61A Lecture 12

Monday, September 24

Dictionaries

```
{'Dem': 0}
```

Restrictions on Dictionaries

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- Two **keys cannot be equal.** There can be at most one value for a given key.

This first restriction is tied to Python's underlying implementation of dictionaries.

The second restriction is an intentional consequence of the dictionary abstraction.

Sharing and Identity

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demo = []
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A function that takes any argument and returns a function that returns that arg

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def delay(arg):
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Names in nested def statements can refer to their enclosing scope

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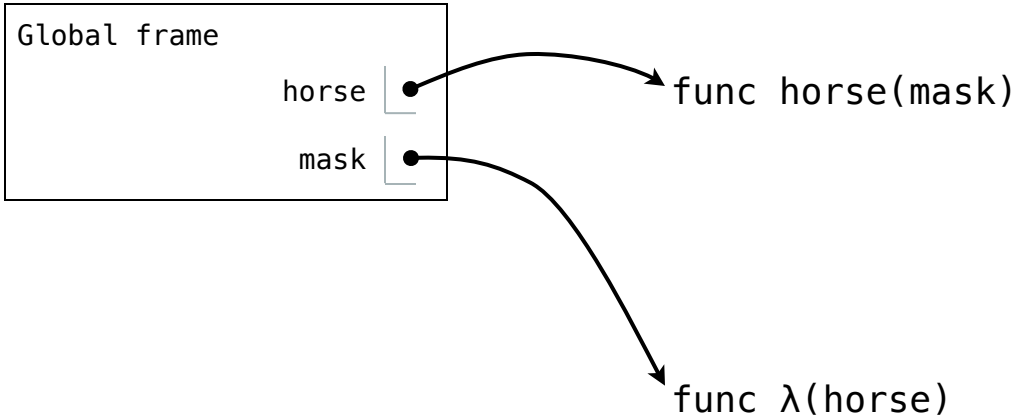

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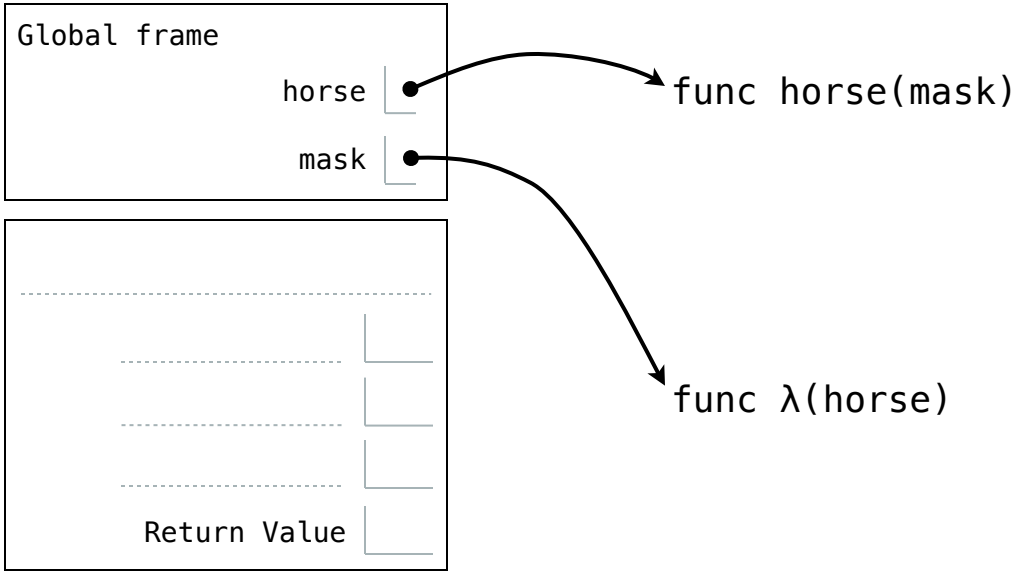

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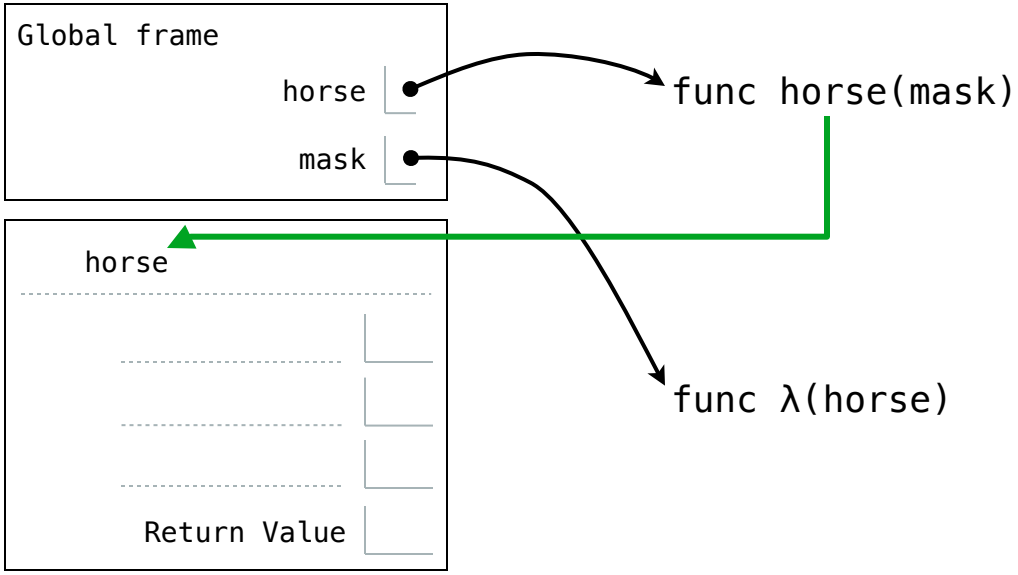

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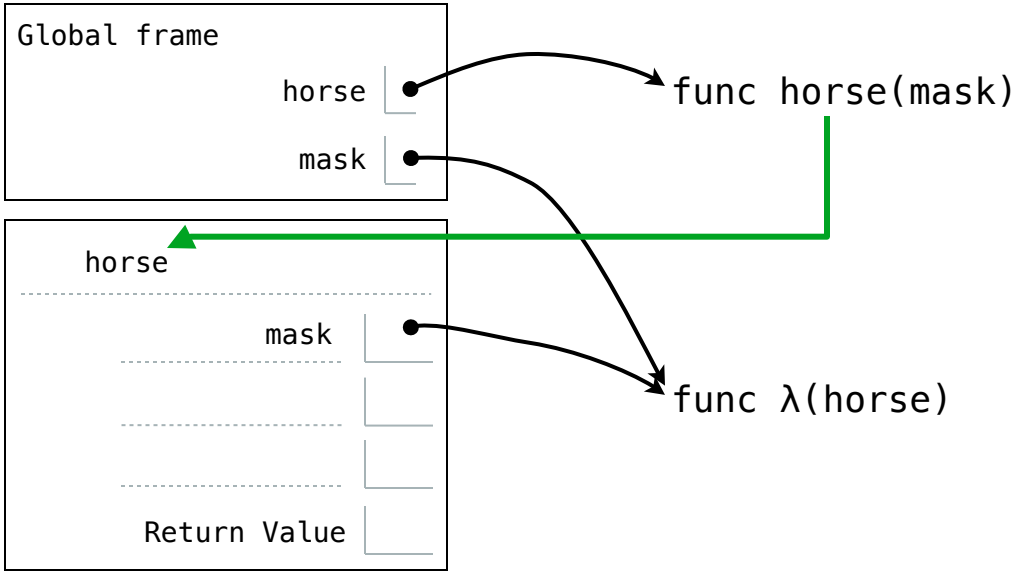

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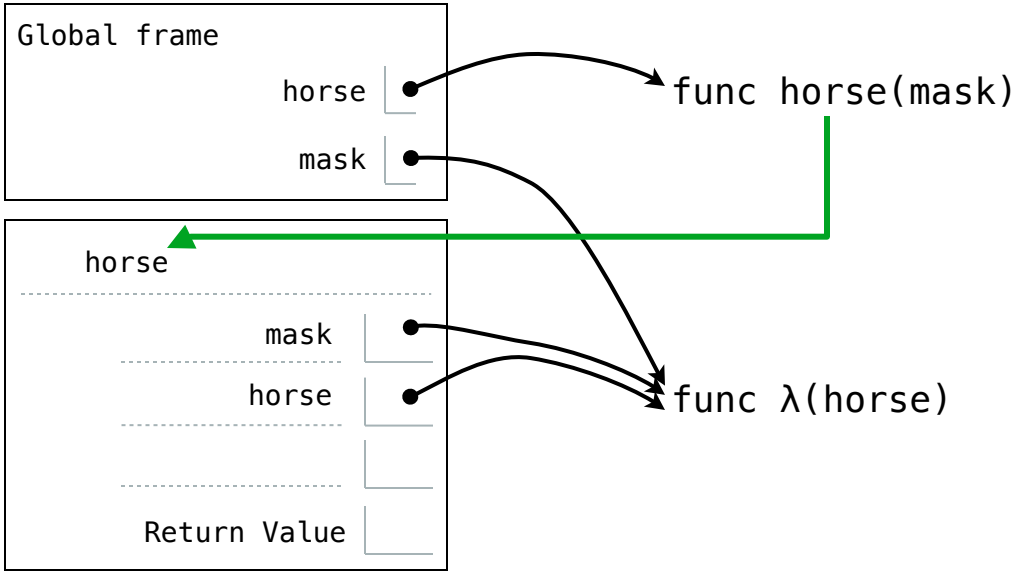

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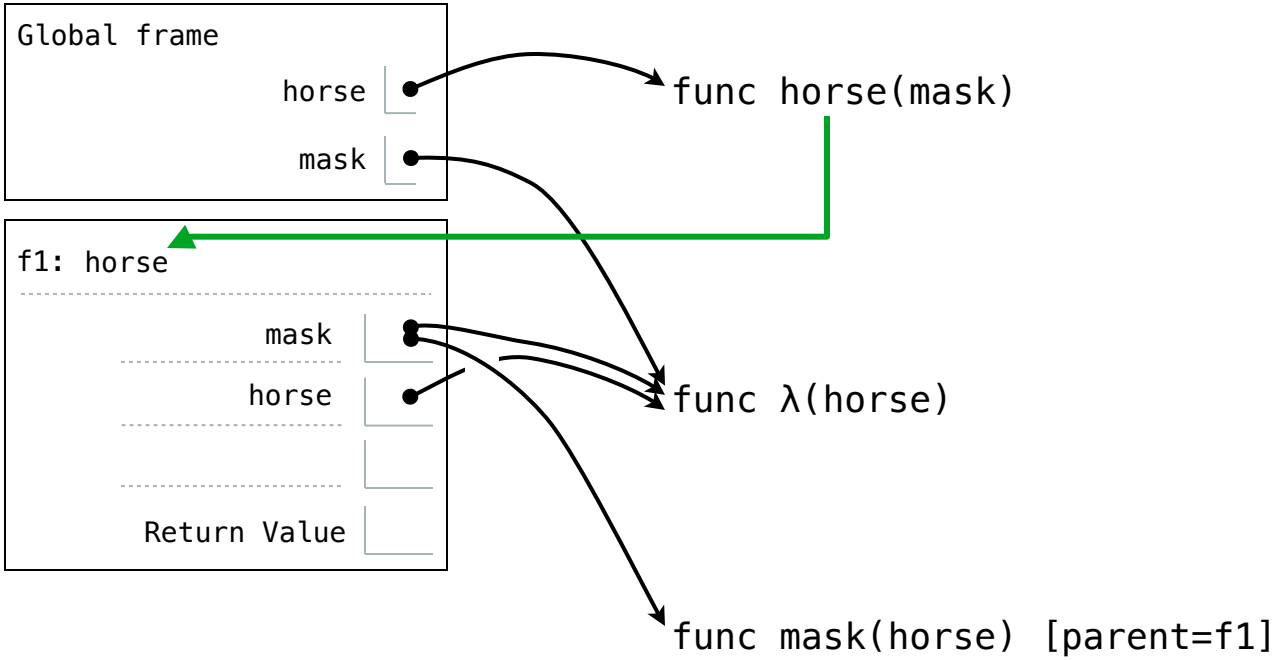

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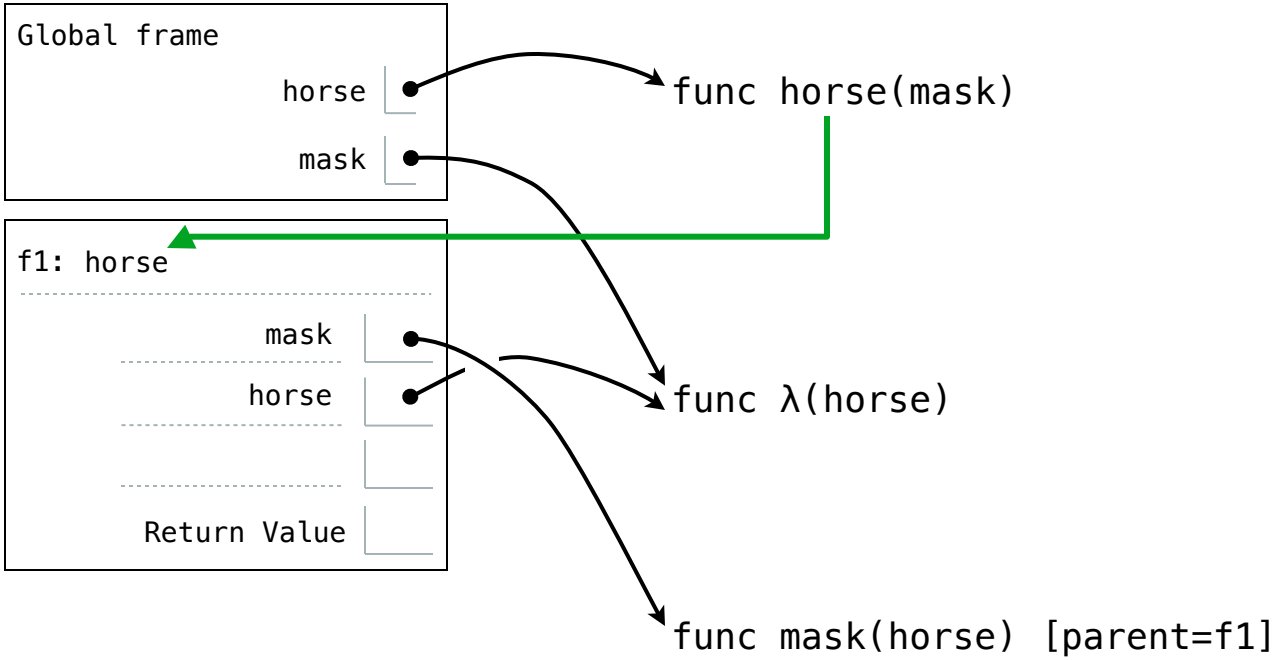

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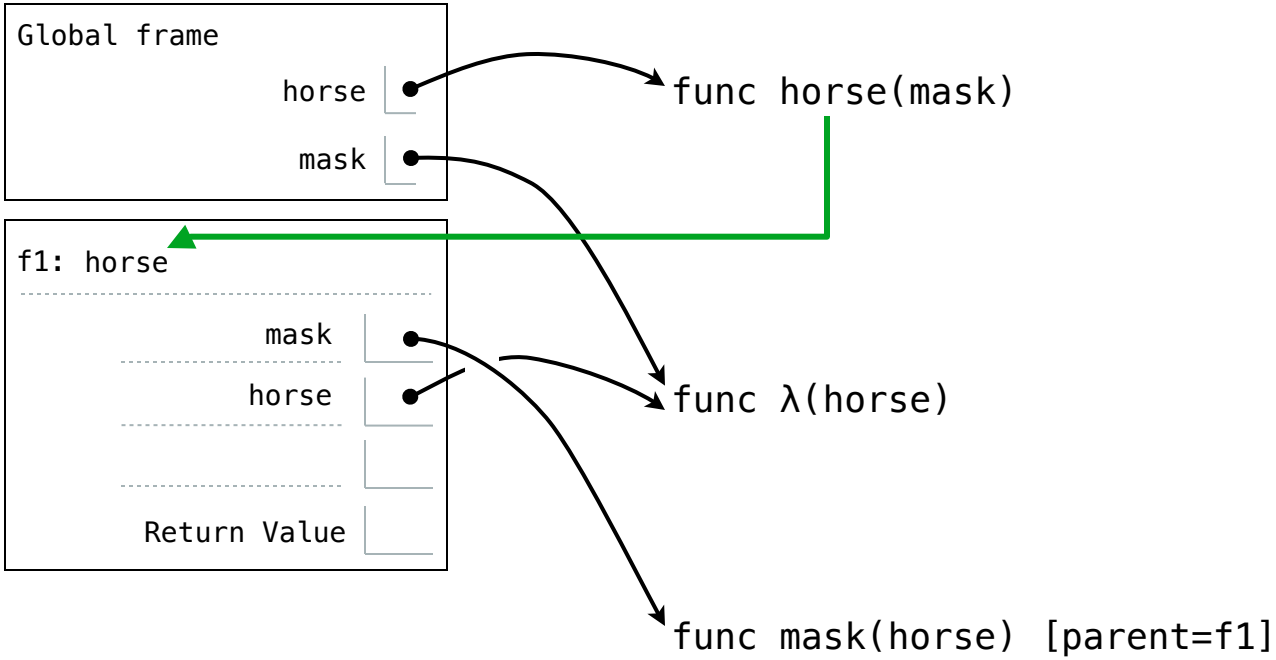

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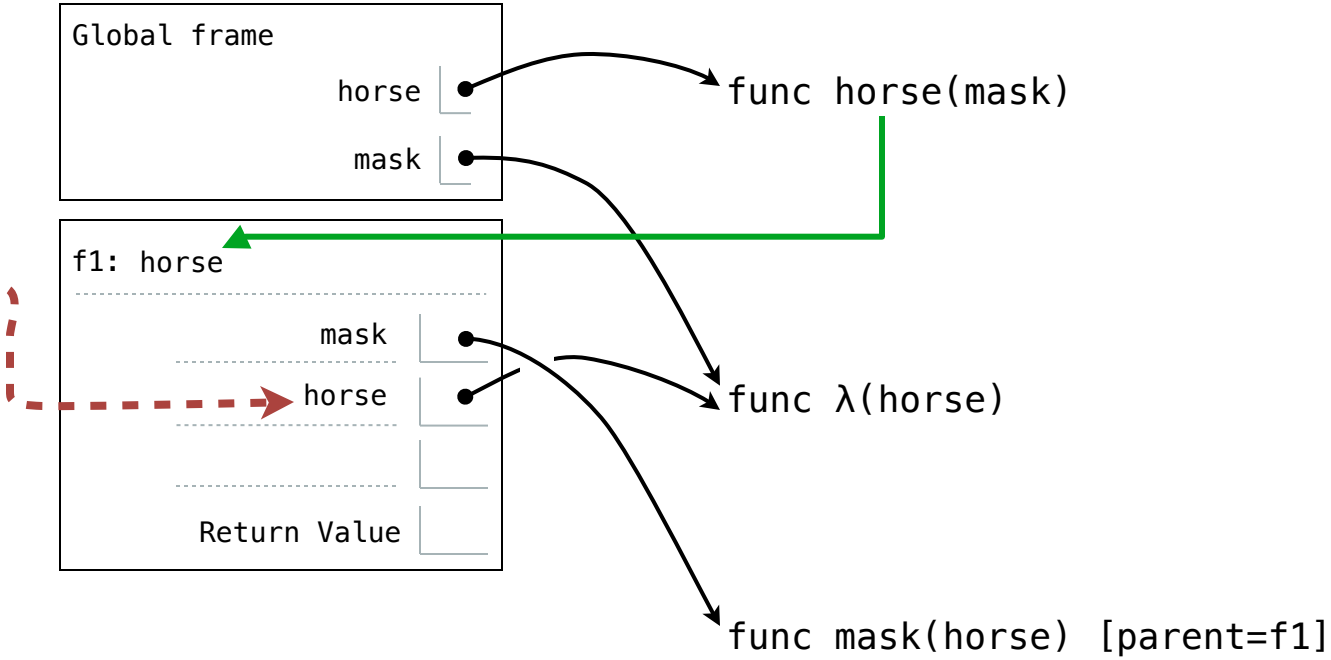

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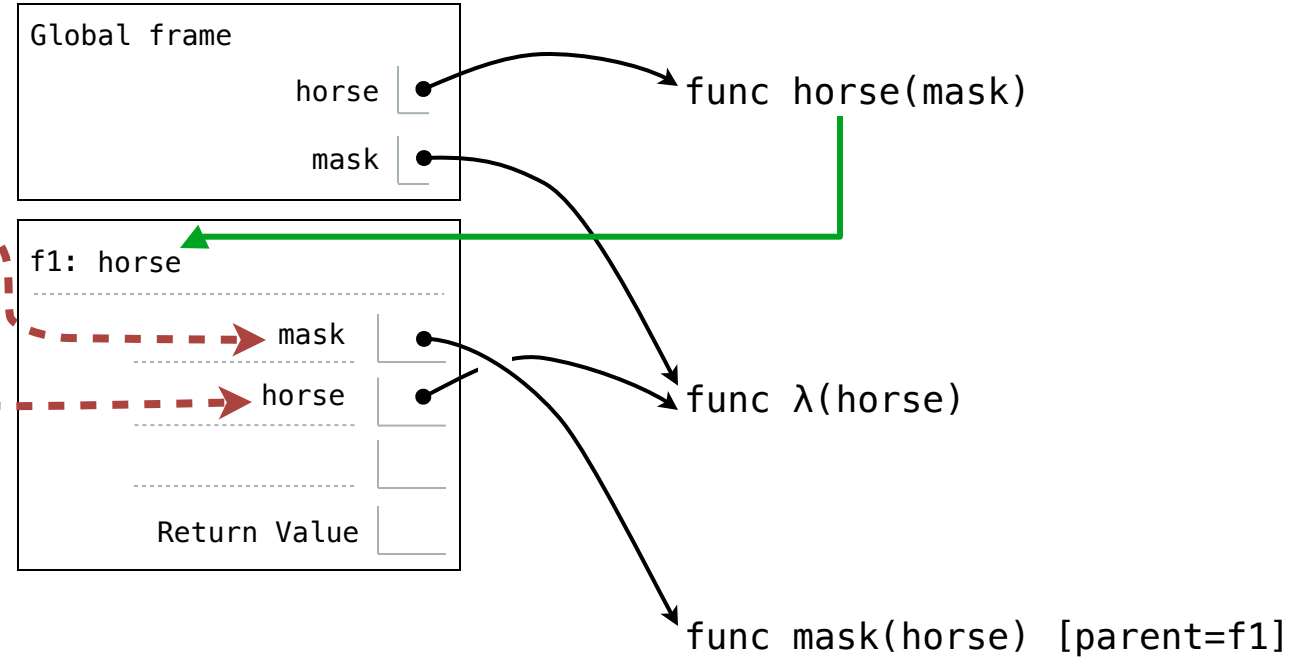

