61A Lecture 13

Wednesday, September 26

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```
>>> withdraw(25)
```

Argument:
amount to withdraw

A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value:
remaining balance

```
>>> withdraw(25)
75
```

Argument:
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Second withdrawal
of the same amount

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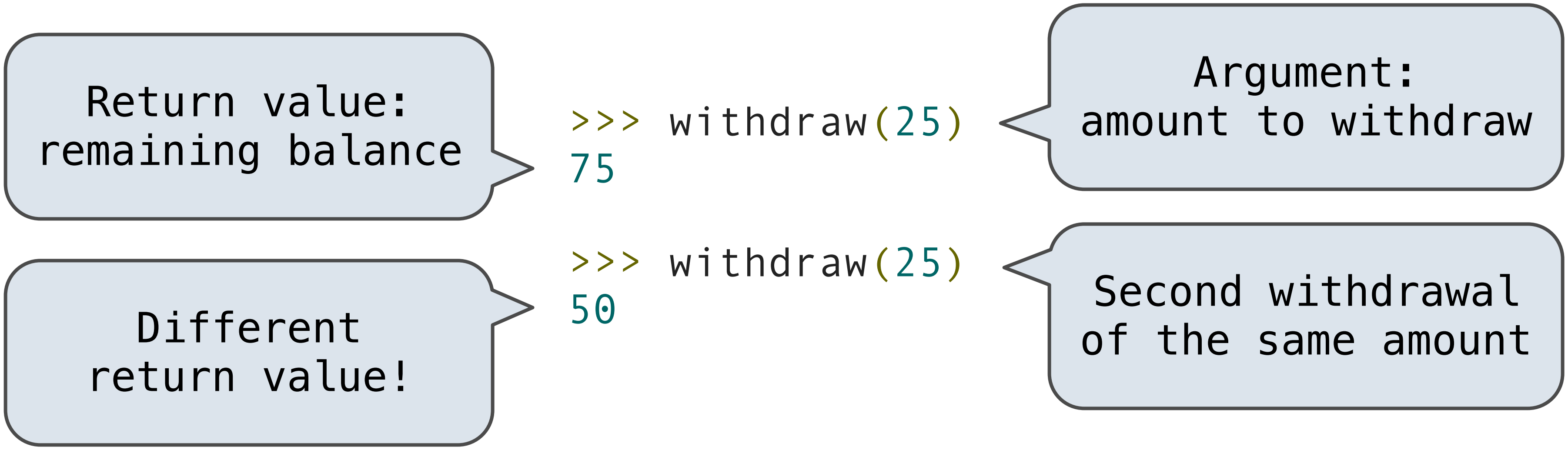

Return value: remaining balance

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Different return value!

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>>> withdraw(25)
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Second withdrawal of the same amount

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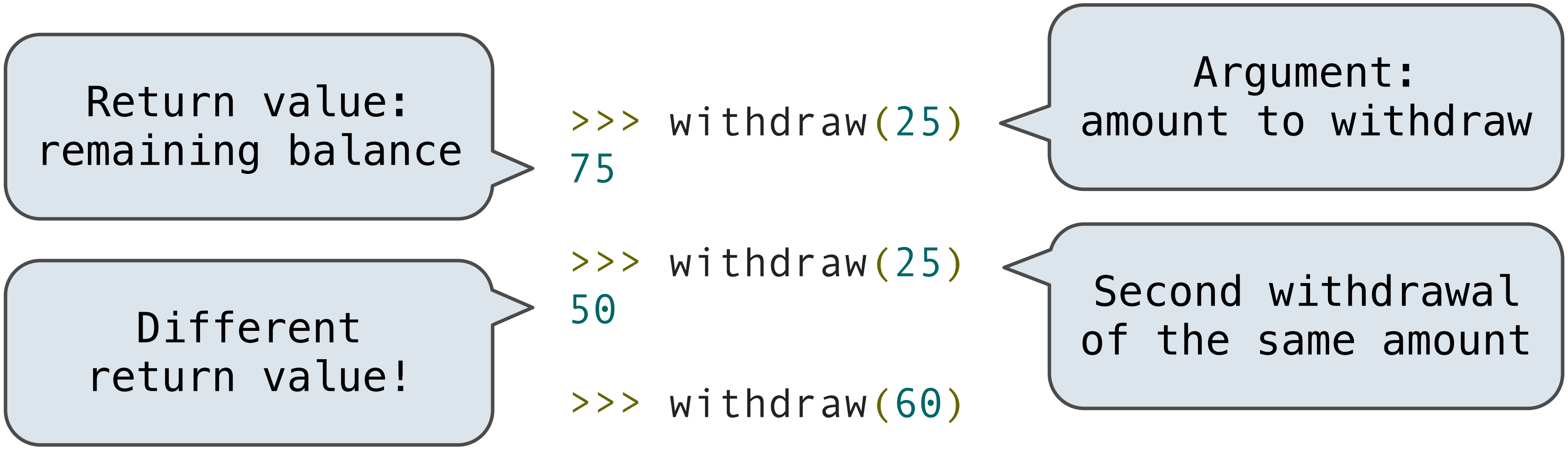

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>>> withdraw(25)
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Return value: remaining balance

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Argument: amount to withdraw

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Where's this
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Where's this balance stored?

```
>>> withdraw = make_withdraw(100)
```

A Function with Behavior That Varies Over Time

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Where's this balance stored?

