61A Lecture 6

Friday, September 7

Lambda Expressions

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Lambda expressions are rare in Python, but important in general

Lambda Expressions Versus Def Statements

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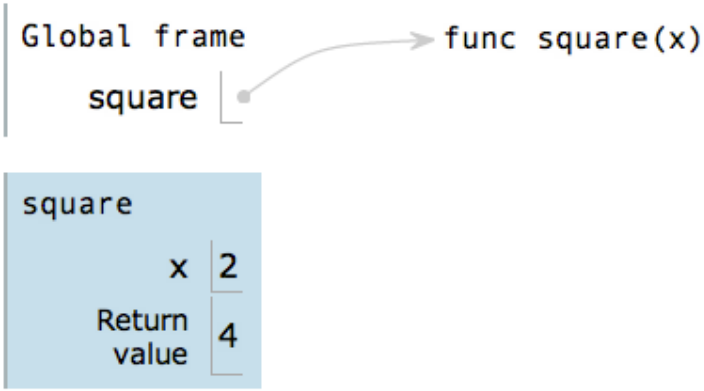

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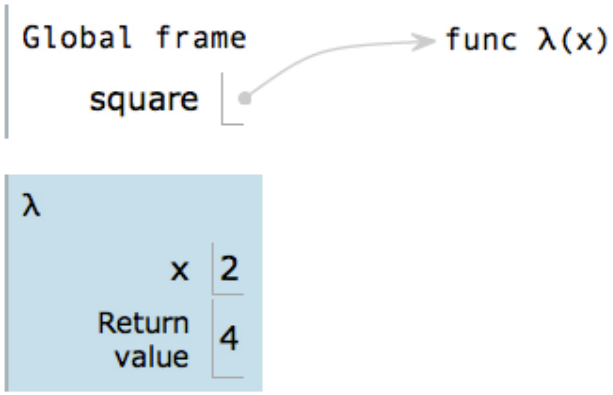

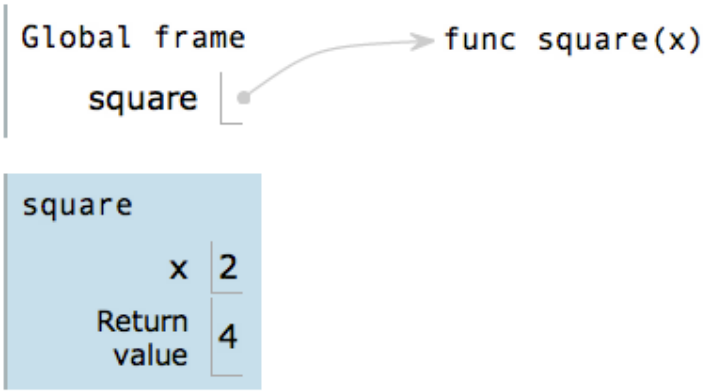

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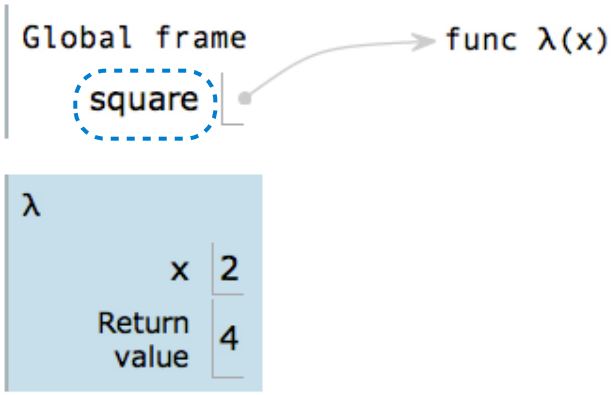

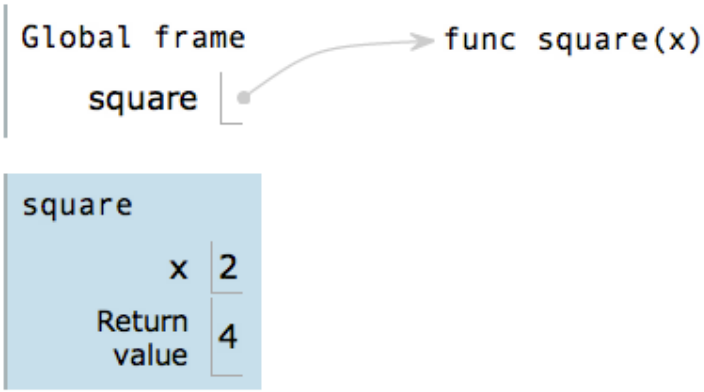

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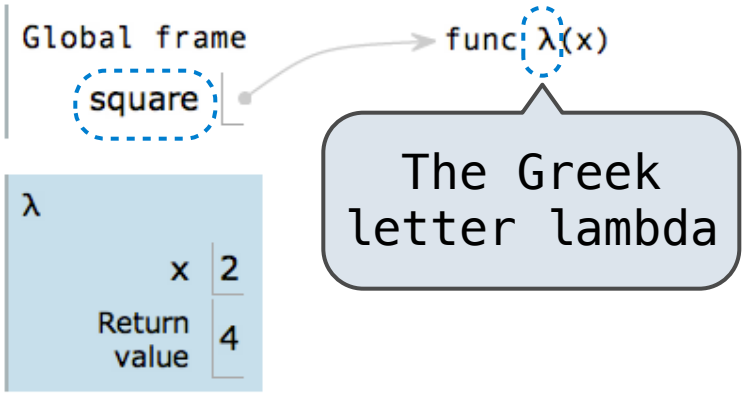

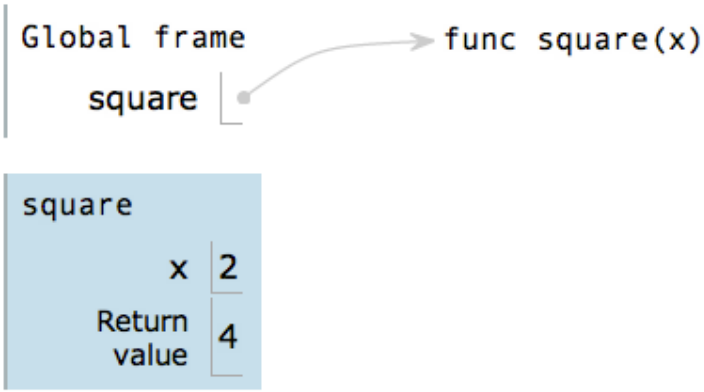

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Schönfinkeling?