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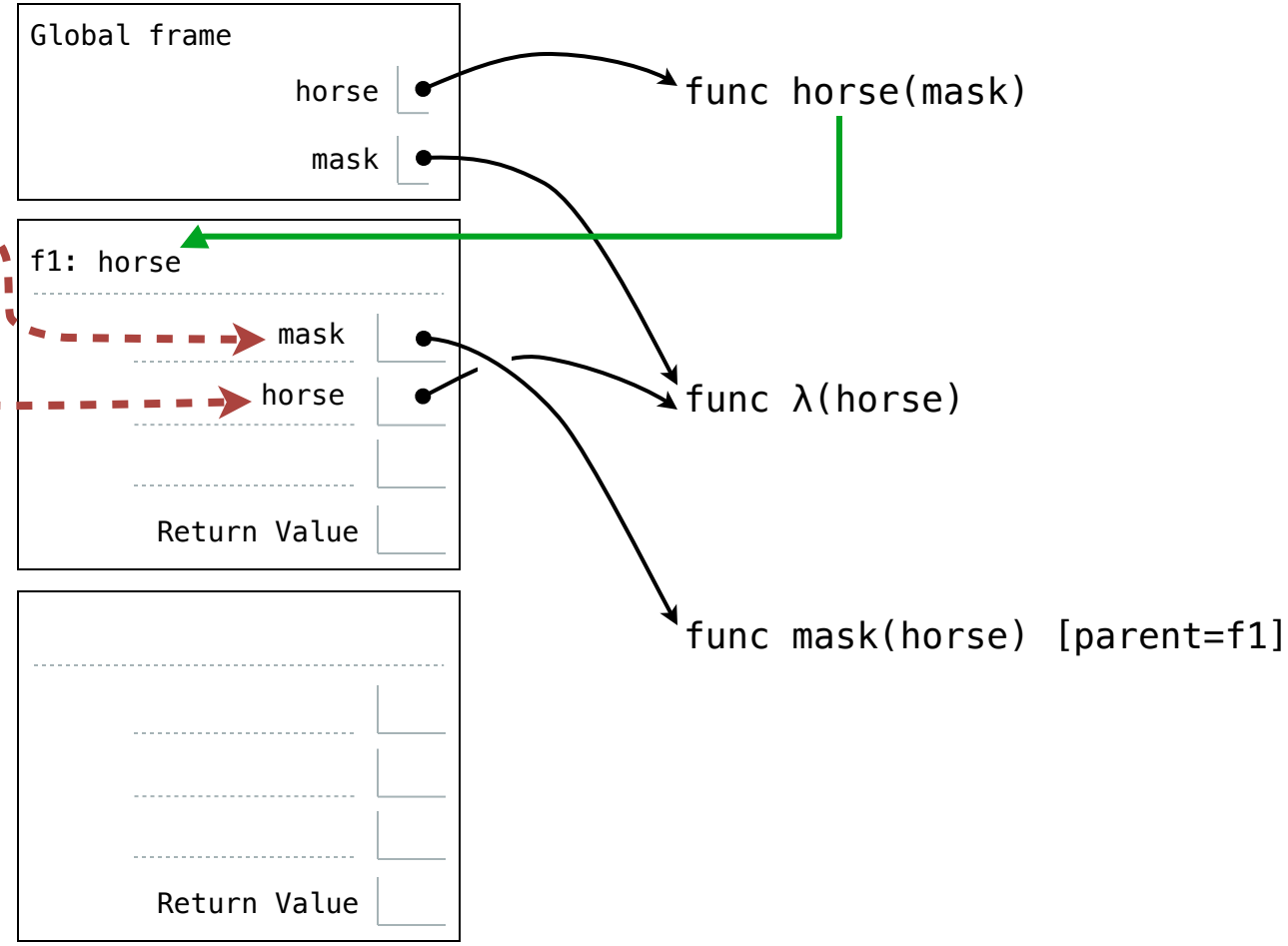

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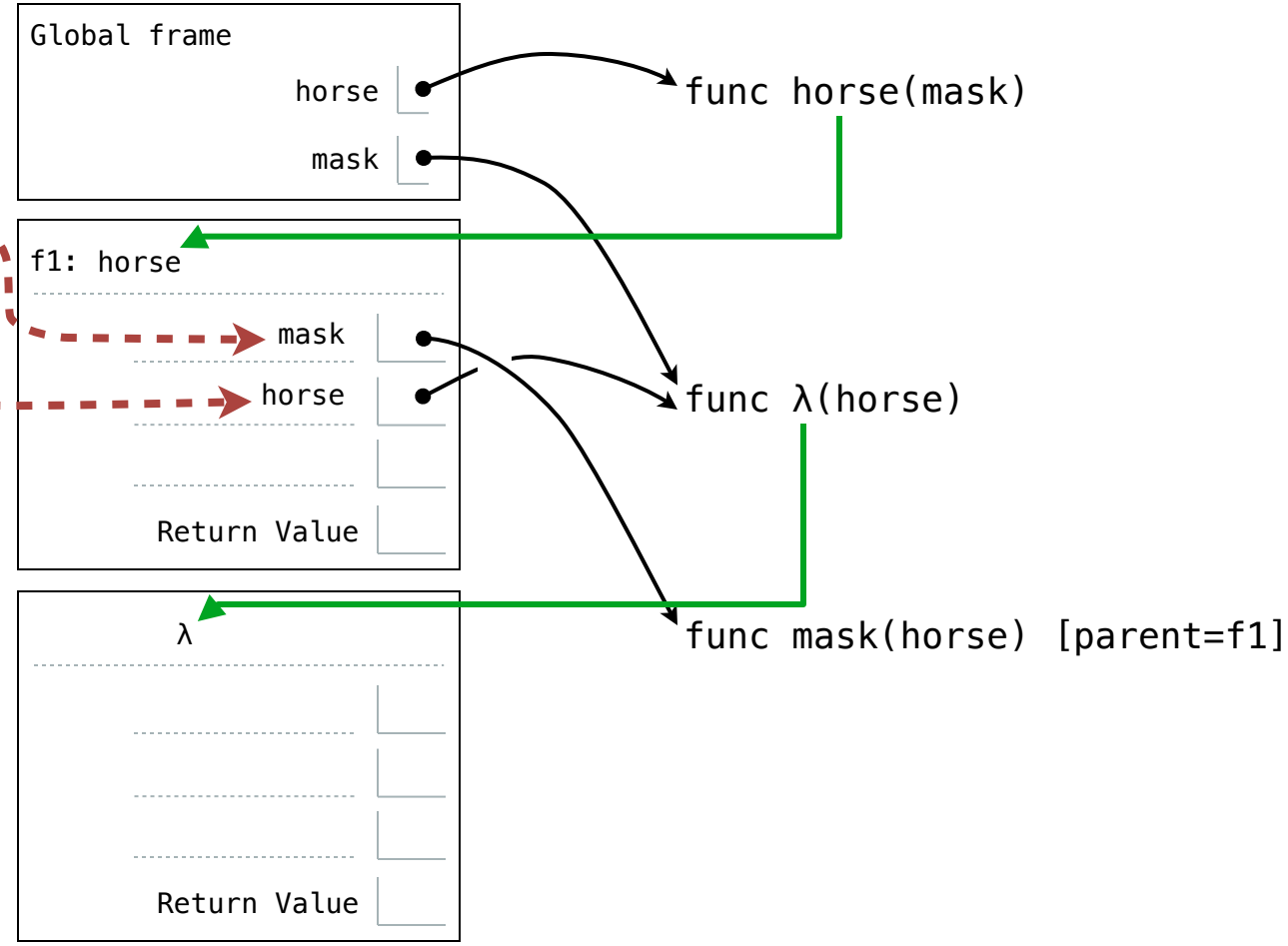

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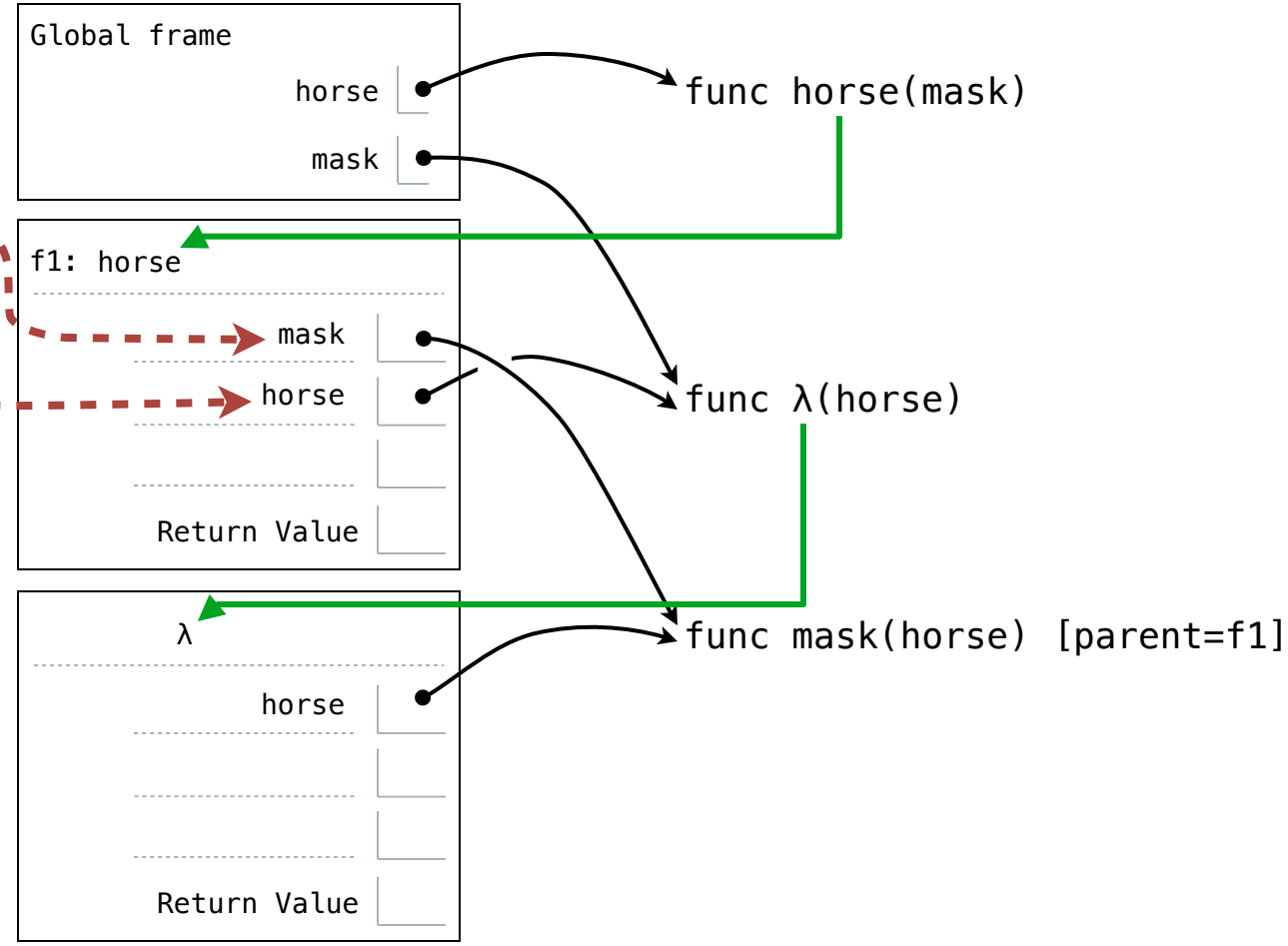

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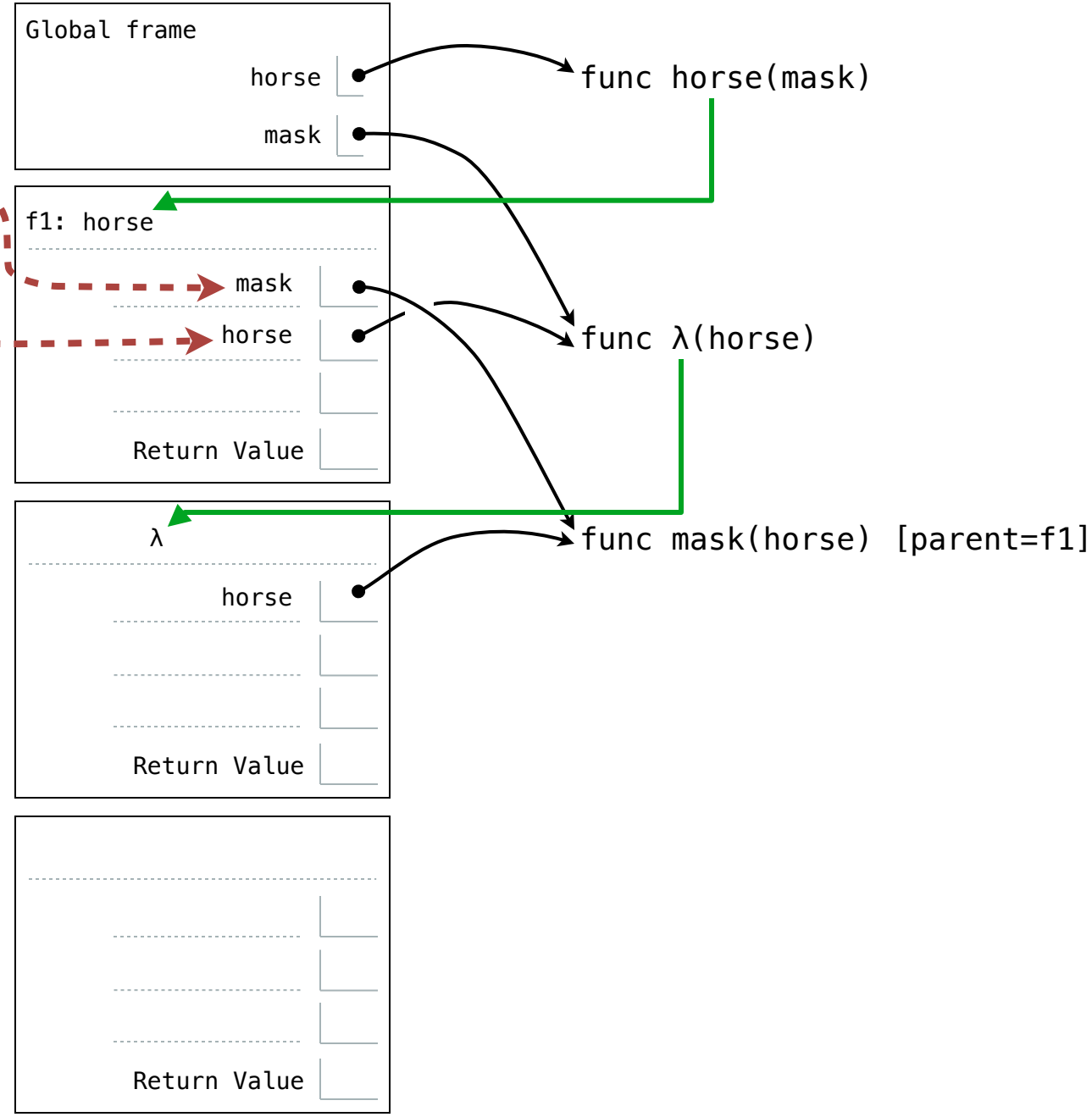

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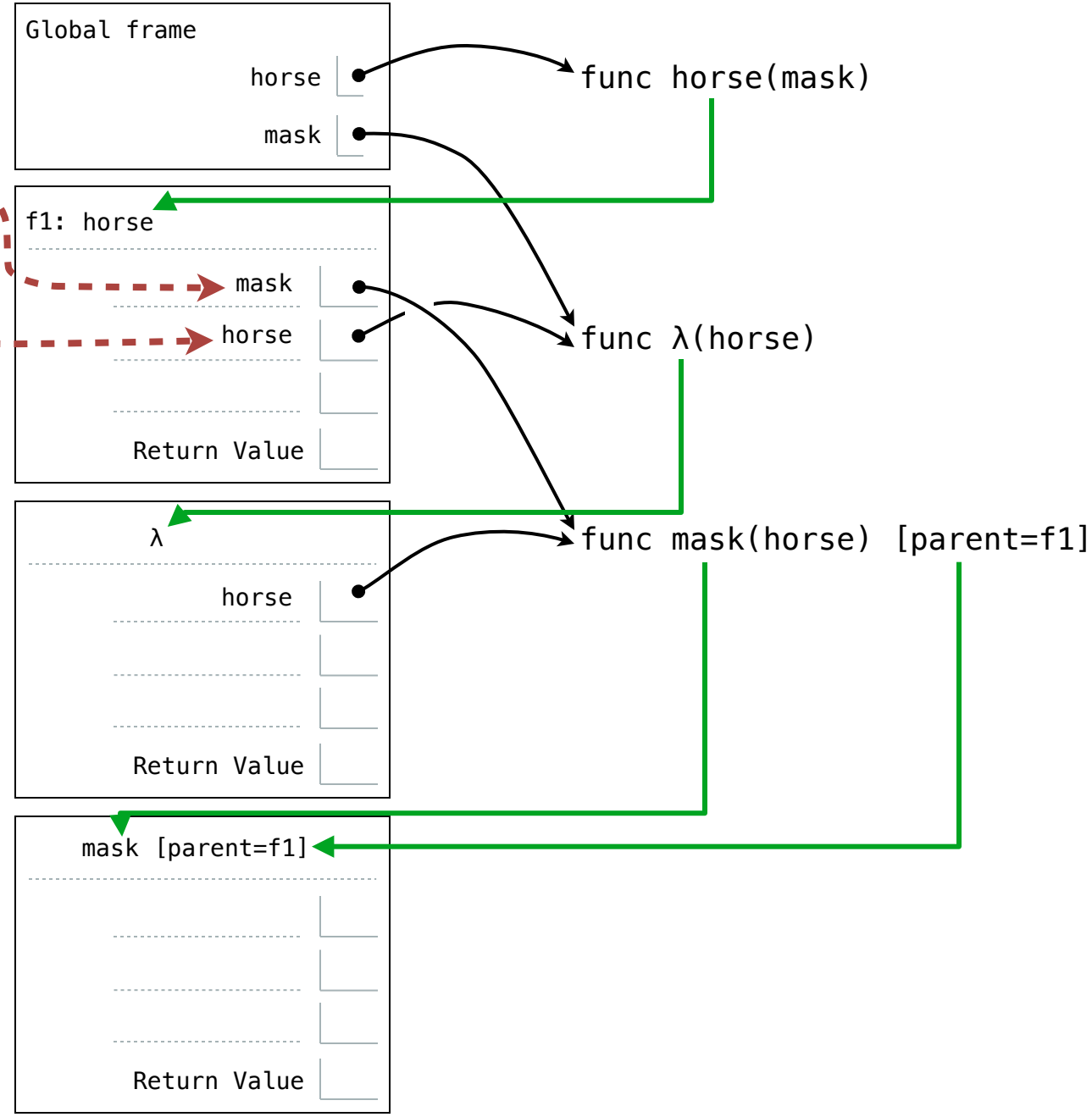

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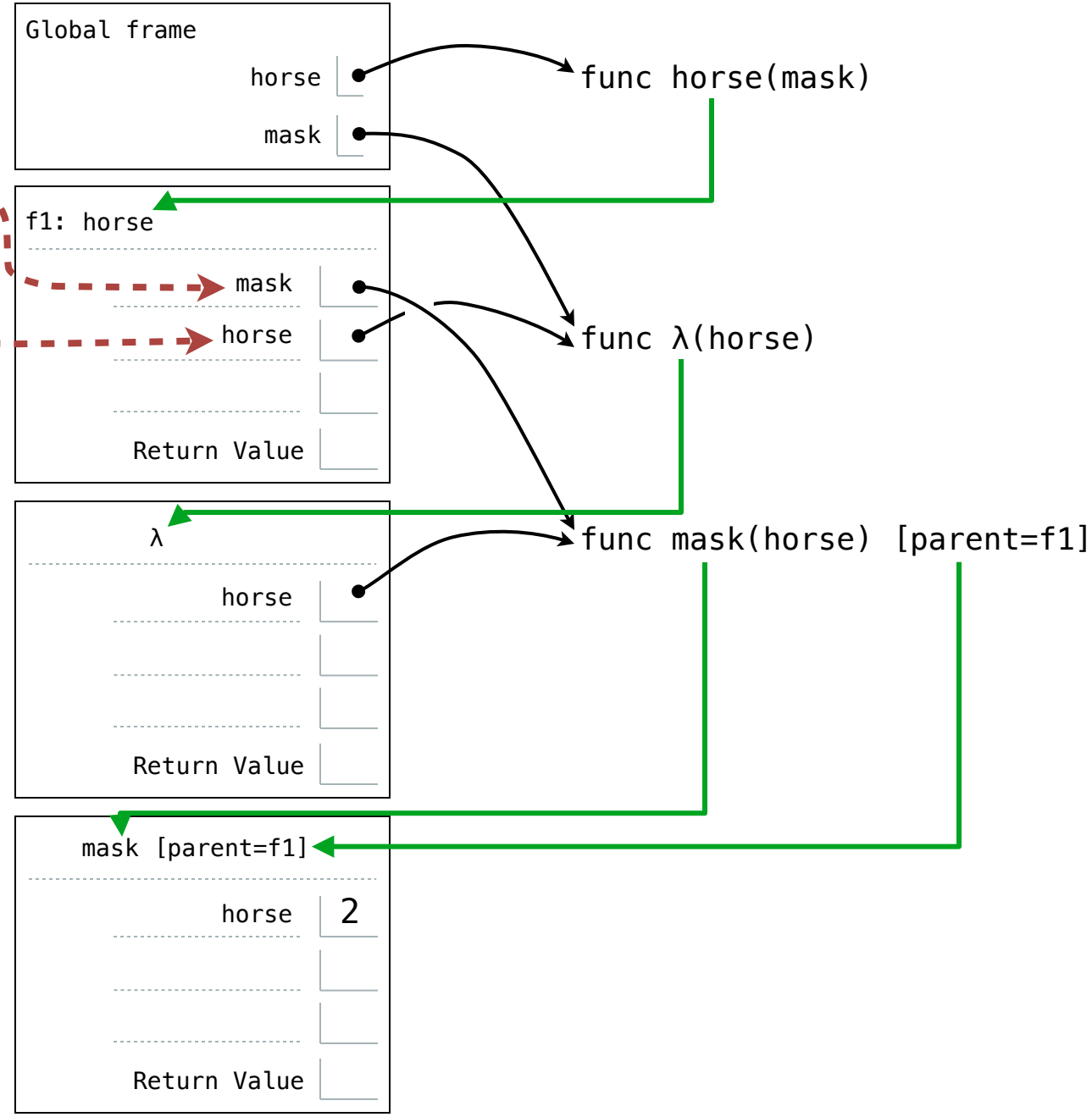