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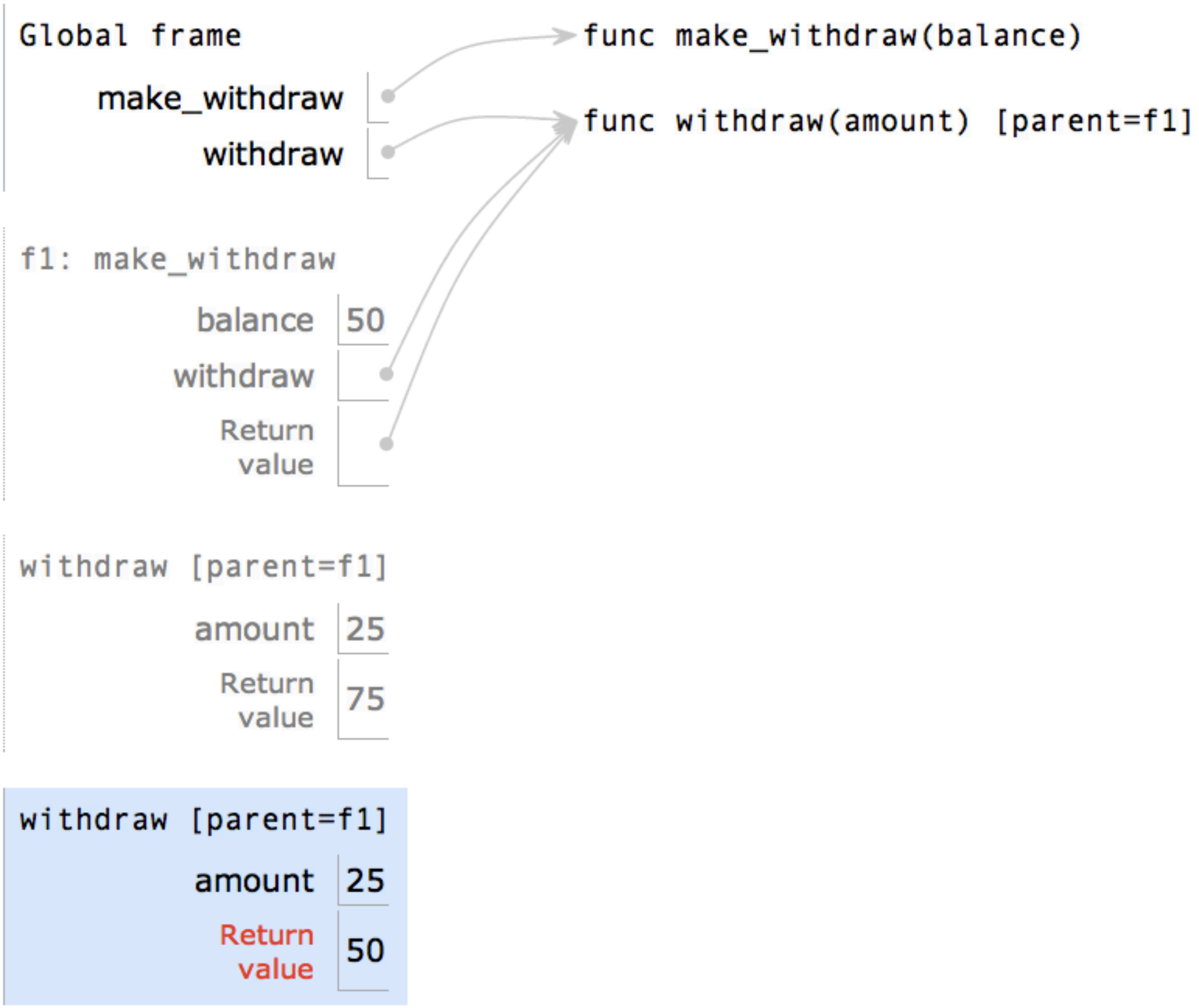
Within the function!

Persistent Local State



Global frame

make_withdraw

withdraw

func make_withdraw(balance)

func withdraw(amount) [parent=f1]

f1: make_withdraw

balance 50

withdraw

Return value

withdraw [parent=f1]

amount 25

Return value 75

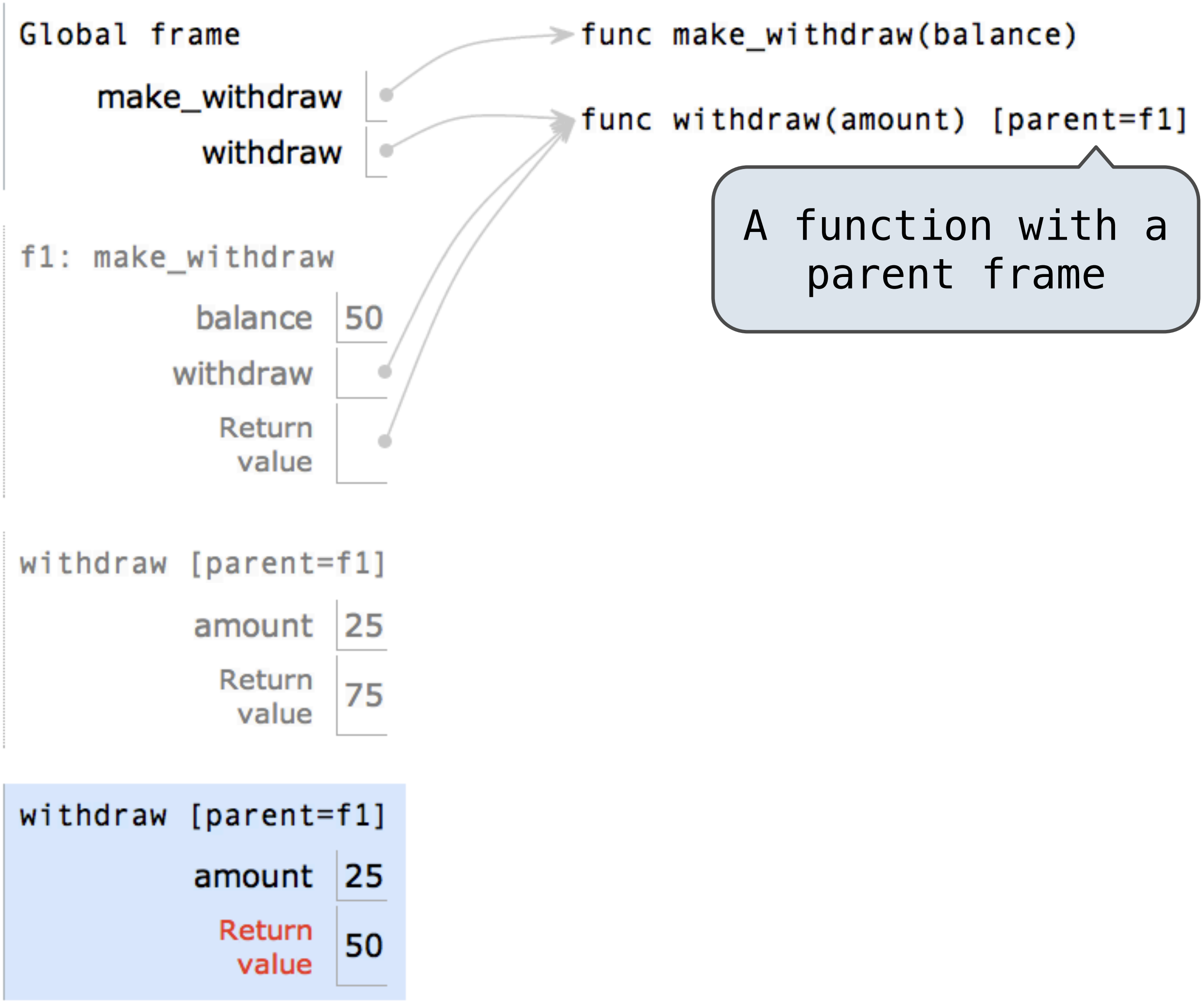
withdraw [parent=f1]

amount 25

Return value 50

http://goo.gl/StRZP

Persistent Local State



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withdraw

Return value