Newton's Method Background

Finds approximations to zeroes of differentiable functions

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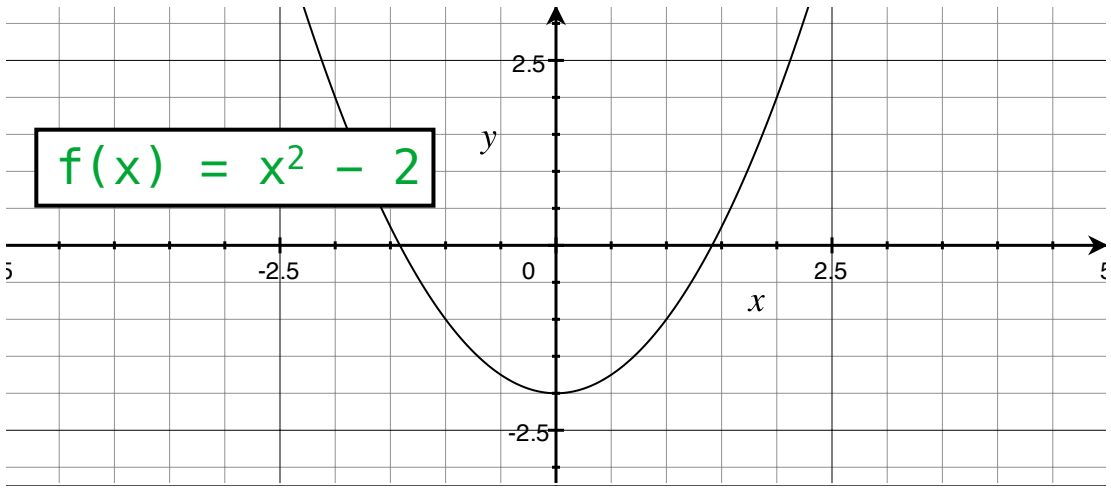

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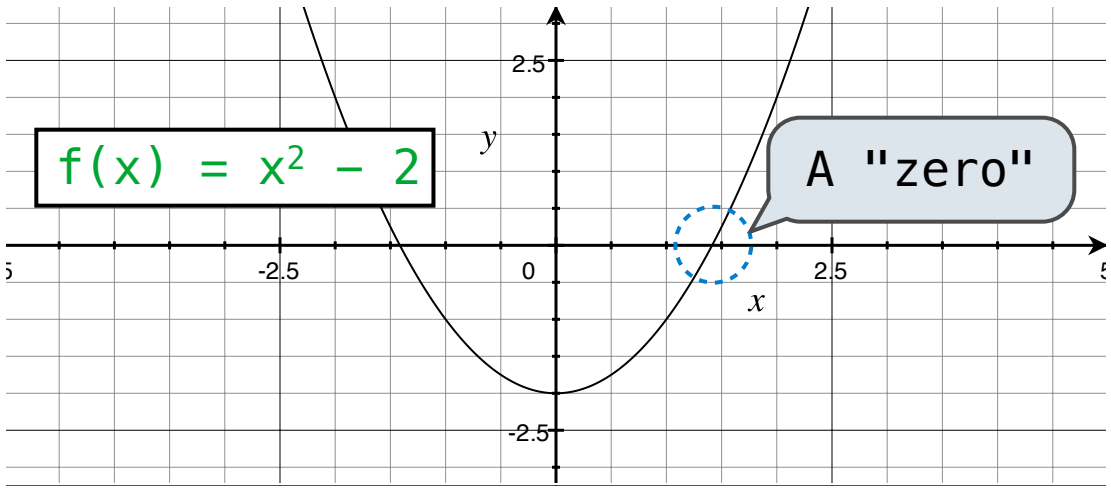

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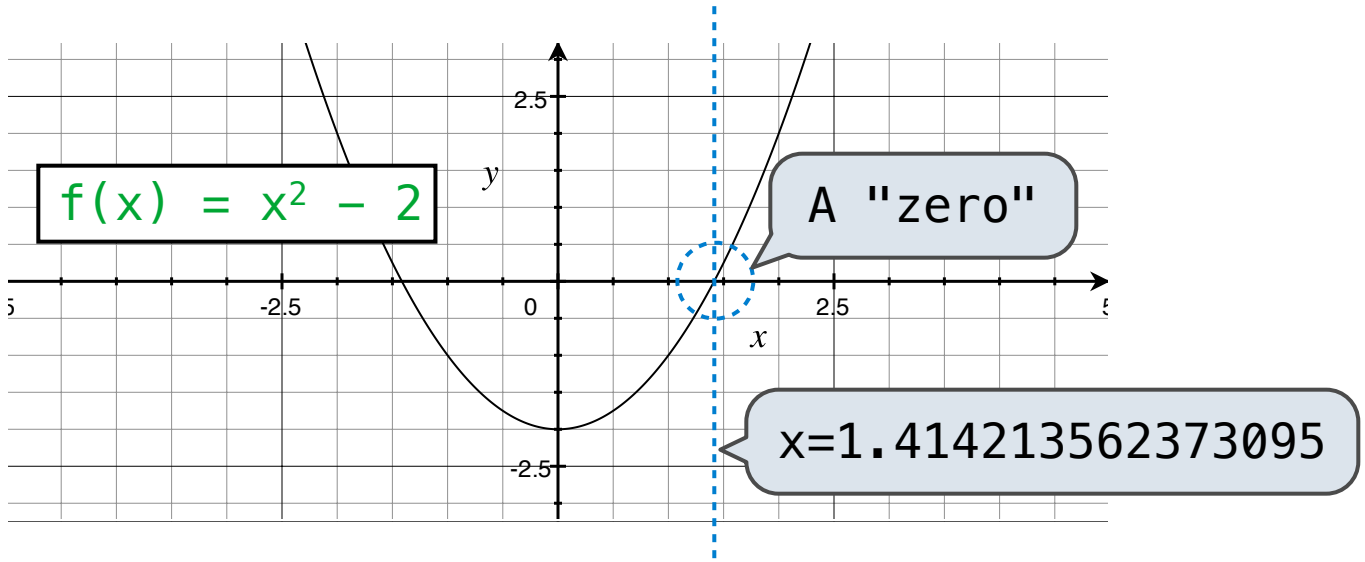

