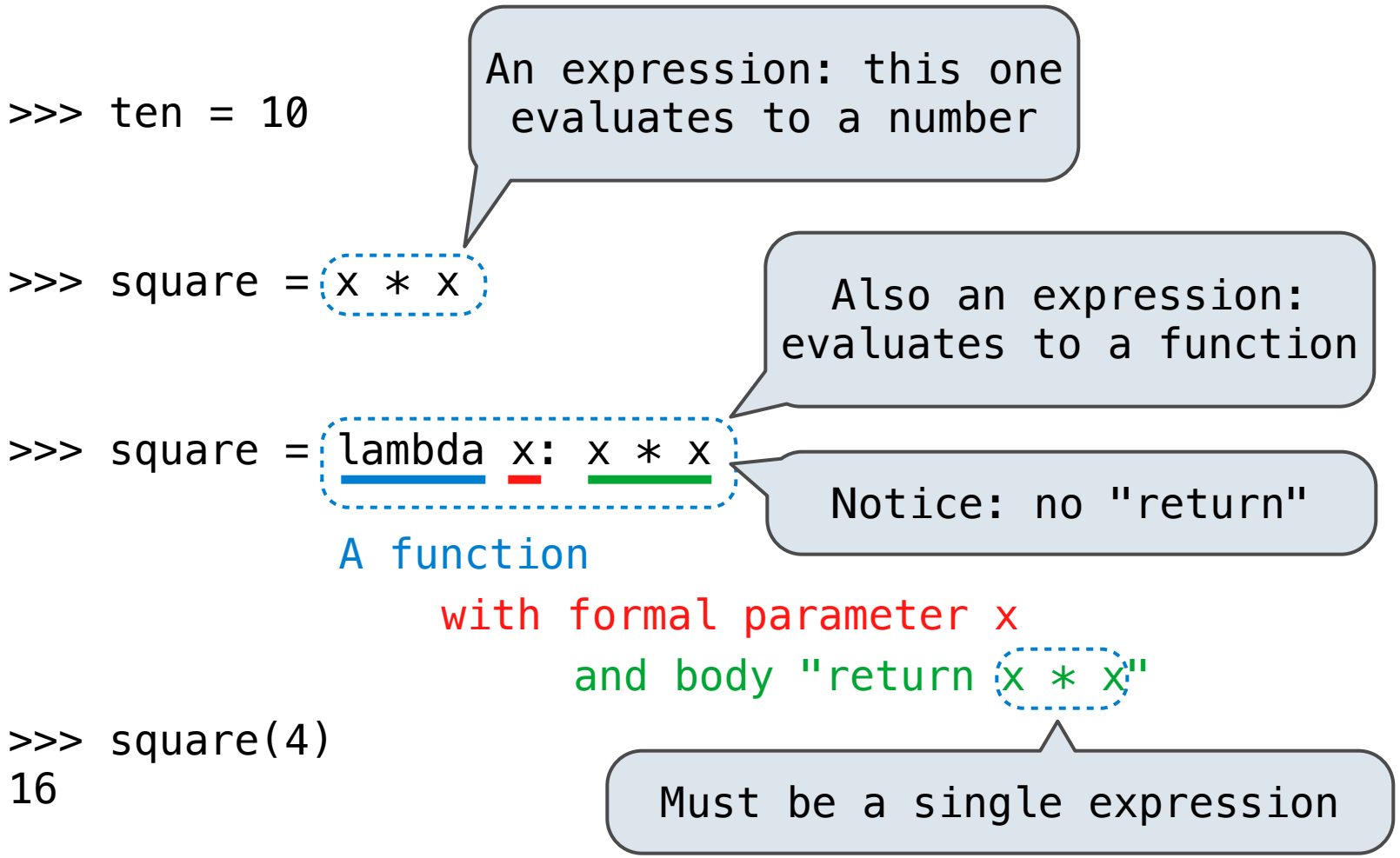

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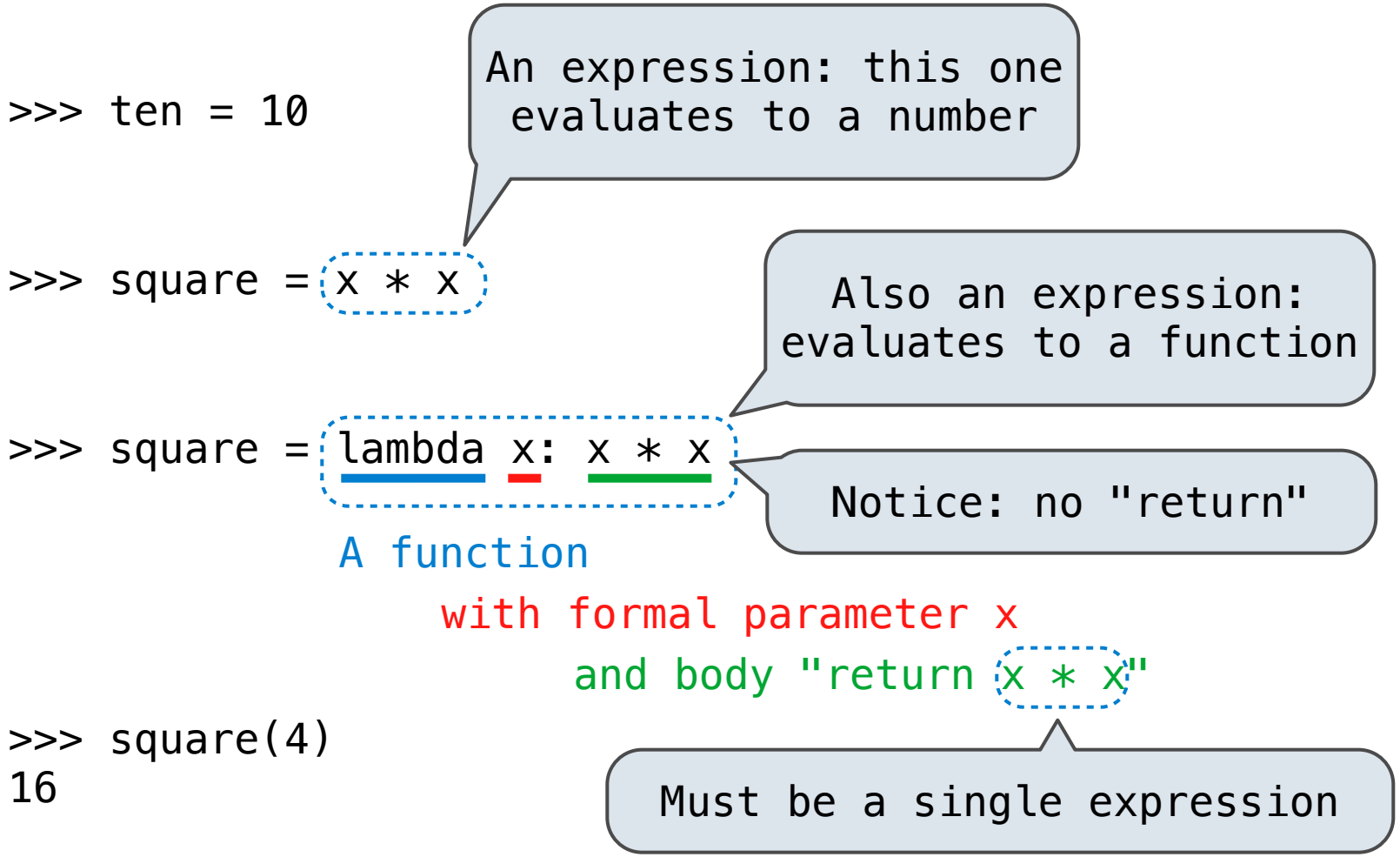

Lambda expressions are rare in Python, but important in general

More Higher-Order Function Examples

(Demo)