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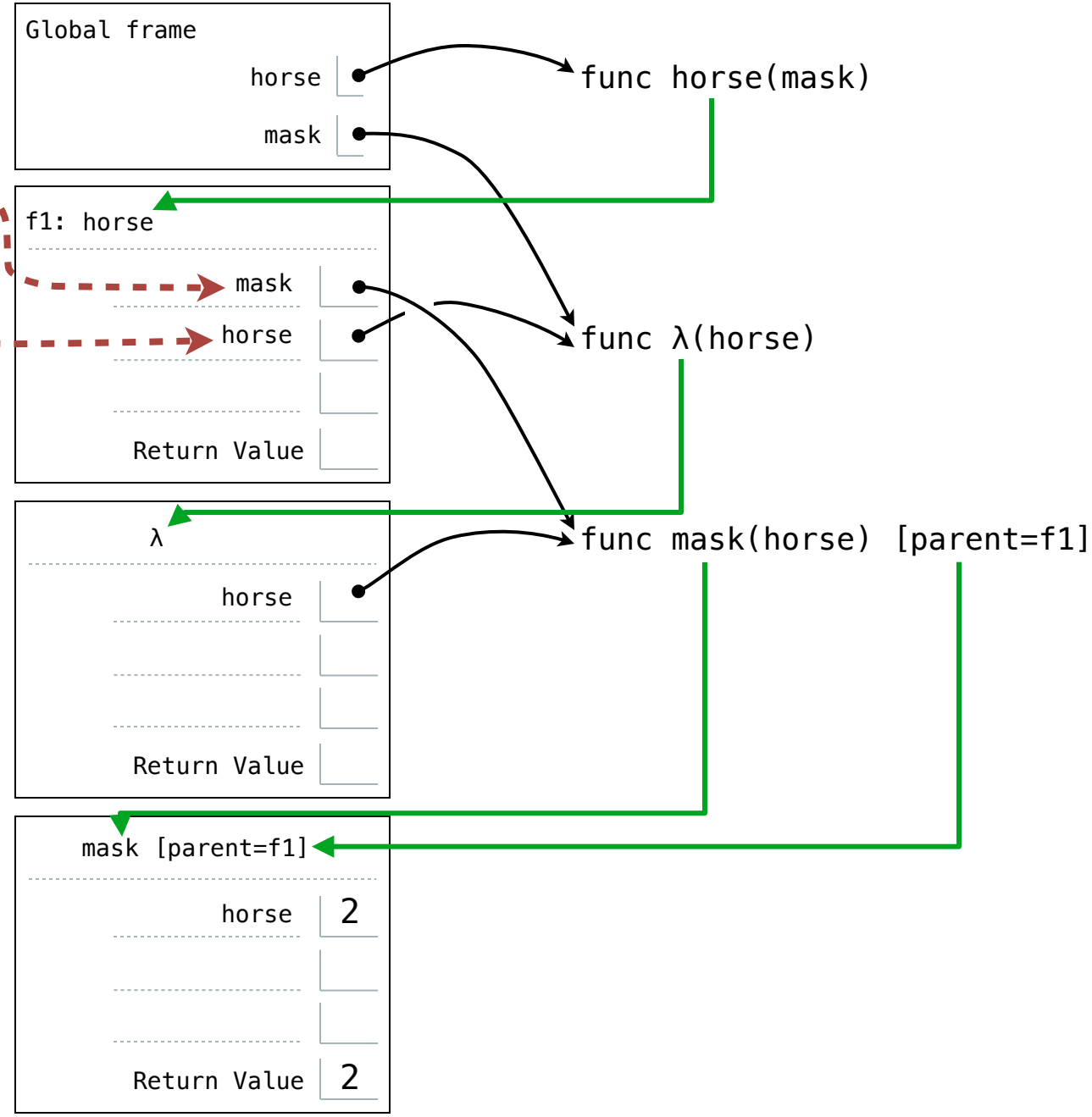

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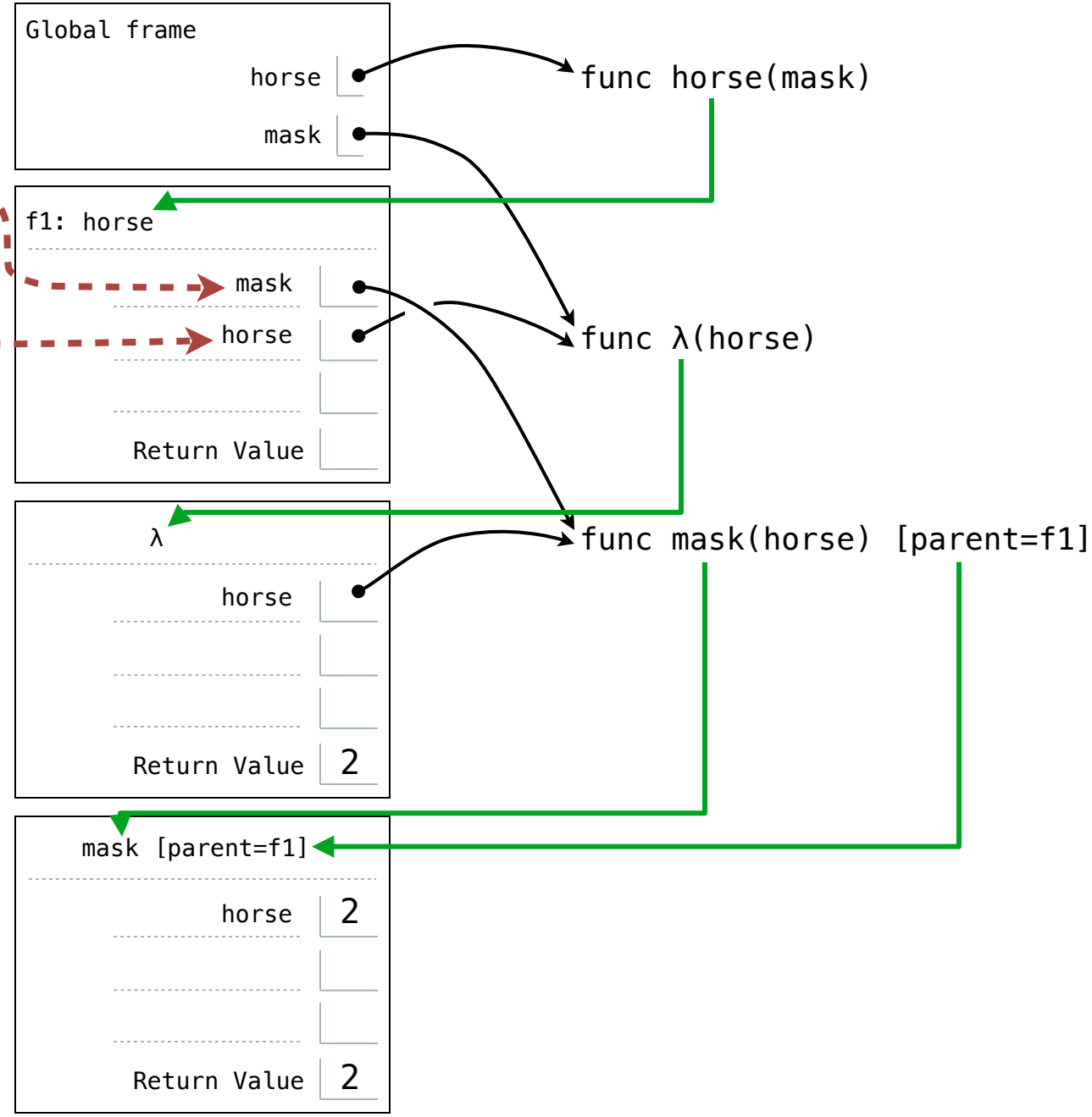

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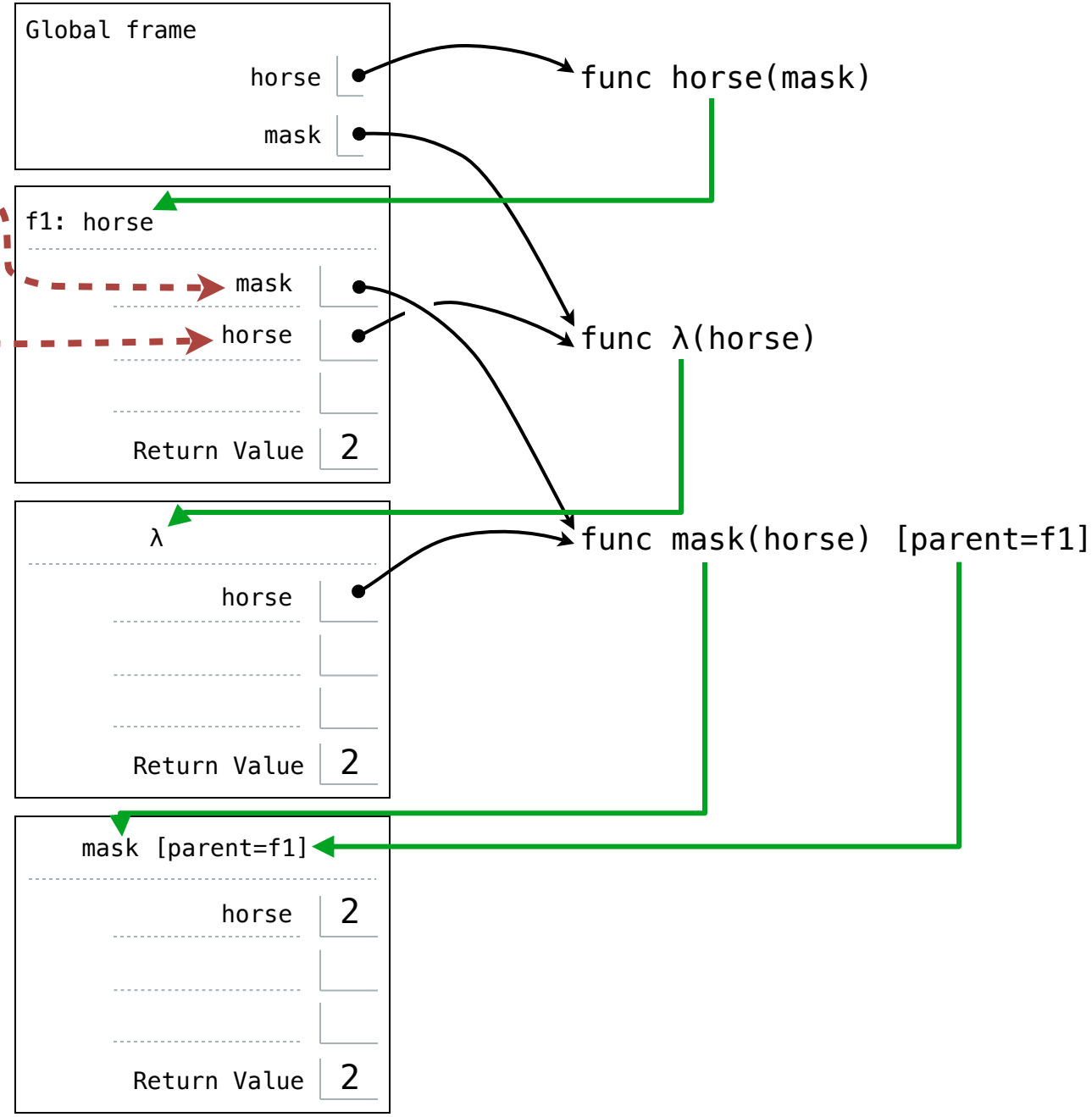

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    nam = style(p)
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def style(s):
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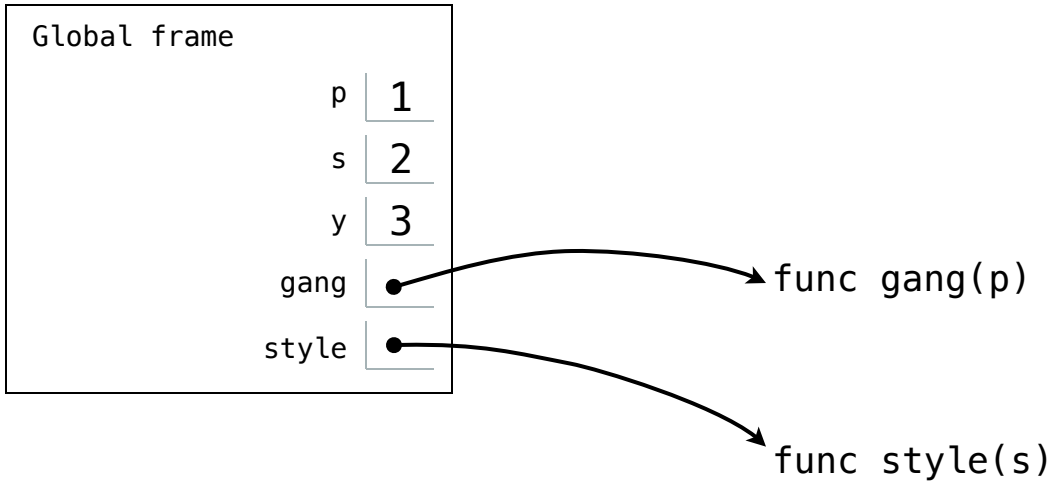

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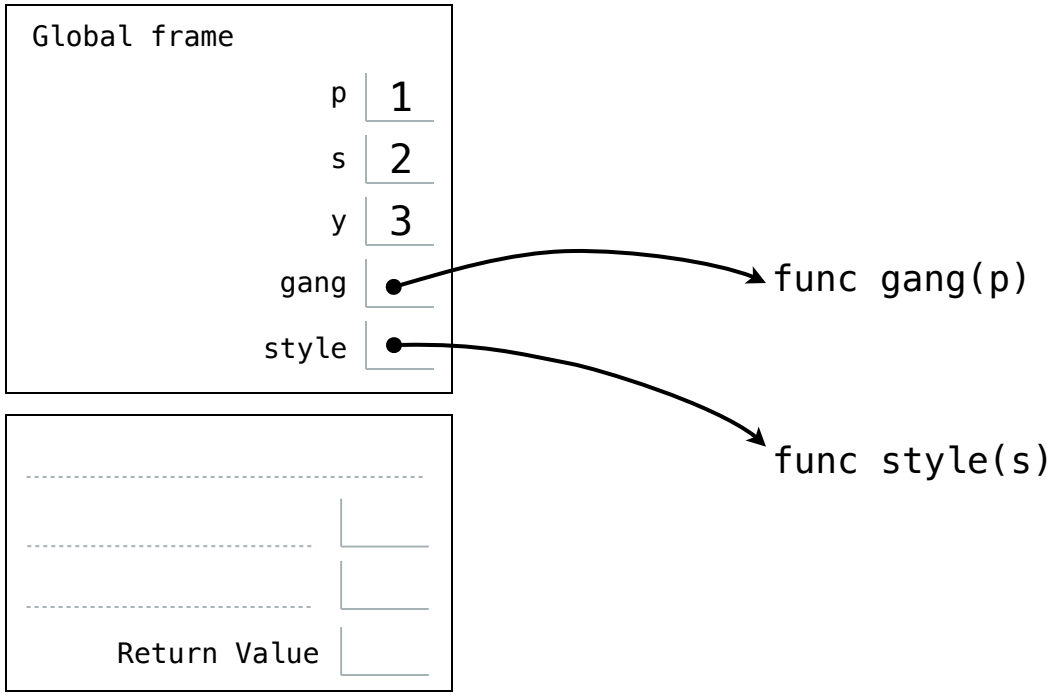

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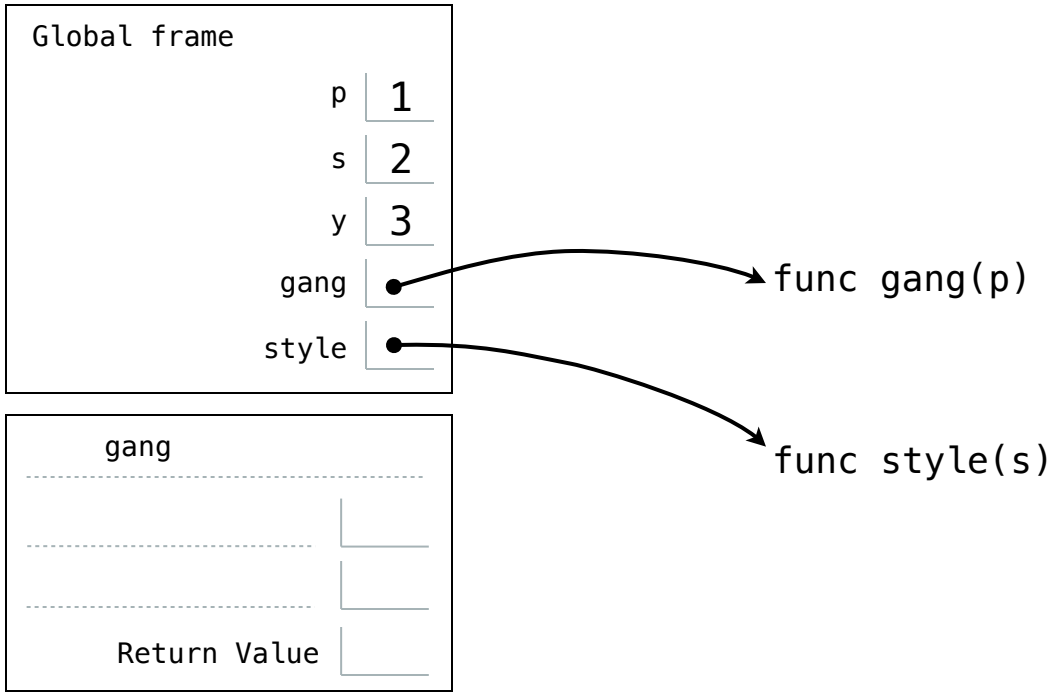

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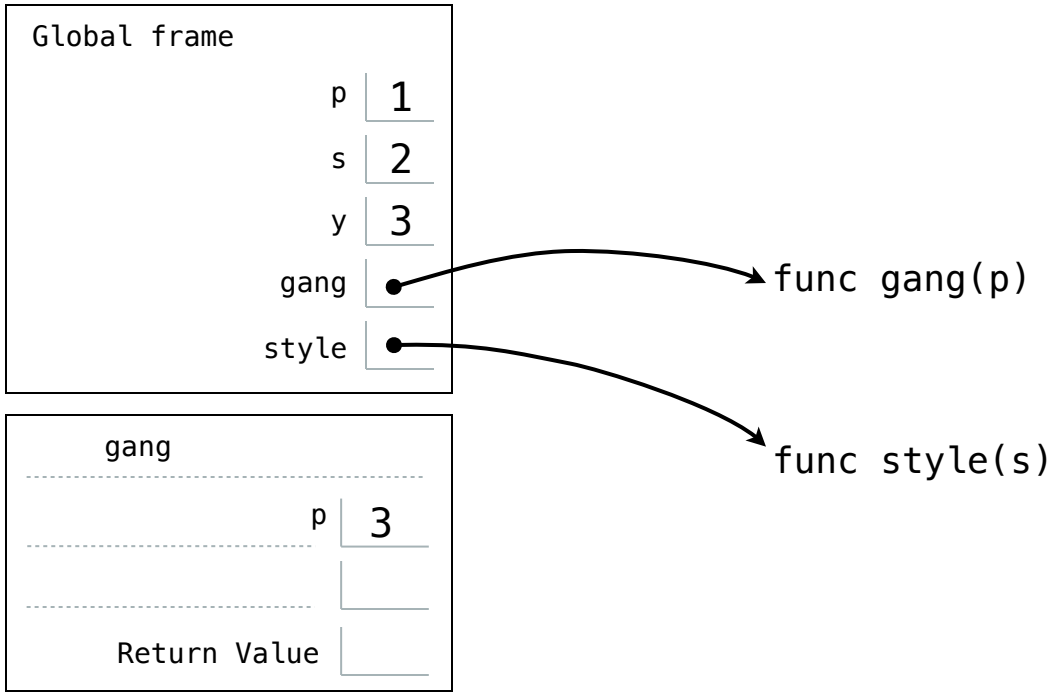

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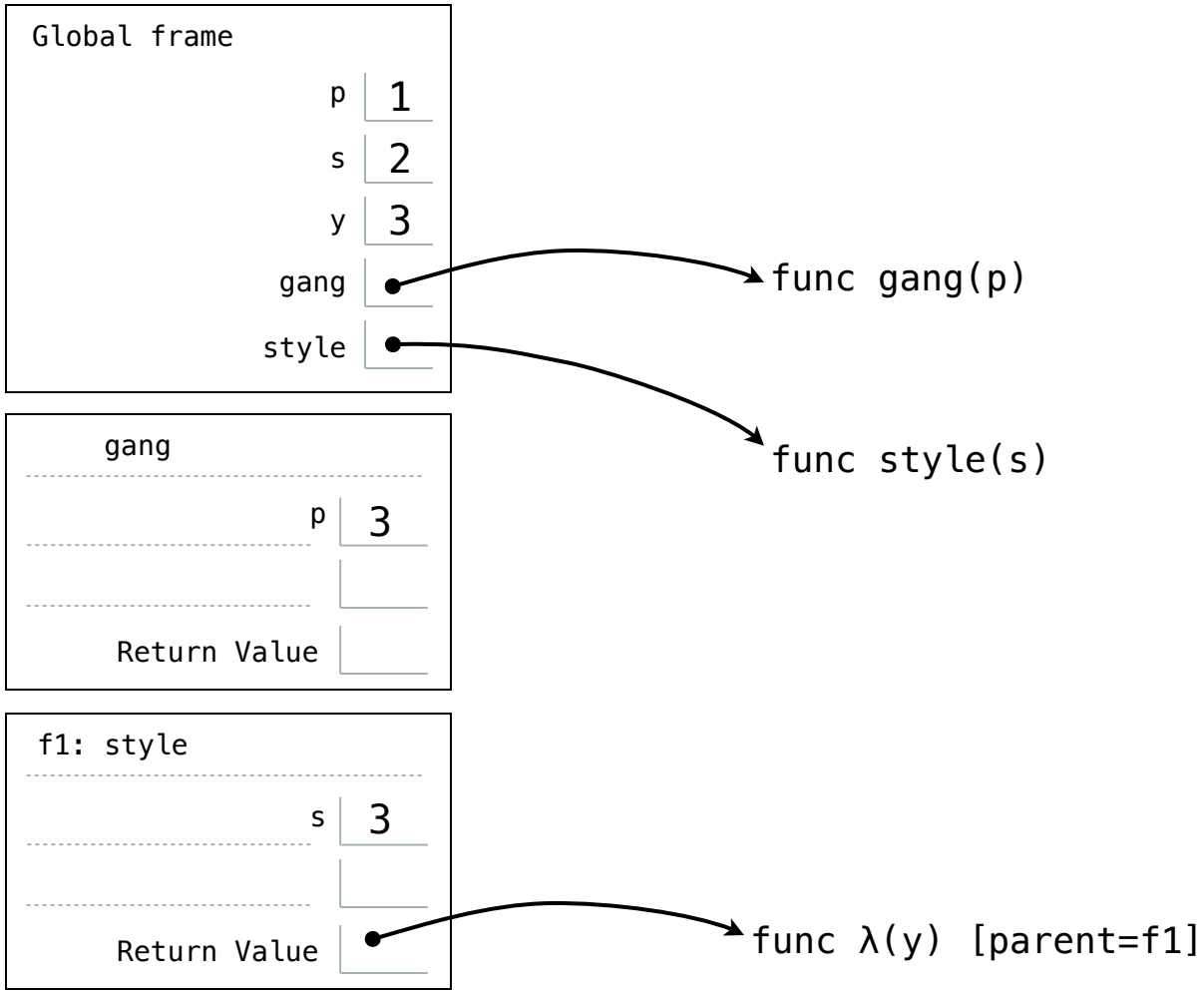