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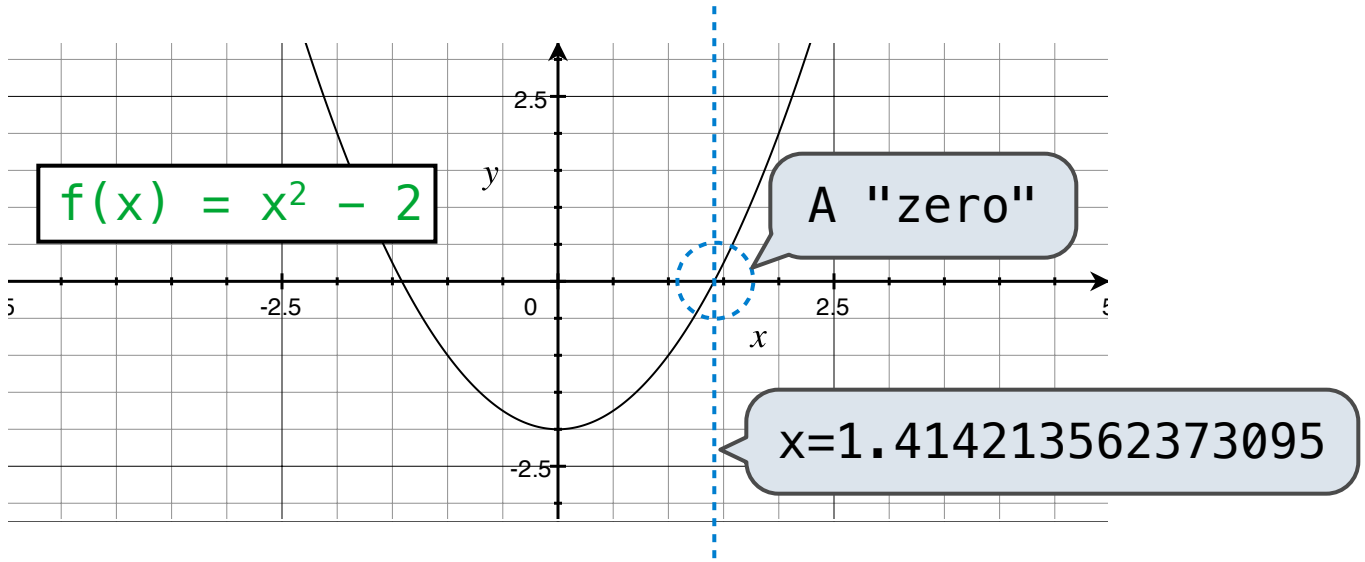

Application: a method for (approximately) computing square roots, using only basic arithmetic.

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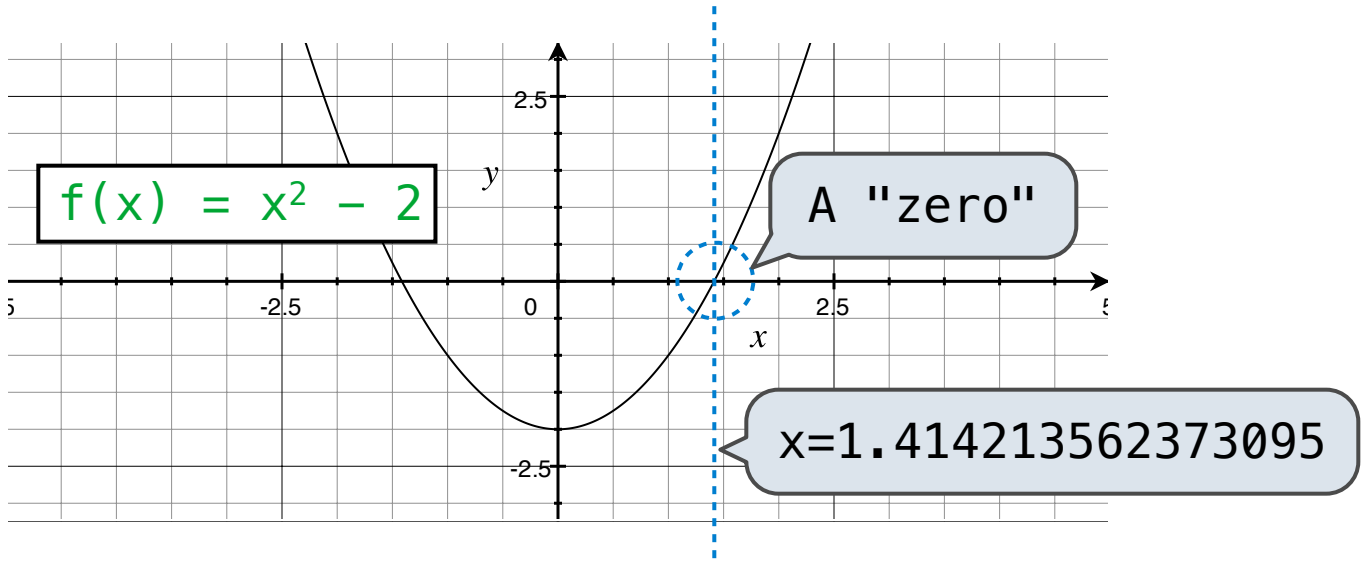

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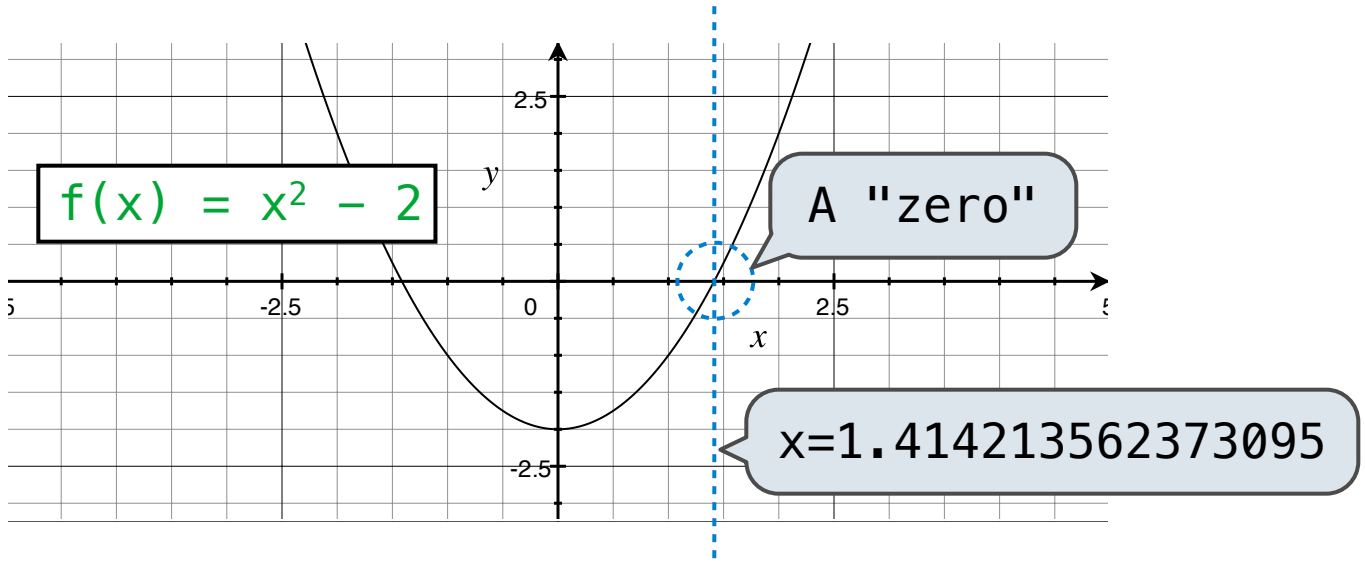