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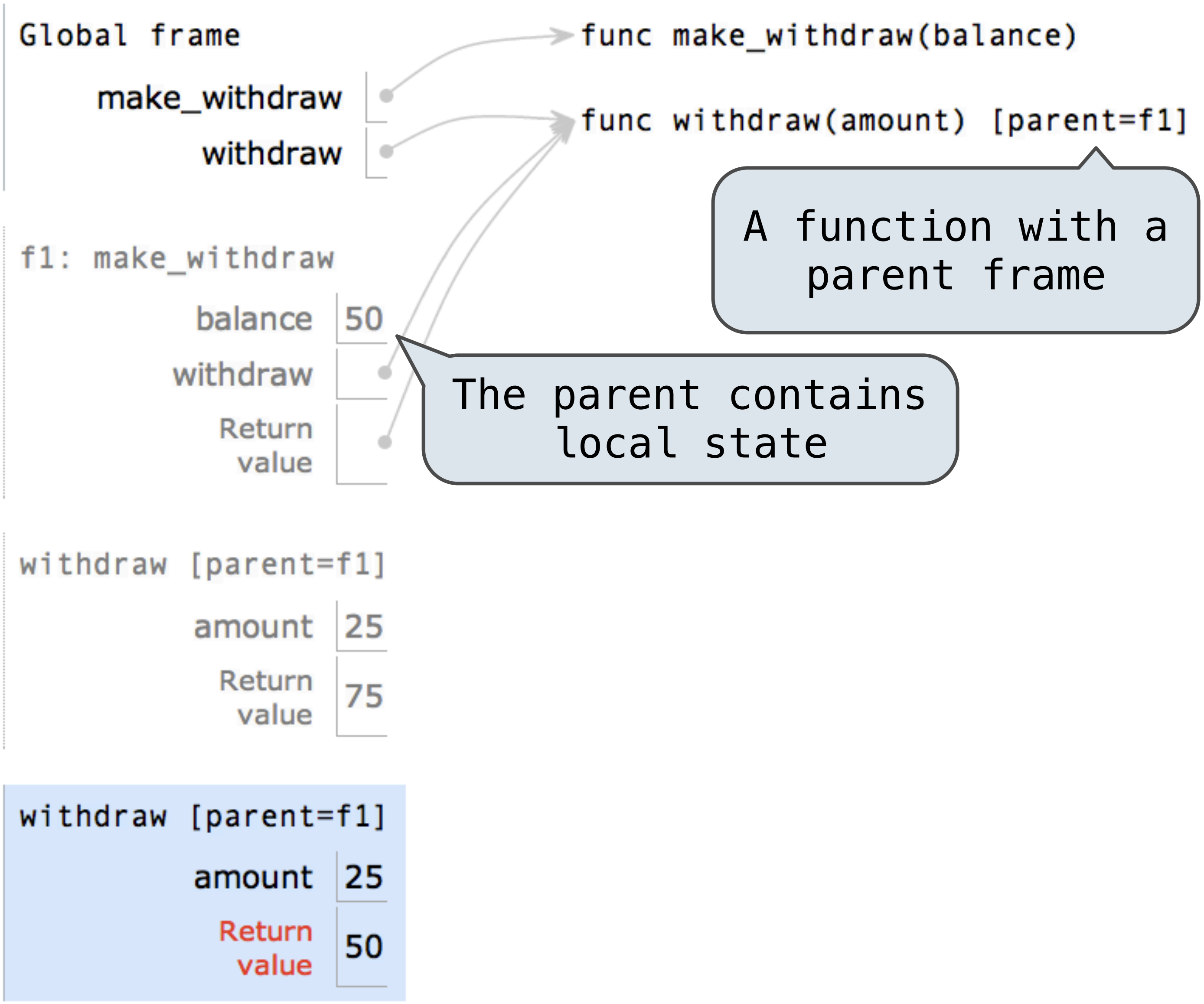
Return value 50

A function with a parent frame

http://goo.gl/StRZP

Persistent Local State



Global frame

make_withdraw

withdraw

func make_withdraw(balance)

func withdraw(amount) [parent=f1]

f1: make_withdraw

balance	50
withdraw	
Return value	

withdraw [parent=f1]

amount	25
Return value	75

withdraw [parent=f1]

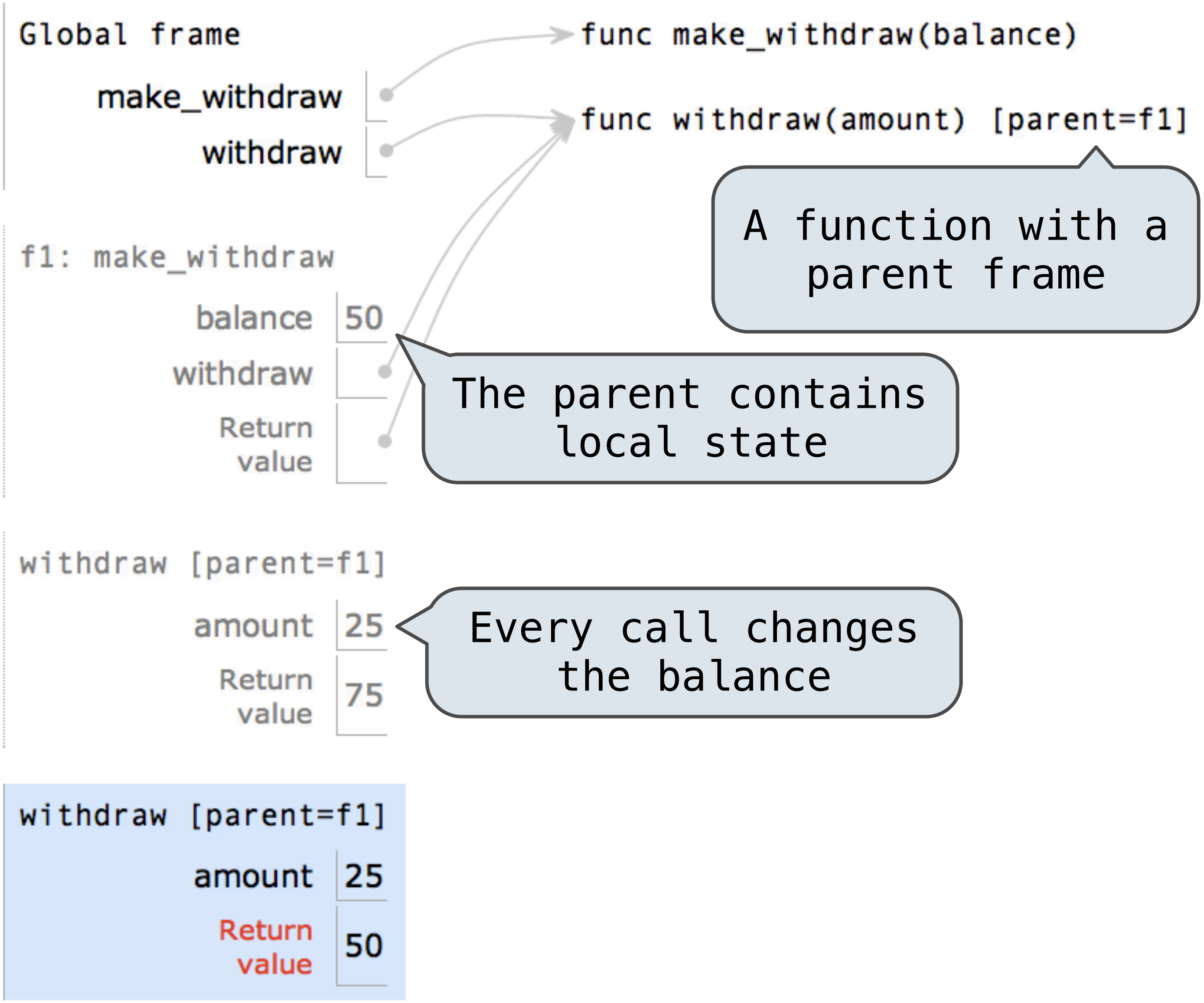
amount	25
Return value	50

A function with a parent frame

The parent contains local state

http://goo.gl/StRZP

Persistent Local State



A function with a parent frame

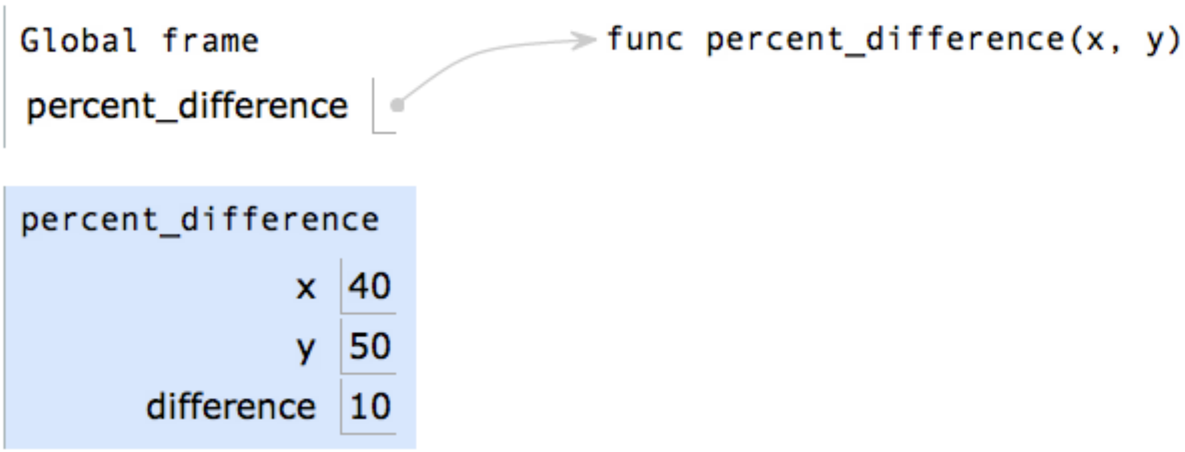