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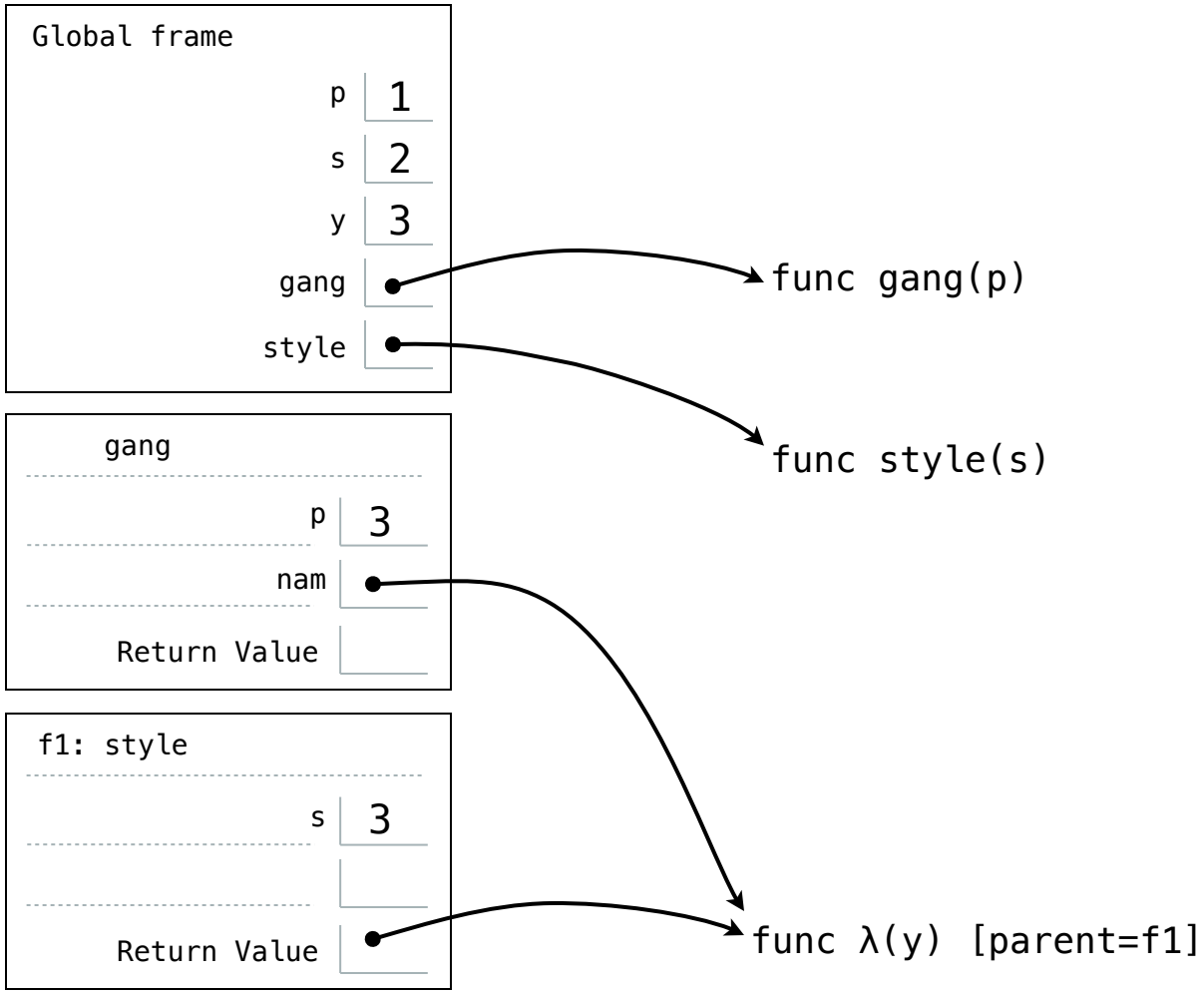

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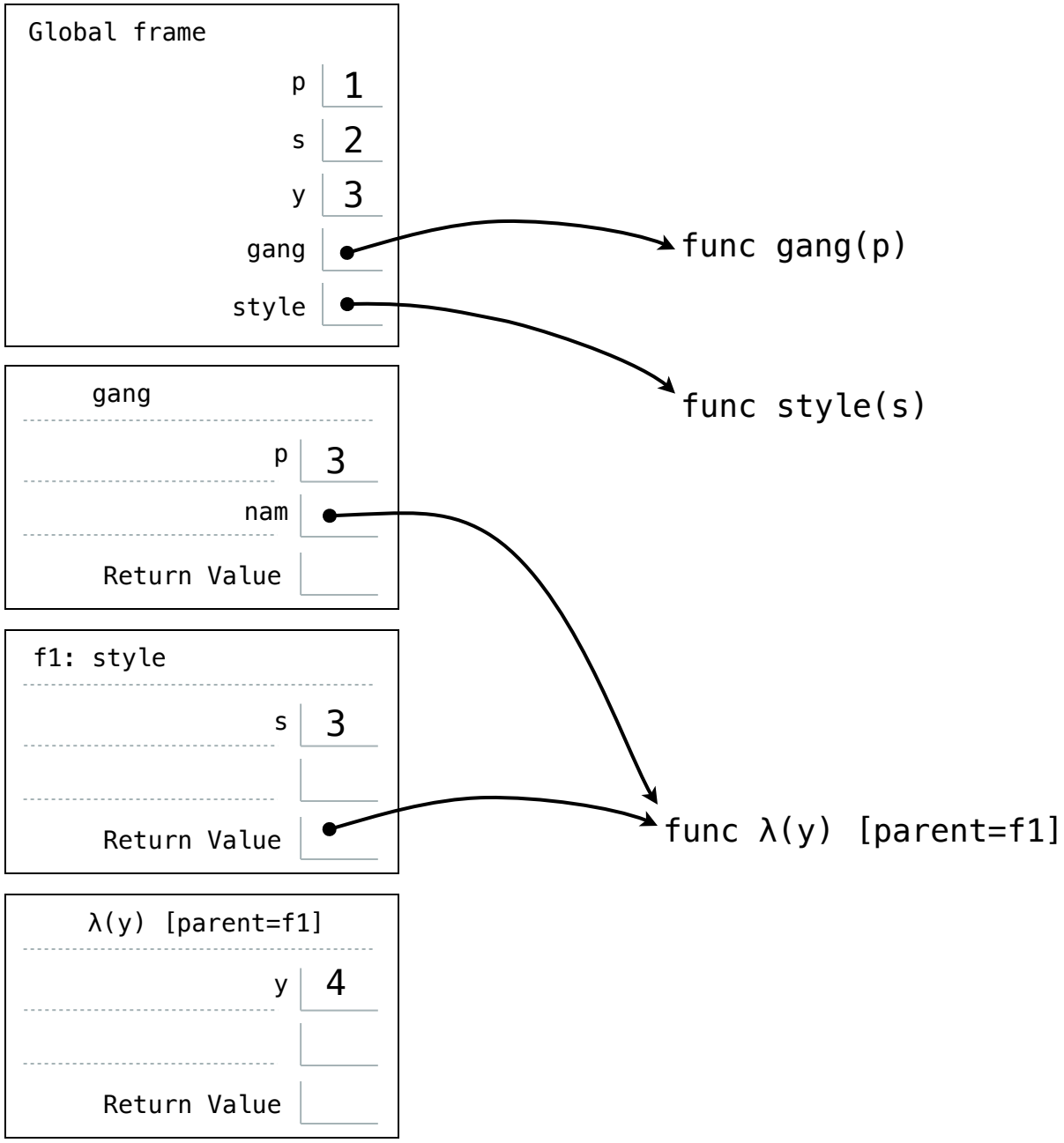

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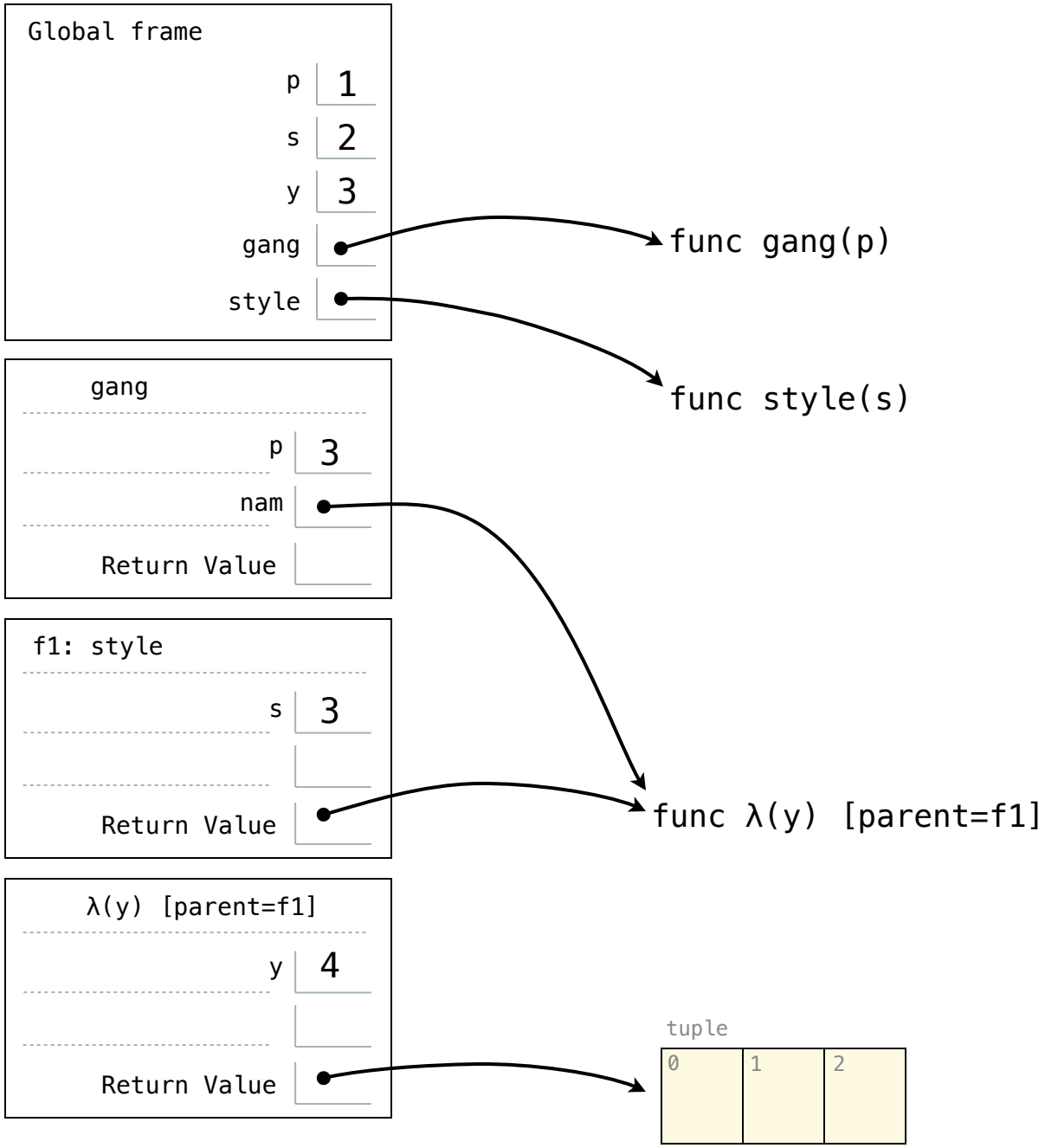

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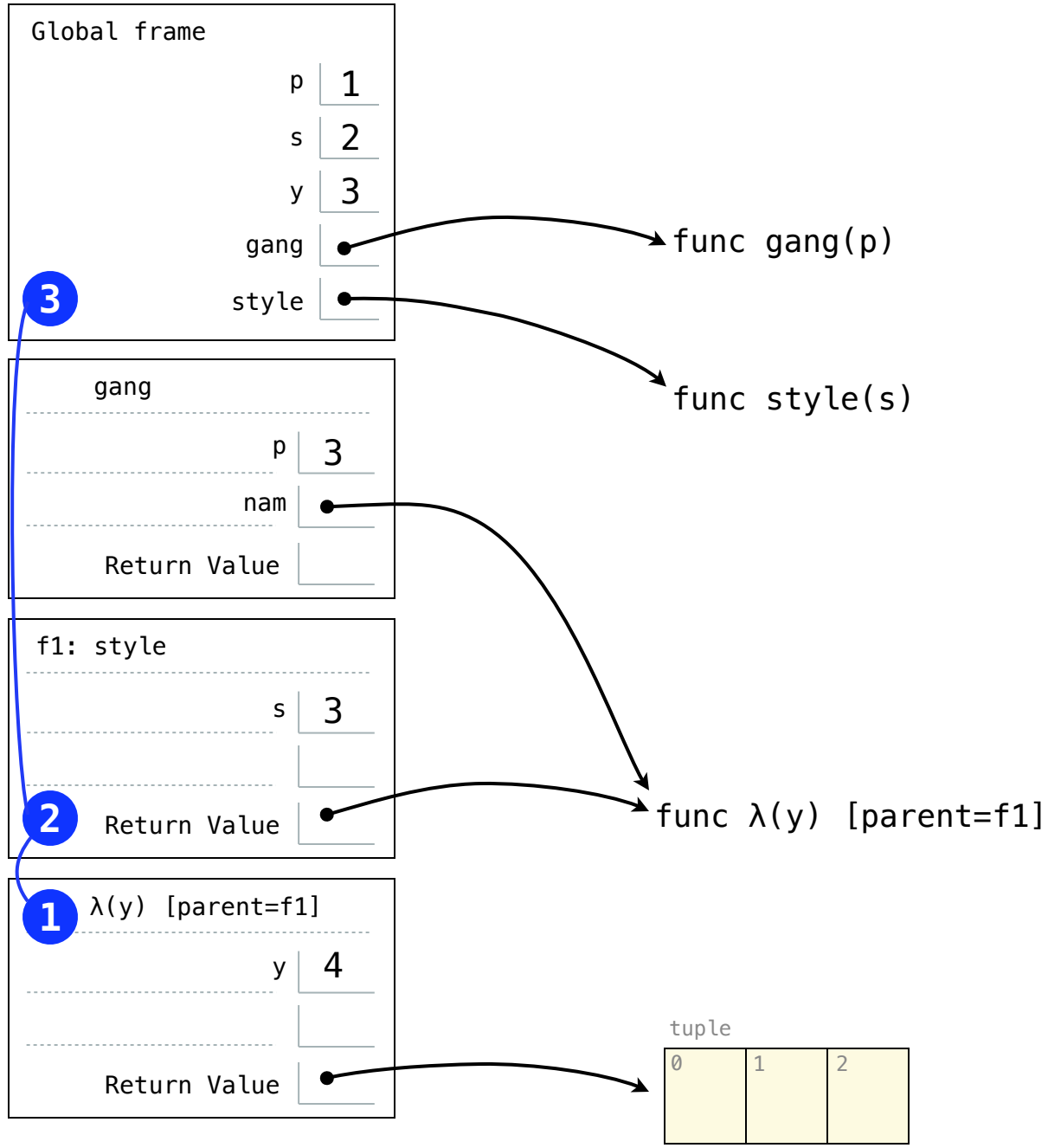

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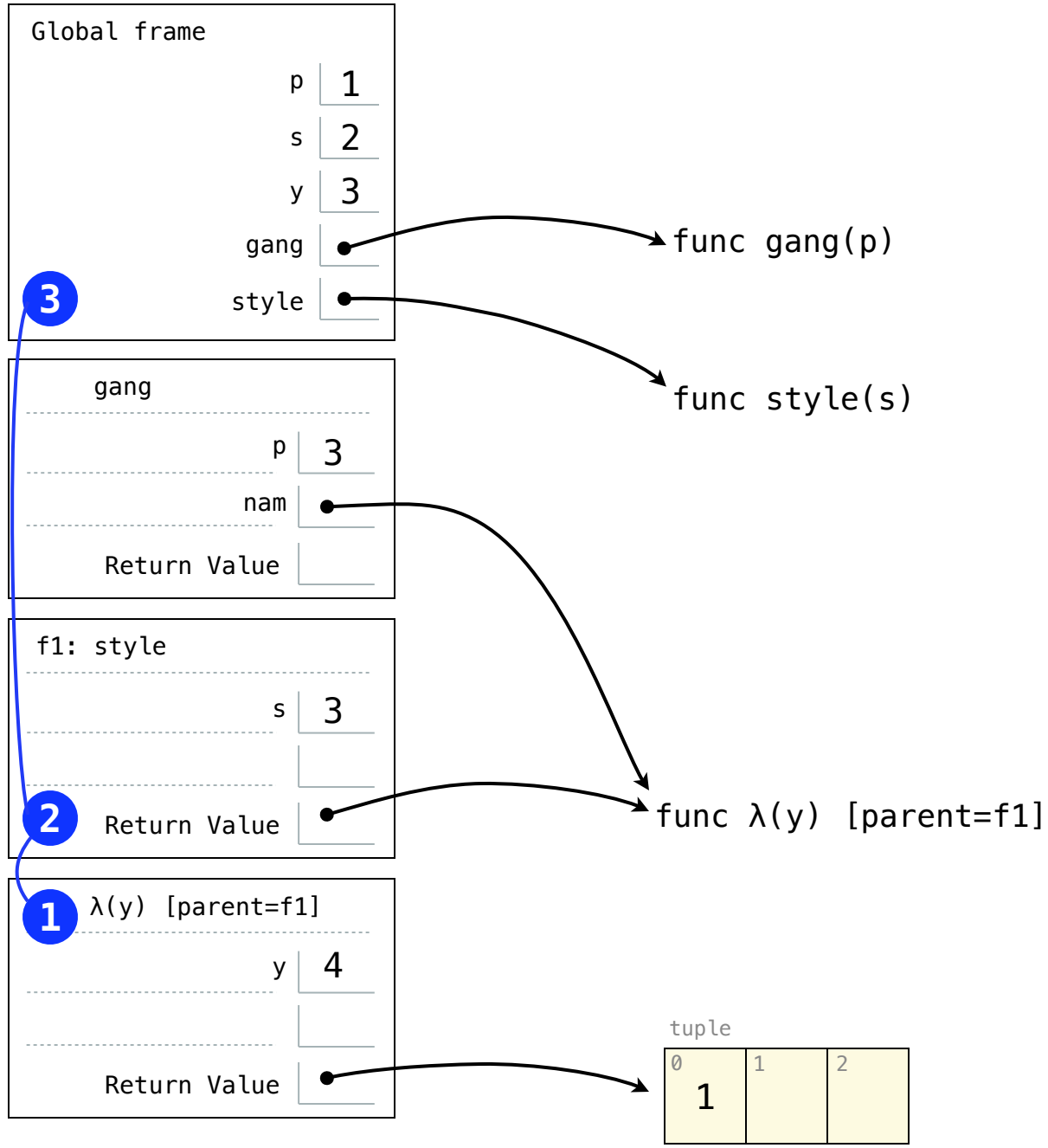

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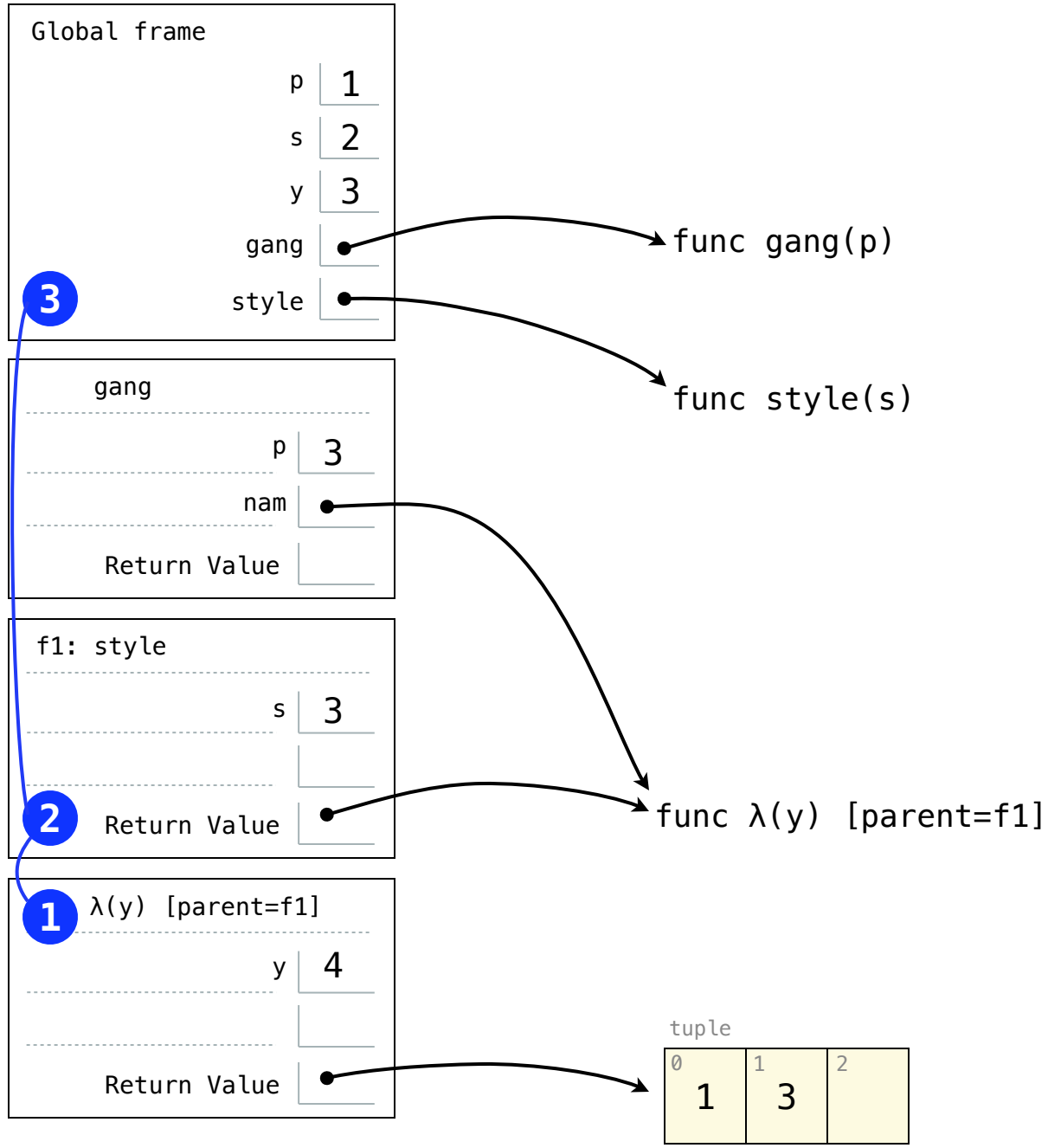