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Begin with a function f and

an initial guess x

$$x - \frac{f(x)}{f'(x)}$$

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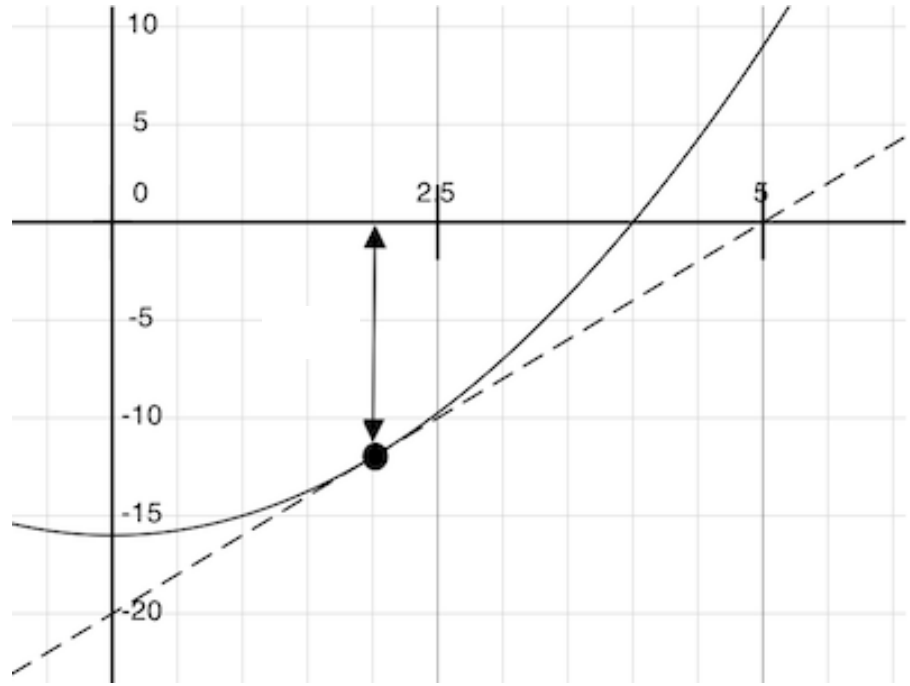

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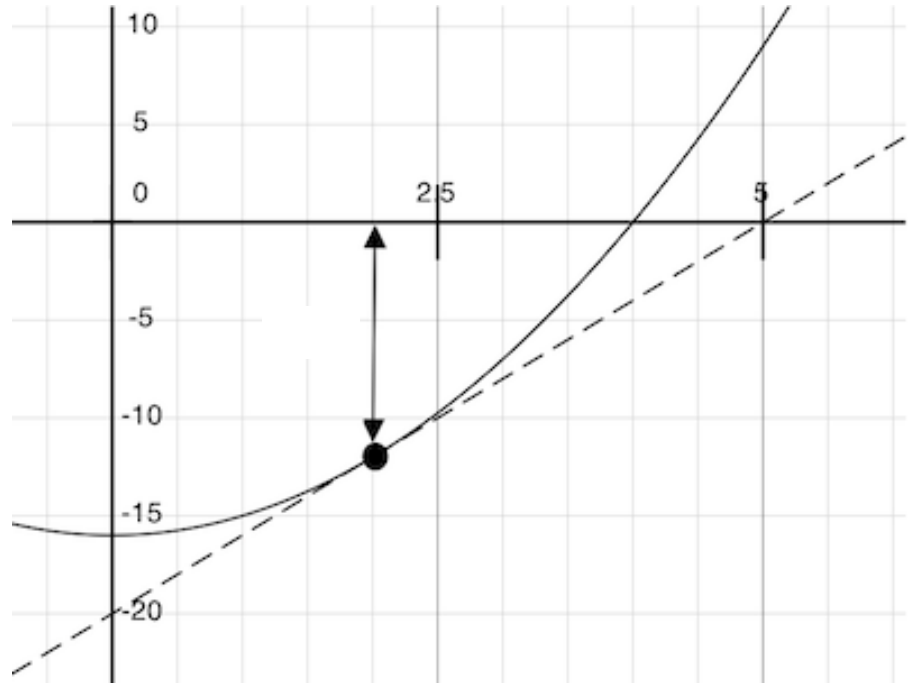

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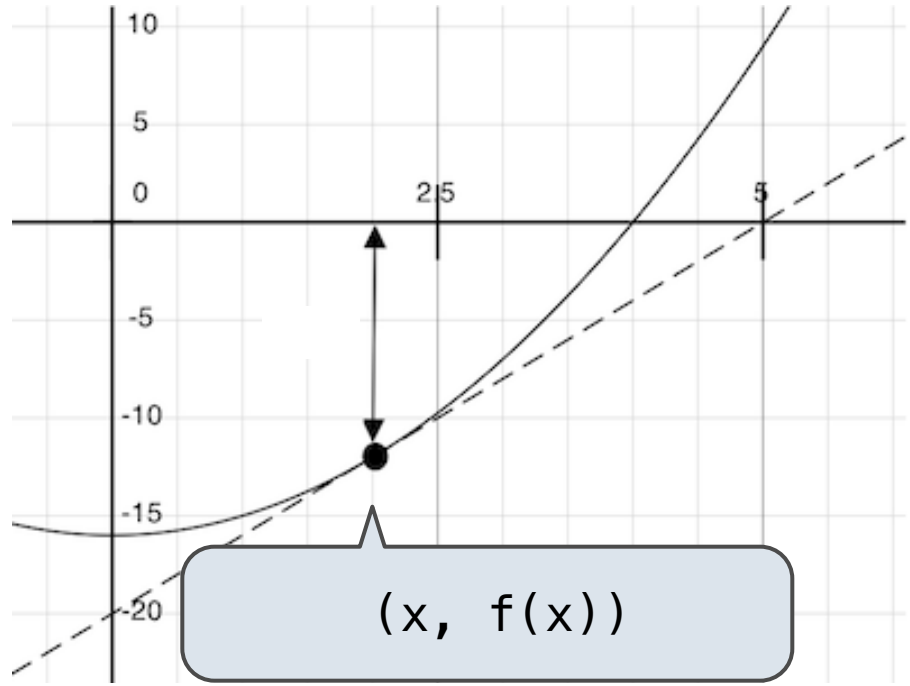