The parent contains local state

Every call changes the balance

http://goo.gl/StRZP

Reminder: Local Assignment

```
def percent_difference(x, y):
        difference = abs(x-y)
        return 100 * difference / x
diff = percent_difference(40, 50)
```



Global frame

percent_difference

func percent_difference(x, y)

percent_difference

x 40

y 50

difference 10

Example: http://goo.gl/wcF71

Reminder: Local Assignment

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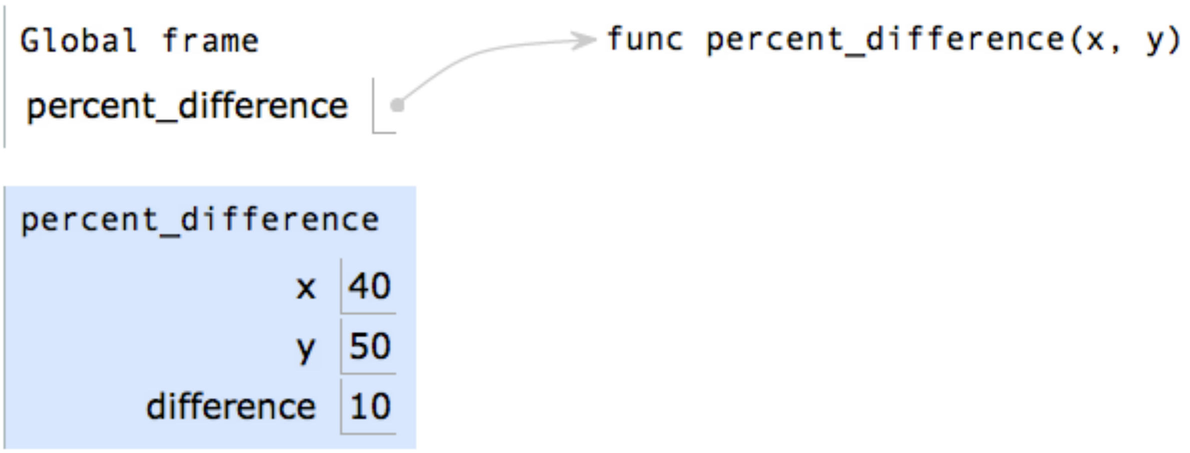
Assignment binds names to values in the current local frame