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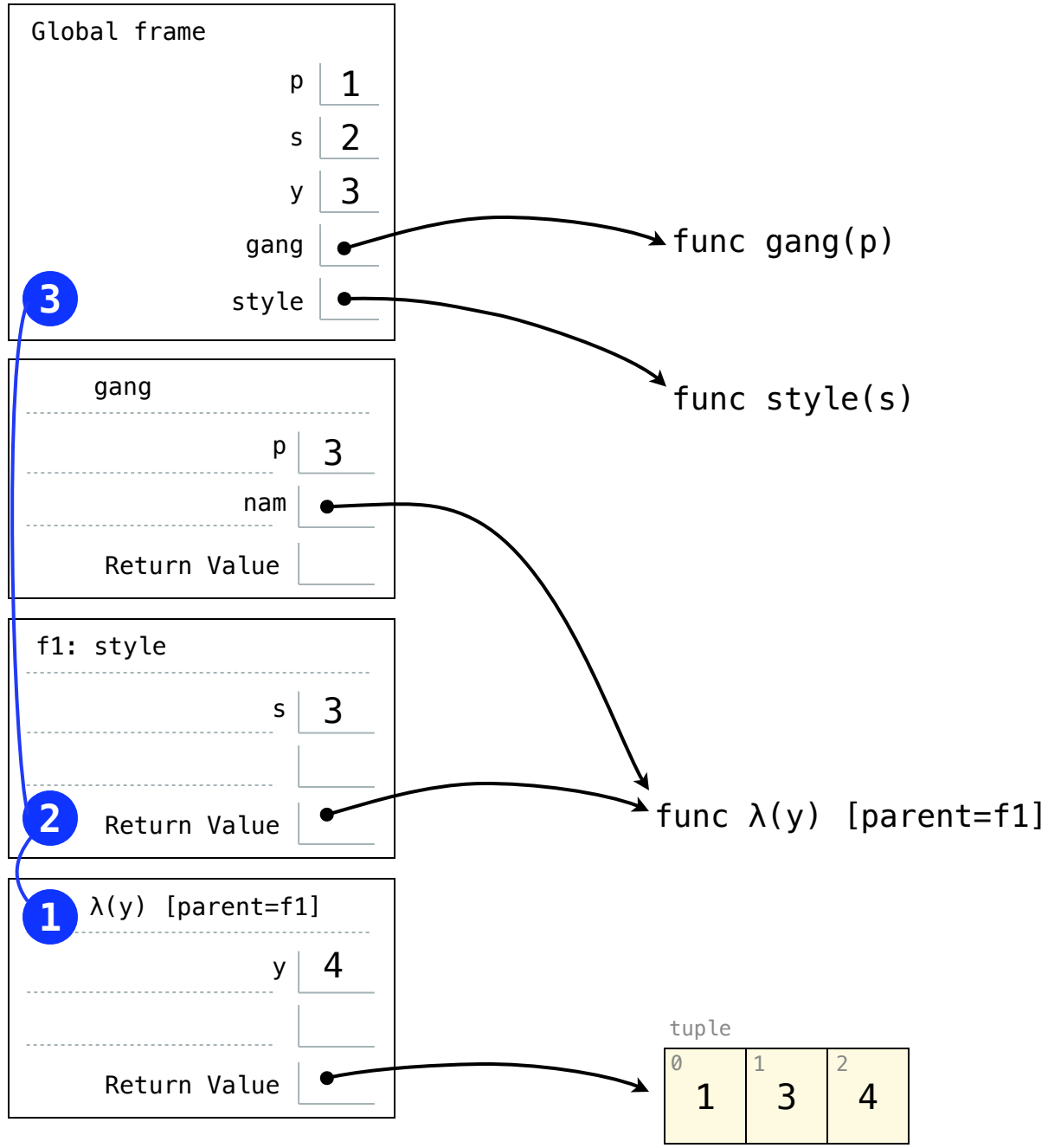

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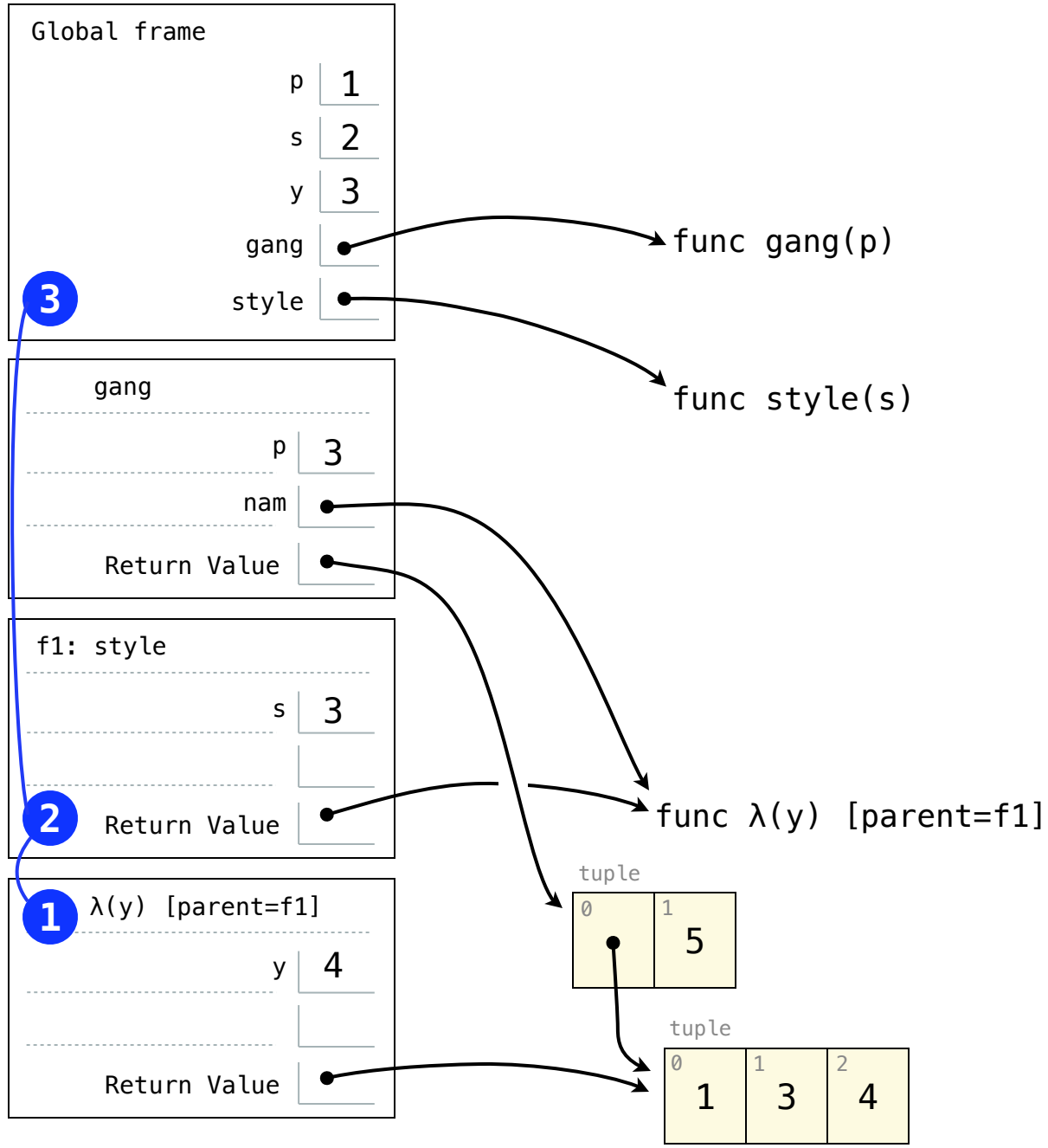

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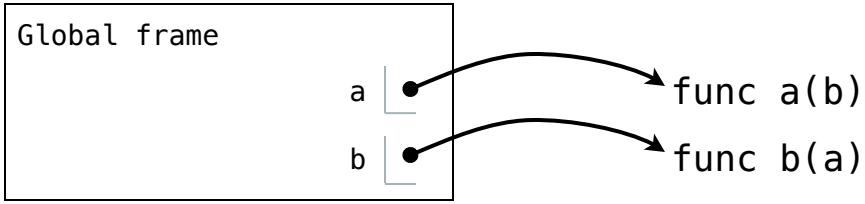

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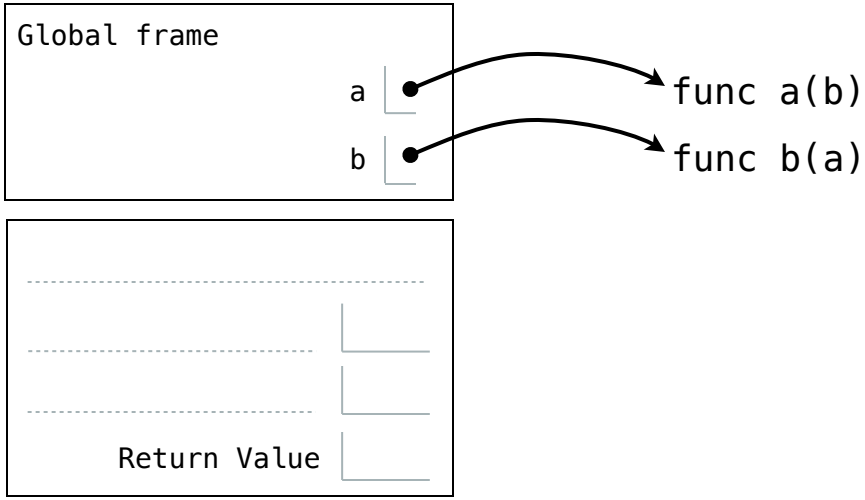

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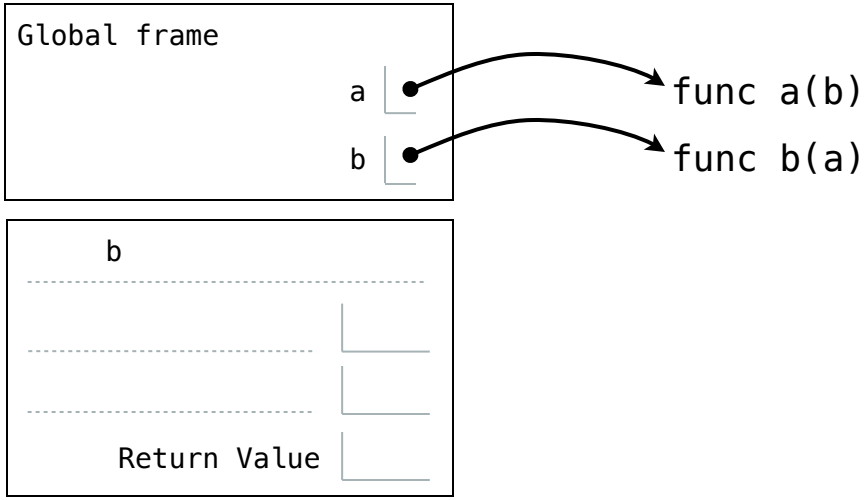

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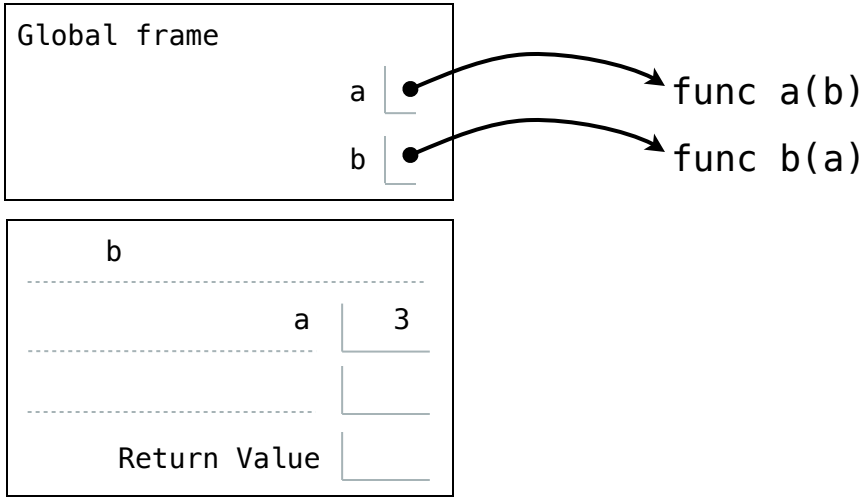

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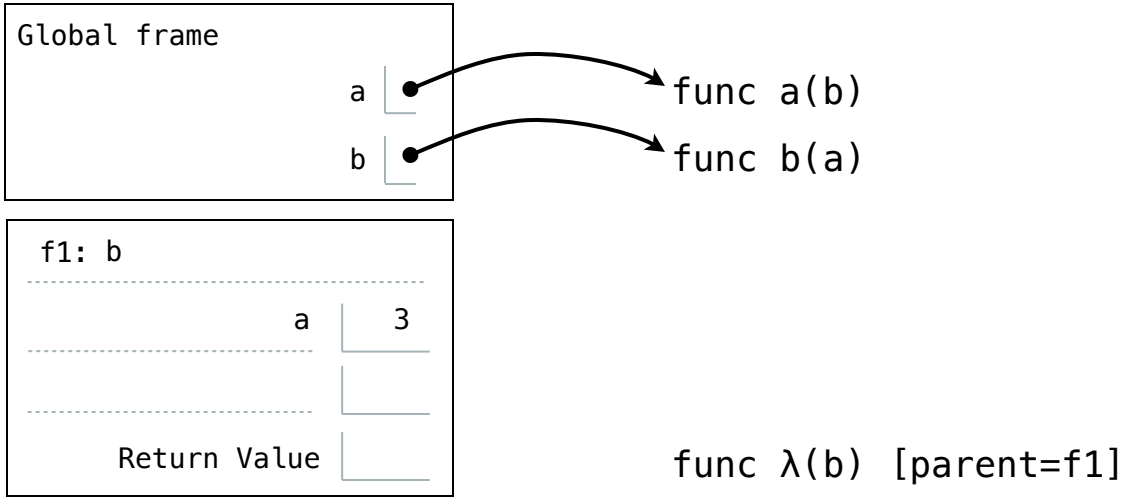

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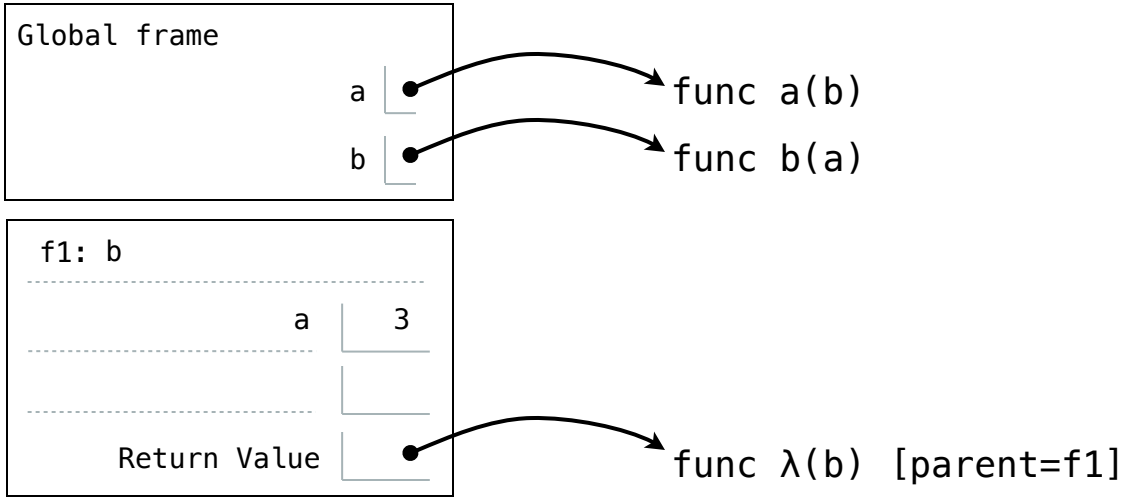

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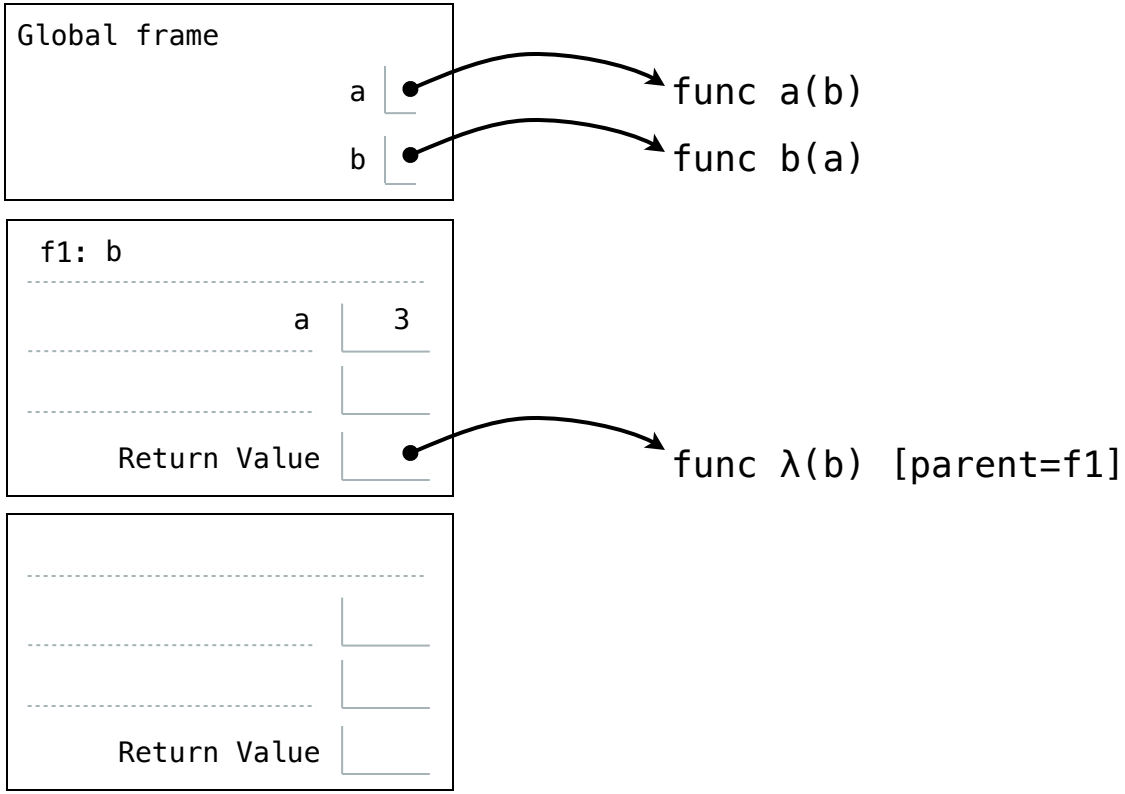

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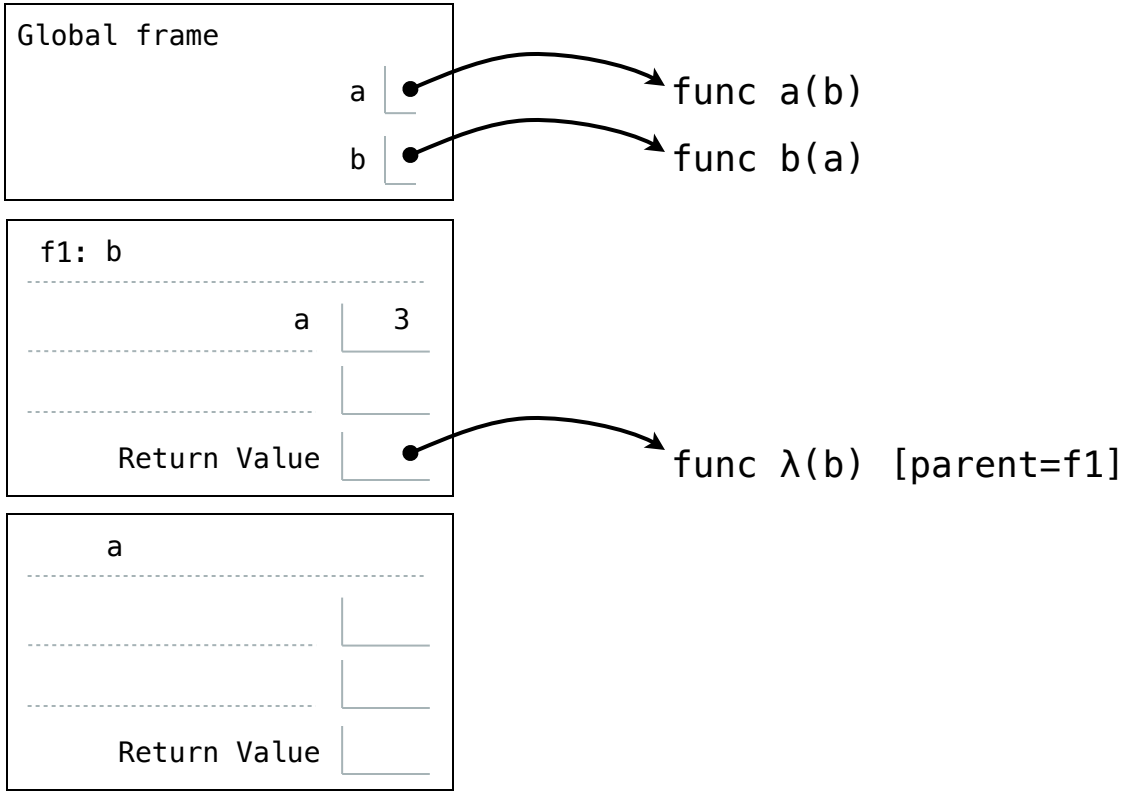

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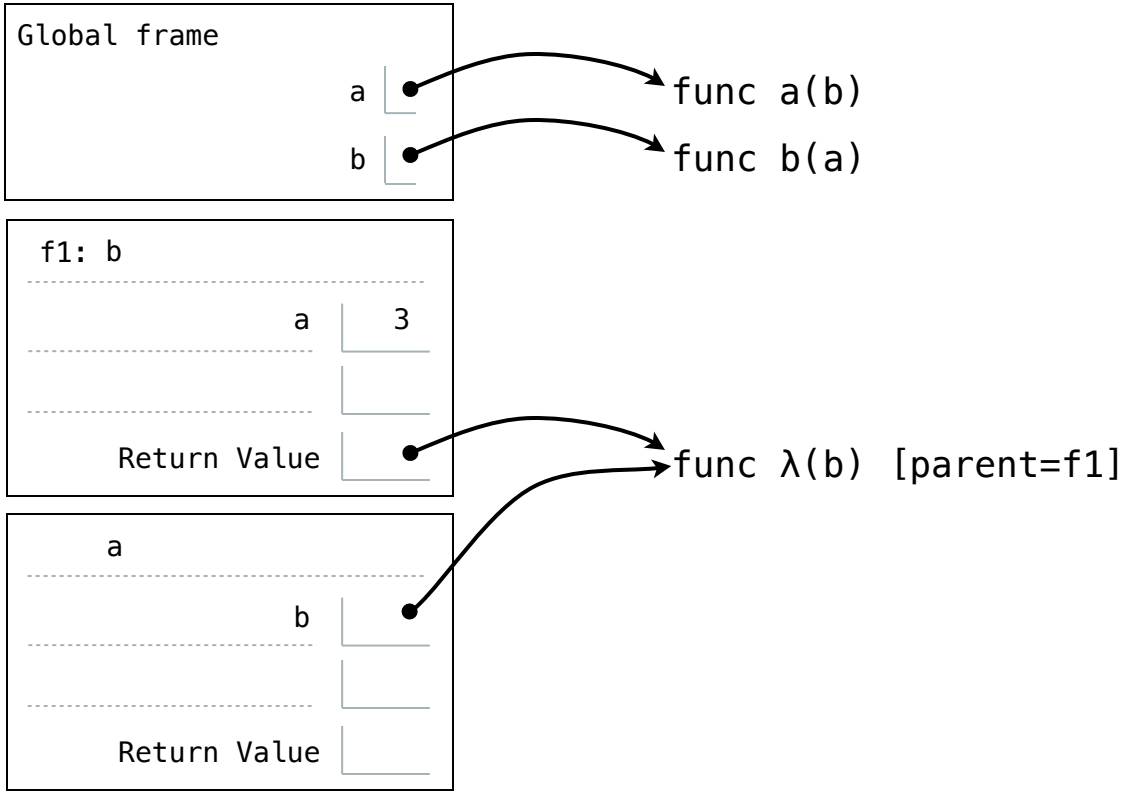