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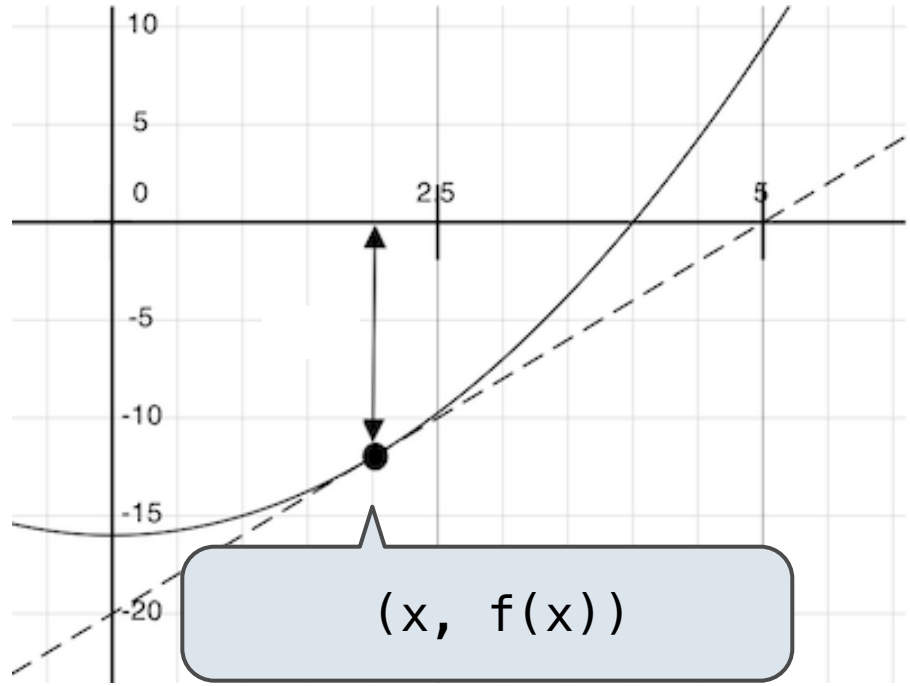

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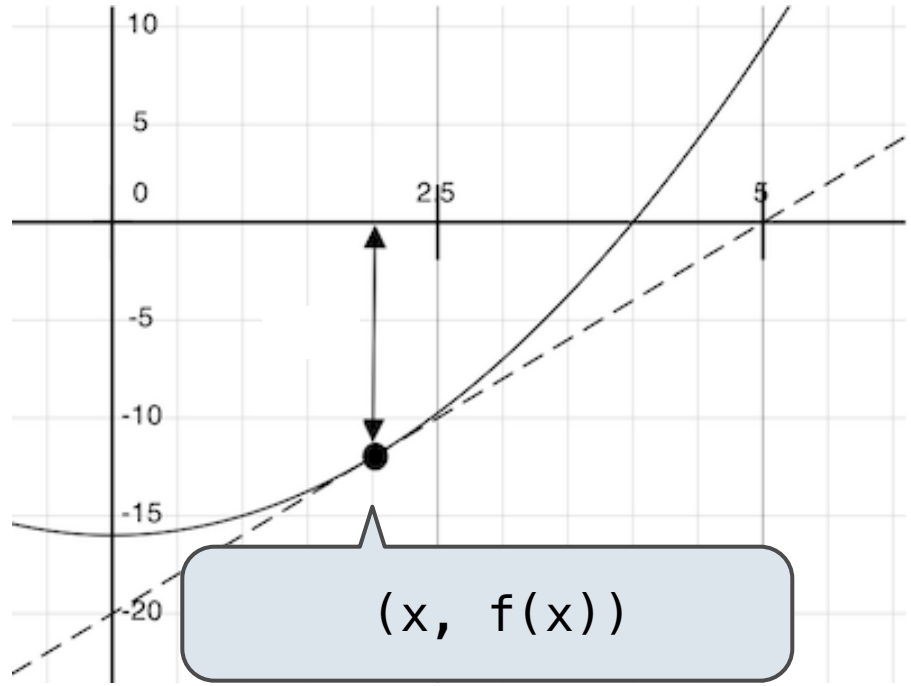

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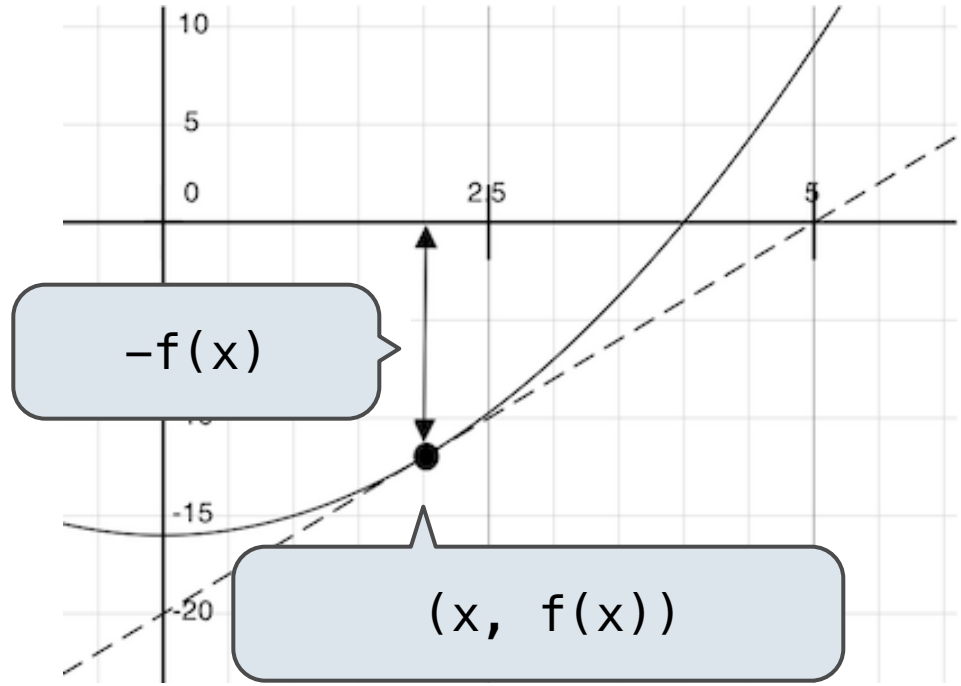

