61A Lecture 8

Wednesday, September 12

Data Abstraction

Data Abstraction

- `Compound objects combine primitive objects together`

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- A date: a year, a month, and a day

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All Programmers

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All Programmers

Great Programmers

Rational Numbers

Rational Numbers

$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

- `rational(n, d)` *returns a rational number* `x`

Rational Numbers

numerator
―――――――――
denominator

Exact representation of fractions

A pair of integers

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

Constructor

- rational(n, d) *returns a rational number* x

Selectors

- numer(x) *returns the numerator of* x
- denom(x) *returns the denominator of* x

Rational Number Arithmetic

Example:

General Form:

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5}$$

General Form:

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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General Form:

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General Form:

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

Rational Number Arithmetic

Example:

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5}$$

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Rational Number Arithmetic

Example:

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Rational Number Arithmetic Implementation

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- `rational(n, d)` *returns a rational number* `x`
- `numer(x)` *returns the numerator of* `x`
- `denom(x)` *returns the denominator of* `x`

Rational Number Arithmetic Implementation

Wishful thinking

- `rational(n, d)` *returns a rational number* `x`
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Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))
```

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- rational(n, d) *returns a rational number* x
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Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))
```

Constructor

Selectors

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def eq_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Wishful thinking

- `rational(n, d)` *returns a rational number* `x`
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Tuples

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```
>>> pair = (1, 2)
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A tuple literal:
Comma-separated expression

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>>> x
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)


>>> x, y = pair
>>> x
1
>>> y
```

A tuple literal:
Comma-separated expression

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

```
>>> x, y = pair
>>> x
1
>>> y
2
```

A tuple literal:
Comma-separated expression

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a tuple

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a tuple

```
>>> pair[0]
```

Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)
```

A tuple literal:
Comma-separated expression

```
>>> x, y = pair
>>> x
1
>>> y
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"Unpacking" a tuple

```
>>> pair[0]
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Tuples

```
>>> pair = (1, 2)
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A tuple literal:
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>>> x, y = pair
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```
>>> pair[0]
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>>> pair = (1, 2)
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```
>>> pair[0]
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>>> from operator import getitem
```

Tuples

```
>>> pair = (1, 2)
>>> pair
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A tuple literal:
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>>> pair[0]
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>>> from operator import getitem
>>> getitem(pair, 0)
```

Tuples

```
>>> pair = (1, 2)
>>> pair
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A tuple literal:
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>>> x
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"Unpacking" a tuple

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Element selection

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Element selection

More tuples next lecture

Representing Rational Numbers

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```

Construct a tuple

```
from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)
```

Representing Rational Numbers

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def rational(n, d):
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Construct a tuple

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from operator import getitem

def numer(x):
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Select from a tuple

Reducing to Lowest Terms

Example:

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Example:

$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} \quad * \quad \frac{5}{3} \quad = \quad \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \boxed{\frac{5}{2}} \qquad\qquad \frac{2}{5} + \frac{1}{10}$$

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```
from fractions import gcd
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Greatest common divisor

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```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return (n//g, d//g)
```

Abstraction Barriers

Rational numbers as whole data values

`add_rationals` `mul_rationals` `eq_rationals`

Rational numbers as numerators & denominators

`rational` `numer` `denom`

Rational numbers as tuples

`tuple` `getitem`

However tuples are implemented in Python

Violating Abstraction Barriers

```
add_rational( (1, 2), (1, 4) )

def divide_rational(x, y):
    return (x[0] * y[1], x[1] * y[0])
```

Violating Abstraction Barriers

Does not use constructors

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Twice!

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No selectors!

And no constructor!

Violating Abstraction Barriers

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- If behavior conditions are met, the representation is valid.

You can recognize data types by behavior, not by bits

Behavior Conditions of a Pair

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Constructors, selectors, and behavior conditions:

If a pair p was constructed from elements x and y, then

- getitem_pair(p, 0) returns x, and
- getitem_pair(p, 1) returns y.

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Behavior Conditions of a Pair

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Generally true of *container types*.

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If a pair p was constructed from elements x and y, then

- getitem_pair(p, 0) returns x, and
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Together, selectors are the inverse of the constructor

Generally true of *container types*.

Not true for rational numbers because of GCD

Functional Pair Implementation

Functional Pair Implementation

```
def pair(x, y):
    """Return a functional pair."""
    def dispatch(m):
        if m == 0:
            return x
        elif m == 1:
            return y
    return dispatch
```

Functional Pair Implementation

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This function represents a pair

Constructor is a higher-order function

```
def getitem_pair(p, i):
    """Return the element at index i of pair p."""
    return p(i)
```

Functional Pair Implementation

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def pair(x, y):
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Constructor is a higher-order function

```
def getitem_pair(p, i):
    """Return the element at index i of pair p."""
    return p(i)
```

Selector defers to the object itself

Using a Functionally Implemented Pair

```
>>> p = pair(1, 2)

>>> getitem_pair(p, 0)
1

>>> getitem_pair(p, 1)
2
```

Using a Functionally Implemented Pair

```
>>> p = pair(1, 2)

>>> getitem_pair(p, 0)
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```

As long as we do not violate
the abstraction barrier,
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pairs are just functions

Using a Functionally Implemented Pair

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>>> getitem_pair(p, 0)
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>>> getitem_pair(p, 1)
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```

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- getitem_pair(p, 0) returns x, and
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This pair representation is valid!