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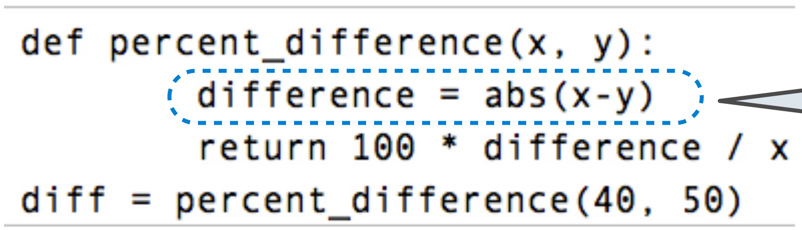

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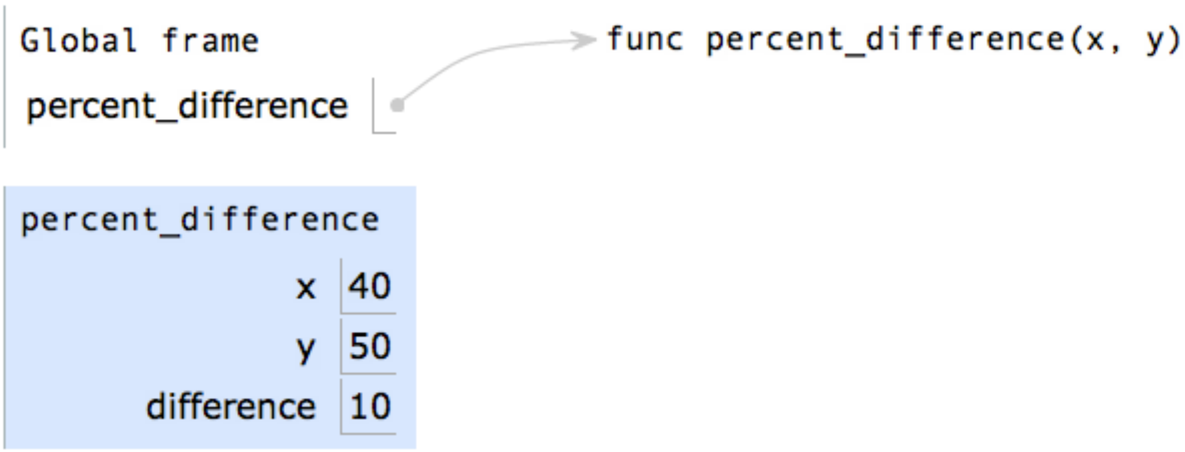

Execution rule for assignment statements:

Example: http://goo.gl/wcF71

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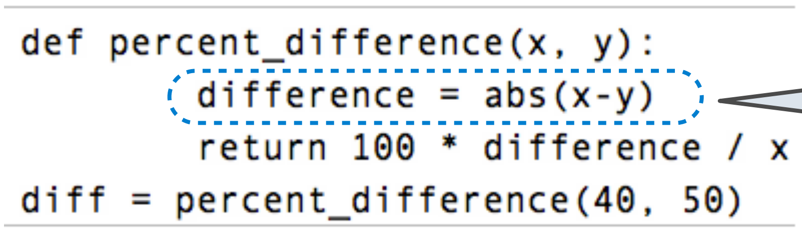

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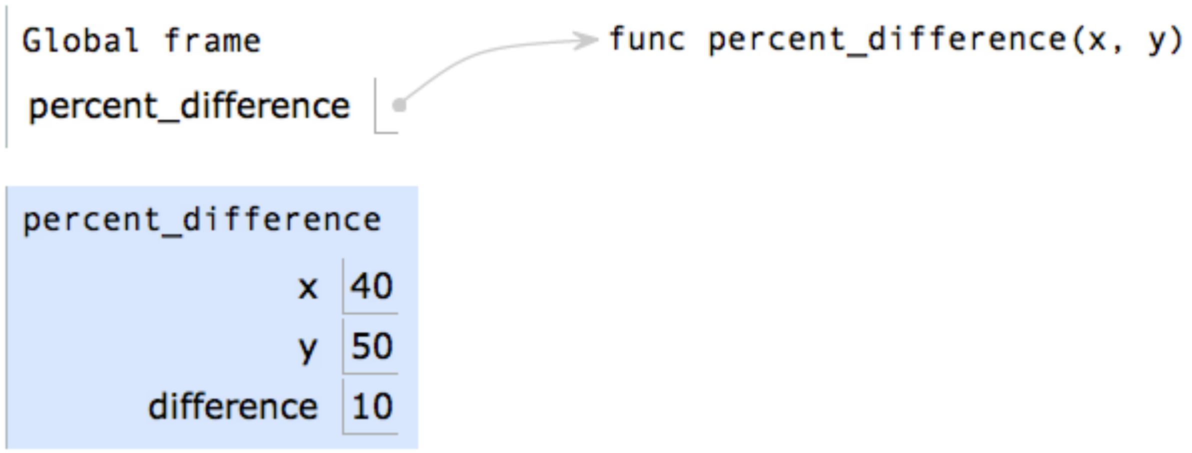

Global frame

percent_difference → func percent_difference(x, y)

percent_difference

x 40

y 50

difference 10

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right.