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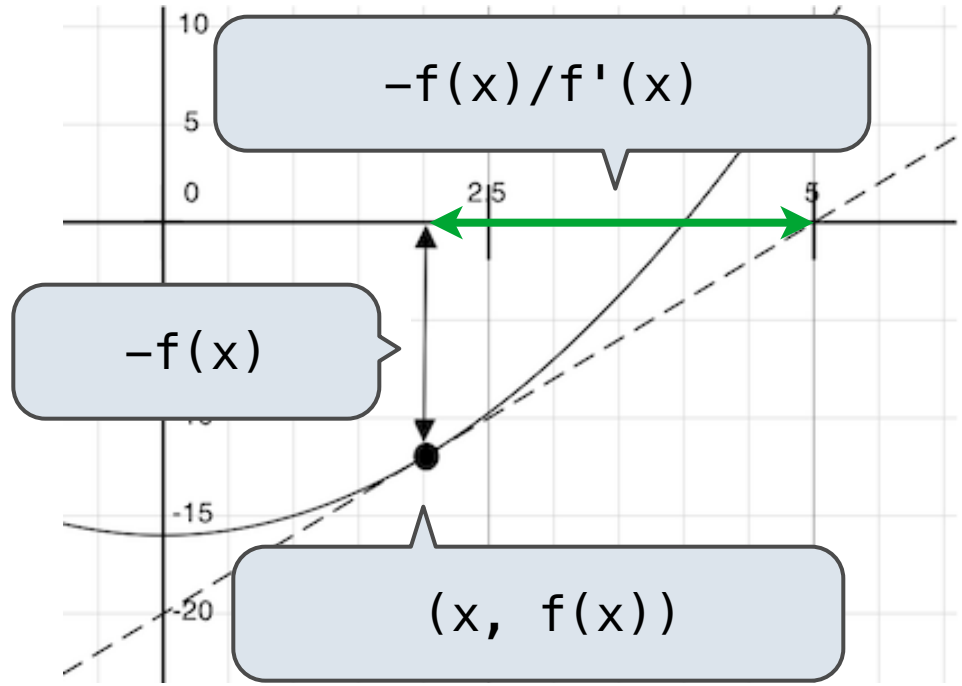

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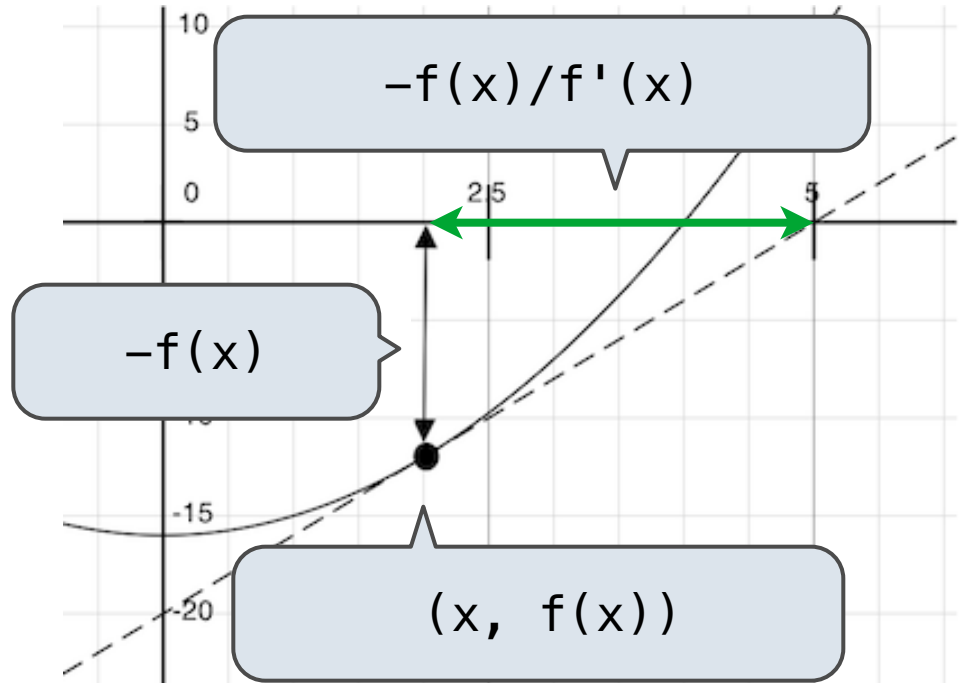

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Visualization of Newton's Method

(Demo)

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

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How to find the **square root** of 2?

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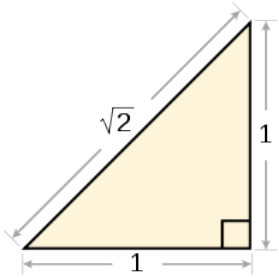


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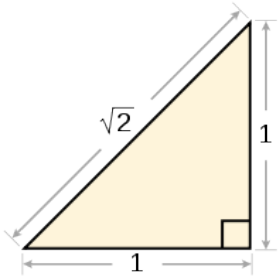


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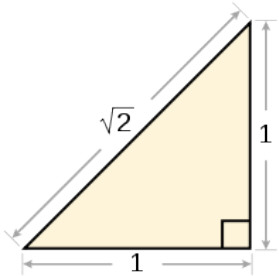


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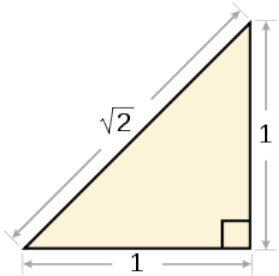


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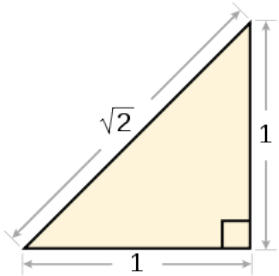


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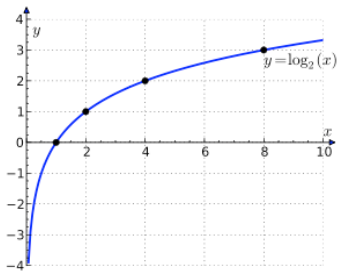


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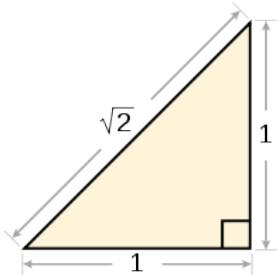


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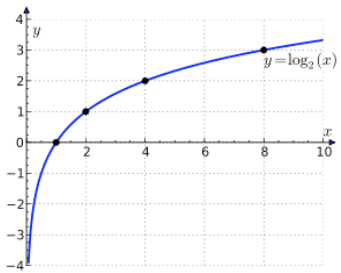