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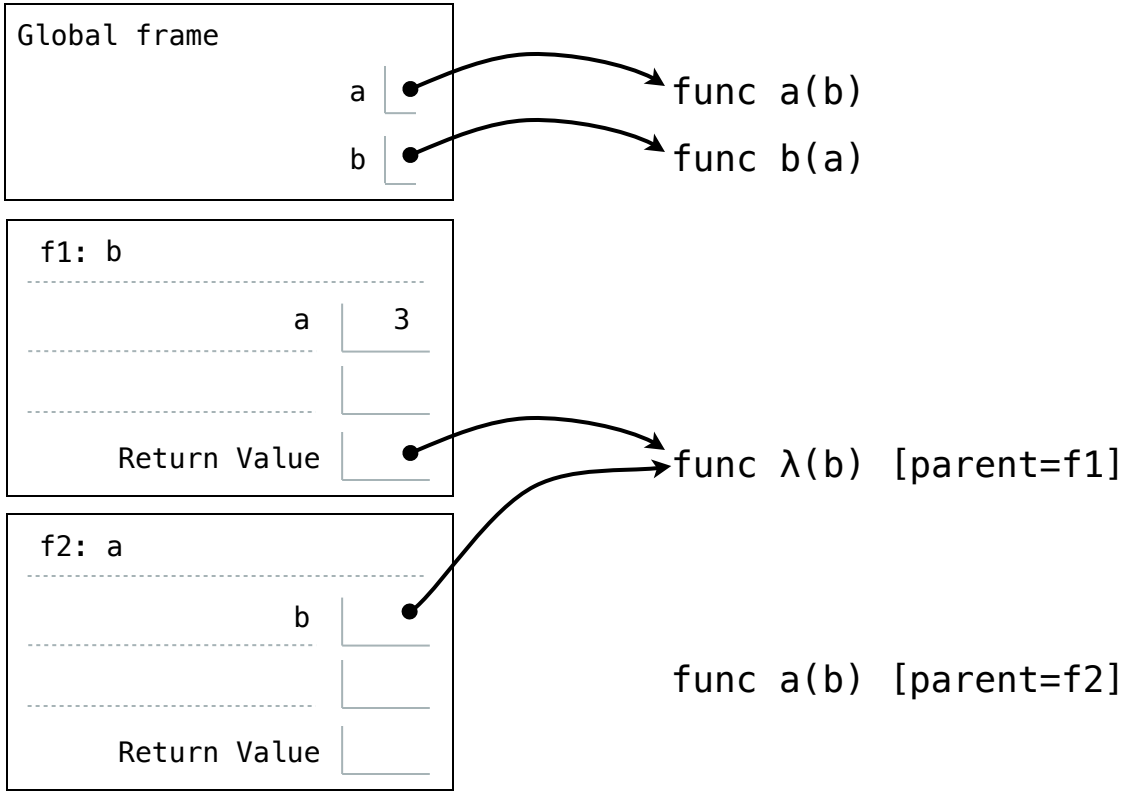

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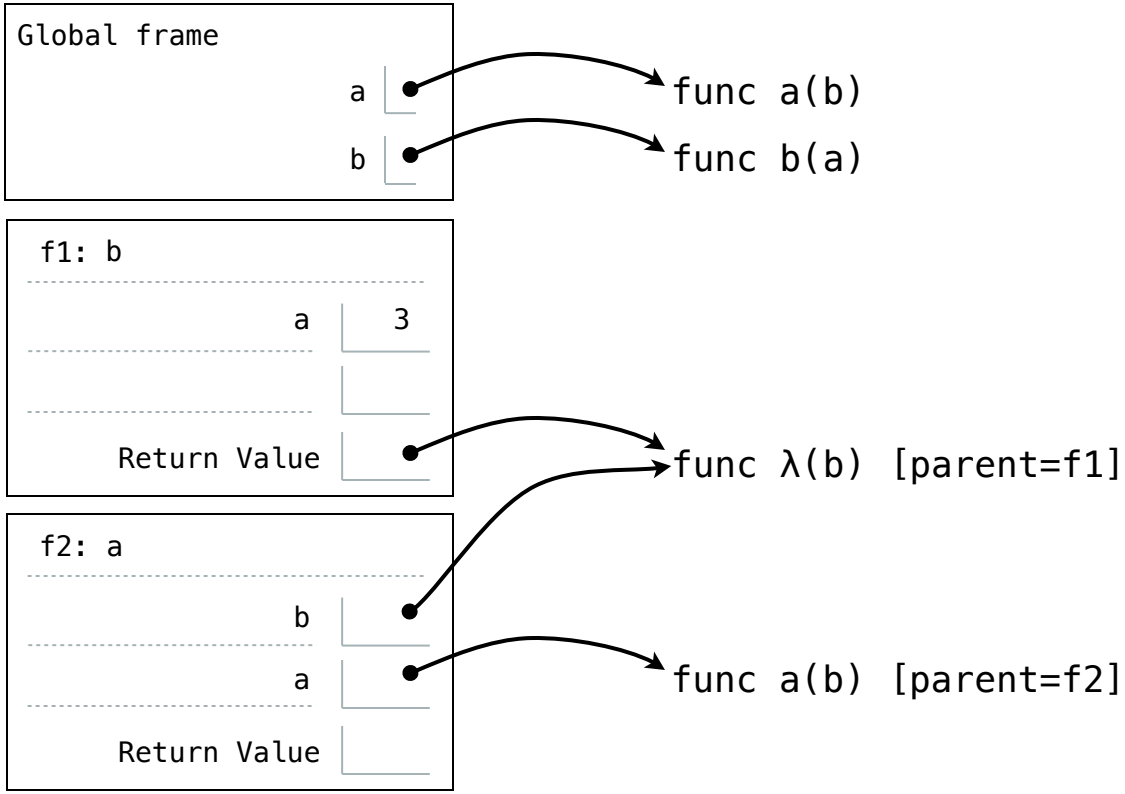

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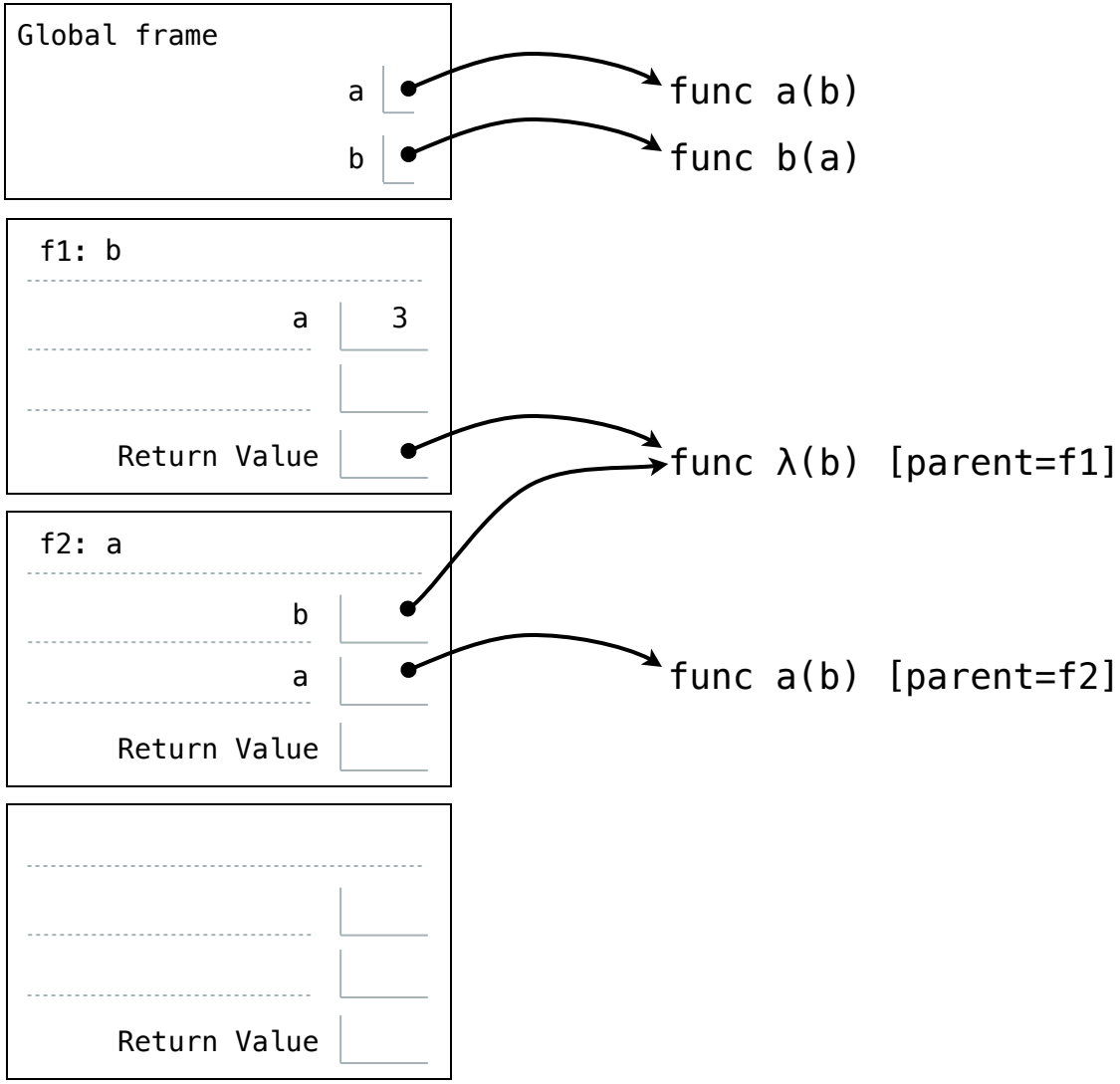

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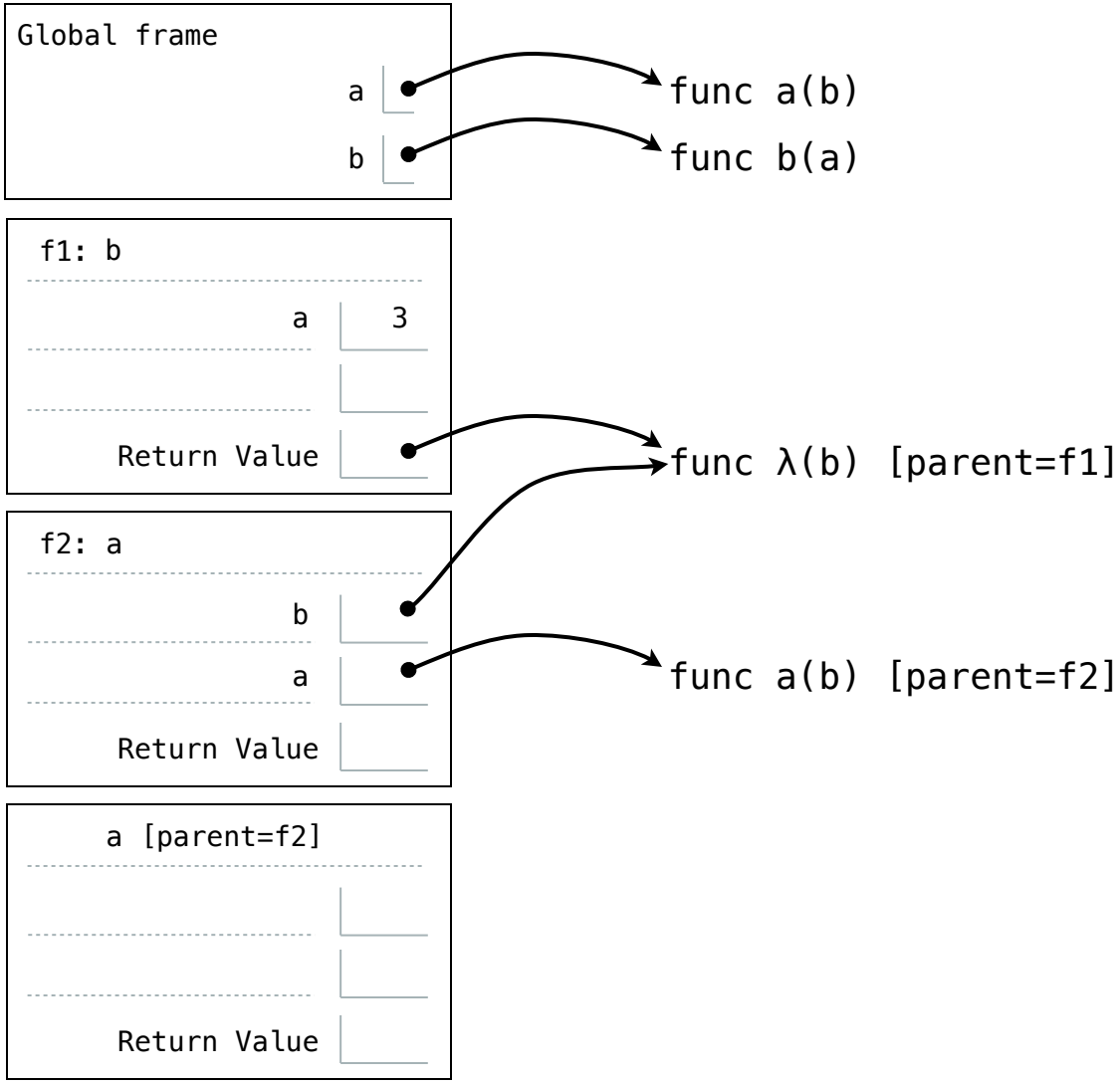

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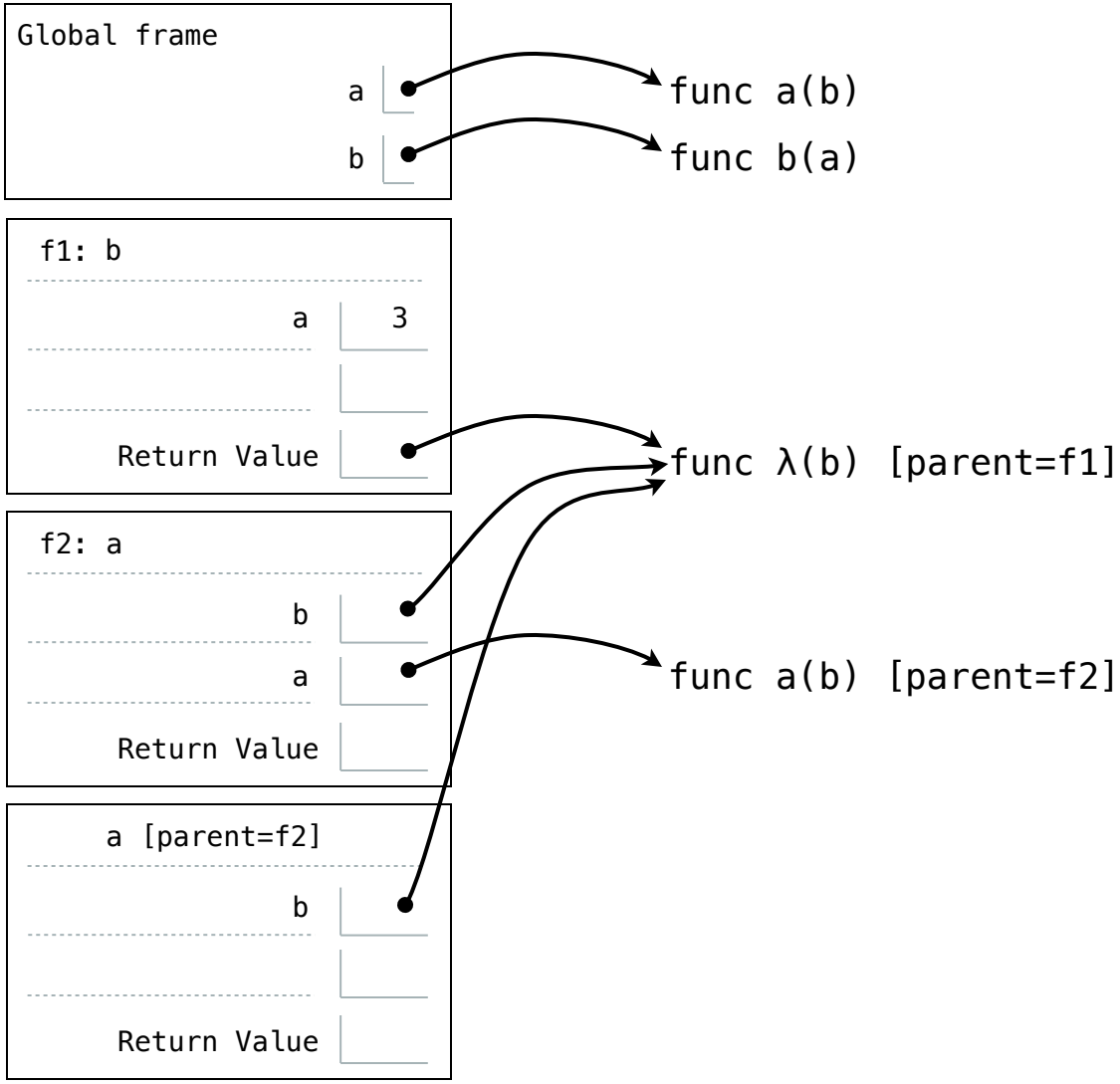

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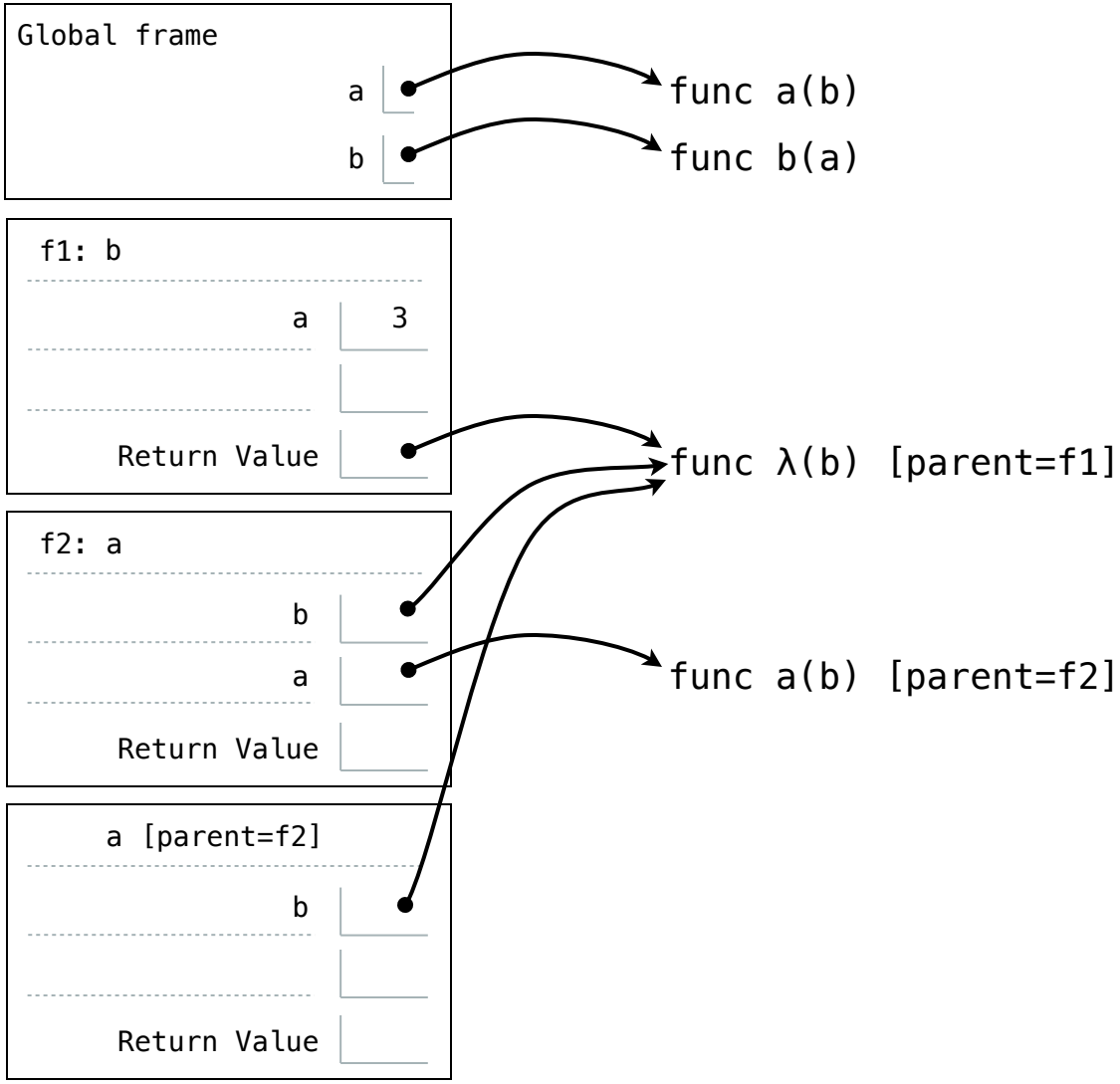

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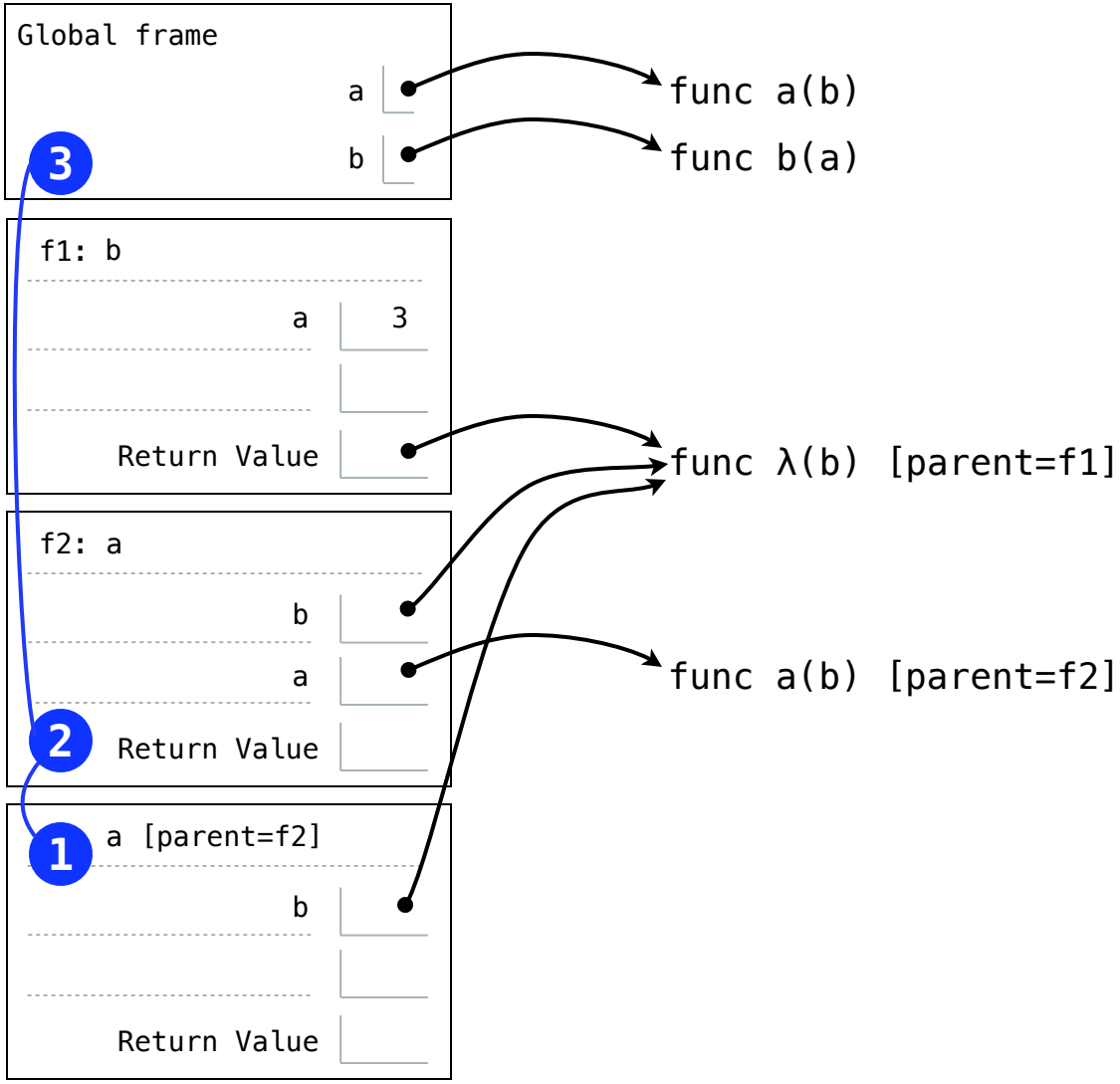