2. Bind the names on the left the resulting values in the **first frame** of the current environment.

Example: http://goo.gl/wcF71

Non-Local Assignment & Persistent Local State

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```
def make_withdraw(balance):
```

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Declare the name "balance" nonlocal

Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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        if amount > balance:
```

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Non-Local Assignment & Persistent Local State

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def make_withdraw(balance):
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Demo

The Effect of Nonlocal Statements

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nonlocal <name>
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Effect: Future references to that name refer to its pre-existing binding in the **first non-local frame** of the current environment in which that name is bound.

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http://www.python.org/dev/peps/pep-3104/

The Many Meanings of Assignment Statements

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• nonlocal x • "x" **is not** bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found

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Python Particulars

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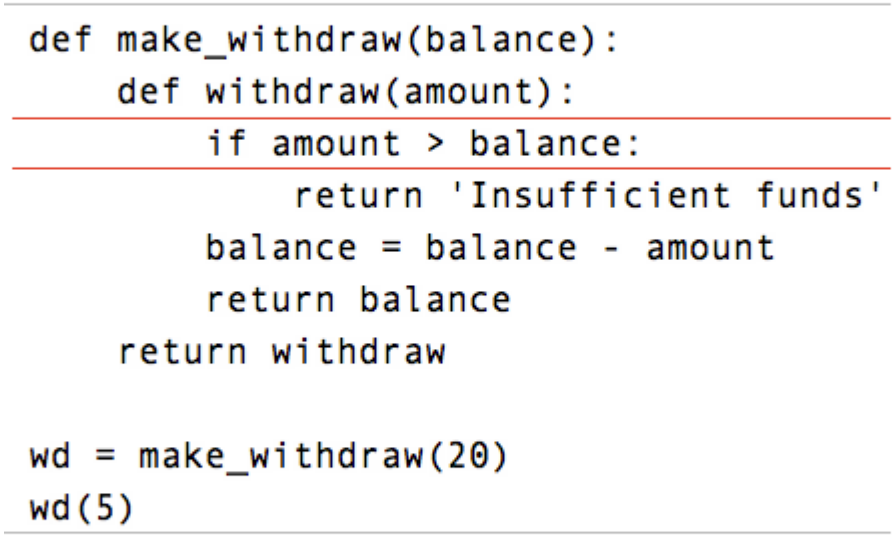

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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
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wd = make_withdraw(20)
wd(5)
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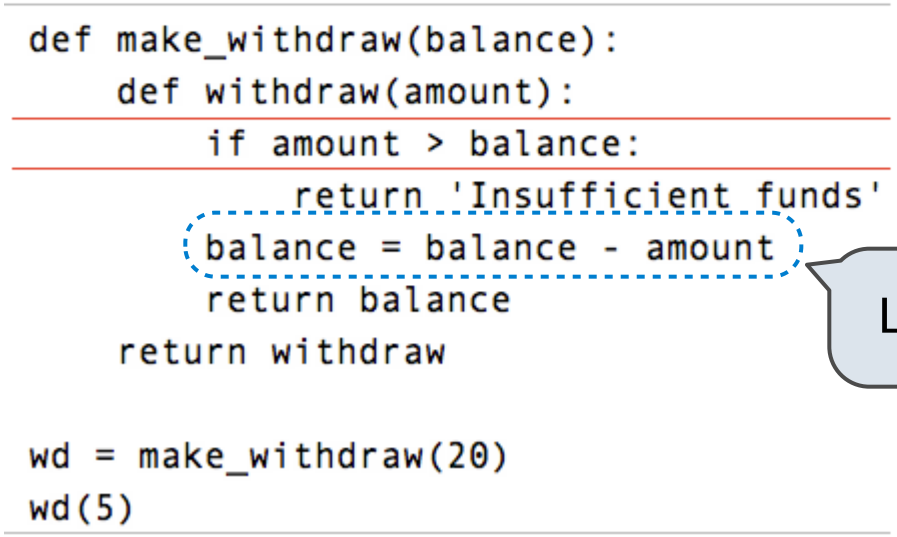

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Local assignment

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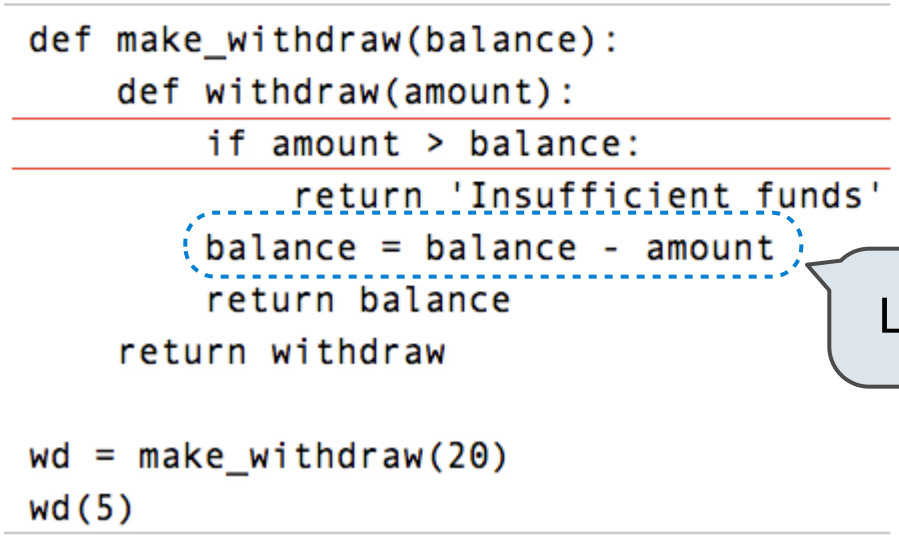

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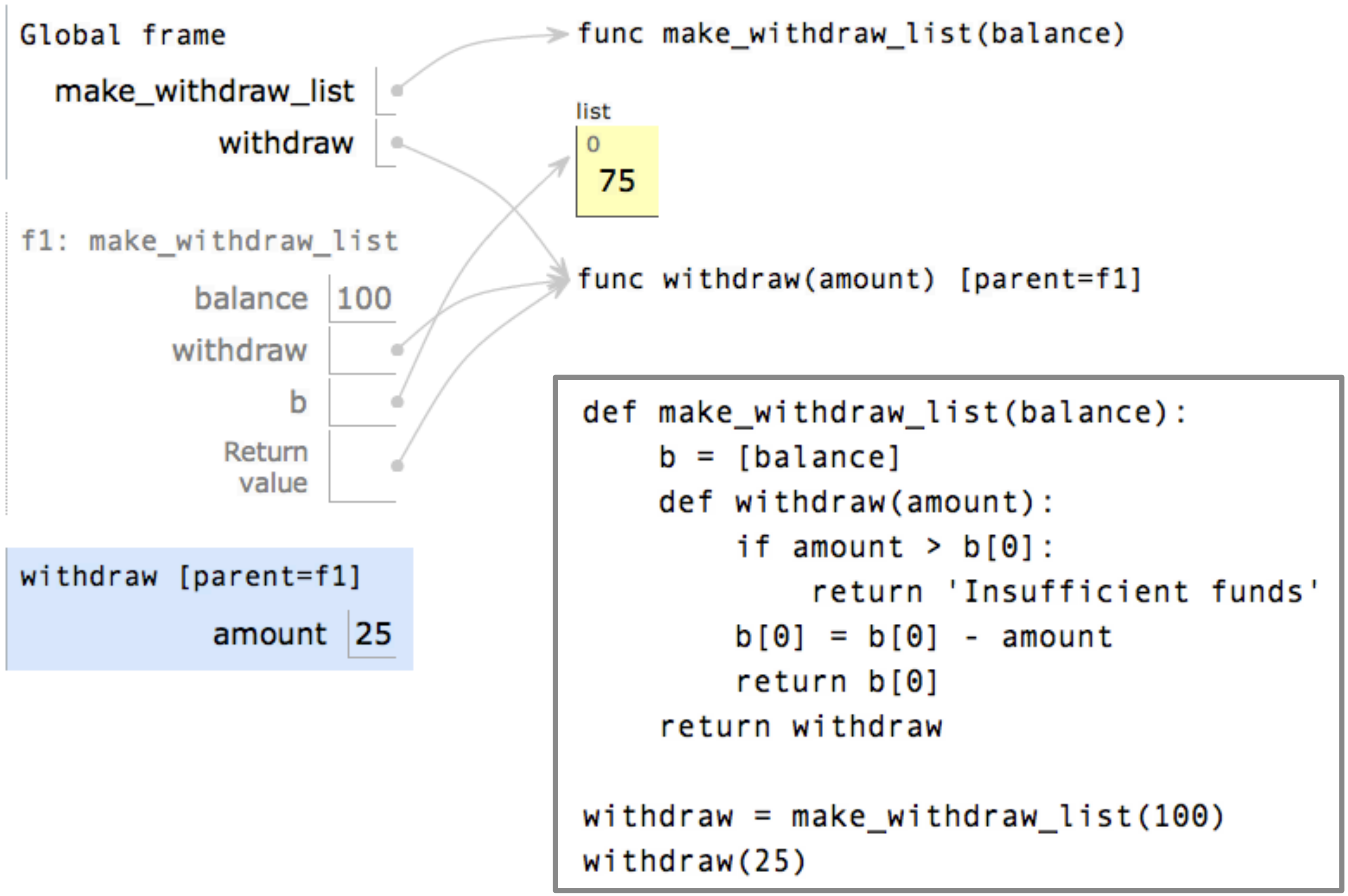
Local assignment

UnboundLocalError: local variable 'balance' referenced before assignment

Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.



```
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
```

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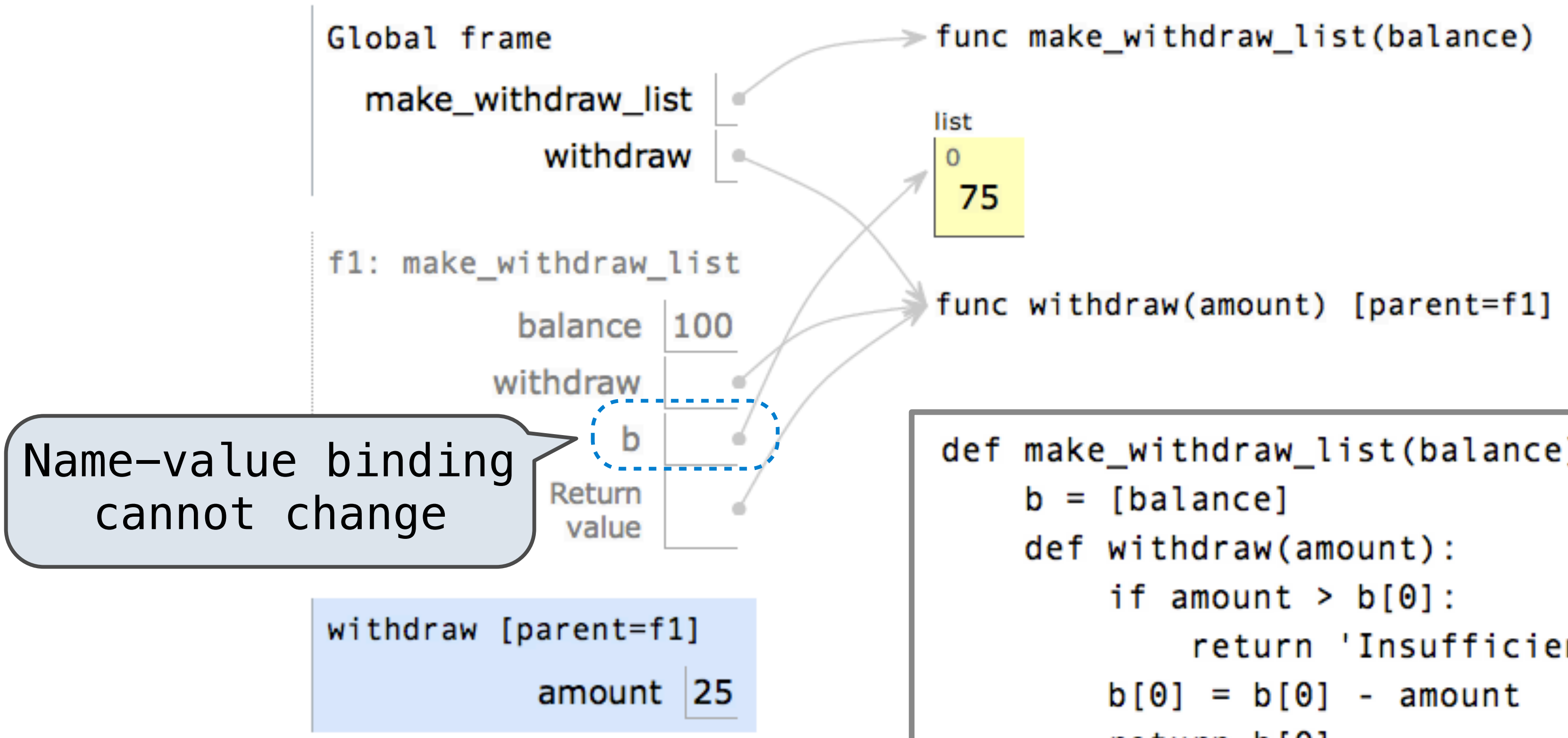

Name-value binding cannot change