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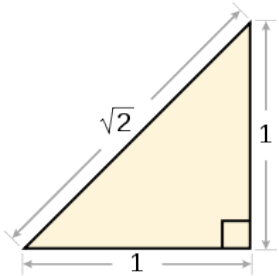


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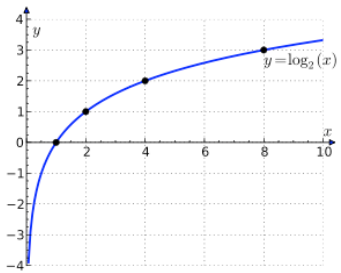


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Using Newton's Method

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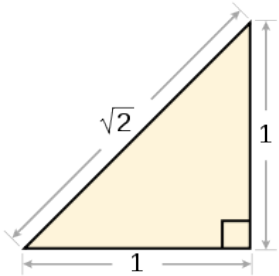


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>>> f = lambda x: x*x - 2
>>> find_zero(f)
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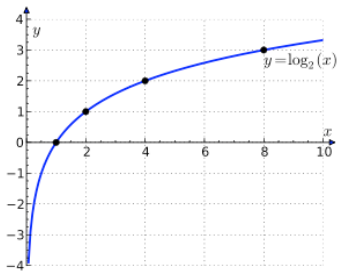
```

$f(x) = x^2 - 2$

How to find the **log base 2** of 1024?

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>>> g = lambda x: pow(2, x) - 1024
>>> find_zero(g)
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```

$g(x) = 2^x - 1024$

What number is one less than its square?

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>>> h = lambda x: x*x - (x+1)
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$h(x) = x^2 - (x+1)$

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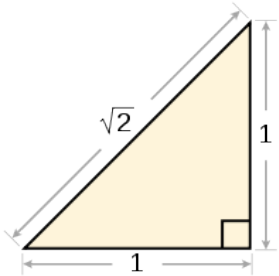


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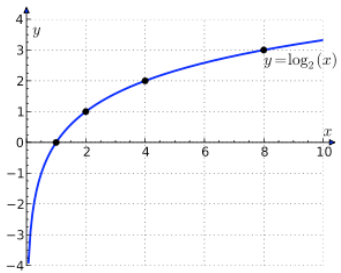


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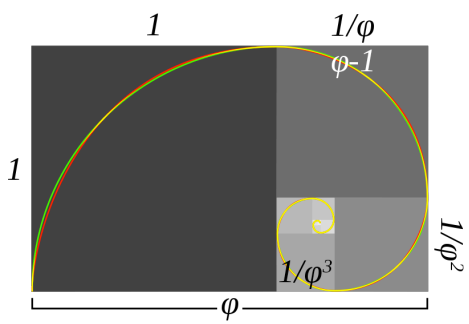


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Iterative Improvement

(Demo)

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def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done returns a true value.

    guess -- An initial guess
    update -- A function from guesses to guesses; updates the guess