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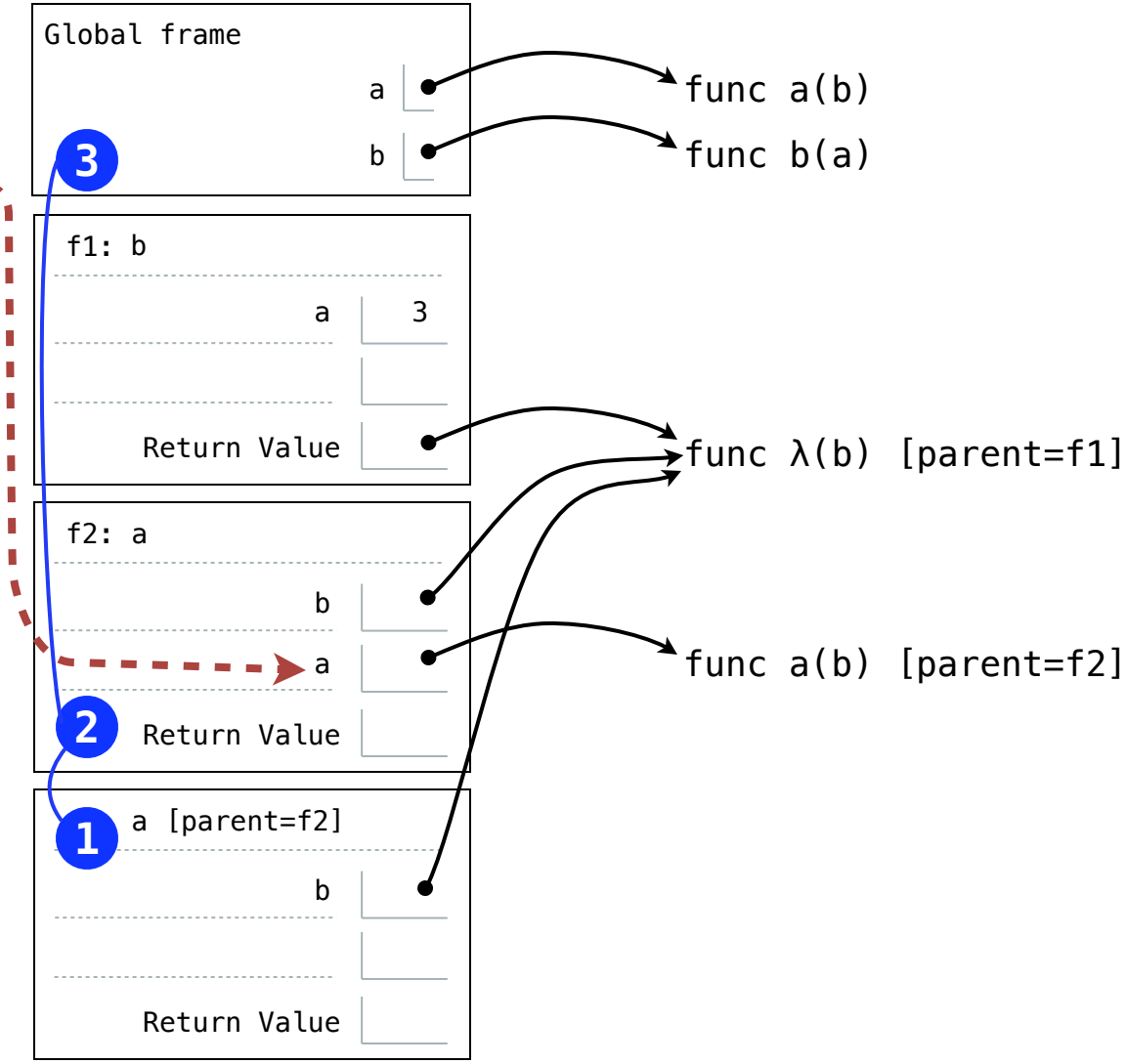

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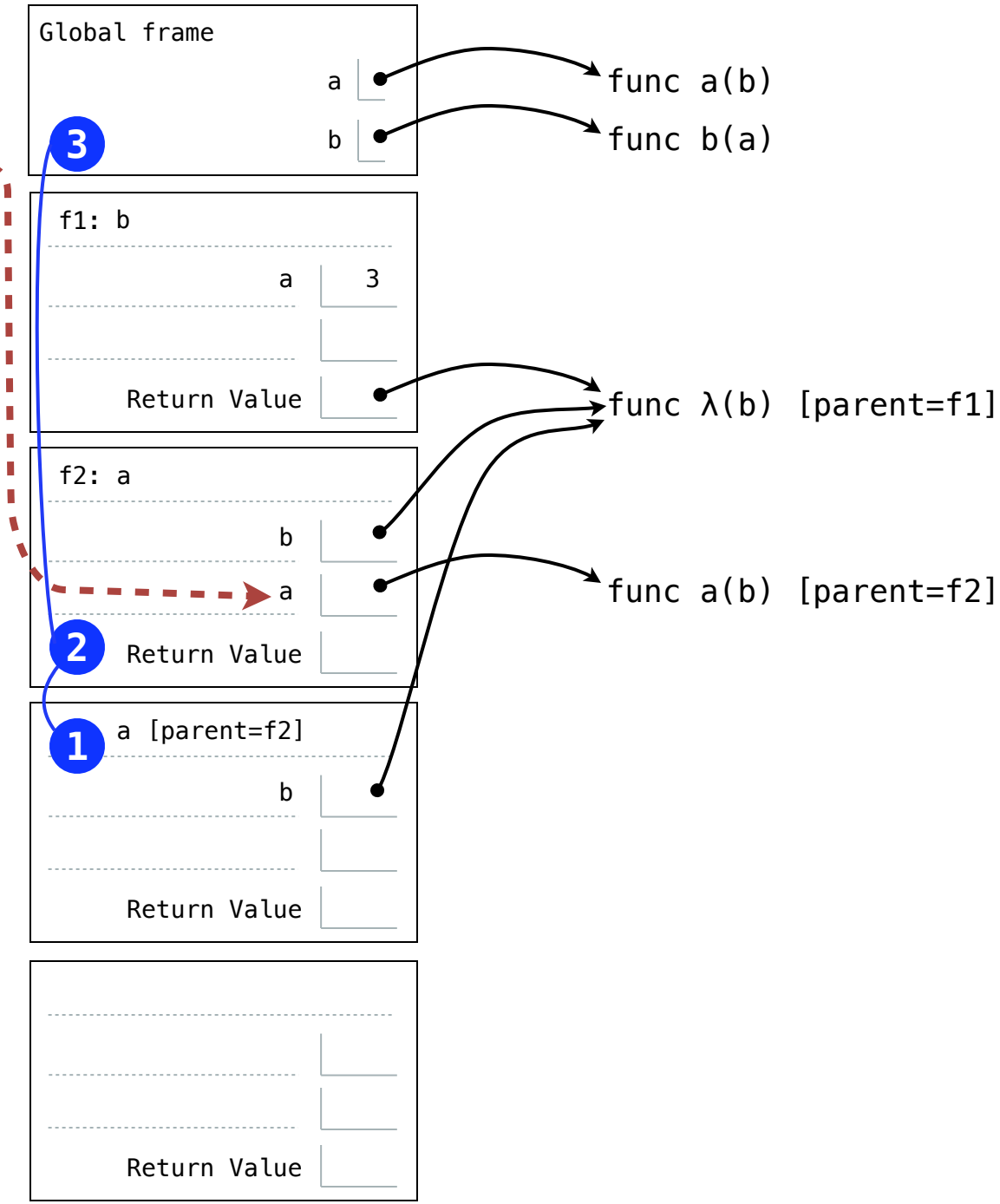

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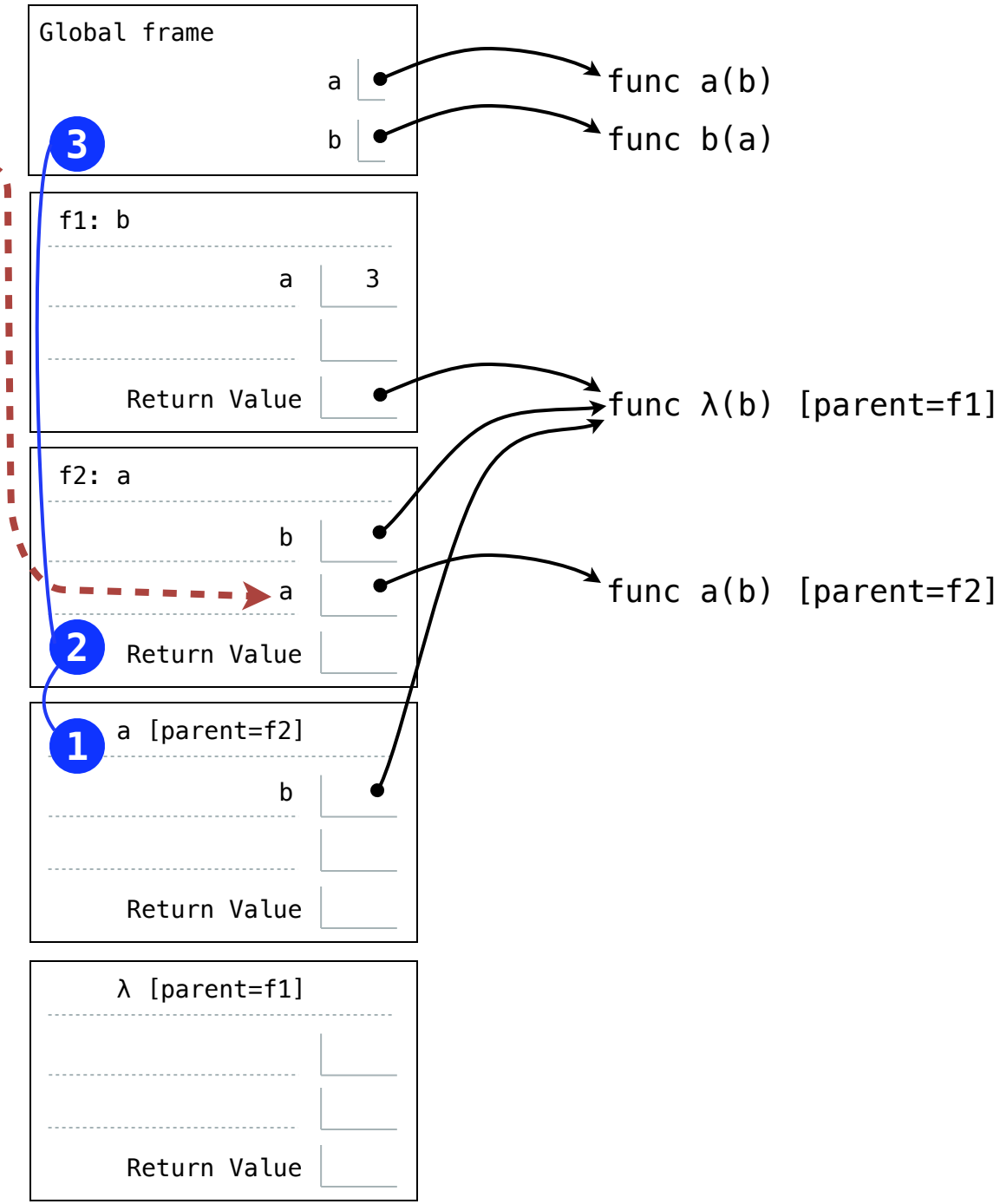

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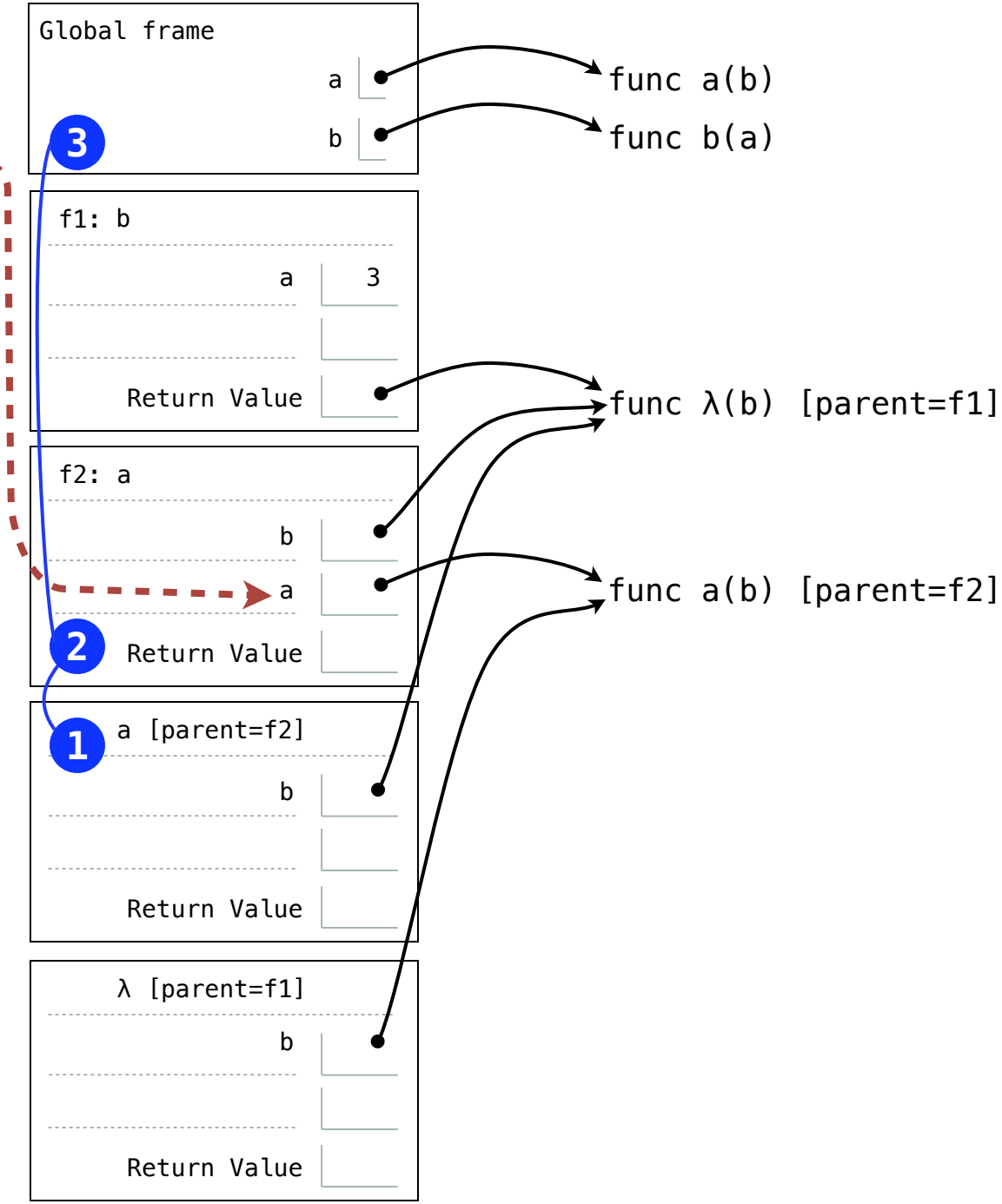

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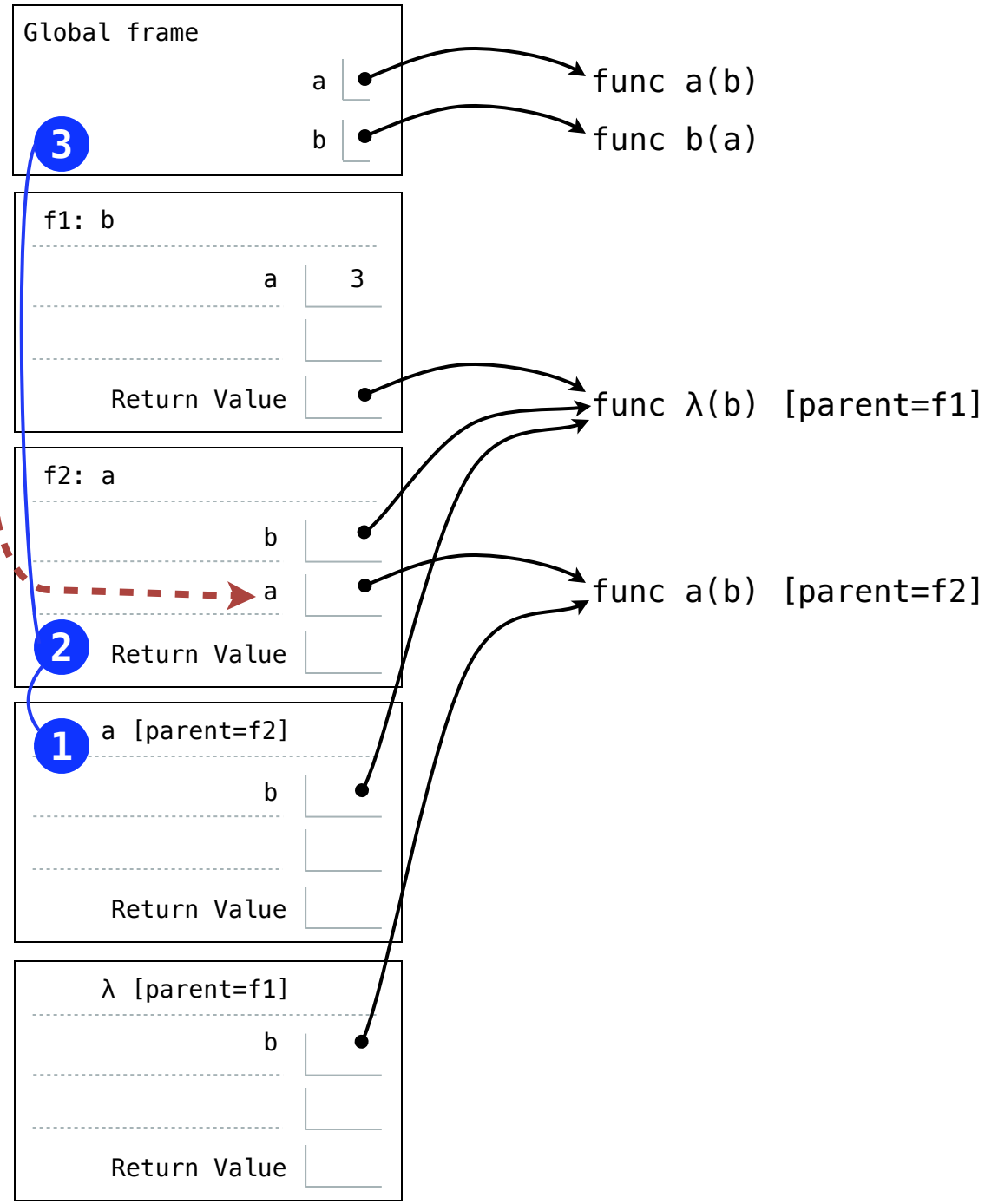

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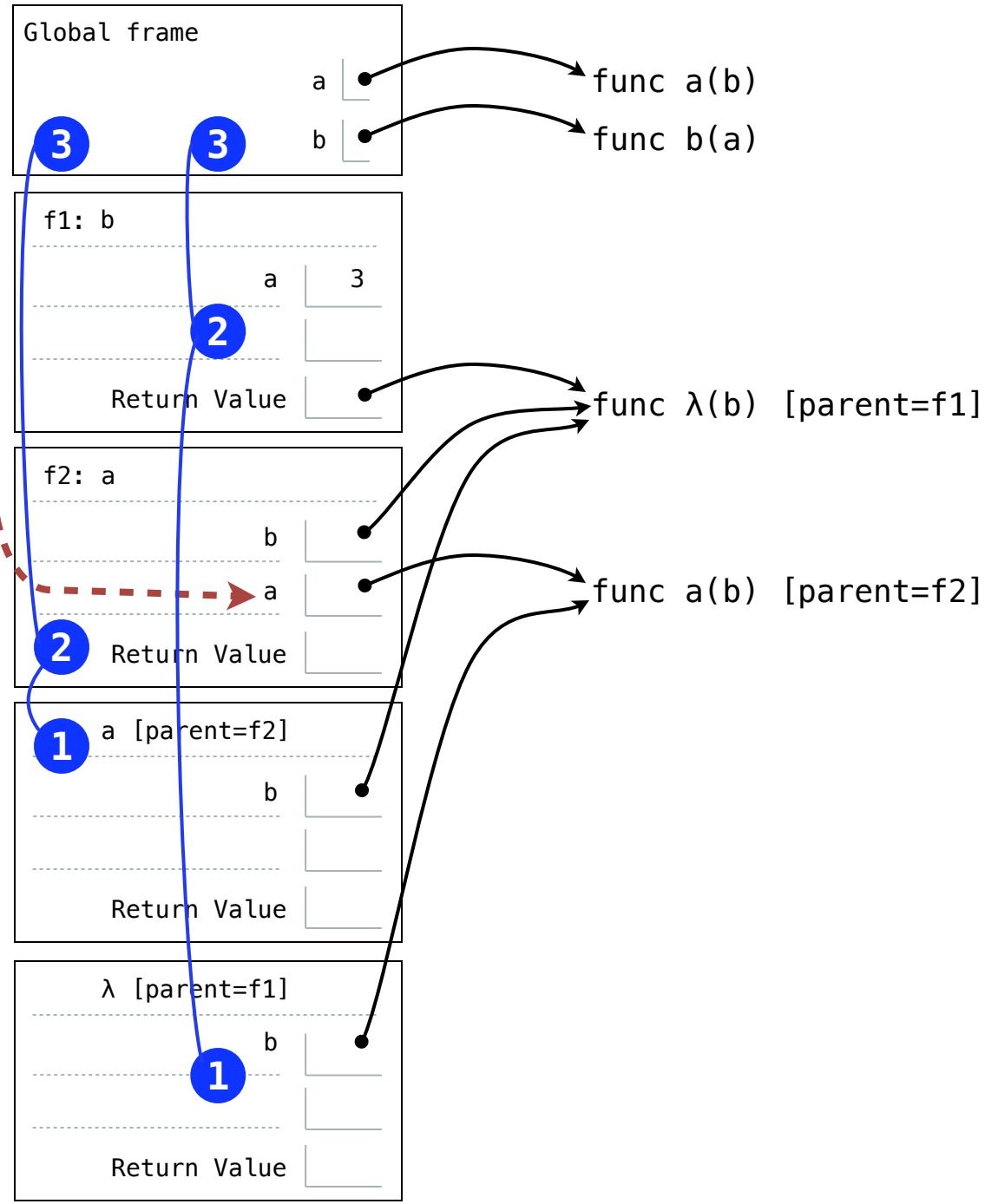