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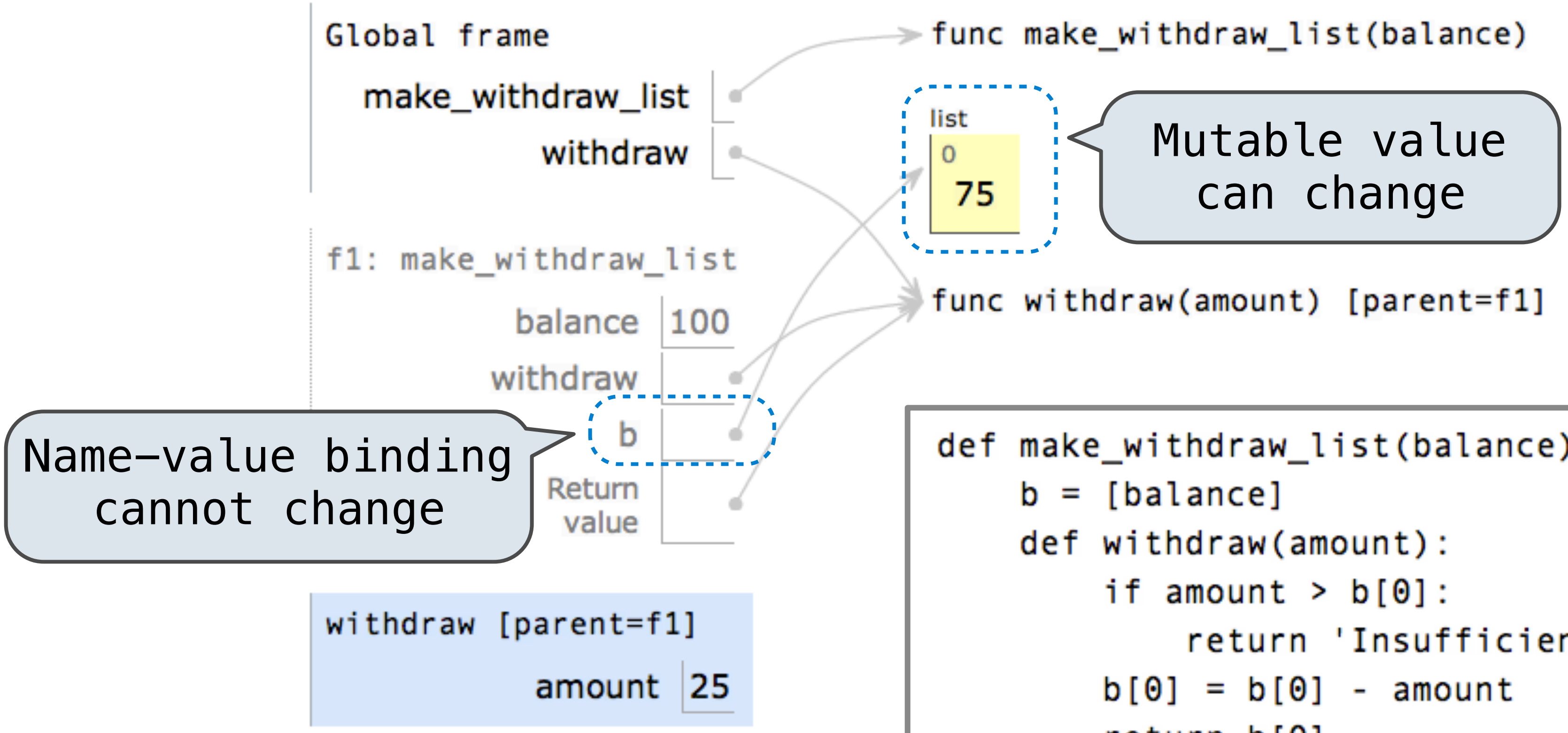

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Creating Two Different Withdraw Functions

Demo

The Benefit of Non-Local Assignment

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- Ability to **maintain some state** that is **local** to a function, but **evolves** over successive calls to that function.

The Benefit of Non-Local Assignment

- Ability to **maintain some state** that is **local** to a function, but **evolves** over successive calls to that function.

- The binding for balance in the first non-local frame of the environment associated with an instance of withdraw is **inaccessible to the rest of the program.**

The Benefit of Non-Local Assignment

- Ability to **maintain some state** that is **local** to a function, but **evolves** over successive calls to that function.

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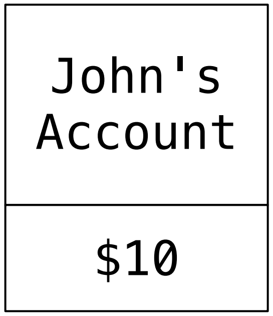

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Multiple References to a Single Withdraw Function

Demo

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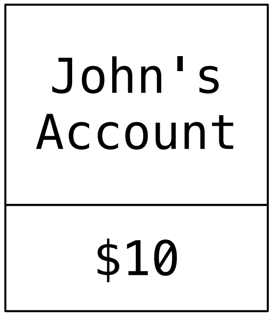

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