    done -- A function from guesses to boolean values; tests if guess is good

    >>> iter_improve(golden_update, golden_test)
    1.618033988749895
    """
    k = 0
    while not done(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
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def golden_update(guess):
    return 1/guess + 1
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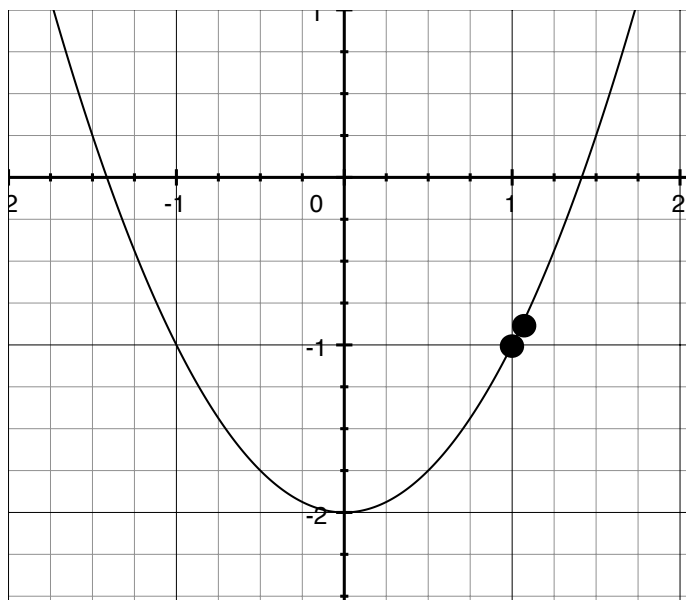

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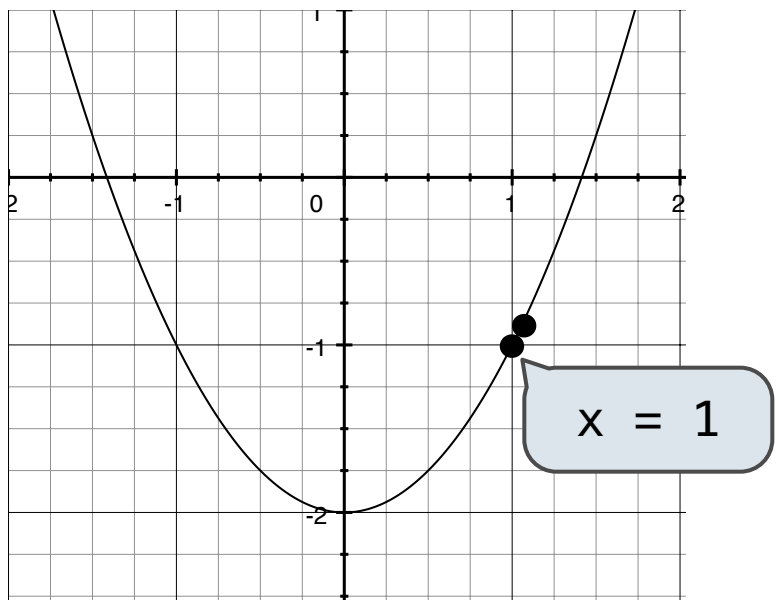

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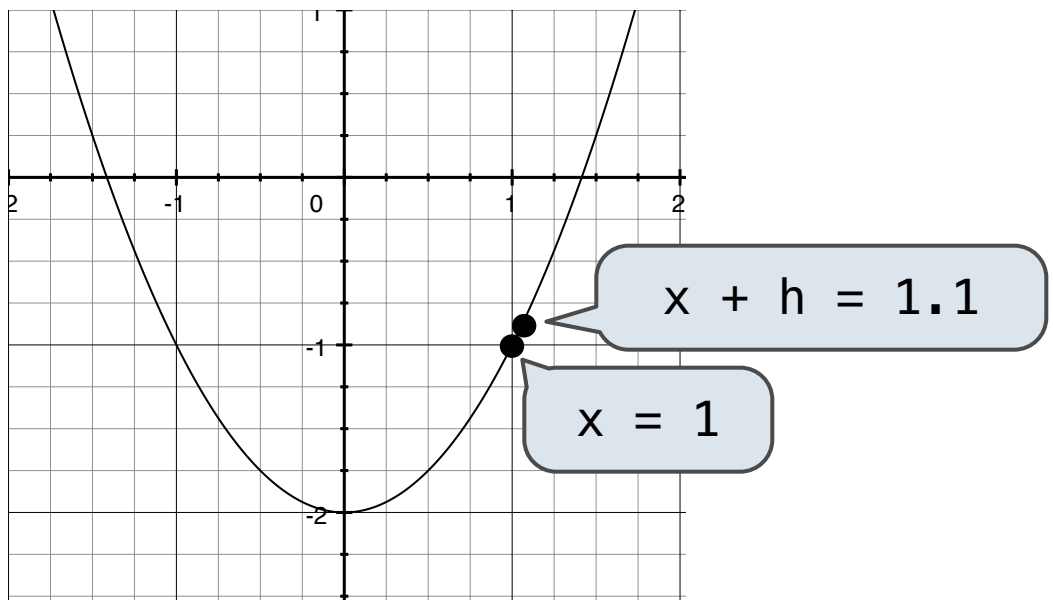

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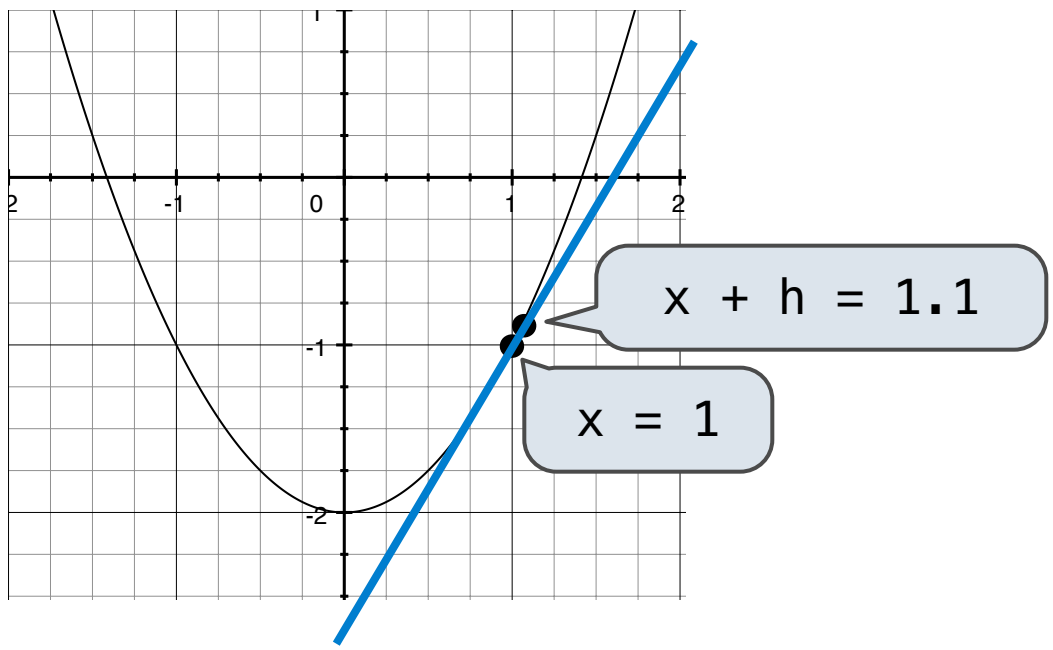

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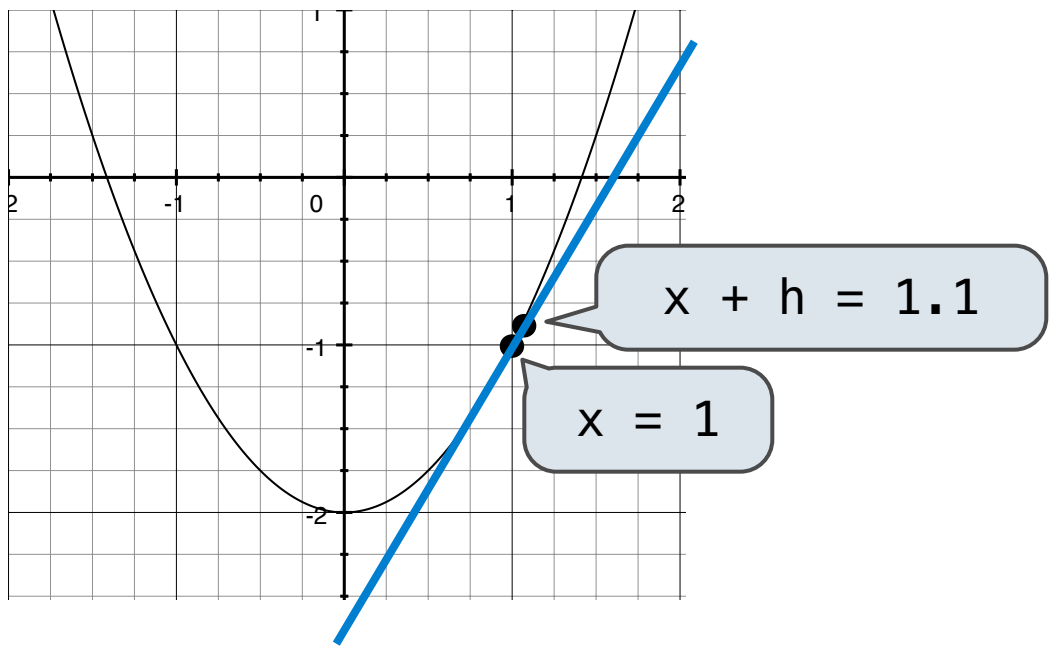

(Demo)

http://en.wikipedia.org/wiki/File:Graph_of_sliding_derivative_line.gif

Approximating Derivatives

(Demo)

Implementing Newton's Method

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def newton_update(f):
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    def update(x):
        return x - f(x) / approx_derivative(f, x)
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def find_root(f, guess=1):
    """Return a guess of a zero of the function f, near guess.

    >>> from math import sin
    >>> find_root(lambda y: sin(y), 3)
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    """
    return iter_improve(newton_update(f), lambda x: f(x) == 0, guess)
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Definition of a function zero