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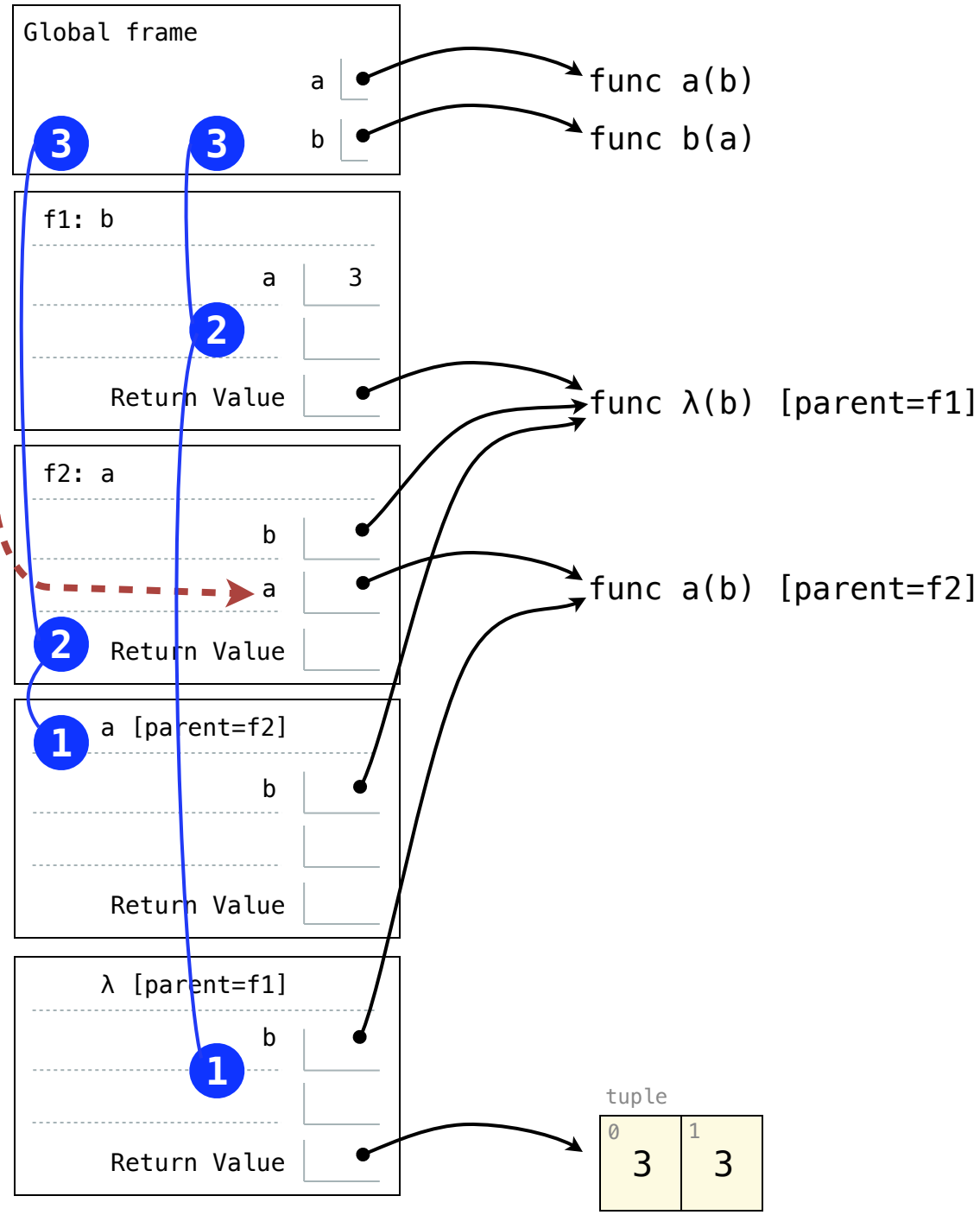

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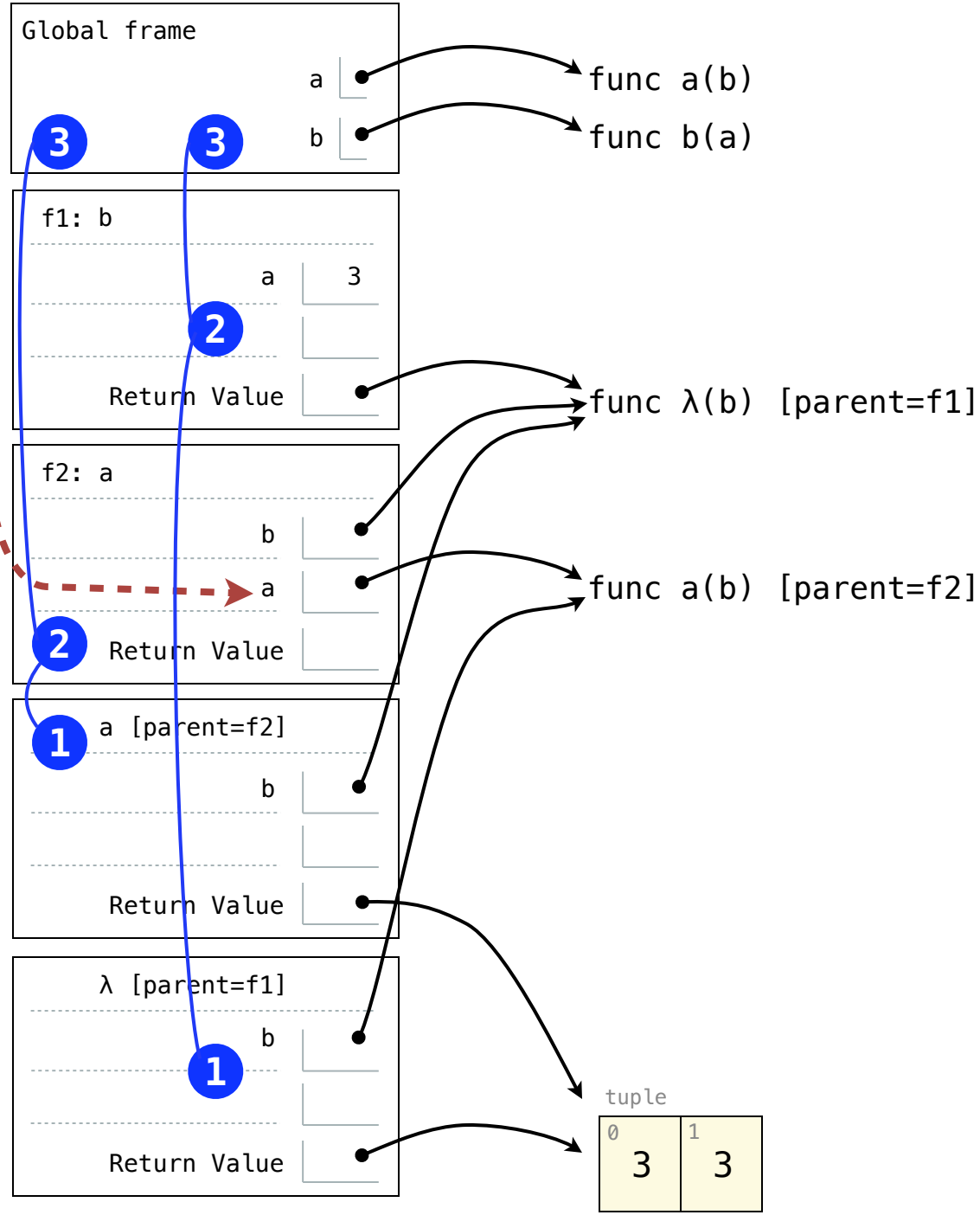

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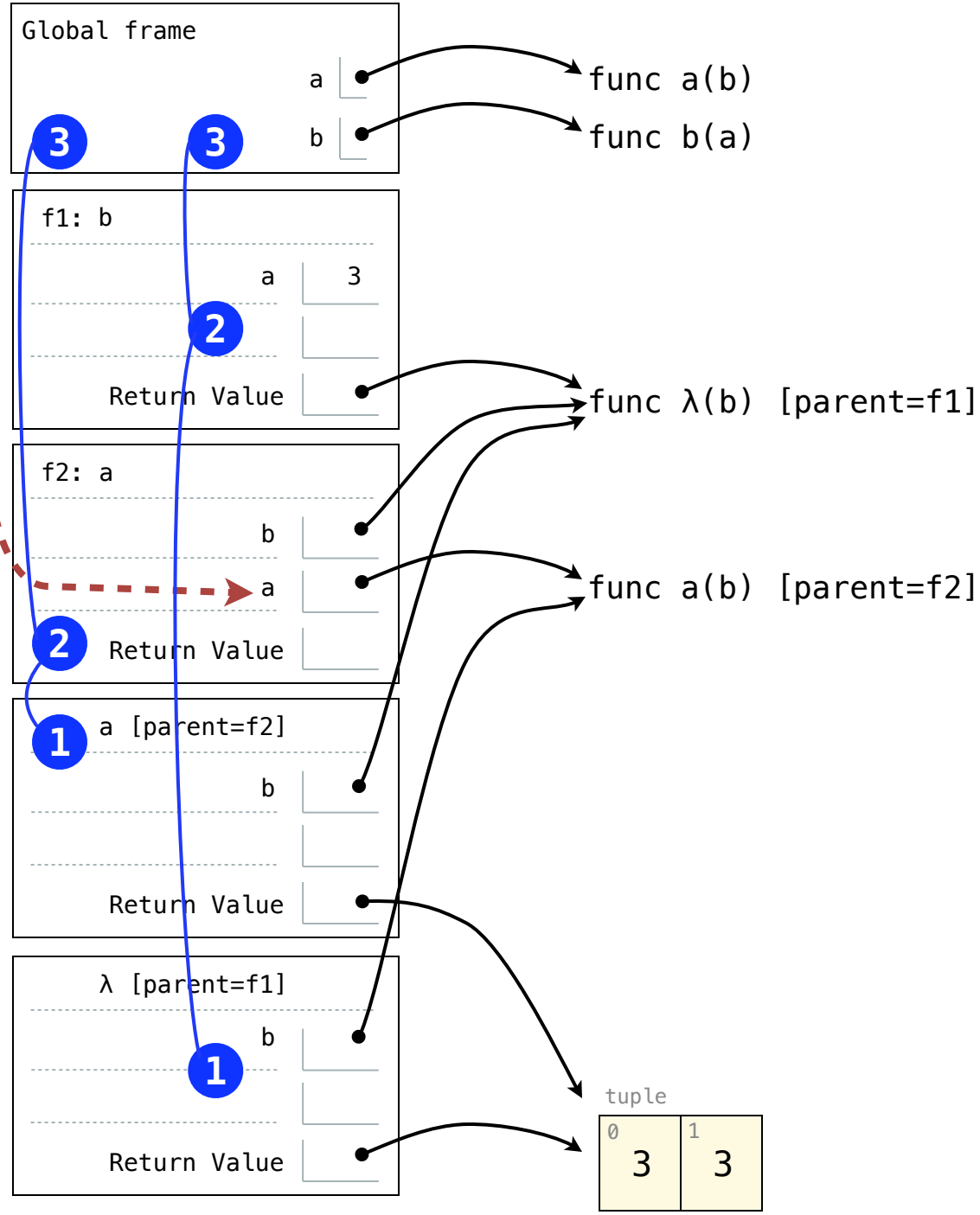

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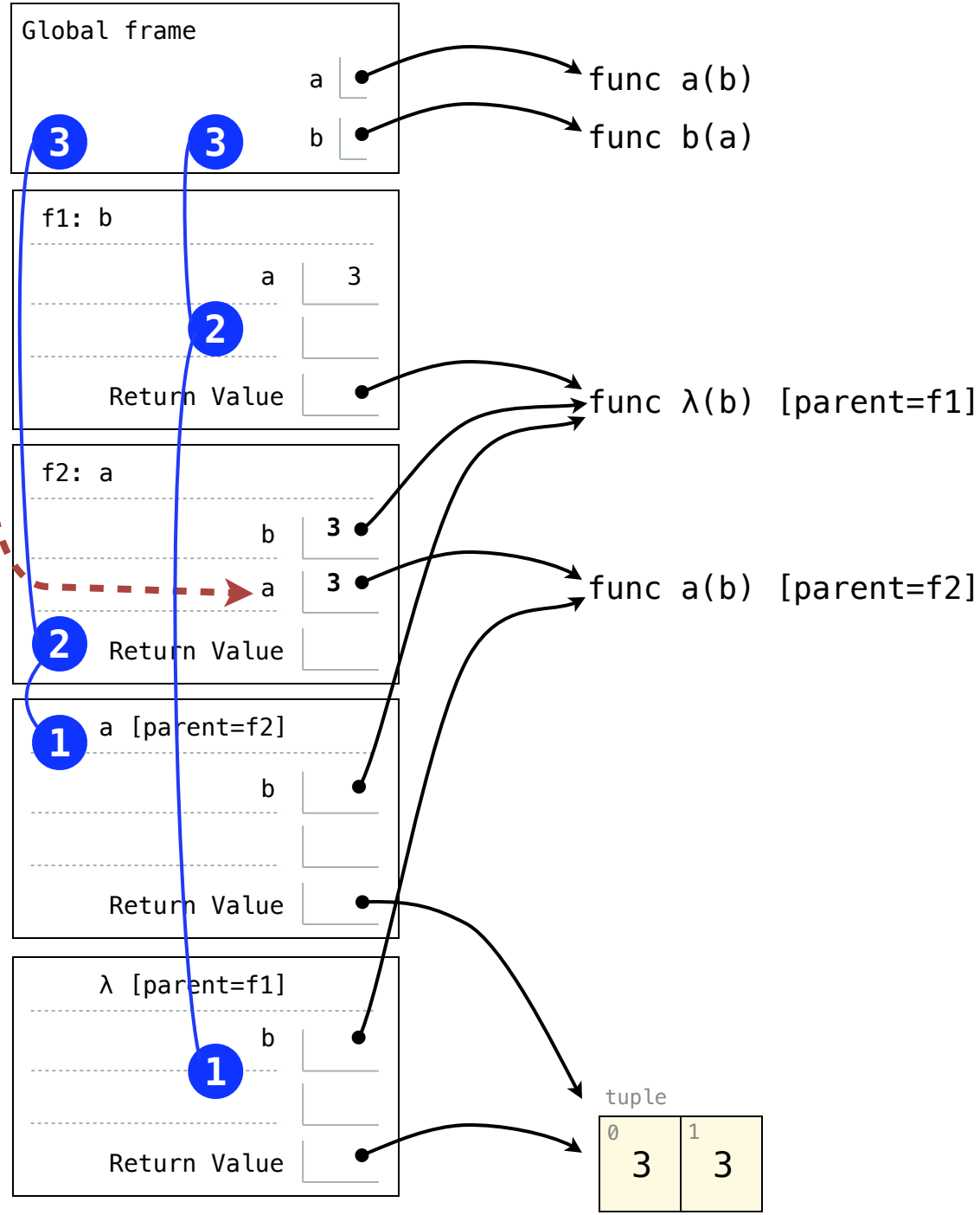

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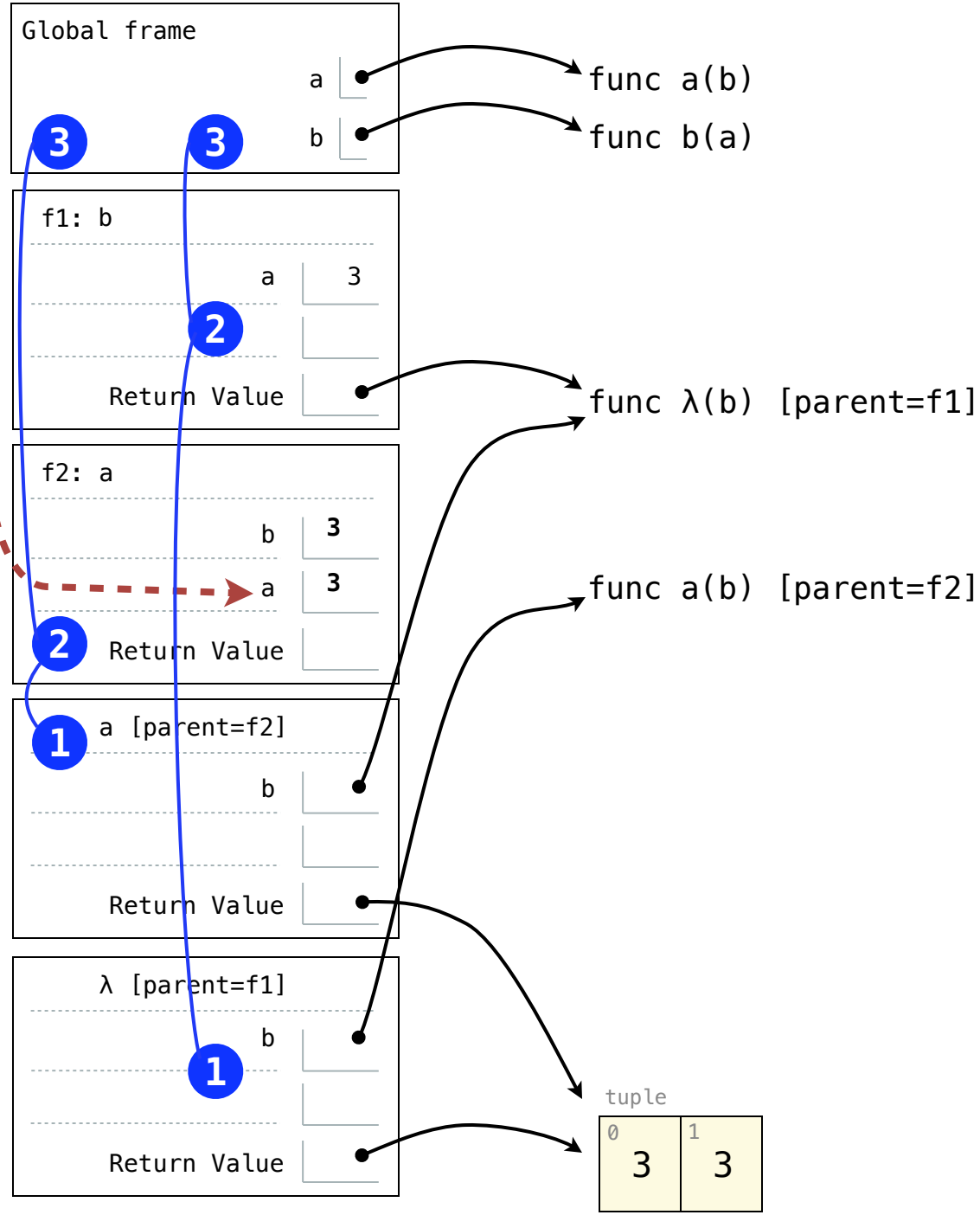

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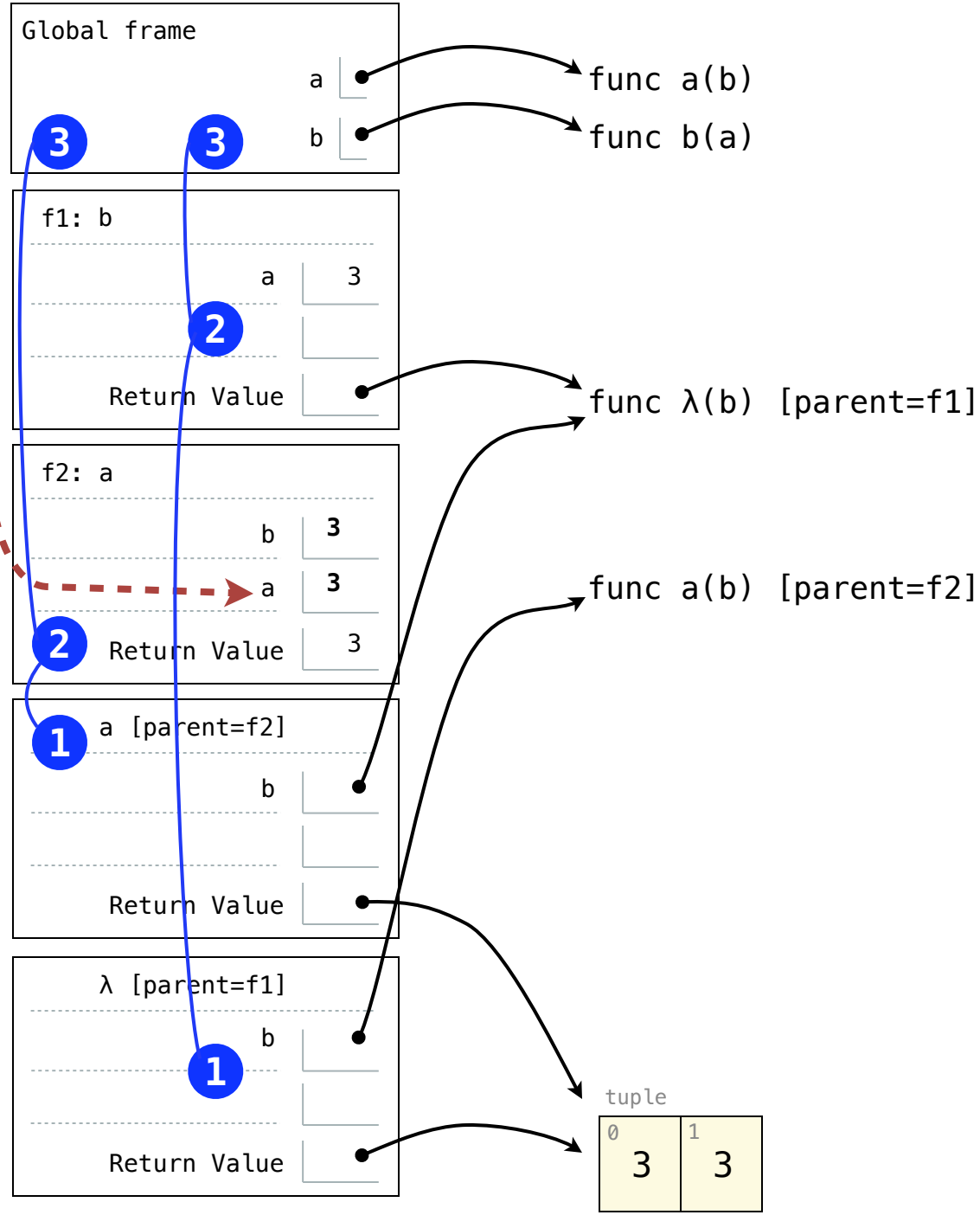

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For variable y
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